WAVE ON A STRING

Contents

Syllabus

Wave Motion (Plane waves Only), longitudinal and transverse waves, superposition of waves ; Progressive and stationary waves ; Vibration of strings and air columns; Resonance.

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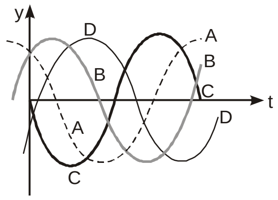
EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* ***Marked Questions are having more than one correct option.***

SECTION (A) : DEFINITION OF WAVES AND EQUATION OF WAVES

A-1. Which of the following curves represents correctly the oscillation given $y=y_0\sin(\omega t - \phi)$, where $0 < \phi < 90$:



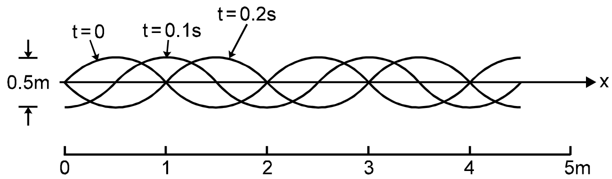
(A) A (B) B (C) C (D) D

A-2. Which of the following functions represent a practically possible wave (with finite displacement at all points)?

(A) $(x - vt)^2$ (B) $ln\ (x + vt)$ (C) $e^{-(x-vt)^2}$ (D) $\dfrac{1}{x+vt}$

A-3.* A wave is represented by the equation
$y = (0.001\ mm)\ \sin[(50\ s^{-1})t + (2.0\ m^{-1})x]$.
(A) The wave velocity = 100 m/s. (B) The wavelength = 2.0 m.
(C) The frequency = $25/\pi$ Hz. (D) The amplitude = 0.001 mm.

A-4. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)



(A) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(B) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(C) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(D) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

A-5. A transverse wave on a string is described by the equation y(x, t) = (2.20 cm) sin [(130 rad/s) t (15rad/m)x].
(i) Find the approximate maximum transverse speed of a point on the string.
(A) 1.2 m/s (B) 1.7 m/s (C) 2.9 m/s (D) 3.4 m/s

(ii) Find the approximate maximum transverse acceleration of a point on the string.
(A) 300 m/s^2 (B) 372 m/s^2 (C) 410 m/s^2 (D) 450 m/s^2

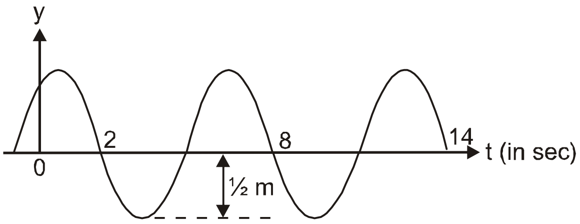
(iii) Find the approximate speed of wave moving along the string.
(A) 4.2 m/s (B) 5.6 m/s (C) 7.4 m/s (D) 8.7 m/s

A-6. A surface seismic wave moving along the –x-axis with amplitude 2.0 cm, period 4.0 s, and wavelength 4.0 km. Assume the wave is harmonic, x is measured in m, and t is measured in s.
(i) The maximum speed of the ground as the wave moves by,
(A) 0.015 m/s (B) 0.031 m/s (C) 0.052 m/s (D) 0.076 m/s
(ii) The wave speed is
(A) 1.0 km/s (B) 2.0 km/s (C) 3.0 km/s (D) 4.0 km/s

A-7. A transverse wave of amplitude 0.50m, wavelength 1m and frequency 2 Hz is propagating on a string in the negative x-direction. The expression form of the wave is
(A) $y(x, t) = 0.5 \sin (2\pi x - 4\pi t)$ (B) $y(x, t) = 0.5 \cos (2\pi x + 4\pi t)$
(C) $y(x, t) = 0.5 \sin (\pi x - 2\pi t)$ (D) $y(x, t) = 0.5 \cos (2\pi x - 2\pi t)$

A-8. Consider a function $y = 10\sin^2(100\pi t+5\pi z)$ where y, z are in cm and t is in seconds.
(A) the function represents a travelling, periodic wave propagating in (–z) direction with speed 20m/s.
(B) the function does not represent a travelling wave.
(C) the amplitude of the wave is 5 cm.
(D) the amplitude of the wave is 10 cm.

A-9. The sketch in the figure shows displacement time curve of a sinusoidal wave at x = 8 m. Taking velocity of wave v = 6m/s along positive x-axis, write the equation of the wave.



(A) $0.5 \sin\left(\frac{\pi}{3}t + \frac{\pi}{18}x + \frac{7\pi}{9}\right)$ (B) $0.5 \sin\left(\frac{\pi}{3}t - \frac{\pi}{18}x + \frac{11\pi}{9}\right)$
(C) $0.5 \sin\left(\frac{\pi}{3}t + \frac{\pi}{18}x + \frac{11\pi}{9}\right)$ (D) $0.5 \sin\left(\frac{\pi}{3}t - \frac{\pi}{18}x + \frac{7\pi}{9}\right)$

A-10.* The plane wave represented by an equation of the form y = f(x – v t) implies the propagation along the positive x-axis without change of shape with constant velocity v :
(A) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$ (B) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial^2 y}{\partial x^2}\right)$ (C) $\frac{\partial^2 y}{\partial t^2} = -v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$ (D) $\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$

SECTION (B) :SPEED OF A WAVE, ENERGY , POWER TRANSFER, INTENSITY

B-1. The speed of waves in a stretched string depends on which one of the following?
(A) The tension in the string (B) The amplitude of the waves
(C) The wavelength of the waves (D) The gravitational field strength

B-2. The higher the frequency of a wave,
(A) the smaller its speed. (B) the shorter its wavelength.
(C) the greater its amplitude. (D) the longer its period.

B-3. Of these properties of a wave, the one that is independent of the others is its
(A) amplitude. (B) wavelength (C) speed. (D) frequency.

B-4. A transverse wave travels on a string of mass m, length L, and tension F. Which statement here is correct?

(A) The energy of the wave is proportional to the square root of the wave amplitude.

(B) The speed of a moving point on the string is the same as the wave speed.

(C) The wave speed is determined by the values of m, L, and F.

(D) The wavelength of the wave is proportional to L.

B-5. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. The following conclusions are drawn. Mark the one which is correct.

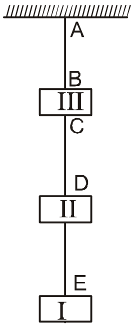
(A) doubling the frequency doubles the rate at which energy is carried along the string

(B) if the amplitude were doubled, the rate at which energy is carried would be halved

(C) if the amplitude were doubled, the rate at which energy is carried would be doubled

(D) the rate at which energy is carried is directly proportional to the velocity of the wave.

B-6. Three blocks I, II, & III having mass of 1.6 kg, 1.6 kg and 3.2 kg respectively are connected as shown in the figure. The linear mass density of the wire AB, CD and DE are 10 g/m, 8 g/m and 10 g/m respectively. The speed of a transverse wave pulse produced in AB, CD and DE are : (g = 10 m/sec^2)



(A) $20\sqrt{10}$ m/s, 80 m/s, 40 m/s

(B) 80 m/s, $20\sqrt{10}$ m/s, 40 m/s

(C) $20\sqrt{10}$ m/s in all

(D) 80 m/s in all

B-7. The average power transmitted through a given point on a string supporting a sine wave is. 0.40 watt when the amplitude of wave is 2 mm. What average power will transmitted through this point its amplitude is increased to 4 mm.

(A) 1.6 watt (B) 2.12 watt (C) 2.18 watt (D) 2.24 watt

B-8. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$):

(A) 45 Hz (B) 50 Hz (C) 30 Hz (D) 62 Hz

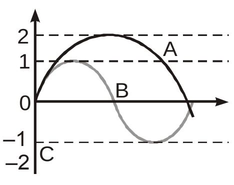
B-9. A 6.00 m segment of a long string has a mass of 180 g. A high-speed photograph shows that the segment contains four complete cycles of a wave. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" displacement is the vertical distance from the farthest positive displacement to the farthest negative displacement.) The average power being supplied to the string will be(approximately) :

(A) 200 W (B) 420 W (C) 625 W (D) 845 W

B-10. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 g/m kept under a tension of 60 N. The average power transmitted across a given point on the string will be :

(A) 0.23 W (B) 0.47 W (C) 0.82 W (D) 1.24 W

B-11. The displacement-time graphs for two waves A and B are shown in the figure, then the ratio of their intensities I_A/I_B is equal to



(A) 1 : 4 (B) 1 : 16 (C) 1 : 2 (D) 1 : 1

B-12. A wave moving with constant speed on a uniform string passes the point x = 0 with amplitude A_0, angular frequency ω_0 and average rate of energy transfer P_0. As the wave travels down the string it gradually loses energy and at the point $x = \ell$, the average rate of energy transfer becomes $\frac{P_0}{2}$. At the point $x = \ell$, angular frequency and amplitude are respectively :

(A) ω_0 and $A_0/\sqrt{2}$ (B) $\omega_0/\sqrt{2}$ and A_0 (C) less than ω_0 and A_0 (D) $\omega_0/\sqrt{2}$ and $A_0/\sqrt{2}$

SECTION (C) : INTERFERENCE & REFLECTION OF WAVES

C-1. Two waves of amplitude A_1, and A_2 respectively and equal frequency travel towards same point. The amplitude of the resultant wave is

(A) $A_1 + A_2$ (B) $A_1 - A_2$
(C) between $A_1 - A_2$ and $A_1 + A_2$ (D) Can not say

C-2. The resultant amplitude due to superposition of two waves $y_1 = 5\sin(wt - kx)$ and $y_2 = -5\cos(wt - kx - 150°)$

(A) 5 (B) $5\sqrt{3}$ (C) $5\sqrt{2-\sqrt{3}}$ (D) $5\sqrt{2+\sqrt{3}}$

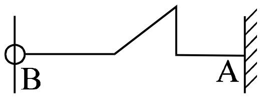
C-3. Consider two waves passing through the same string. Principle of superposition for displacement says that the net displacement of a particle on the string is sum of the displacements produced by the two waves individually. Suppose we state similar principles for the net velocity of the particle and the net kinetic energy of the particle. Such a principle will be valid for

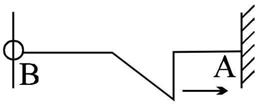
(A) both the velocity and the kinetic energy (B) the velocity but not for the kinetic energy
(C) the kinetic energy but not for the velocity (D) neither the velocity nor the kinetic energy.

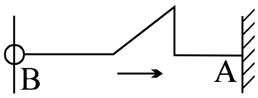
C-4. Two wave pulses travel in opposite directions on a string and approach each other. The shape of one pulse is inverted with respect to the other.

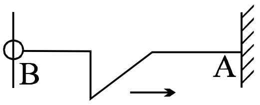
(A) The pulses will collide with each other and vanish after collision.
(B) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(C) The pulses will pass through each other but their shapes will be modified.
(D) The pulses will pass through each other without any change in their shapes.

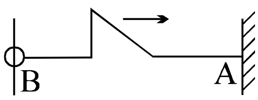
C-5. A pulse shown here is reflected from the rigid wall A and then from free end B. The shape of the string after these 2 reflections will be



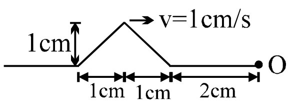
(A) 

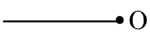
(B) 

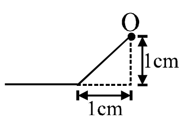
(C) 

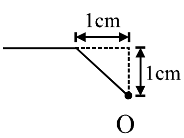
(D) 

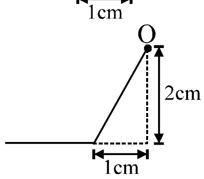
C-6. A wave pulse on a string has the dimension shown in figure. The wave speed is v = 1 cm/s. If point O is a free end. The shape of wave at time t = 3 s is :



(A) 

(B) 

(C) 

(D) 

C-7. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has speed v and the transmitted wave v′,

(A) v′ > v (B) v′ = v (C) v′ < v
(D) nothing can be said about the relation of v and v′.

C-8. A composite string is made up by joining two strings of different masses per unit length μ and 4μ. The composite string is under the same tension. A transverse wave pulse : Y = (6 mm) sin(5t + 40x), where 't' is in seconds and 'x' in meters, is sent along the lighter string towards the joint. The joint is at x = 0. The equation of the wave pulse reflected from the joint is

(A) (2 mm) sin(5t – 40x) (B) (4 mm) sin(40x – 5t)
(C) – (2 mm) sin(5t – 40x) (D) (2 mm) sin (5t – 10x)

C-9. In the previous question, the percentage of power transmitted to the heavier string through the joint is approximately

(A) 33% (B) 89% (C) 67% (D) 75%

C-10. Three waves of equal frequency having amplitude 10 μm, 4 μm and 7 μm arrive at a given point with successive phase difference of $\pi/2$. The amplitude of the resulting wave in μm is given by

(A) 7 (B) 6 (C) 5 (D) 4

SECTION (D) :STANDING WAVES

D-1. Mark out the correct options.

(A) The energy of any small part of a string remains constant in a travelling wave.
(B) The energy of any small part of a string remains constant in a standing wave.
(C) The energies of all the small parts of equal length are equal in a travelling wave.
(D) The energies of all the small parts of equal length are equal in a standing wave.

D-2.* In a stationary wave,

(A) all the particles of the medium vibrate in phase
(B) all the antinodes vibrate in phase
(C) the alternate antinodes vibrate in phase
(D) all the particles between consecutive nodes vibrate in phase.

D-3. The transverse displacement of a string fixed at both ends is given by $y = 0.06 \sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$ where x and y are in metres and t is in seconds. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. What is the tension in the string ?

(A) 648 N (B) 724 N (C) 832 N (D) 980 N

D-4. A standing wave pattern of amplitude A in a string of length L shows 2 nodes (plus those at two ends). If one end of the string corresponds to the origin and v is the speed of progressive wave, the disturbance in the string, could be represented (with appropriate phase) as:

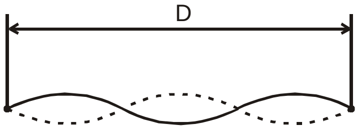
(A) $y(x,t) = A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi vt}{L}\right)$ (B) $y(x,t) = A\cos\left(\frac{3\pi x}{L}\right)\sin\left(\frac{3\pi vt}{L}\right)$

(C) $y(x,t) = A\cos\left(\frac{4\pi x}{L}\right)\cos\left(\frac{4\pi vt}{L}\right)$ (D) $y(x,t) = A\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi vt}{L}\right)$

D-5. A violin string of length L is fixed at both ends. Which one of these is not a wavelength of a standing wave on the string?

(A) L/2 (B) L/3 (C) 2L/3 (D) 3L/2

D-6. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in figure.



Calculate

(i) The speed of the traveling waves whose superposition gives this standing wave

(A) $\frac{125}{\sqrt{3}}$ m/s (B) $\frac{500}{\sqrt{3}}$ m/s (C) $\frac{250}{\sqrt{3}}$ m/s (D) $100\sqrt{3}$ m/s

(ii) The wavelength of the traveling waves whose superposition gives this standing wave.

(A) 20 cm (B) 40.0 cm (C) 60.0 cm (D) 80.0 cm

(iii) The frequency of the traveling waves whose superposition gives this standing wave.

(A) $\frac{1000}{3\sqrt{3}}$ Hz (B) $\frac{1250}{3\sqrt{3}}$ Hz (C) $\frac{1500}{3\sqrt{3}}$ Hz (D) $\frac{1750}{3\sqrt{3}}$ Hz

SECTION (E) :SONOMETER

E-1. A chord attached to a vibrating form divides it into 6 loops, when its tension is 36 N. The tension at which it will vibrate in 4 loops is

(A) 24 N (B) 36 N (C) 64 N (D) 81 N

E-2. If n_1, n_2, n_3.... are the frequencies of segments of a stretched string, the frequency n of the string is given by:

(A) $n = n_1 + n_2 + n_3 + ...$ (B) $n = \sqrt{n_1 \times n_2 \times n_3 \times ....}$

(C) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + ...$ (D) none of the above

E-3. A sonometer wire has a total length of 1 m between the fixed ends. Two wooden bridges are placed below the wire at a distance 1/7 m from one end and 4/7 m from the other end. The three segments of the wire have their fundamental frequencies in the ratio :

(A) 1 : 2 : 3 (B) 4 : 2 : 1 (C) 1 : 1/2 : 1/3 (D) 1 : 1 : 1

E-4. A sonometer wire of length l vibrates in fundamental mode when excited by a tuning fork of frequency 416 Hz. If the length is doubled keeping other things same, the string will

(A) vibrate with a frequency of 416 Hz (B) vibrate with a frequency of 208 Hz

(C) vibrate with a frequency of 832 Hz (D) stop vibrating.

E-5. A string vibrates in 4 loops with a frequency of 400 Hz .

(i) What is its fundamental frequency ?

(A) 25 Hz (B) 50 Hz (C) 100 Hz (D) 200 Hz

(ii) What frequency will cause it to vibrate into 7 loops .

(A) 100 Hz (B) 400 Hz (C) 700 Hz (D) 1400 Hz

E-6. A tuning fork of frequency 480 Hz is used to vibrate a sonometer wire having natural frequency 240 Hz. The wire will vibrate with a frequency of

(A) 240 Hz (B) 480 Hz (C) 720 Hz (D) will not vibrate.

E-7. A steel wire 0.5 m long has a total mass of 0.01 kg and is stretched under a tension of 800 N. The frequency of vibration of string in its fundamental mode is :

(A) 2 Hz (B) 4 Hz (C) 100 Hz (D) 200 Hz

E-8. Three resonant frequencies of a string are 90, 150 and 210 Hz.

(i) The highest possible fundamental frequency of vibration of this string.

(A) 15 Hz (B) 30 Hz (C) 45 Hz (D) 90 Hz

(ii) Which overtone are these frequencies.

(A) 1st, 3rd, 5th (B) 2nd, 4th, 6th (C) 3rd, 5th, 7th (D) None of these

(iii) If the length of the string is 80 cm, what would be the speed of a transverse wave on this string ?

(A) 48 m/sec (B) 96 m/sec (C) 120 m/sec (D) 140 m/sec

E-9. A rope, under a tension of 200 N and fixed at both ends, oscillates in a second-harmonic standing wave pattern. The displacement of the rope is given by :

$$Y = (0.10\text{ m})(\sin \pi x/2)\sin 12\pi t$$

where x = 0 at one end of the rope, x is in meters and t is in seconds.

(i) The length of the rope is:
(A) 4 m (B) 6 m (C) 8 m (D) 10 m

(ii) The speed of the progressive waves on the rope is:
(A) 20 m/s (B) 24 m/s (C) 36 m/s (D) 48 m/s

(iii) The mass of the rope is:
(A) 20/18 kg (B) 35/18 kg (C) 25/18 kg (D) 15/18 kg

(iv) If the rope oscillates in a third-harmonic standing wave pattern, the period of oscillation will be:
(A) 2/9 sec (B) 4/9 sec (C) 5/9 sec (D) 1/9 sec

E-10. A stone is hung in air from a wire which is stretched over a sonometer. The bridge of the sonometer are L cm apart when the wire is in unison with a tuning fork of frequency N. When the stone is completely immersed in water, the length between the bridge is l cm for re-establishing unison, the specific gravity of the material of the stone is

(A) $\frac{L^2}{L^2+l^2}$ (B) $\frac{L^2-l^2}{L^2}$ (C) $\frac{L^2}{L^2-l^2}$ (D) $\frac{L^2+l^2}{L^2}$

E-11. Two uniform strings A and B formed of steel are made to vibrate under the same tension. If the first overtone of A is equal to the second overtone of B and the radius of A is twice that of B, the ratio of the lengths of the strings is :
(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 1 : 5

E-12. An aluminium wire of cross-sectional area 1×10^{-6} m^2 is joined to a steel wire of same cross-sectional area. This compound wire is stretched on a sonometer, pulled by a weight of 10 kg. The total length of the compound wire between the bridges is 1.5 m, of which the aluminium is 0.6 m and the rest is steel wire. Transverse vibrations are set up in the wire by using an external force of variable frequency. The lowest frequency of excitation for which standing waves are formed such that the joint in the wire is a node. (density of aluminium is 2.6×10^3 kg/m^3 and that of steel is 1.04×10^4 kg/m^3).
(A) 27 vibrations/sec (B) 81 vibrations/sec (C) 162 vibrations/sec (D) 324 vibrations/sec

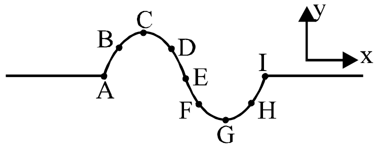
E-13. In the previous question, the total number of nodes observed at this frequency, excluding the two at the ends of the wire is :
(A) 2 (B) 3 (C) 5 (D) 7

PART - II : MISLLANEOUS QUESTIONS

1. COMPREHENSION

COMPREHENSION # 1

A progressive wave pulse is generated on a string is traveling in –ve X direction as shown in figure.



1. Kinetic energy is maximum for the particle
(A) D (B) E (C) F (D) G

2. Potential energy is maximum for the particle
(A) D (B) E (C) F (D) G

3. Particle B and D are moving respectively
(A) ↑↑ (B) ↓↓ (C) ↑↓ (D) ↓↑

COMPREHENSION # 2

A standing wave exists in a string of length 150 cm. and is fixed at both ends. The displacement amplitude of a point at a distance of 10cm from one of the ends is $5\sqrt{3}$ mm. The distance between the two nearest points, with in the same loop and having displacement amplitude equal to $5\sqrt{3}$ mm, is 10cm.

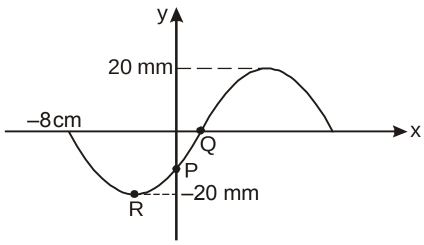
4. The maximum displacement amplitude of the particle in the string is :
(A) 10mm (B) $20/\sqrt{3}$ mm (C) $10\sqrt{3}$ mm (D) 20 mm

5. The mode of vibration of the string i.e. the overtone produced is :
(A) 2 (B) 3 (C) 4 (D) 6

6. At what minimum distance from one end, is the potential energy of string zero :
(A) $10\sqrt{3}$ cm (B) 15 cm (C) 20 cm (D) 30 cm

COMPREHENSION # 3

A plane sinusoidal sound wave is propagating along +x-axis with speed 120cm/s and having frequency 5Hz. At t = 0, the displacement S of the particles is plotted as function of position x as shown in figure.



7. In figure, what is the distance of point Q from the origin
(A) 2 cm (B) 3 cm (C) 4 cm (D) 1cm

8. Velocity of the particle P in figure is

(A) $10\sqrt{3}\pi$ cm/s upward (B) 10π cm/s downward

(C) $10\sqrt{3}\pi$ cm/s right (D) 10π cm/s left

9. What is the velocity of particle R in figure when particle P reach its mean position?

(A) $10\sqrt{3}\pi$ cm/s , upward (B) 10π cm/s , downward

(C) $10\sqrt{3}\pi$ cm/s , right (D) 10π cm/s , left

COMPREHENSION # 4

A block of mass 2m is hanging at the lower end of a rope of mass m and length the ℓ, the other end being fixed to the ceiling. A pulse of wavelength λ_0 is produced at the lower end of the rope.

10. The wavelength of the pulse when it reaches the other end of the rope is

(A) $\sqrt{3}\lambda_0$ (B) $\sqrt{\frac{3}{2}}\lambda_0$ (C) λ_0 (D) $\frac{\lambda_0}{2}$

11. The speed of the pulse at the mid point of rope is :

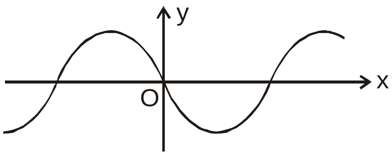
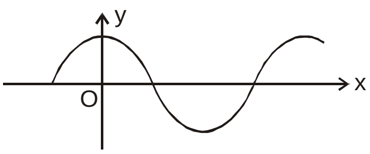
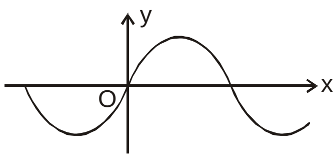
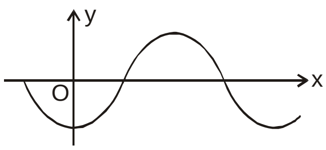
(A) $\sqrt{\frac{5}{2}g\ell}$ (B) $\sqrt{\frac{5}{3}g\ell}$ (C) $\sqrt{\frac{2}{5}g\ell}$ (D) $\sqrt{\frac{g\ell}{2}}$

12. The time taken by the pulse to reach the other end of the rope is

(A) $2\sqrt{\frac{\ell}{g}}\left(\sqrt{3}-1\right)$ (B) $2\sqrt{\frac{\ell}{g}}\left(\sqrt{3}-2\right)$ (C) $2\sqrt{\frac{\ell}{g}}$ (D) $2\sqrt{\frac{\ell}{g}}\left(\sqrt{3}-\sqrt{2}\right)$

2. MATCH THE COLUMN

11. For four sine waves, moving on a string along positive x direction, displacement-distance curves (y-x curves) are shown at time t = 0. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y-x curves with corresponding equations.

Column – I	Column – II
(A) 	(P) y = A cos (4t – kx)
(B) 	(Q) y = – A cos (kx –4t)
(C) 	(R) y = A sin (4t – kx)
(D) 	(S) y = A sin (kx – 4t)

12. A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.00 cm . The motion is continuous and is repeated regularly 120 times per second. The string has linear density 90 gm/m and is kept under a tension of 900 N.

	Column – I		**Column – II**
(A)	The maximum value of the transverse component of the tension (in Newtons)	(P)	$12.96\,\pi^2$
(B)	The transverse displacement y (in cm) when this maximum value of the tension occurs	(Q)	0.5
(C)	The maximum power (in watts) transferred along the string .	(R)	$10.8\,\pi$
(D)	The transverse displacement y (in cm) when the minimum power transfer occurs	(S)	0

13. In case of mechanical wave a particle oscillates and during oscillation its kinetic energy and potential energy changes.Match the statements in column-I with the statements in column-II.

	Column I		**Column II**
(A)	When particle of travelling wave is passing through mean position.	(P)	Kinetic energy is maximum
(B)	When particle of travelling wave is at extreme position.	(Q)	Potential energy is maximum
(C)	When particle between node and antinode in standing wave is passing through mean position	(R)	Kinetic energy is minimum
(D)	When particle between node and antinode in standing wave is at extreme position	(S)	Potential energy is minimum

3. ASSERTION / REASON

14. **Statement-1 :** When a pulse on string reflects from free end, the resultant pulse is formed in such a way that slope of string at free end is zero.
Statement-2 : Zero resultant slope ensures that there is no force component perpendicular to string.
(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

15. **Statement-1 :** Transverse mechanical waves travel through air in a hollow pipe.
Statement-2 : Air possesses only volume elasticity.
(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

16. **Statement-1 :** Transverse mechanical waves are not produced in liquids and gases.
Statement-2 : Light waves are transverse waves.
(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

17. **Statement-1 :** Particle velocity and wave velocity both are independent of time.
Statement-2 : For the propagation of wave motion, the medium must have the properties of elasticity and inertia.
(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

18. **Statement-1 :** Velocity of particles, while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.

Statement-2 : Amplitude of vibration at antinodes is maximum and at nodes, the amplitude is zero and all particles between two successive nodes cross the mean position together.

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

4. TRUE OR FALSE

19. In a stretched string the frequency of third overtone is three times that of fundamental.

20. In a small segment of string carrying sinusoidal wave, total energy is conserved.

21. Two waves moving in a uniform string having uniform tension cannot have different velocities.

22. The particle speed can never be equal to the wave speed in sine wave if the amplitude is less then wavelength divided by 2π

23. Standing waves are not produced when two identical waves move in the same direction with a phase difference of π.

5. FILL IN THE BLANKS

24. A transverse wave is described by the equation $y = x_0 \cos 2\pi (vt - x/\lambda)$. The maximum particle velocity is two times the wave velocity provided λ = ...

25. A travelling wave in a stretched string is described by the equation $y = A \sin (kx - \omega t)$. The maximum particle velocity = ...

26. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m & initial phase zero propagates along the (–ve) x–direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is ____ & the corresponding amplitude difference is ____ m.

27. The equation of transverse wave in a vibrating string is $y = 0.021 \sin (x + 30t)$, where the distances are in meter and time is in second. If the linear density of the string is 1.3×10^{-4} kg/ m, then the tension in the string in newton = ...

28. Out of the following three wave forms ;

(A) $2A \cos kx \sin \omega t$

(B) $2A \cos (\Delta\omega/2) t \cos (\omega t - kx)$ &

(C) $2A \cos (\phi/2) \sin (\omega t - kx + \theta)$ .... represent the phenomenon of stationary wave.

PART - I : MIXED OBJECTIVE

* ***Marked Questions are having more than one correct option.***

SINGLE CORRECT ANSWER TYPE

1. A motion is described by $y = 3e^x.e^{-3t}$ where y, x are in metre and t is in second.
(A) This represents equation of progressive wave propagating along – x direction with 3 ms^{-1}
(B) This represents equation of progressive wave propagating along + x direction with 3 ms^{-1}
(C) This does not represent a progressive wave equation.
(D) Date is insufficient to arrive at any conclusion of this sort.

2. Transverse wave of amplitude 0.1 m is generated at one end (x = 0) of a long string by a tuning fork of frequency 500 Hz. At a certain instant of time, the displacement of particle A at x = 1.0 m is – 0.05 m and of particle B at x = 2.0 m is + 0.05 m. How long does the wave take to travel from A to B ?
(A) 1 s (B) $\frac{1}{10}$ s (C) $\frac{1}{100}$ s (D) $\frac{1}{1000}$ s

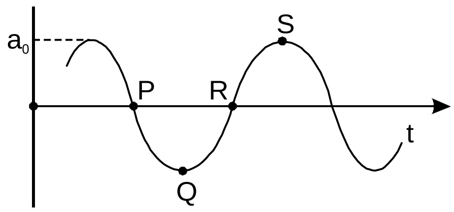
3. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment t = T/6 (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is
(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m

4. The linear density of a vibrating string is 10^{-4} kg/m. A transverse wave is propagating in the string, which is described by the equation y = 0.02 sin (x + 30*t*) were x and y are in metres and time *t* in seconds. The tension in the string is :
(A) 0.09 N (B) 0.36 N (C) 0.9 N (D) 3.6 N.

5. The equation $y = a\cos^2(2\pi nt - 2\pi x/\lambda)$ represents a wave with
(A) amplitude a, frequency n and wavelength λ
(B) amplitude a, frequency 2n and wavelength 2λ
(C) amplitude a/2, frequency 2n and wavelength λ
(D) amplitude a/2, frequency 2n and wavelength $\lambda/2$

6. The period of oscillations of a point is 0.04 sec. and the velocity of propagation of oscillation is 300m/sec. The difference of phases between the oscillations of two points at distances 10m and 16m respectively from the source of oscillations is
(A) 2π (B) $\pi/2$ (C) $\pi/4$ (D) π

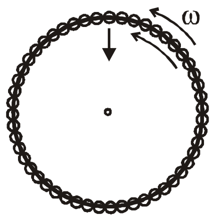
7. A wave motion has the function $y = a_0 \sin(\omega t - kx)$. The graph in figure shows how the displacement y at a fixed point varies with time t. Which one of the labelled points shows a displacement equal to that at the position $x = \frac{\pi}{2k}$ at time t =0



(A) P (B) Q (C) R (D) S

8. A string 1m long is drawn by a 300Hz vibrator attached to its end. The string vibrates in 3 segments. The speed of transverse waves in the string is equal to
(A) 100 m/s (B) 200 m/s (C) 300 m/s (D) 400 m/s

9. **(i)** A circular loop of rope of length L rotates with uniform angular velocity ω about an axis through its centre on a horizontal smooth platform. Velocity of pulse (with respect to rope) produced due to slight radial displacement is given by



(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) $\frac{\omega L}{4\pi^2}$

(ii) In the above question if the motion of the pulse and rotation of the loop, both are in same direction then the velocity of the pulse w.r.t. to ground will be :

(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) $\frac{\omega L}{4\pi^2}$

(iii) In the above question if both are in opposite direction then the velocity of the pulse w.r.t. to ground will be:

(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) 0

10. If the length of a stretched string is shortened by 40% and the tension is increased by 44% then the ratio of final and initial fundamental frequencies is :
(A) 2 : 1 (B) 3 : 2 (C) 3 : 4 (D) 1 : 3

11. Two uniform strings A and B formed of steel are made to vibrate under the same tension. If the first overtone of A is equal to the second overtone of B and the radius of A is twice that of B, the ratio of the lengths of the strings is :
(A) 1 : 2 (B) 1 : 3 (C) 1 : 4 (D) 1 : 5

12. A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is :
(A) 144 m/s (B) 216 m/s (C) 108 m/s (D) 72 m/s

13. Two wires of the same material and radii r and 2r are welded together end to end. The combination is used as a sonometer wire and kept under tension T. The welded point is mid-way between the two bridges. When stationary waves are set up in the composite wire, the joint is a node. Then the ratio of the number of loops formed in the thinner to thicker wire is.
(A) 2 : 3 (B) 1 : 2 (C) 2 : 1 (D) 5 : 4

14. A 20 cm long rubber string fixed at both ends obeys Hooke's law. Initially when it is stretched to make its total length of 24 cm, the lowest frequency of resonance is υ_0. It is further stretched to make its total length 26 cm. The lowest frequency of resonance will now be :
(A) the same as υ_0 (B) greater than υ_0 (C) lower than υ_0 (D) None of these

15. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation $y = 4 \sin\left(\frac{\pi x}{15}\right)$ $\cos(96\pi t)$ where x and y are in centimetres and t is in seconds. The equations of the component waves whose superposition gives the above equation are.
(A) $y_1 = 2\sin(\pi x/15 - 96\pi t)$ and $y_2 = 2\sin(\pi x/15 + 96\pi t)$
(B) $y_1 = 2\sin \pi x/15 \cos 96\pi t)$ and $y_2 = 2\sin \pi x/15 \sin 96\pi t$
(C) $y_1 = 4\sin(2\pi x/15 - 96\pi t)$ and $y_2 = 4\sin(2\pi x/15 + 96\pi t)$
(D) $y_1 = 4\cos(2\pi x/15 - 48\pi t)$ and $y_2 = 4\sin(2\pi x/15 + 48\pi t)$

16. A string of length 0.4 m & mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is :
(A) 0.05 s (B) 0.10 s (C) 0.20 s (D) 0.40 s

17. A standing wave y = A sin $\left(\frac{20}{3}\pi x\right)$ cos (1000πt) is maintained in a taut string where y and x are expressed in meters. The distance between the successive points oscillating with the amplitude A/2 across a node is equal to

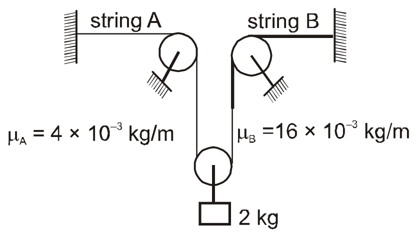
(A) 2.5cm (B) 25cm (C) 5cm (D) 10cm

18. A string of length 1m and linear mass density 0.01kgm^{-1} is stretched to a tension of 100N. When both ends of the string are fixed, the three lowest frequencies for standing wave are f_1, f_2 and f_3. When only one end of the string is fixed, the three lowest frequencies for standing wave are n_1, n_2 and n_3. Then

(A) $n_3 = 5n_1 = f_3 = 125$ Hz (B) $f_3 = 5f_1 = n_2 = 125$ Hz

(C) $f_3 = n_2 = 3f_1 = 150$ Hz (D) $n_2 = \frac{f_1 + f_2}{2} = 75$ Hz

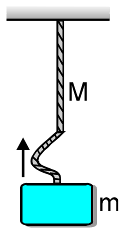
19. The velocity of wave in strings A & B is respectively.



(A) 50 m/sec, 20 m/sec (B) 40 m/sec, 25 m/sec

(C) 50 m/sec, 25 m/sec (D) 40 m/sec, 20 m/sec

20. A uniform rope of length ℓ and mass M hangs vertically from a rigid support. A block of mass m is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse, when it reaches the top of the rope, is



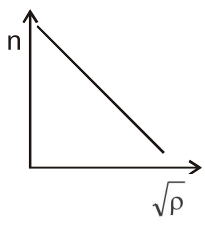
(A) $\lambda\sqrt{\frac{M-m}{m}}$ (B) $\lambda\frac{M+m}{m}$ (C) $\lambda\sqrt{\frac{m}{M+m}}$ (D) $\lambda\sqrt{\frac{M+m}{m}}$

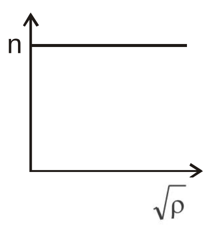
21. In a demonstration experiment on standing waves, it is desired to show one loop, two loops and three loops on a string of length 7.3 m on which tensions of F_1, F_2 and F_3 N respectively are applied. The linear density of the string is 7.47 × 10^{-3} kg/m and the frequency of the vibrator to which the string is attached is 20 Hz. The values of F_1, F_2 and F_3 corresponding to one loop, two loops and three loops are.

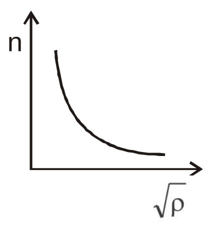
(A) 320 N, 140 N and 80 N (B) 640 N, 240 N and 142 N

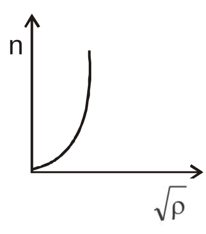
(C) 640 N, 160 N and 71 N (D) 320 N, 80 N and 35 N

22. The correct graph between the frequency n and square root of density (ρ) of a wire, keeping its length, radius and tension constant, is

(A) 

(B) 

(C) 

(D) 

23. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of half of its volume is submerged. The new fundamental frequency in Hz is :

(A) 300 $\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (B) 300 $\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ (C) 300 $\left(\frac{2\rho}{2\rho-1}\right)$ (D) 300 $\left(\frac{2\rho-1}{2\rho}\right)$.

24. An iron load of 2 kg is suspended in air from the free end of a sonometer wire of length 1 m. Tuning fork of frequency 256 Hz is in resonance with $\left(\frac{1}{\sqrt{7}}\right)$ times the length of the sonometer wire. If the load is immersed in water; the length of the wire in metre that will be in resonance with the same tuning fork is (specific gravity of iron = 8) :

(A) $\sqrt{8}$ (B) $\sqrt{6}$ (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{\sqrt{8}}$

25. A string 120 cm in length and fixed at both ends sustains a standing wave, with the consecutive points of the string at which the displacement amplitude is equal to 3.5 mm not maximum being separated by 15.0 cm.

(i) Find the maximum displacement amplitude.

(A) $\frac{7}{\sqrt{2}}$ mm (B) $7\sqrt{2}$ mm (C) 7 mm (D) $\frac{7}{2}$ mm

(ii) To which overtone do these oscillations correspond ?

(A) 2nd (B) 3rd (C) 4th (D) 5th

26. A wave equation is represented as $r = A\sin\left[\alpha\left(\frac{x-y}{2}\right)\right]\cos\left[\omega t-\alpha\left(\frac{x+y}{2}\right)\right]$, where x and y are in metre and t is in second. Then,

(A) the wave is a stationary wave

(B) the wave is a progressive wave propagating along +x axis

(C) the wave is a progressive wave propagating at right angle to +x axis

(D) all points lying on line : $y = x + \frac{4\pi}{\alpha}$ are always at rest

27. A stone hangs from the free end of a sonometer wire whose vibrating length, when tuned to a tuning fork, is 40 cm. When the stone hangs wholly immersed in water, the resonant length is reduced to 30 cm. The relative density of the stone is

(A) 16/9 (B) 16/7 (C) 16/5 (D) 16/3

28. Transverse waves are generated in two uniform wires A and B of same material by attaching their free ends to a vibrating source of frequency 200 Hz. The cross-sectional area of A is half that of B while the tension on A is twice that on B. The ratio of the wavelengths of the transverse waves in A and B is :

(A) $1 : \sqrt{2}$ (B) $\sqrt{2} : 1$ (C) 1 : 2 (D) 2 : 1.

29. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz, there are no other resonant frequencies between these two. Then the lowest resonance frequency for the string is :

(A) 105 Hz (B) 1.05 Hz (C) 1050 Hz (D) 10.5 Hz

MULTIPLE CORRECT ANSWER(S) TYPE QUESTIONS

30. The effects are produced at a given point in space by two waves described by the equations, $y_1 = y_m \sin\omega t$ and $y_2 = y_m \sin(\omega t + \phi)$ where y_m is the same for both the waves and ϕ is a phase angle. Then :

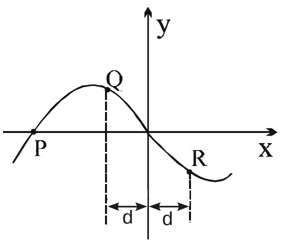
(A) The maximum intensity that can be achieved at a point is twice the intensity of either wave and occurs if $\phi = 0$

(B) The maximum intensity that can be achieved at a point is four times the intensity of either wave and occurs if $\phi = 0$

(C) The maximum amplitude that can be achieved at the point is twice the amplitude of either wave and occurs at $\phi = 0$

(D) When the intensity is zero, the net amplitude is zero, and at this point $\phi = \pi$.

31. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(A) v_P is upwards

(B) $v_Q = -v_R$

(C) $|v_P| > |v_Q| = |v_R|$

(D) $v_Q = v_R$

32. One end of a string of length L is tied to the ceiling of a lift accelerating upwards with an acceleration 2g. The other end of the string is free. The linear mass density of the string varies linearly from 0 to λ from bottom to top.

(A) The velocity of the wave in the string will be 0.

(B) The acceleration of the wave on the string will be 3g/4 every where.

(C) The time taken by a pulse to reach from bottom to top will be $\sqrt{8L/3g}$.

(D) The time taken by a pulse to reach from bottom to top will be $\sqrt{4L/3g}$.

33. A plane wave : $y = A \sin \omega\left(t - \frac{x}{v}\right)$ undergoes a normal incidence on a plane boundary eparating medium M_1 and M_2 and splits into a reflected and transmitted wave having speeds v_1 and v_2 then

(A) for all values of v_1 and v_2 the phase of transmitted wave is same as that of incident wave

(B) for all values of v_1 and v_2 the phase of reflected wave is same as that of incident wave

(C) the phase of transmitted wave depends upon v_1 and v_2

(D) the phase of reflected wave depends upon v_1 and v_2

34. A wave given by $\xi = 10 \sin[80\pi t - 4\pi x]$ propagates in a wire of length 1m fixed at both ends. If another wave is superimposed on this wave to produce a stationary wave, then :

(A) another wave is $\xi = 10 \sin[80\pi t + 4\pi x]$

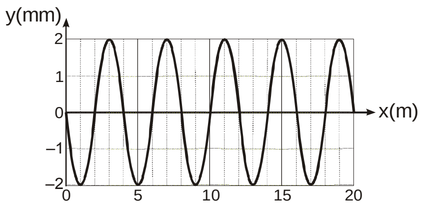
(B) the amplitude of the stationary wave is 20 m.

(C) the wave length of the wave is 0.5 m.

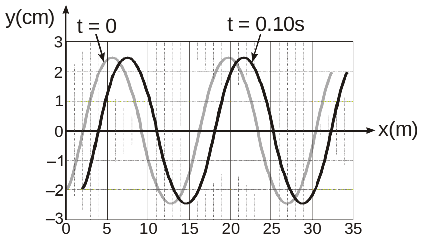
(D) the number of total nodes produced in the wire are 3.

PART - II : SUBJECTIVE QUESTIONS

1. The drawing shows a snapshot of a transverse wave traveling along a string at 10.0 m/s. The equation for the wave is $y(x, t) = A \cos(\omega t + kx)$. The numerical value of ω is $n\pi$ rad/s. Find the value of n.



2. A sine wave is traveling to the right on a cord. The lighter line in the figure represents the shape of the cord at time t = 0; the darker line represents the shape of the cord at time t = 0.10 s. (Note that the horizontal and vertical scales are different.) The speed of the wave is given as 5x m/sec and the frequency is given as $\frac{10}{P}$ Hz , where the valuesof x and P are



3. A uniform string of length 10.0 m and weight 0.25 N is attached to the ceiling. A weight of 1.00 kN hangs from its lower end. The lower end of the string is suddenly displaced horizontally. If it takes $y\sqrt{10}$ ms by the resulting wave pulse to travel to the upper end then find the value of y. Take the weight of the string negligible in comparison with that of the hanging mass. ($g = 10$ m/s^2)

4. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to 8 z kg. Find the value of z.

5. The longest "string" (a thick metal wire) on a particular piano is 2.0 m long and has a tension of 300.0 N. It vibrates with a fundamental frequency of 27.5 Hz. If the total mass of the wire is nearly x/ 100 kg, then the value of x is.

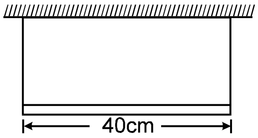
6. A 1.6 m long string fixed at both ends vibrates at resonant frequencies of 780 Hz and 1040 Hz, with no other resonant frequency between these values. (A) If the fundamental frequency of this string is 130 x Hz, the value of x is. (B) If the tension in the string is 1200 N, the total mass of the string is approximately 0.4 y grams, where the value of y is :

7. A steel wire of length $50\sqrt{3}$ cm is connected to an aluminium wire of length 60 cm and stretched between two fixed supports. The tension produced is 104 N, if the cross section area of each wire is 1mm^2. If a transverse wave is set up in the wire, the lowest frequency for which standing waves with node at the joint are produced is 1000/y Hz. The value of y is. (density of aluminium = 2.6 gm/cm^3 and density of steel = 7.8 gm/cm^3)

8. A man generates a symmetrical pulse in a string by moving his hand up and down. At t = 0 the point in his hand moves downward. The pulse travels with speed 3 m/s on the string & his hands pass 6 times in each second from the mean position. The point on the string at a distance 3m will reach its upper extreme first time at time t = 5/x sec. The value of x is.

9. In an experiment of standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. The string must be under a tension of 17.82 X N (weights are attached to the other end) if it is to oscillate in four loops. The value of x is.

10. A uniform horizontal rod of length 40 cm and mass 1.2 kg is supported by two identical wires as shown in figure. At what distance x(in cm) from the left end of the rod should a mass of 4.8 kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take g = 10 m/s^2.



11. A parabolic pulse given by equation y (in cm) = $0.3 - 0.1(x - 5t)^2$ ($y \geq 0$) x in meter and t in second travelling in a uniform string. The pulse passes through a boundary beyond which its velocity becomes 2.5 m/s. If the amplitude of pulse in this medium after transmission is 0.1y cm, then the value of y is :

12. Two metallic strings A and B of different materials are connected in series forming a joint. The strings have similar corss-sectional area. The length of A is ℓ_A = 0.2 m and that of B is ℓ_B = 0.75 m. One end of the combined string is tied with a support rigidly and the other end is loaded with a block of mass m passing over a frictionless pulley. Transverse waves are set up in the combined string using an external source of variable frequency. Standing waves are observed at lowest frequency such that the joint is a node. Calculate the total number of antinodes at this frequency.(The densities of A and B are 6.3×10^3 kg m^{-3} and 2.8×10^3 kg m^{-3} respectively).

13. A string of length 50 cm is vibrating with a fundamental frequency of 400 Hz in horizontal position. In the fundamental mode the maximum displacement at the middle is 2 cm from equilibrium position and the tension in the string is 10 N. The maximum value of the vertical component of force on the end supporting is π/x N. Find the value of x.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

* ***Marked Questions are having more than one correct option.***

1. A linearly polarised transverse wave is propagating in z-direction through a fixed point P in space. At time t_0, the x-component E_x and the y-component E_y of the displacement at P are 3 and 4 units respectively. At a later time t_1, if E_x at P is 2 units, the value of E_y will be : **[REE- 96]**
(A) 5 units (B) 8/3 units (C) 3/8 units (D) 1/3 units

2. When the tension in a stretched string is quadrupled, the velocity of the transverse wave is
(A) greater than twice the original velocity (B) twice the original velocity
(C) less than twice the original velocity (D) not changed **[REE- 97]**

3. The (x , y) co–ordinates of the corners of a square plate are (0, 0) (L, 0) (L, L) & (0, L). The edges of the plate are clamped & transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is/are :
(a = positive constant) **[JEE - 98,2]**

(A) $a \cos\left(\frac{\pi x}{2L}\right) \cos\left(\frac{\pi y}{2L}\right)$ (B) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

(C) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right)$ (D) $a \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

4.* A transverse sinusoidal wave of amplitude a, wavelength λ & frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is speed of propagation of the wave. If a = 10^{-3} m & v = 10 ms^{-1}, then λ & f are given by **[JEE - 98]**

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-2}$ m (C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

5. A cork floats on the water surface. A wave given by

$$y = 0.1 \sin 2\pi (0.1x - 2t)$$

passes over the water surface. Due to passage of the wave, the cork moves up and down. The maximum velocity of the cork, in ms^{-1}, is **[REE- 98]**

(A) 0.1 (B) 0.1π (C) $0.4\,\pi$ (D) π

6*. $y(x, t) = 0.8/[(4x + 5t)^2 + 5]$ represents a moving pulse, where x & y are in meter and t in second. Then : **[JEE - 99, 3/200]**

(A) pulse is moving in +x direction (B) in 2s it will travel a distance of 2.5 m
(C) its maximum displacement is 0.16 m (D) it is a symmetric pulse

7*. Standing waves can be produced : **[JEE - 99, 3/200]**

(A) on a string clamped at both the ends
(B) on a string clamped at one end and free at the other
(C) when incident wave gets reflected from a wall
(D) when two identical waves with a phase difference of π are moving in same direction

8. Two metallic strings A and B of different materials are connected in series forming a joint. The strings have similar cross–sectional area. The length of A is $\ell_A = 0.3$m and that of B is $\ell_B = 0.75$m. One end of the combined string is tied with a support rigidly and the other end is loaded with a block of mass *m* passing over a frictionless pulley. Transverse waves are set up in the combined string using an external source of variable frequency. Calculate

(i) the lowest frequency for which standing waves are observed such that the joint is a node and
(ii) the total number of anti–nodes at this frequency. The densities of A & B are 6.3×10^3kg m^{-3} and 2.8×10^3kg m^{-3} respectively. **[REE- 99]**

9. Two vibrating strings of the same material but lengths L & 2 L have radii 2 r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1/f_2 is given by : **[JEE - 2000 Screening, 1/100]**

(A) 2 (B) 4 (C) 8 (D) 1

10. A wave pulse starts propagating in the + x direction along a non–uniform wire of length 10 m with mass per unit length given by $\mu = \mu_0 + \alpha x$ and under a tension of 100 N . Find the time taken by the pulse to travel from the lighter end (x = 0) to the heavier end. ($\mu_0 = 10^{-2}$ kg/m and $\alpha = 9 \times 10^{-3}$ kg/m^2) **[REE - 2000 Mains, 6]**

11. Two sinusoidal waves with same wavelengths and amplitude travel in opposite directions along a string with a speed 10 ms^{-1}. If the minimum time interval between instants when the string is flat is 0.5s, the wavelength of the waves is : **[REE - 2000]**

(A) 25 m (B) 20 m (C) 15 m (D) 10 m

12. A longitudinal travelling wave transports **[REE - 2000]**

(A) energy and linear momentum (B) energy and angular momentum
(C) energy and torque (D) angular momentum and torque

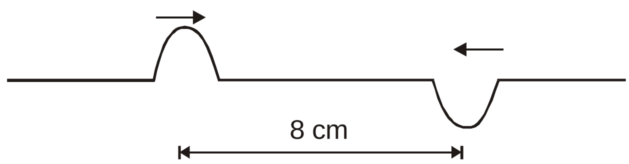
13. The intensity of a progressing plane wave in loss-free medium is **[REE - 2000]**

(A) directly proportional to the square of amplitude of the wave
(B) directly proportional to the velocity of the wave
(C) directly proportional to the square of frequency of the wave
(D) inversely proportional to the density of the medium.

14. The ends of a stretched wire of length L are fixed at x = 0 & x = L . In one experiment the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ & energy is E_1 and in other experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2 . Then : **[JEE - 2001 Screening, 2/200]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

15. Two symmetrical and identical pulses in a stretched string, whose centers are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be : **[JEE - 2001 Screening, 2/200]**



(A) zero (B) purely kinetic
(C) purely potential (D) partly kinetic and partly potential

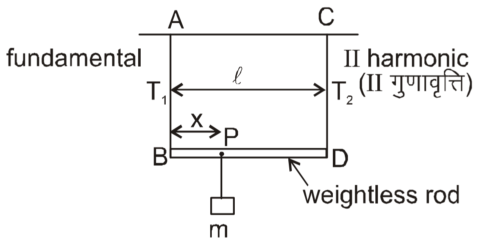
16. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire . When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is : **[JEE - 2002 Screening, 3/300]**

(A) 25 kg (B) 5 kg (C) 12.5 kg (D) 1/25 kg

17. A string of mass ' m ' and length 'ℓ', fixed at both ends is vibrating in its fundamental mode.The maximum amplitude is ' a ' and the tension in the string is ' T '. Find the energy of vibrations of the string. **[JEE - 2003_mains , 4/60]**

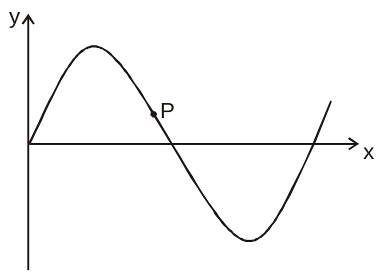
18. A transverse wave travelling in a string produces maximum transverse velocity of 3 m/s and maximum transverse acceleration 90 m/s^2 in a particle. If the velocity of wave in the string is 20 m/s. Determine the equation of the wave ? **[JEE - 2005_mains , 4/60]**

19. A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass 'm' is suspended point P such that BP is equal to 'x', if the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is : **[JEE - 2006_mains, 3/184]**



(A) l/5 (B) l/4 (C) 4l/5 (D) 3l/4

20. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **[JEE - 2008, 3/163]**
Figure :



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

21. A 20cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string. **[JEE - 2009, 4/160, –1]**

22. When two progressive waves y_1 = 4 sin (2x – 6t) and y_2 = 3 sin $\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is : **[JEE- 2010, 3/163]**

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

* ***Marked Questions are having more than one correct option.***

1. A wave y = a sin (ωt - kx) on a string meets with another wave producing a node at x = O. Then the equation of the unknown wave is : **[AIEEE - 2002 4/300]**
(1) y = a sin (ωt + kx) (2) y = -a sin (ωt + kx) (3) y = a sin (ωt - kx) (4) y = -a sin (ωt - kx)

2. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is - **[AIEEE - 2002 4/300]**
(1) 20 (2) 80 (3) 40 (4) 120

3. The displacement y of a wave travelling in the x-direction is given by **[AIEEE 2003 4/300]**

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$

where x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is :
(1) 300 (2) 600 (3) 1200 (4) 200

4. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is : **[AIEEE 2003 4/300]**
(1) 50 Hz (2) 100 Hz (3) 200 Hz (4) 25 Hz

5. The displacement y of a particle in a medium can be expressed as : **[AIEEE 2004 4/300]**

$y = 10^{-6} \sin(100t + 20x + \frac{\pi}{4})$m, where t is in second and x in metre. The speed of the wave is:
(1) 2000 m/s (2) 5 m/s (3) 20 m/s (4) 5π m/s

6. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[AIEEE 2006 3/180]**
(1) 10.5 Hz (2) 105 Hz (3) 1.05 Hz (4) 1050 Hz

7. A wave travelling along the x- axis is described by the equation y(x,t) = 0.005 cos (αx –βt). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(1) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (2) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ **[AIEEE 2008 3/105, –1]**

(3) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ (4) α = 25.00 π, β = π

8. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02 (m) sin

$\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE 2010 144/4 –1]**

(1) 4.0 N (2) 12.5 N (3) 0.5 N (4) 6.25 N

EXERCISE # 4

NCERT QUESTIONS

1. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other and?

2. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wage on the wire equals the speed of sound in dry air at 20 ^{o}C = 343 m s^{-1}.

3. You have learnt that a travelling wave in one dimension is represented by a function y = $f(x,t)$ where x and t must appear in the combination x - υ t or $x + \upsilon t$, i.e. y = F (x ± υ t). Is the converse true ? Examine if the following functions for y can possibly represent a travelling wave:
(a) $(x - \upsilon t)^2$, (b) log $[(x + \upsilon t)/x_o]$, (c) exp $[-(x + \upsilon t)/x_o]$, (d) $1/(x + \upsilon t)$

4. A transverse harmonic wave on a string is described by y (x, t) = 3.0 sin (36 t + 0.018 x + π/4), where x and y are in cm. and t in s. The positive direction of x is from left to right.
(a) Is this a travelling wave or a stationary wave ?
(b) What are its amplitude and frequency ?
(c) What is the initial phase at the origin ?
(d) What is the least distance between two successive crests in the wave ?

5. For the travelling harmonic wave

$$y(x,t) = 2.0 \cos 2\pi\ (10t - 0.0080\, x + 0.35)$$

where x and y are in cm and t in s. Calculate tha phase difference between oscillatory motion of two points separated by a distance of
(a) 4 m
(b) 0.5 m,
(c) $\lambda/2$,
(d) $3\lambda/4$

6. The transverse displacement of string (clamped at its both ends) is given by

$$y(x,t) = 0.06 \sin\ \cos (120\pi t)$$

where and y are in m and t in s. The length os the string is 1.5 m and its mass is 3.0 x 10^{-2} kg.
Answer the following :
(a) Does the function represent a travelling wave or a stationary wave?
(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength, frequency, and speed of each wave ?
(d) Determine the tension in the string.

7. (i) For the wave on a string described in Exercise, do all the points on the string oscillate with the same (a) frequency, (b) phases, (c) amplitude ? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end ?

8. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 x 10-2 kg and its linear density is 4.0 x 10-2 kg m-1. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

9. One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 mm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

ANSWERS

Exercise # 1

PART-I

A-1. (D) **A-2.** (C) **A-3.*** (CD) **A-4.** (C) **A-5.** (i) (C), (ii) (B), (iii) (D)

A-6. (i) (B), (ii) (A) **A-7.** (B) **A-8.** (C) **A-9.** (D) **A-10.*** (AD) **B-1.** (A)

B-2. (B) **B-3.** (A) **B-4.** (C) **B-5.** (D) **B-6.** (B) **B-7.** (A) **B-8.** (C)

B-9. (C) **B-10.** (B) **B-11.** (D) **B-12.** (A) **C-1.** (C) **C-2.** (A) **C-3.** (B)

C-4. (D) **C-5.** (A) **C-6.** (D) **C-7.** (C) **C-8.** (C) **C-9.** (B) **C-10.** (C)

D-1. (B) **D-2.*** (CD) **D-3.** (A) **D-4.** (D) **D-5.** (D) **D-6.** (i) (C), (ii) (C), (iii) (B)

E-1. (D) **E-2.** (C) **E-3.** (B) **E-4.** (A) **E-5.** (i) (C), (ii) (C) **E-6.** (B)

E-7. (D) **E-8.** **(i)** (B), **(ii)** (B), **(iii)** (A) **E-9.** **(i)** (A), **(ii)** (B), **(iii)** (C), **(iv)** (D)

E-10. (C) **E-11.** (B) **E-12.** (C) **E-13.** (B)

PART-II

1. (B) **2.** (B) **3.** (C) **4.** (A) **5.** (C) **6.** (D) **7.** (C)

8. (B) **9.** (A) **10.** (B) **11.** (A) **12.** (D)

11. (A) R (B) P (C) S (D) Q **12.** (A) – R, (B) – S, (C) – P, (D) – Q

13. (A) – P, Q (B) – R, S (C) – P, S (D) – Q, R **14.** (A) **15.** (D) **16.** (B)

17. (D) **18.** (A) **19.** F **20.** F **21.** F **22.** T **23.** T

24. πx_0 **25.** $A\omega$ **26.** π rad , 0 m **27.** 0.117 **28.** (A)

Exercise # 2

PART-I

1. (B) **2.** (D) **3.** (B) **4.** (A) **5.** (D) **6.** (D) **7.** (B)

8. (B) **9.** **(i)** (B), **(ii)** (C), **(iii)** (D) **10.** (A) **11.** (B) **12.** (A)

13. (B) **14.** (B) **15.** (A) **16.** (B) **17.** (C) **18.** (D) **19.** (C)

20. (D) **21.** (C) **22.** (C) **23.** (A) **24.** (D) **25.** **(i)** (A), **(ii)** (B)

26. (D) **27.** (B) **28.** (D) **29.** (A) **30.** (BCD) **31.** (CD) **32.** (BC)

33. (AD) **34.** (ABC)

PART-II

1.	n = 5	**2.**	x = 4, P = 2	**3.**	y = 5	**4.**	z = 2
5.	x = 5	**6.**	x = 2, y = 7	**7.**	y = 3	**8.**	x = 4
9.	X = 2	**10.**	x = 5	**11.**	y = 2	**12.**	8
13.	x = 4						

Exercise # 3

PART-I

1. (B) **2.** (B) **3.** (C) **4.*** (AC) **5.** (C) **6.*** (BCD) **7.*** (ABC)

8. (i) $\frac{5}{3}\sqrt{\frac{m}{70\,S}}$, where, S = area of cross section of wire, (ii) 8 **9.** (D)

10. $\frac{1}{15\,\alpha}\,[(\mu_0 + a\,\ell)^{3/2} - (\mu_0)^{3/2}] = \frac{10\sqrt{10}-1}{135}$ s **11.** (D) **12.** (A) **13.** (ABC)

14. (C) **15.** (B) **16.** (A) **17.** $\frac{\pi^2 a^2 T}{4L}$

18. Equation of wave in string y = 0.1 sin $\left(30t \pm \frac{3}{2}x + \phi\right)$ [where ϕ is initial phase]

19. (A) **20.** (A) **21.** 5 **22.** 5

PART-II

1. (2) **2.** (2) **3.** (1) **4.** (2) **5.** (2) **6.** (2) **7.** (4)
8. (4)

Exercise # 4

1. 0.5 s **2.** 0.5 s

3. The converse is not true. An obvious requirement for an acceptable function for a travelling wave is that it should be finite everywhere and at all times. Only function (c) satisfies this condition, the remaining functions cannot possibly represent a travelling wave.

4. (a) A travelling wave. It travels from right to left with a speed of 20 ms^{-1}.
(b) 3.0 cm, 5.7 Hz (c) $\pi/4$ (d) 3.5 m

5. (a) 6.4 π rad, (b) 0.8 π rad, (c) π rad, (d) (p/2) rad

6. (a) Stationary wave
(b) ℓ = 3 m, n = 60 Hz, and u= 180 m s^{-1} for each wave
(c) 648 N

7. (a) All the points except the nodes on the string have the same frequency and phase but not the same amplitude.
(b) 0.042 m

8. (a) 79 m s^{-1} (b) 248 m s^{-1}

9. y = 0.005 sin ($\omega t \pm kx$); here $\omega = 1.61 \times 10^3\ s^{-1}$, k = 4.84 m^{-1} ; x and y are in m.