SIMPLE HARMONIC MOTION

Contents

Syllabus

Linear and angular simple harmonic motions.

Name : ______________________________ **Contact No.** ____________________

ETOOS ACADEMY Pvt. Ltd

F-106, Road No.2 Indraprastha Industrial Area, End of Evergreen Motor, BSNL Lane, Jhalawar Road, Kota, Rajasthan (324005)

Tel. : +91-744-242-5022, 92-14-233303

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

* ***Marked Questions are having more than one correct option.***

SECTION (A) : EQUATION OF SHM

A-1. For a particle executing simple harmonic motion, the acceleration is proportional to
(A) displacement from the mean position (B) distance from the extreme position
(C) distance travelled since t = 0 (D) speed

A-2. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

A-3. A simple harmonic motion having an amplitude A and time period T is represented by the equation : $y = 5 \sin \pi(t + 4)$ m
Then the values of A (in m) and T (in sec) are :
(A) A = 5; T = 2 (B) A = 10 ; T = 1 (C) A = 5 ; T = 1 (D) A = 10 ; T = 2

A-4. The displacement of a particle in simple harmonic motion in one time period is
(A) A (B) 2A (C) 4A (D) zero

A-5. The maximum acceleration of a particle in SHM is made two times, keeping the maximum speed to be constant. It is possible when
(A) amplitude of oscillation is doubled while frequency remains constant
(B) amplitude is doubled while frequency is halved
(C) frequency is doubled while amplitude is halved
(D) frequency is doubled while amplitude remains constant

A-6. A particle is executing S.H.M. between $x = \pm a$. The time taken to go from 0 to $\frac{a}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2, then

(A) $T_1 < T_2$ (B) $T_1 > T_2$ (C) $T_1 = T_2$ (D) $T_1 = 2T_2$

A-7. A particle executing SHM. Its time period is equal to the smallest time interval in which particle acquires a particular velocity $\vec{v}$, the magnitude of $\vec{v}$ may be :

(A) Zero (B) V_{max} (C) $\frac{V_{max}}{2}$ (D) $\frac{V_{max}}{\sqrt{2}}$

A-8. The displacement of a body executing SHM is given by $x = A \sin (2\pi t + \pi/3)$. The first time from t = 0 when the speed is maximum is
(A) 0.33 sec (B) 0.16 sec (C) 0.25 sec (D) 0.5 sec

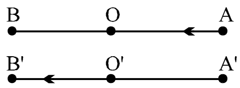
A-9. If the displacement (x) and velocity v of a particle executing simple harmonic motion are related through the expression $4v^2 = 25 - x^2$ then its time period is

(A) π (B) 2π (C) 4π (D) 6π

A-10. Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

A-11. Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time



(A) T/8 (B) 3T/8 (C) T/6 (D) T/3

A-12. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

SECTION (B) : ENERGY

B-1. A body executes simple harmonic motion. The potential energy (PE), kinetic energy (KE) and total energy (TE) are measured as a function of displacement x. Which of the following statements is true?

(A) TE is zero when x = 0

(B) PE is maximum when x = 0

(C) KE is maximum when x = 0

(D) KE is maximum when x is maximum

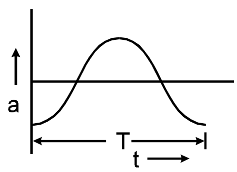
B-2. A particle starts oscillating simple harmonically from its equilibrium position then, the ratio of kinetic energy and potential energy of the particle at the time T/12 is : (T = time period)

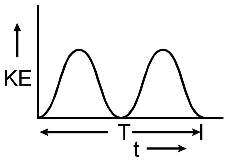
(A) 2 : 1 (B) 3 : 1 (C) 4 :1 (D) 1 : 4

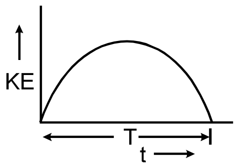
B-3. In SHM particle oscillates with frequency υ then find the frequency of oscillation of its kinetic energy.

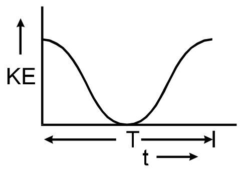
(A) υ (B) $\upsilon/2$ (C) 2υ (D) zero

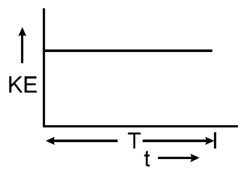
B-4. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(A) 

(B) 

(C) 

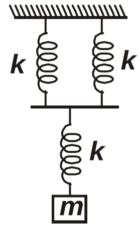
(D) 

B-5. The total mechanical energy of a particle of mass m executing SHM is $E = 1/2m\omega^2A^2$. If the particle is replaced by another particle of mass m/2 while the amplitude A remains same. (force constant of S.H.M. remain same) New mechanical energy will be :

(A) $\sqrt{2}$ E (B) 2E (C) E/2 (D) E

SECTION (C) : SPRING MASS SYSTEM

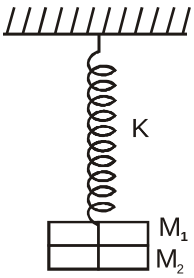
C-1. A block of mass *m* hangs from three light springs having same spring constant *k*. If the mass is slightly displaced vertically, time period of oscillation will be



(A) $2\pi\sqrt{\frac{m}{3k}}$ (B) $2\pi\sqrt{\frac{3m}{2k}}$

(C) $2\pi\sqrt{\frac{2m}{3k}}$ (D) $2\pi\sqrt{\frac{3k}{m}}$

C-2. Two masses M_1 and M_2 are suspended from the ceiling by a massless spring of force constant K. Initially the system is at equilibrium. Now M_1 is gently removed, then amplitude of vibration of the system will be:



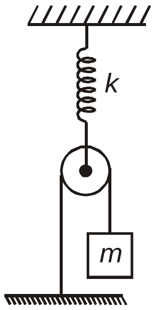
(A) $\frac{(M_1+M_2)g}{K}$

(B) $\frac{M_1g}{K}$

(C) $\frac{M_2g}{K}$

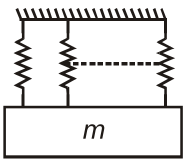
(D) $\frac{(M_2-M_1)g}{K}$

C-3. In given figure. If spring is light and pulley is massless, then time period of oscillations of block of mass *m* suspended will be



(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $4\pi\sqrt{\frac{m}{k}}$ (C) $\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{2m}{k}}$

C-4. A spring of spring constant *k* is cut into *n* equal parts. A block of mass *m* is attached to these parts of the spring as shown



Time period of the oscillation of the block will be

(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{nk}}$ (C) $\frac{2\pi}{n}\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{nm}{k}}$

C-5. A uniform spring whose unstressed length is ℓ, has a force constant K. The spring is cut into two pieces of unstressed length ℓ_1 and ℓ_2, where $\ell_2 = n\ell_1$, n being an integer. Now a mass m is made to oscillate with first spring. The time period of its oscillation would be

(A) $T = 2\pi\sqrt{\dfrac{mn}{K(n+1)}}$ (B) $T = 2\pi\sqrt{\dfrac{m}{nK}}$

(C) $T = 2\pi\sqrt{\dfrac{m}{K(n+1)}}$ (D) $T = 2\pi\sqrt{\dfrac{m(n+1)}{nK}}$

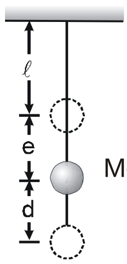
C-6. A horizontal spring–block system of mass 2kg executes S.H.M. When the block is passing through its equilibrium position, an object of mass 1kg is put on it and the two move together. The new amplitude of vibration is (A being its initial amplitude):

(A) $\sqrt{\dfrac{2}{3}}A$ (B) $\sqrt{\dfrac{3}{2}}A$ (C) $\sqrt{2}\,A$ (D) $\dfrac{A}{\sqrt{2}}$

C-7. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is

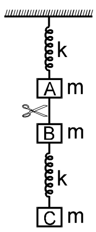
(A) $2\pi\sqrt{\dfrac{m(2k)}{k^2}}$ (B) $\dfrac{1}{2\pi}\sqrt{\dfrac{m(2k)}{k^2}}$ (C) $2\pi\sqrt{\dfrac{m}{k}}$ (D) $2\pi\sqrt{\dfrac{m}{k+k}}$

C-8. An elastic string of length ℓ supports a heavy particle of mass m and the system is in equilibrium with elongation produced being e as shown in figure. The particle is now pulled down below the equilibrium position through a distance d (<e) and released. The angular frequency and maximum amplitude for SHM is



(A) $\sqrt{\dfrac{g}{e}}$, e (B) $\sqrt{\dfrac{g}{\ell}}$, 2e (C) $\sqrt{\dfrac{g}{d+e}}$, d (D) $\sqrt{\dfrac{g}{e}}$, 2d

C-9. The spring block system as shown in figure is in equilibrium. The string connecting blocks A and B is cut. The mass of all the three blocks is m and spring constant of both the spring is k. The amplitude of resulting oscillation of block A is :



(A) $\dfrac{mg}{k}$ (B) $\dfrac{2mg}{k}$ (C) $\dfrac{3mg}{k}$ (D) $\dfrac{4mg}{k}$

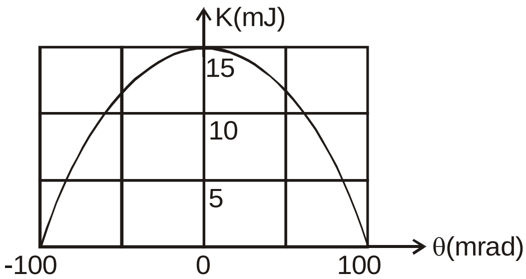
Section (D) : Simple Pendulum

D-1. A simple pendulum suspended from the ceiling of a stationary lift has period T_0. When the lift descends at steady speed, the period is T_1. When it descends with constant downward acceleration, the period is T_2. Which one of the following is true?

(A) $T_0 = T_1 = T_2$ (B) $T_0 = T_1 < T_2$

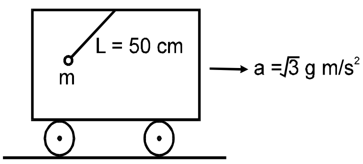
(C) $T_0 = T_1 > T_2$ (D) $T_0 < T_1 < T_2$

D-2. Figure shows the kinetic energy K of a simple pendulum versus its angle θ from the vertical. The pendulum bob has mass 0.2 kg. The length of the pendulum is equal to (g = 10 m/s^2).



(A) 2.0 m (B) 1.8 m (C) 1.5 m (D) 1.2 m

D-3. A simple pendulum 50 cm long is suspended from the roof of a cart accelerating in the horizontal direction with constant acceleration $\sqrt{3}$ g m/s^2. The period of small oscillations of the pendulum about its equilibrium position is (g = π^2 m/s^2) :



(A) 1.0 sec (B) $\sqrt{2}$ sec (C) 1.53 sec (D) 1.68 sec

SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

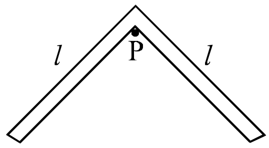
E-1. A ring of diameter 2m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre move in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is

(A) 2m (B) 4m (C) 1.5m (D) 3m

E-2. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then maintain the orientation. If the sphere is then released, its time period of the oscillation will be :

(A) π second (B) $\sqrt{2}\pi$ second (C) 2π second (D) 4π second

E-3. A system (L-shaped) of two identical rods of mass m and length ℓ are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:



(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$ (C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

E-4. A rod whose ends are A & B and of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9 : 4 at points A and B. Find the distance of the centre of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

E-5. A circular disc has a tiny hole in it, at a distance z from its center. Its mass is M and radius R (R>z). A horizontal shaft is passed through the hole and held fixed so that the disc can freely swing in the vertical plane. For small disturbance, the disc performs SHM whose time period is minimum for z =

(A) R / 2 (B) R / 3 (C) $R/\sqrt{2}$ (D) $R/\sqrt{3}$

SECTION (F) : SUPERPOSITION OF SHM

F-1. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(0.5t)\sin(1000\,t)$. The given expression is composed by minimum :

(A) four SHMs (B) three SHMs (C) one SHM (D) None of these

F-2. A particle is subjected to two mutually perpendicular simple harmonic motions such that its x and y coordinates are given by

$$x = 2\sin\omega t \;;\; y = 4\sin\left(\omega t + \frac{\pi}{2}\right)$$

The path of the particle will be :

(A) an ellipse (B) a straight line (C) a parabola (D) a circle

F-3. The amplitude of the vibrating particle due to superposition of two SHMs,

$$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right) \text{ and } y_2 = \sin\omega t \text{ is :}$$

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

F-4. Two simple harmonic motions $y_1 = A\sin\omega t$ and $y_2 = A\cos\omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is:

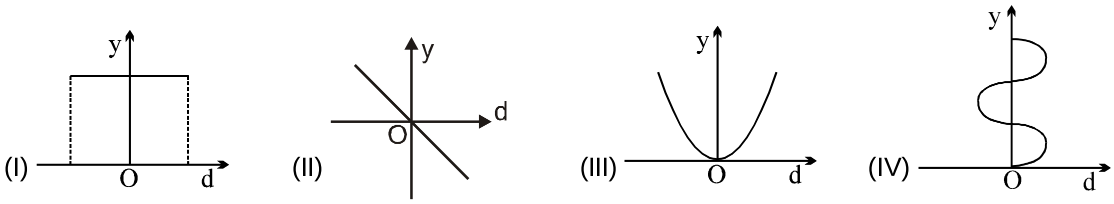
(A) $\frac{1}{2}m\omega^2A^2$ (B) $m\omega^2A^2$ (C) $\frac{1}{4}m\omega^2A^2$ (D) zero

PART - II : MISLLANEOUS QUESTIONS

1. COMPREHENSION

COMPREHENSION # 1

The graphs in figure show that a quantity y varies with displacement d in a system undergoing simple harmonic motion.



Which graphs best represents the relationship obtained when y is

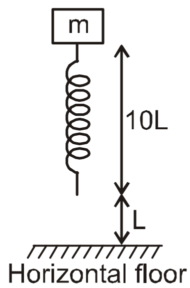
1. The total energy of the system
(A) I (B) II (C) III (D) IV

2. The time
(A) I (B) II (C) III (D) IV

3. The unbalanced force acting on the system.
(A) I (B) II (C) III (D) None

COMPREHENSION # 2

A small block of mass m is fixed at upper end of a massless vertical spring of spring constant $K = \frac{4mg}{L}$ and natural length '10L'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown. The spring is initially unstressed and the spring-block system is released from rest in the shown position.



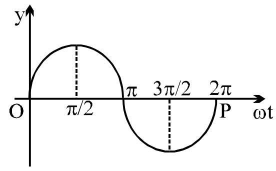
4. At the instant speed of block is maximum, the magnitude of force exerted by spring on the block is
(A) $\frac{mg}{2}$ (B) mg (C) Zero (D) None of these

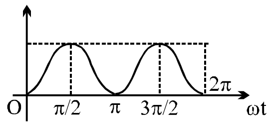
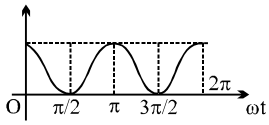
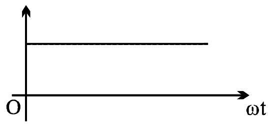
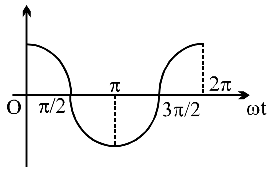
5. As the block is coming down, the maximum speed attained by the block is
(A) $\sqrt{gL}$ (B) $\sqrt{3gL}$ (C) $\frac{3}{2}\sqrt{gL}$ (D) $\sqrt{\frac{3}{2}gL}$

6. Till the block reaches its lowest position for the first time, the time duration for which the spring remains compressed is
(A) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$ (B) $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$
(C) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$ (D) $\frac{\pi}{2}\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$

2. MATCH THE COLUMN

7. The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown below. Then the best matching is



Column A	Column B
(A) K.E. versus phase angle curve	(i) 
(B) P.E. versus phase angle curve	(ii) 
(C) T.E. versus phase angle curve	(iii) 
(D) Velocity versus phase angle curve	(iv) 

8. In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

Column-I	Column-II
(A) A simple pendulum of length 'ℓ' oscillating with small amplitude in a lift moving down with retardation g/2.	(p) $T = 2\pi\sqrt{\frac{2\ell}{3g}}$
(B) A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a lift, stretches the spring by length 'ℓ' in equilibrium. It's time period when lift moves up with an acceleration g/2 is	(q) $T = 2\pi\sqrt{\frac{\ell}{g}}$
(C) The time period of small oscillation of a uniform rod of length 'ℓ' smoothly hinged at one end. The rod oscillates in vertical plane.	(r) $T = 2\pi\sqrt{\frac{2\ell}{g}}$
(D) A cubical block of edge 'ℓ' and specific density $\rho/2$ is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is	(s) $T = 2\pi\sqrt{\frac{\ell}{2g}}$

3. ASSERTION / REASON

9. **Statement-1 :** A particle is moving along x-axis. The resultant force F acting on it at position x is given by $F = -ax - b$. Where a and b are both positive constants. The motion of this particle is not SHM.
Statement-2 : In SHM restoring force must be proportional to the displacement from mean position.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

10. **Statement-1 :** For a particle performing SHM, its speed decreases as it goes away from the mean position.
Statement-2 : In SHM, the acceleration is always opposite to the velocity of the particle.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

11. **Statement-1 :** Motion of a ball bouncing elastically in vertical direction on a smooth horizontal floor is a periodic motion but not an SHM.
Statement-2 : Motion is SHM when restoring force is proportional to displacement from mean position.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

12. **Statement-1 :** A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, will perform SHM only if the two SHM's are in the same direction.
Statement-2 : A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, perpendicular to each other the particle can be in uniform circular motion.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true

13. **Statement-1 :** In case of oscillatory motion the average speed for any time interval is always greater than or equal to its average velocity.
Statement-2 : Distance travelled by a particle cannot be less than its displacement.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

4. TRUE/FALSE

14. A particle is moving on an elliptical path with constant angular velocity. Its projection on major axis is simple harmonic motion.

15. Suppose that a system consists of a block of unknown mass and a spring of unknown force constant. It is possible to calculate the period of oscillation of this block- spring system simply by measuring the extension of the spring produced by attaching the block with the spring kept in vertical equilibrium position and by knowing value of gravitational acceleration at that place.

16. Two simple harmonic motions are represented by the equations
$x_1=5\sin[2\pi t+\pi/4]$ and $x_2=5\sqrt{2}(\sin 2\pi t+\cos 2\pi t)$ their amplitudes are in the ratio 1:2.

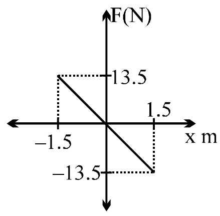
EXERCISE # 2

PART - I : MIXED OBJECTIVE

* ***Marked Questions are having more than one correct option.***

SINGLE CORRECT ANSWER TYPE

1. A particle of mass 1 kg is undergoing S.H.M., for which graph between force and displacement (from mean position) as shown. Its time period, in seconds, is:

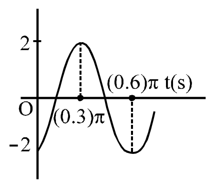


(A) $\pi/3$ (B) $2\pi/3$ (C) $\pi/6$ (D) $3/\pi$

2. The motion of a particle is given by $y = A \sin \omega t + B \cos \omega t$. The motion of the particle is

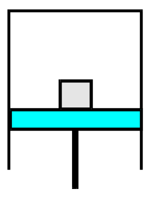
(A) Not simple harmonic
(B) Simple harmonic with amplitude $A + B$
(C) Simple harmonic with amplitude $\frac{A+B}{2}$
(D) Simple harmonic with amplitude $\sqrt{A^2+B^2}$

3. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is



(A) y = 4 cos (0.6t)
(B) y = 2 sin $\left(\frac{10}{3}t - \frac{\pi}{2}\right)$
(C) y = 4 sin $\left(\frac{10}{3}t + \frac{\pi}{2}\right)$
(D) y = 2 cos $\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

4. A block of mass m is resting on a piston as shown in figure which is moving vertically with a SHM of period 1 s. The minimum amplitude of motion at which the block and piston separate is ($g = \pi^2$):



(A) 0.25 m (B) 0.52 m (C) 2.5 m (D) 0.15 m

5. A body performs simple harmonic oscillations along the straight line ABCDE with C as the midpoint of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If AE = 2R, the distance between B and D is

(A) $\frac{\sqrt{3}R}{2}$ (B) $\frac{R}{\sqrt{2}}$ (C) $\sqrt{3}$ R (D) $\sqrt{2}$ R

6. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement:

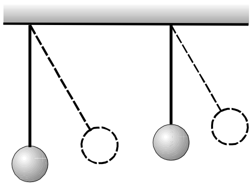
(A) The amplitude of the block is $\frac{v_0}{\omega_0}$

(B) The initial phase of the block is π.

(C) The equation of motion for the block is $\frac{v_0}{\omega_0} \sin \omega_0 t$.

(D) The maximum speed of the block is v_0.

7. Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) :



(A) 1 vibration (B) 3 vibrations (C) 4 vibrations (D) 5 vibrations

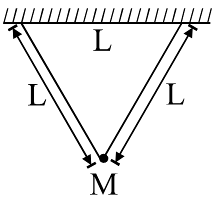
8. A particle is made to under go simple harmonic motion. Find its average acceleration in one time period.

(A) $\omega^2 A$ (B) $\frac{\omega^2 A}{2}$ (C) $\frac{\omega^2 A}{\sqrt{2}}$ (D) zero

9. The magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is :

(A) $\frac{2A\omega^2}{\pi}$ (B) $\frac{A\omega^2}{2\pi}$ (C) $\frac{A\omega^2}{\sqrt{2}\pi}$ (D) Zero

10. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is



(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$

(C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

11. Equation of SHM is $x = 10 \sin 10\pi t$. Find the distance between the two points where speed is 50π cm/sec. x is in cm and t is in seconds.
(A) 10 cm (B) 20 cm (C) 17.32 cm (D) 8.66 cm.

12. Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is
(A) 0° (B) 120° (C) 180° (D) 135°

13. A simple pendulum with a metallic bob has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is 1/4 that of metal, then the time period of the pendulum will be

(A) $\frac{T}{\sqrt{3}}$ (B) $\frac{2T}{\sqrt{3}}$ (C) $\frac{4}{3}T$ (D) $\frac{2}{3}T$

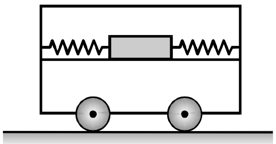
14. A particle performing SHM is found at its equilibrium at t = 1sec. and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m (C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$

15. For a particle acceleration is defined as $\vec{a} = \frac{-5x\hat{i}}{|x|}$ for $x \neq 0$ and $\vec{a} = 0$ for x = 0. If the particle is initially at rest at point (a, 0) what is period of motion of the particle.

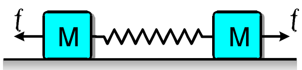
(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec. (C) $2\sqrt{2a/5}$ sec. (D) cannot be determined

16. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is :



(A) $T = 2\pi\sqrt{\frac{Mm}{2k}}$ (B) $T = 2\pi\sqrt{\frac{(M+m)}{kmM}}$ (C) $T = 2\pi\sqrt{\frac{mM}{2k(M+m)}}$ (D) $T = 2\pi\frac{(M+m)^2}{k}$

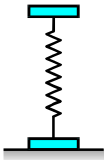
17. A spring of force constant α has two blocks of same mass M connected to each end of the spring as shown in figure. Same force f extends each end of the spring. If the masses are released, then period of vibration is :



(A) $2\pi\sqrt{\frac{M}{2\alpha}}$ (B) $2\pi\sqrt{\frac{M}{\alpha}}$ (C) $2\pi\sqrt{\frac{2\alpha M}{\alpha^2}}$ (D) $2\pi\sqrt{\frac{M\alpha^2}{2\alpha}}$

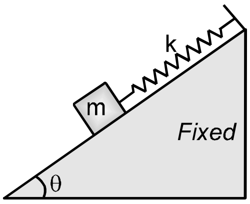
18. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitude is decreased by 5%?
(A) 6 % (B) 3 % (C) 1.5 % (D) 0 %

19. Two plates of same mass are attached rigidly to the two ends of a spring as shown in figure. One of the plates rests on a horizontal surface and the other results a compression y of the spring when it is in equilibrium state. The further minimum compression required, so that when the force causing compression is removed the lower plate is lifted off the surface, will be :



(A) 0.5 y (B) 3y (C) 2y (D) y

20. In the figure shown, the time period and the amplitude respectively when m is released from rest when the spring is relaxed is: (the inclined plane is smooth)



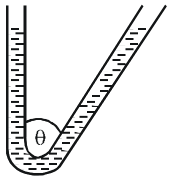
(A) $2\pi\sqrt{\frac{m}{k}}$, $\frac{mg\sin\theta}{k}$

(B) $2\pi\sqrt{\frac{m\sin\theta}{k}}$, $\frac{2\,mg\sin\theta}{k}$

(C) $2\pi\sqrt{\frac{m}{k}}$, $\frac{mg\cos\theta}{k}$

(D) none of these

21. The period of oscillation of mercury of mass m and density ρ poured into a bent tube of cross sectional area S whose right arm forms an angle θ with the vertical as shown in figure is :



(A) $2\pi\sqrt{\frac{m}{\rho S(1+\sin\theta)g}}$

(B) $2\pi\sqrt{\frac{m}{\rho S\sin\theta g}}$

(C) $2\pi\sqrt{\frac{m}{\rho S(1+\cos\theta)g}}$

(D) $2\pi\sqrt{\frac{m}{\rho S\cos\theta g}}$

22. Which of the following expressions does not represent SHM :

(A) $A\cos\omega t$

(B) $A\sin 2\omega t$

(C) $A\sin\omega t + B\cos\omega t$

(D) $Ae^{\sin\omega t}$.

23. The bob in a simple pendulum of length ℓ is released at t = 0 from the position of small angular displacement θ. Linear displacement of the bob at any time t from the mean position is given by

(A) $\ell\theta\cos\sqrt{\frac{g}{\ell}}\,t$

(B) $\ell\sqrt{\frac{g}{\ell}}\,t\cos\theta$

(C) $\ell g\sin\theta$

(D) $\ell\theta\sin\sqrt{\frac{g}{\ell}}\,t$

24. The period of small oscillations of a simple pendulum of length ℓ if its point of suspension O moves a with a constant acceleration $\alpha = \alpha_1\hat{i} - \alpha_2\hat{j}$ with respect to earth is

(A) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_2)^2+\alpha_1^2\}^{1/2}}}$

(B) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_1)^2+\alpha_2^2\}^{1/2}}}$

(C) $T = 2\pi\sqrt{\frac{\ell}{g}}$

(D) $T = 2\pi\sqrt{\frac{\ell}{(g^2+\alpha_1^2)^{1/2}}}$

25. A cylindrical cork piece of density σ, base area A and height h floats in a liquid of density ρ. The cork is slightly depressed and then released. The cork oscillates

(A) Simple harmonically with time period $2\pi\sqrt{\frac{\sigma h}{\rho g}}$

(B) Simple harmonically with time period $2\pi\sqrt{\frac{\rho h}{\sigma g}}$

(C) With time period $2\pi\sqrt{\frac{\sigma h}{\rho g}}$, non harmonically

(D) With time period $2\pi\sqrt{\frac{\rho h}{\sigma g}}$, non harmonically

26. A particle moves along the X-axis according to the equation $x = 10\sin^3(\pi t)$. The amplitudes and frequencies of component SHMs are
(A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2
(B) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2
(C) amplitude 10, 10 ; frequencies 1/2, 1/2
(D) amplitude 30/4, 10 ; frequencies 3/2, 2

MULTIPLE CORRECT ANSWER(S) TYPE QUESTIONS

27. Which of the following is/are the characteristics(s) of SHM?

(A) projection of uniform circular motion on any straight line

(B) periodic nature

(C) displacement time graph is a sine curve

(D) acceleration is zero at the mean position

28. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.

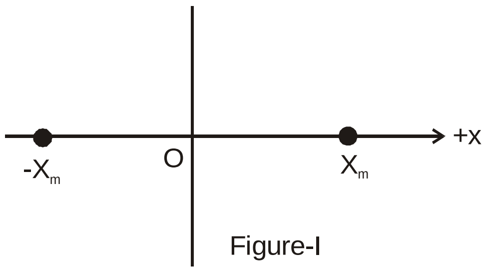
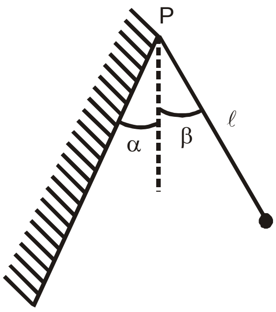


Figure-I

(A) At time t_1, it is going towards X_m.
(B) At time t_1, its speed is decreasing.
(C) At time t_2, its position lies in between $-X_m$ and O.
(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

29. The displacement of a particle varies according to the relation $x = 3\sin 100t + 8\cos^2 50t$. Which of the following is/are correct about this motion.
(A) the motion of the particle is not S.H.M.
(B) the amplitude of the S.H.M. of the particle is 5 units
(C) the amplitude of the resultant S.H. M. is $\sqrt{73}$ units
(D) the maximum displacement of the particle from the origin is 9 units.

30. A ball is hung vertically by a thread of length 'ℓ' from a point 'P' of an inclined wall that makes an angle 'α' with the vertical. The thread with the ball is then deviated through a small angle 'β' ($\beta > \alpha$) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are



(A) $2\sqrt{\frac{\ell}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(B) $2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

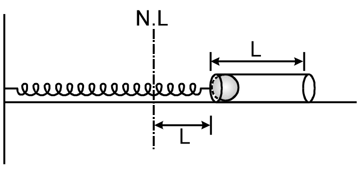
(C) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(D) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\alpha}{\beta}\right)\right]$

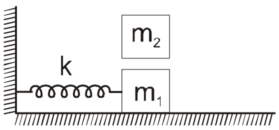
31. The potential energy of a particle of mass 0.1kg, moving along x-axis, is given by U = 5x(x-4)J where x is in metres. It can be concluded that
(A) the particle is acted upon by a constant force.
(B) the speed of the particle is maximum at x = 2 m
(C) the particle executes simple harmonic motion
(D) the period of oscillation of the particle is $\pi/5$ s.

PART - II : SUBJECTIVE QUESTIONS

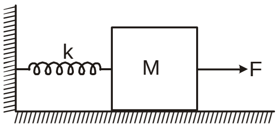
1. A hollow cylinder (closed at both ends) of length L and mass m is attached with a spring of force constant K. An equal mass with negligible radius is kept inside the cylinder. Initially the spring is stretched by a distance L. Find out the time period of the resultant motion. Is this SHM? If all collisions are assumed elastic. There is no friction any where. Initially mass is touching the left side of the cylinder.



2. A block of mass 4kg attached with spring of spring constant 100 N/m executing SHM of amplitude 0.1m on smooth horizontal surface as shown in figure. If another block of mass 5 kg is gently placed on it, at the instant it passes through the mean position then find the frequency and amplitude of the motion assuming that two blocks always move together.



3. A spring mass system is shown in figure .spring is initially unstretched. A man starts pulling the block with constant force F. Find
(A) The amplitude and the time period of motion of the block
(B) The K.E. of the block at mean position
(C) The energy stored in the spring when the block passes through the mean position



4. A 0.1kg ball is attached to a string 1.2m long and suspended as a simple pendulum. At a point 0.2 m below the point of suspension a peg is placed, which the string hits when the pendulum comes down. If the mass is pulled a small distance to one side and released what will be the time period of the motion.

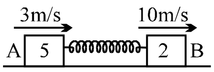
5. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the co-ordinates (3, 0) and (0, 4) respectively. At t = 0, B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. Find the maximum and minimum distances between A and B.

6. A particle of mass 'm' moves on a horizontal smooth line AB of length 'a' such that when particle is at any general point P on the line two forces act on it. A force $\frac{mg(AP)}{a}$ towards A and another force $\frac{2mg(BP)}{a}$ towards B.

(i) Show that particle performs SHM on the line when released from rest from mid-point of line AB.
(ii) Find its time period and amplitude.
(iii) Find the minimum distance of the particle from B during the motion.
(iv) If the force acting towards A stops acting when the particle is nearest to B then find the velocity with which it crosses point B.

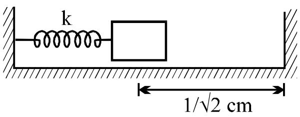
7. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then

(A) find the maximum extension of the spring.



(B) when does the first maximum compression occurs after start.

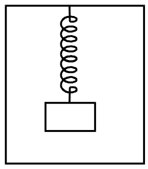
8. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. the approximate value of k is 100 X N/m. Find the value of X. ($\pi^2 \approx 10$)



9. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by, U = ($x^2 - 4x + 3$). All units are in S.I.

if speed of the body at equilibrium position is $2\sqrt{2}$ m/s then the amplitude of oscillations is X m. Find the value of X.

10. A spring mass system is hanging from the ceiling of an elevator in equilibrium. Elongation of spring is ℓ. The elevator suddenly starts accelerating downwards with acceleration g/3 then the frequency of osscilation is $\frac{1}{2\pi}\sqrt{\frac{xg}{\ell}}$. Find the value of X.



11. In the above questions the amplitude of the resulting SHM is $\frac{\ell}{X}$ m. Find the value of X.

12. For euqation

$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{4}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta)$. The value of A' is $\frac{3}{8}\sqrt{X}$ A. Find the value of X.

13. In the above question the phase of the vibrations δ is $\tan^{-1}\left(\frac{1}{X}\right)$. Find the value of X.

EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

* ***Marked Questions are having more than one correct option.***

1. A particle free to move along the x-axis has potential energy given by $U(x) = k[1 - e^{-x^2}]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then
(A) at points away from the origin, the particle is in unstable equilibrium.
(B) for any finite non-zero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic. **[JEE - 99, 2/200]**

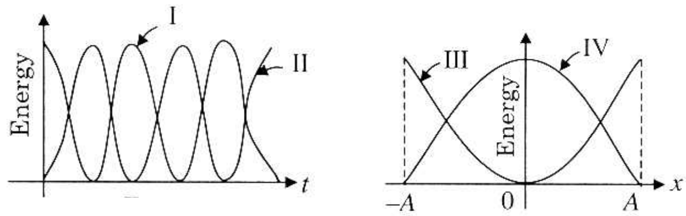
2. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^0, then, **[IIT- 1999, 3/100]**
(A) the resultant amplitude is $(1+\sqrt{2})a$
(B) the phase of the resultant motion relative to the first is 90^0.
(C) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.
(D) the resulting motion is not simple harmonic.

3. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination α, is given by **[I.I.T. (Scr.) 2000, 1/35]**

(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$ (C) $2\pi\sqrt{\frac{L}{g}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

4. A particle executes simple harmonic motion between X = – A and x = +A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2, then **[I.I.T. (Scr.) 2001, 1/35]**
(A) $T_1 < T_2$ (B) $T_1 > T_2$ (C) $T_1 = T_2$ (D) $T_1 = 2T_2$

5. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graphs which represents the variation of potential energy (P.E.) as a function of time t and displacement x :



[JEE (Scr.) 2003, 3/84,–1]

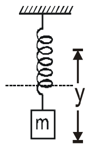
(A) I and III (B) II and IV (C) II and III (D) I and IV

6. A solid sphere of radius R is half submerged in a liquid of density ρ. If the sphere is slightly pushed down and released, find the frequency of small oscillations. **[JEE (Mains) - 2004, 2/60]**

7. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$ ($k = 1\ m/s^2$) where y is the vertical displacement, the time period now becomes T_2. The ratio of $\left(\frac{T_1}{T_2}\right)^2$ is : ($g = 10\ m/s^2$) **[JEE (Scr.) 2005, 3/84,–1]**

(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) 1 (D) $\frac{4}{5}$

8. A block is performing SHM of amplitude 'A' in vertical direction. When block is at 'y' (measured from mean position), it detaches from spring, so that spring contracts and does not affect the motion of the block. Find 'y' such that block attains maximum height from the mean position. (Given $A\omega^2 > g$) **[JEE (Mains) 2005, 4/60]**



9. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM **[JEE 2006, 5/184,–1]**

(A) for any value of A,B and C (except C = 0) (B) If A = – B, C = 2B , amplitude = $|B\sqrt{2}|$

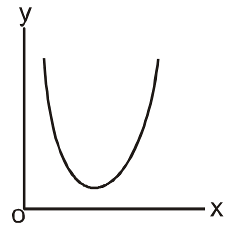
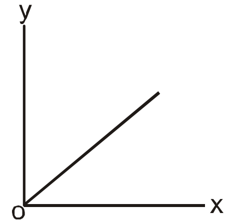
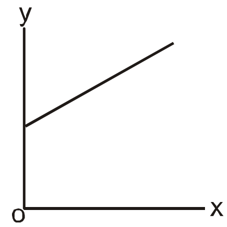
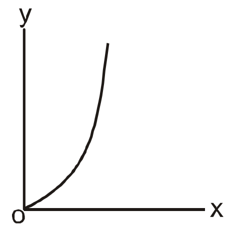
(C) If A = B ; C = 0 (D) If A = B ; C = 2B , amplitude = |B|

10. **Column I** describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II**

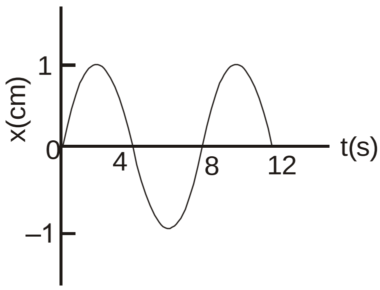
[IIT-JEE 2007, 6/162]

Column I		**Column II**	
(A)	The object moves on the x–axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p)	The object executes a simple harmonic motion.
(B)	The object moves on the x–axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q)	The object does not change its direction,
(C)	The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r)	The kinetic energy of the object keeps on decreasing.
(D)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s)	The object can change its direction only once.

11. **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graphs given in **Column II**. **[IIT-JEE 2008, 6/163]**

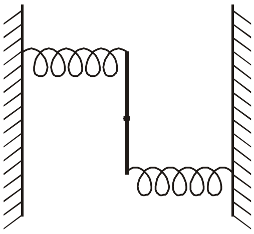
Column I		Column II	
(A)	Potential energy of a simple pendulum (y–axis) as a function of displacement (x–axis)	(p)	
(B)	Displacement (y–axis) as a function of time (x–axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x–direction.	(q)	
(C)	Range of a projectile (y–axis) as a function of its velocity (x–axis) when projected at a fixed angle.	(r)	
(D)	The square of the time period (y–axis) of a simple pendulum as a function of its length (x–axis).	(s)	

12. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is : **[IIT-JEE 2009, 3/160, –1]**



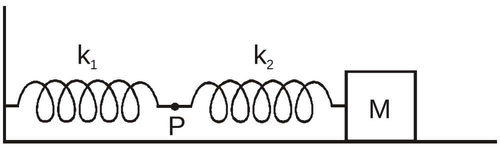
(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (B) $\frac{-\pi^2}{32}$ cm/s^2 (C) $\frac{\pi^2}{32}$ cm/s^2 (D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

13. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is : **[IIT-JEE 2009, 3/160, –1]**



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

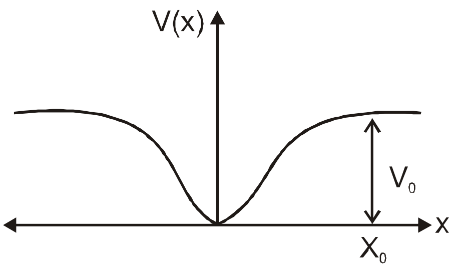
14. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is : **[JEE 2009, 3/160, –1]**



(A) $\frac{k_1A}{k_2}$ (B) $\frac{k_2A}{k_1}$ (C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

Comprehension :

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[JEE 2010, 3/160, –1]**



15. If the total energy of the particle is E, it will perform periodic motion only if : **[JEE 2010, 3/160, –1]**

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

16. For periodic motion of small amplitude A, the time period T of this particle is proportional to :

[JEE 2010, 3/160, –1]

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

17. The acceleration of this particle for $|x| > X_0$ is : **[JEE 2010, 3/160, –1]**

(A) proportional to V_0

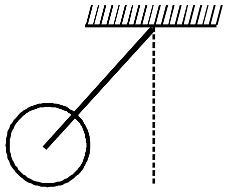
(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) zero

18. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is : **[JEE 2010, 3/160, –1]**

19. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached : (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are true ?

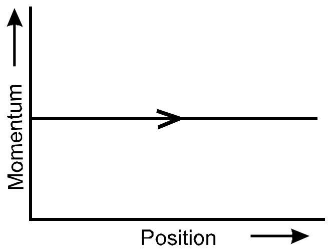


[JEE 2011, 4/160,]

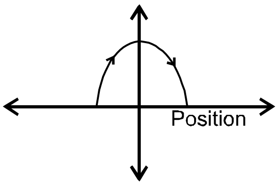
(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A < Restoring torque in cae B
(C) Angular frequency for case A > Angular frequency for case B
(D) Angular frequency for case A < Angular frequency for case B.

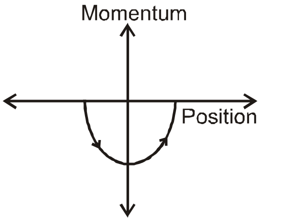
Paragraph for Question Nos. 20 to 22

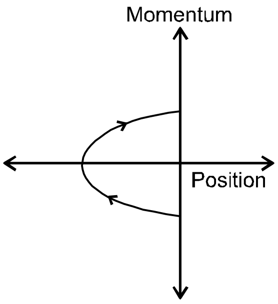
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. momentum are changed. here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicated the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards or to right is positive and downwards (or to left) is negative.

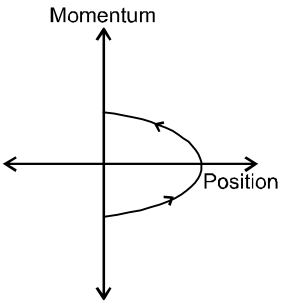


20. The phase space diagram for a ball thrown vertically up from ground is : **[JEE 2011, 3/160, – 1]**

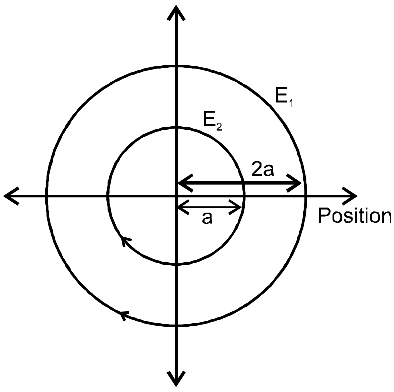
(A) 

(B) 

(C) 

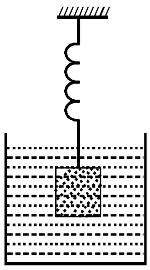
(D) 

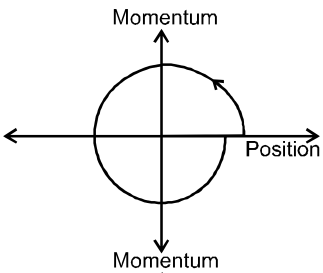
21. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then **[JEE 2011, 3/160, – 1]**

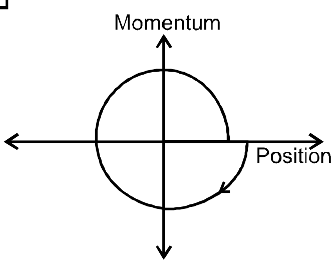


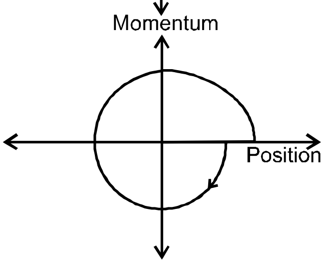
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$ (C) $E_1 = 4E_2$ (D) $E_1 = 16\ E_2$

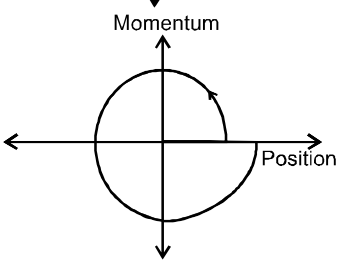
22. Consider the spring - mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is **[JEE 2011, 3/160, – 1]**



(A) 

(B) 

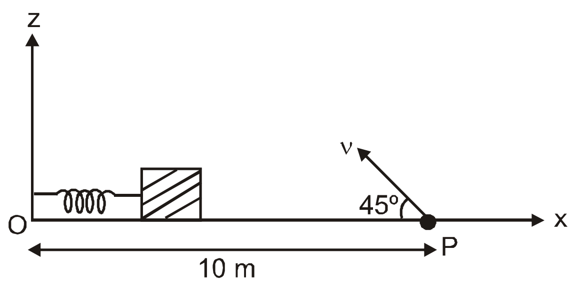
(C) 

(D) 

23. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin \left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin (\omega t + \phi)$ brings the mass to a complete rest. The value of B and ϕ are : **[JEE 2011, 3/160, – 1]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

24. A small block is connected to one end of a massless spring of un-stretched length 4.9 m.The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P an angle of 45° as shown in figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2) **[JEE 2012 (3, –1)/136]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ M/S

25. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontal from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision, **[JEE Advanced (P-2) 2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0.

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

*** Marked Questions are having more than one correct option.**

1. In a simple harmonic oscillator, at the mean position: **[AIEEE 2002; 4/300]**
(1) kinetic energy is minimum, potential energy is maximum
(2) both kinetic and potential energies are maximum
(3) kinetic energy is maximum, potential energy is minimum
(4) both kinetic and potential energies are minimum

2. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. Then the ratio of m/M is : **[AIEEE 2003; 4/300]**
(1) 3/5 (2) 25/9 (3) 16/9 (4) 5/3

3. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is : **[AIEEE 2003, 4/300]**
(1) 11% (2) 21% (3) 42% (4) 10.5%

4. The displacement of a particle varies according to the relation x = 4 (cos πt + sin πt). The amplitude of the particle is : **[AIEEE 2003; 4/300]**
(1) –4 (2) 4 (3) $4\sqrt{2}$ (4) 8

5. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statements is true?
(1) K.E. is maximum when x = 0 (2) T.E. is zero when x = 0 **[AIEEE 2003; 4/300]**
(2) K.E. is maximum when x is maximum (4) P.E. is maximum when x = 0

6. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. Which relationship between t and t_0 is true? **[AIEEE 2004]**
(1) $t = t_0$ (2) $t = t_0/2$ (3) $t = 2t_0$ (4) $t = 4t_0$

7. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then : **[AIEEE 2004]**
(1) $T = t_1 + t_2$ (2) $T^2 = t_1^2 + t_2^2$ (3) $T^{-1} = t_1^{-1} + t_2^{-1}$ (4) $T^{-2} = t_1^{-2} + t_2^{-2}$

8. The total energy of a particle, executing simple harmonic motion is : Where x is the displacement from the mean position. **[AIEEE 2004]**
(1) $\propto x$ (2) $\propto x^2$ (3) independent of x (4) $\propto x^{1/2}$

9. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to : **[AIEEE 2004]**
(1) $\frac{m}{\omega_0^2 - \omega^2}$ (2) $\frac{1}{m(\omega_0^2 - \omega^2)}$ (3) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (4) $\frac{m}{\omega_0^2 + \omega^2}$

10. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then : **[AIEEE 2004]**
(1) $\omega_1 = \omega_2$
(2) $\omega_1 > \omega_2$
(3) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(4) $\omega_1 < \omega_2$

11. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is : **[AIEEE 2005]**

(1) $\frac{2\pi}{\alpha}$ (2) $\frac{2\pi}{\sqrt{\alpha}}$ (3) $2\pi\alpha$ (4) $2\pi\sqrt{\alpha}$

12. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would : **[AIEEE 2005]**

(1) first increase and then decrease to the original value
(2) first decrease and then increase to the original value
(3) remain unchanged
(4) increase towards a saturation value

13. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is : **[AIEEE 2006]**

(1) 100 s (2) 0.01 s (3) 10 s (4) 0.1 s

14. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : **[AIEEE 2006]**

(1) at the highest position of the platform (2) at the mean position of the platform

(3) for an amplitude of $\frac{g}{\omega^2}$ (4) for an amplitude of $\frac{g^2}{\omega^2}$

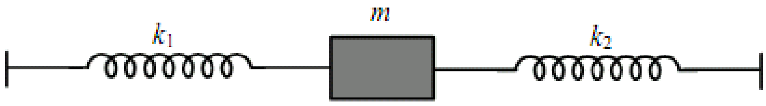
15. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos\pi t$ metres. The time at which the maximum speed first occurs is : **[AIEEE 2007]**

(1) 0.5 s (2) 0.75 s (3) 0.125 s (4) 0.25 s

16. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as , $a = A\cos(\omega t + \delta)$, then : **[AIEEE 2007]**

(1) $A = x_0$, $\delta = -\pi/4$ (2) $A = x_0\omega^2$, $\delta = -\pi/4$ (3) $A = x_0\omega^2$, $\delta = -\pi/4$ (4) $A = x_0\omega^2$, $\delta = 3\pi/4$

17. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes:



[AIEEE 2007]

(1) $f/2$ (2) $f/4$ (3) 4f (4) $2f$

18. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is : **[AIEEE 2007]**

(1) $\pi^2 ma^2\upsilon^2$ (2) $\frac{1}{4}ma^2\upsilon^2$ (3) $4\pi^2 ma^2\upsilon^2$ (4) $2\pi^2 ma^2\upsilon^2$

19. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time? **[AIEEE 2009]**

(1) $\frac{aT}{x}$ (2) $aT + 2\pi v$ (3) $\frac{aT}{v}$ (4) $a^2T^2 + 4\pi^2v^2$

20. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is : **[AIEEE 2011]**

(1) $\frac{M}{M+m}$ (2) $\frac{M+m}{M}$ (3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M+m}{M}\right)^{1/2}$

21. Two particle are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is : **[AIEEE 2011]**

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

22. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = 0s, τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the averatge life time of the pendulum is (assuming damping is small) in seconds :

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$ **[AIEEE 2012]**

23. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2. **[AIEEE 2012]**
Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2
Statement 2 : $k_1 < k_2$
(1) Statement 1 is false, Statement 2 is true.
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

EXERCISE # 4

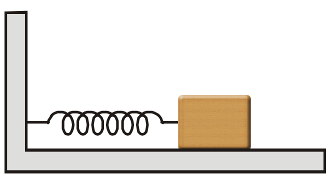
NCERT QUESTIONS

1. Which of the following function of time represent (a) harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion ? Give period for each case of periodic mouton :(ω is any positive constant).
(a) $\sin\omega t - \cos\omega t$ (b) $\sin^3 \omega t$ (c) $3\cos(\pi/4 - 2\omega t)$
(d) $\cos\omega t + \cos 3\omega t + \cos 5\omega t$ (e) $\exp(-\omega^2 t^2)$ (f) $1 + \omega t + \omega^2 t^2$

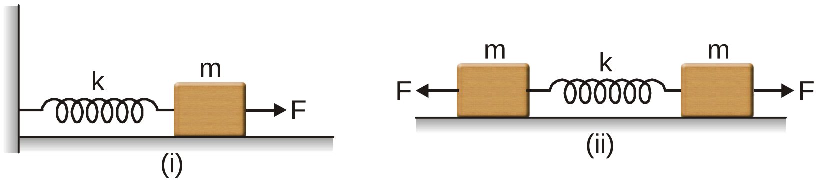
2. A particle is inlinear simple harmonic motin between tow points, A and B, 10 cm apart. Take the directin from A to B as the positive direction and give the signs of velocity, accelerating and force on the particle when it is :
(a) at the end A
(b) at the ent B,
(c) at the mid-point of AB going towards A.
(d) at 2 cm away from b going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from A going towards A.

3. The motion of a particle execution simple harmonic motion is described by the displacement function.
$x(t) = A \cos(\omega t + \theta)$
If the initial (t =0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle ? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B \sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

4. A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in fig. A mass of 3 kg is attached to the free end of the spring. The masses then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass and (iii) the maximum speed of the mass.



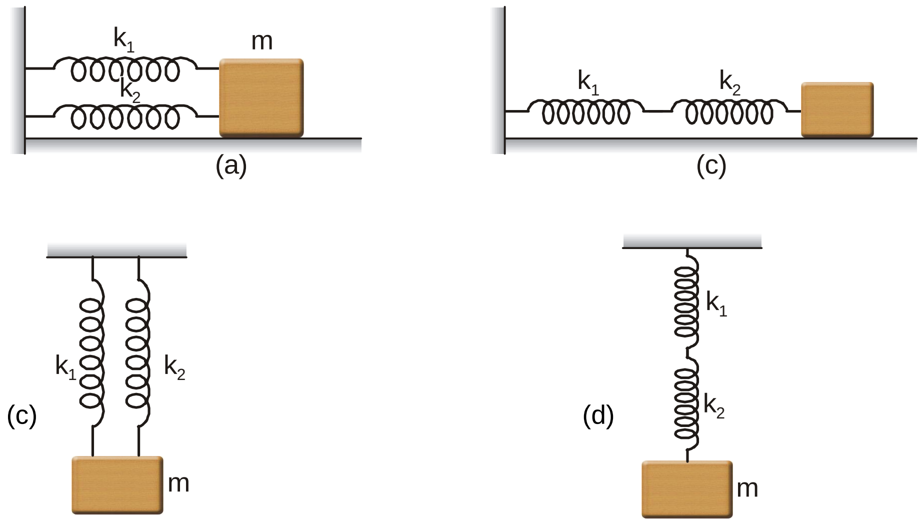
5. Figure (i) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force **F** applied at the free end stretches the spring. Figure (ii) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in figure (ii) is stretched by the same force **F**.



(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in figure (i) and the two masses in Figure (ii) are released free, what is the period of oscillation in each case?

6. Figure 14.31 shows four different spring arrangement. If the mass in each arrangement is displaced form its equilitbrium position and released , what is the resulting frequency of vibration in each case? Neglect the mass of the spring. [Figs. (a) and (b) represent an arrangement of springs in parallel, and (c) and (d) represent 'springs in series']



7. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude of 1.0 m. If the piston moves with simple harmonic motion with angular frequency of 200 rev/ min, what its maximum speed?

8. A simple pendulum of lenght *l* and having a bob of mass *m* is suspended in a car. The car is moving on a circular track of radius *R* with a uniform speed υ. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?

9. A cylindrical piece of cork of base area *A* and height *h* floats in a liquid of density ρ the cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

10. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The amplitude of oscillation 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation . Estimate the values of (a) the spring constant *k* and (b) the damping constant *b* for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

11. Show that for a particle inlinear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

12. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. the period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constat α is defined by the relation $j = -\alpha\,\theta$, where *J* is the restoring couple and θ the angle of twist).

ANSWERS

Exercise # 1

PART-I

A-1.	(A)	**A-2.**	(C)	**A-3.**	(A)	**A-4.**	(D)	**A-5.**	(C)	**A-6.**	(A)	**A-7.**	(B)
A-8.	(A)	**A-9.**	(C)	**A-10.**	(C)	**A-11.**	(B)	**A-12.**	(C)	**B-1.**	(C)	**B-2.**	(B)
B-3.	(C)	**B-4.**	(A)	**B-5.**	(D)	**C-1.**	(B)	**C-2.**	(B)	**C-3.**	(B)	**C-4.**	(C)
C-5.	(C)	**C-6.**	(A)	**C-7.**	(C)	**C-8.**	(A)	**C-9.**	(B)	**D-1.**	(B)	**D-2.**	(C)
D-3.	(A)	**E-1.**	(C)	**E-2.**	(D)	**E-3.**	(B)	**E-4.**	(D)	**E-5.**	(C)	**F-1.**	(B)
F-2.	(A)	**F-3.**	(C)	**F-4.**	(B)								

PART-II

1. (A) **2.** (D) **3.** (B) **4.** (B) **5.** (C) **6.** (B)

7. (A)-(ii), (B)-(i), (C)-(iii) & (D)-(iv) **8.** (A) p (B) q (C) p (D) s **9.** (D)

10. (C) **11.** (A) **12.** (D) **13.** (A) **14.** F **15.** T **16.** T

Exercise # 2

PART-I

1.	(B)	**2.**	(D)	**3.**	(B)	**4.**	(A)	**5.**	(C)	**6.**	(B)	**7.**	(D)
8.	(D)	**9.**	(A)	**10.**	(B)	**11.**	(C)	**12.**	(B)	**13.**	(B)	**14.**	(A)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(D)	**19.**	(C)	**20.**	(A)	**21.**	(C)
22.	(D)	**23.**	(A)	**24.**	(A)	**25.**	(A)	**26.**	(B)	**27.**	(ABCD)	**28.**	(BCD)
29.	(BD)	**30.**	(BD)	**31.**	(BCD)								

PART-II

1. $2 \times \left[\frac{\pi}{2}\sqrt{\frac{m}{k}} + \frac{L}{v_{max}} + \frac{\pi}{2}\sqrt{\frac{m}{k}}\right] = 2\sqrt{\frac{m}{k}}\ [\pi + 1]$

2. $\frac{5}{3\pi}$ Hz, $\frac{2}{30}$ m

3. (A) $\frac{F}{k}$, $2\pi\sqrt{\frac{M}{k}}$, (B) $\frac{F^2}{2k}$ (C) $\frac{F^2}{2k}$

4. $\frac{\pi}{\sqrt{g}}(\sqrt{1.2}+1)$ = 2.1 sec.

5. x = 3 – A cosωt,Y = 4 – A sinωt, Min = 3, Max = 7

6. $T = 2\pi\sqrt{\frac{a}{3g}}$, A = $\frac{a}{6}$, $\frac{a}{6}$, $1/6\sqrt{2ag}$

7. (a) 25cm, (b) 3π/56 seconds **8.** 4 **9.** 2 **10.**

11. (A) 1 (B) 3 **12.** 5 **13.** 2

Exercise # 3

PART-I

1. (D) **2.** (C) **3.** (A) **4.** (A) **5.** (A) **6.** f = $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

7. (B) **8.** y = $\frac{g}{\omega^2}$ **9.** (ABD)

10. (A) $\rightarrow$ (p) ; (B) $\rightarrow$ (q, r) ; (C) $\rightarrow$ (p) ; (D) $\rightarrow$ (q, r)

11. (A) $\rightarrow$ (p) ; (B) $\rightarrow$ (q, s) ; (C) $\rightarrow$ (s) ; (D) $\rightarrow$ (q)

12. (D) **13.** (C) **14.** (D) **15.** (C) **16.** (B) **17.** (D) **18.** 4

19. (AD) **20.** (D) **21.** (C) **22.** (B) **23.** (B) **24.** (A) **25.** (AD)

PART-II

1. (3) **2.** (3) **3.** (4) **4.** (3) **5.** (1) **6.** (3) **7.** (2)

8. (3) **9.** (2) **10.** (1) **11.** (2) **12.** (2) **13.** (2) **14.** (3)

15. (1) **16.** (4) **17.** (4) **18.** (1) **19.** (1) **20.** (4) **21.** (2)

22. (4) **23.** (1)

Exercise # 4

1. (a) Simple harmonic, T = (2π/ω); (b) periodic, T = (2π/ω) but not simple harmonic;
(c) simple harmonic, T = (π/ω); (d) periodic, T= (2π/ω) but not simple harmonic;
(e) non-periodic; (f) non-periodic (physically not acceptable as the function $\rightarrow \infty$ as t$\rightarrow \infty$.

2. (a) 0,+, +; (b) 0,–, –; (c) –, –; (d) –, –, –; (e) +, +, + ; (f) –, +, +.

3. A = $\sqrt{2}$ cm, f = 3p/4; B = $\sqrt{2}$ cm, a = 5π/4.

4. Frequency 3.2 s^{-1} maximum acceleration of the mass 8.0 m s^{-2}; maximum speed of the mass 0.4 m s^{-1}

5. (a) F/k for both (a) and (b). (b) T = $2\pi\sqrt{\frac{m}{k}}$ for (a) and $2\pi\sqrt{\frac{m}{2k}}$ for (b)

6. (a) Hint : If the extension of each spring is x under a stretching force F, $k_1 x + k_2 x = F$.

i.e. the effective spring constant $k = (k_1 + k_2)$, therefore $v = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

(b) $v = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

(c) In this case under a stretching force F, $F = k_1 x : F = k_2 x$. Therefore the effective spring constant $k = F/x = F/(x_1 + x_2)$ or $1/k = x_1/F = 1/k_1 + 1/k_2$.

Consequently $v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ where $k = \frac{k_1 k_2}{k_1+k_2}$

(d) Same as in (c).

7. 100 m / min

8. $T = 2\pi\sqrt{\frac{1}{\sqrt{g^2+v^4/R^2}}}$ .Hint : Effective acceleration due to gravity will get reduced due to radial acceleration mv^2/R acting in the horizontal plane.

9. In equilibrium, weight of the cork equals the up thrust. When the cork is depressed by an amount x, the net upward force is $Ax\rho_\ell g$. Thus the force constant $k = A\rho_\ell g$. Using $m = Ah\rho$,and $T = 2\pi\sqrt{\frac{m}{k}}$ one gets the required formula.

10. (a) 5×10^{-4} N m^{-1} ; (b) 1344.6 kg s^{-1}

11. Average K.E. = $\frac{1}{T}\int_0^T \frac{1}{2}mv^2\, dt$; Average P.E. $\frac{1}{T}\int_0^T \frac{1}{2}kx^2\, dt$

12. The time period of a torsional pendulum is given by $T = 2\pi\sqrt{\frac{I}{\alpha}}$, where I is the moment of inertia about the axis of rotation. In our case $I = \frac{1}{2}MR^2$, where M is the mass of the disk and R its radius, Substituting the given values, α =2.0 N m rad^{-1}.