RIGID BODY DYNAMICS

Contents

Syllabus

Rigid body, moment of inertia, parallel and perpendicular axes theorems, moment of inertia of uniform bodies with simple geometrical shapes ; Angular momentum ; Torque ; Conservation of angular momentum ; Dynamics of rigid bodies with fixed axis of rotation ; Rolling without slipping of rings, cylinders and spheres ; Equilibrium of rigid bodies ; Collision of point masses with rigid bodies.

Name : ______________________________ Contact No. ____________________

ETOOS ACADEMY Pvt. Ltd

F-106, Road No.2 Indraprastha Industrial Area, End of Evergreen Motor, BSNL Lane, Jhalawar Road, Kota, Rajasthan (324005)

Tel. : +91-744-242-5022, 92-14-233303

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

SECTION (A) : KINEMATICS

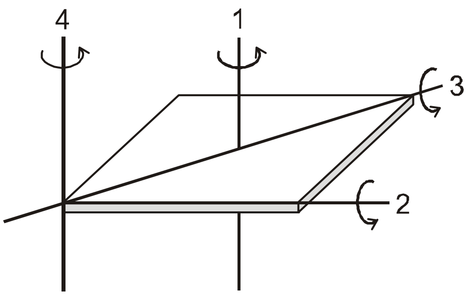
A-1. A car is moving with a speed of 72 Km/hour. The diameter of its wheels is 50cm. If its wheels come to rest after 20 rotations as a result of application of brakes, then the angular retardation produced in the car will be
(A) 25.5 Radians/sec^2 (B) 0.25 Radians/sec^2
(C) 2.55 Radians/sec^2 (D) 0

A-2. A fly wheel rotates about an axis. Due to friction at the axis, it experiences angular retardation proportional to its angular velocity. If its angular velocity falls to half the value while it makes n revolutions, how many more revolutions will it make before coming to rest ?
(A) 2n (B) n (C) n/2 (D) n/3.

SECTION (B) : Moment of inertia

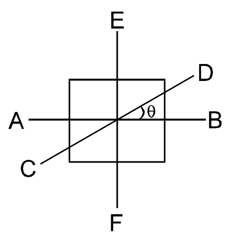
B-1. Particles of masses 1, 2 and 3 kg are respectively arranged at the corners of an equilateral triangle of side 1 m. The M.I. of system about an axis passing through the centroid and perpendicular to the plane of triangle is :
(A) 2 kg-m^2 (B) 3 kg-m^2 (C) 1 kg-m^2 (D) 6 kg-m^2

B-2. As shown in fig., about which axis the rectangular body, has the maximum M.I.-



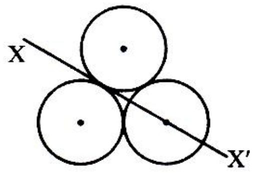
(A) 1 (B) 2 (C) 3 (D) 4

B-3. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to



(A) I (B) I $\sin^2\theta$ (C) I$\cos^2\theta$ (D) I$\cos^2(\theta/2)$

B-4. Three discs each of mass M and radius R are placed in contact with each other as shown in the figure. Then the M I of the system about an XX' is:



(A) $\frac{MR^2}{4}$ (B) $\frac{11MR^2}{4}$ (C) $\frac{2}{3}MR^2$ (D) $7MR^2$

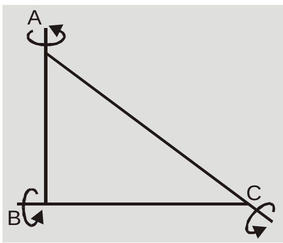
B-5. Three bodies have equal masses m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

B-6. Two rings of the same radius and mass are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to the plane of one of the ring is-
[Mass of the ring = m and radius = r]

(A) $\frac{1}{2}$ mr² (B) mr² (C) $\frac{3}{2}$ mr² (D) 2mr²

B-7. In the adjoining fig. along which axis the M.I. of the triangular lamina will be maximum-
[Given that AB < BC < AC]



(A) AB (B) BC (C) CA (D) For all axis

B-8. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass and I_2 is the moment of inertia of the ring formed by bending the rod, then:

(A) $I_2 = I_1/4\pi^2$ (B) $I_2 = I_1/\pi^2$ (C) $I_2/I_1 = 0.3$ (D) $I_1/I_2 = \pi^2/3$

B-9. The moment of inertia of an elliptical disc of uniform mass distribution of mass 'm', semi major axis 'r', semi minor axis 'd' about its axis is :

(A) $= \frac{mr^2}{2}$ (B) $= \frac{md^2}{2}$ (C) $> \frac{mr^2}{2}$ (D) $< \frac{mr^2}{2}$

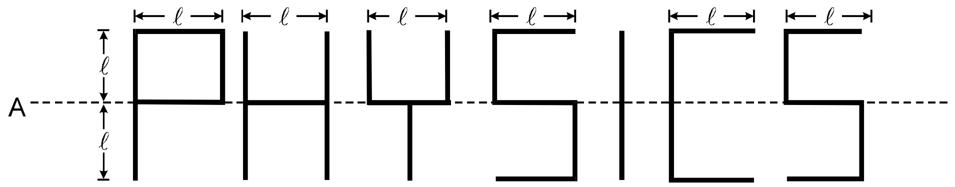
B-10. From the circular disc of radius R and mass M, a concentric circular disc of small radius r is cut and removed, the mass of which is m. the moment of inertia of the angular disc remaining, about its axis perpendicular to the plane and passing through the centre of mass will be

(A) $\frac{1}{2}M\left(R^2+r^2\right)$ (B) $\frac{1}{2}(M-m)\left(R^2+r^2\right)$ (C) $\frac{1}{2}(M-m)\left(R^2-r^2\right)$ (D) $\frac{1}{2}\left(MR^2-mr^2\right)$

B-11. Select the correct alternative(s):
(A) The mass of a body can be taken to be concentrated at its centre of mass for the purpose of calculating its rotational inertia.
(B) Two circular discs of the same mass and thickness are made from metals of different densities, the one with greater density will have less rotational inertia about its central axis.
(C) About an axis passing through diagonally opposite ends of a uniform cube, have its minimum rotational inertia.
(D) None of these.

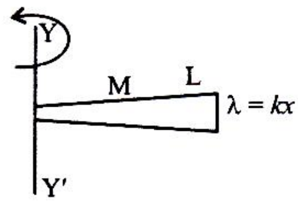
B-12. The ratio of the radii of gyration of a circular disc and a circular ring of the same masses and radii about a tangential axis parallel to their planes is
(A) $\sqrt{6}:\sqrt{5}$ (B) $1:\sqrt{2}$
(C) $\sqrt{5}:\sqrt{6}$ (D) none of these

B-13. Find out the moment of inertia of the following structure (written as PHYSICS) about axis AB made of thin uniform rods of mass per unit length λ.



(A) **5** $\lambda\ell^3$ (B) **13** $\lambda\ell^3$ (C) **10** $\lambda\ell^3$ (D)**12** $\lambda\ell^3$

B-14. Mass M is distributed over the rod of length L. If linear mass density (λ) linearly increases with length as λ = Kx. The M.I. of the rod about one end perpendicular to rod i.e. (YY')



(A) $\frac{ML^2}{3}$ (B) $\frac{ML^2}{12}$
(C) $\frac{2}{3}ML^2$ (D) $\frac{KL^4}{4}$

SECTION (C) : Torque

C-1. Dimensions of torque are -
(A) $M^2L^2T^{-2}$ (B) $M^1L^2T^{-2}$ (C) ML^2T^{-1} (D) $ML^{-1}T^2$

C-2. A uniform circular disc A of radius r is made from a metal plate of thickness t and another uniform circular disc B of radius 4r is made from the same metal plate of thickness t/4. If equal torques act on the discs A and B, initially both being at rest. At a later instant, the angular speeds of a point on the rim of A and another point on the rim of B are ω_A and ω_B respectively. We have
(A) $\omega_A > \omega_B$ (B) $\omega_A = \omega_B$
(C) $\omega_A < \omega_B$ (D) the relation depends on the actual magnitude of the torques.

C-3. A wheel of moment of inertia 5 x 10^{-3} kg-m^2 is making 20rev/s. The torque required to stop it in 10 sec is-
(A) $2\pi \times 10^{-2}$ N-m (B) $2\pi \times 10^{2}$ N-m (C) $\pi \times 10^{-2}$ N-m (D) $4\pi \times 10^{-2}$ N-m

C-4. A force $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ acts at a point (2, –3, 1). Then magnitude of torque of this force about point (0, 0, 2) will be:

(A) 6 (B) $3\sqrt{5}$ (C) $6\sqrt{5}$ (D) none of these

C-5. Let $\vec{F}$ be a force acting on a particle having position vector $\vec{r}$. Let $\vec{\Gamma}$ be the torque of this force about the origin, then

(A) $\vec{r}.\vec{\Gamma} = 0$ and $\vec{F}.\vec{\Gamma} = 0$ (B) $\vec{r}.\vec{\Gamma} = 0$ but $\vec{F}.\vec{\Gamma} \neq 0$

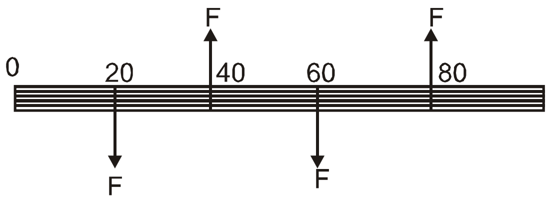
(C) $\vec{r}.\vec{\Gamma} \neq 0$ but $\vec{F}.\vec{\Gamma} = 0$ (D) $\vec{r}.\vec{\Gamma} \neq 0$ and $\vec{F}.\vec{\Gamma} \neq 0$

C-6. In case of torque of a couple if the axis is changed by displacing it parallel to itself, torque will :

(A) increase (B) decrease (C) remain constant (D) None of these

SECTION (D) : Rotational Equilibrium

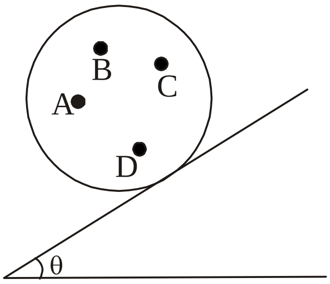
D-1. Four equal and parallel forces are acting on a rod (as shown in figure) at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces the rod :



(A) is at rest (B) experiences a torque

(C) experiences a linear motion (D) experiences a torque and also a linear motion

D-2. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?

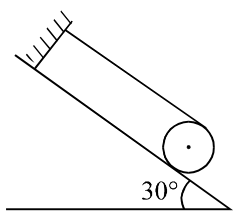


(A) A (B) B (C) C (D) D

D-3. The beam and pans of a balance have negligible mass. An object weighs W_1 when placed in one pan and W_2 when placed in the other pan. The weight W of the object is :

(A) $\sqrt{W_1W_2}$ (B) $\sqrt{(W_1 + W_2)}$ (C) $W_1^2 + W_2^2$ (D) $(W_1^{-1} + W_2^{-1})/2$

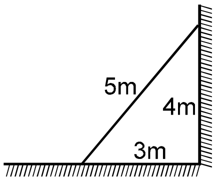
D-4. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration is



(A) $\dfrac{1}{3\sqrt{3}}$ (B) $\dfrac{1}{\sqrt{3}}$

(C) $\dfrac{1}{2}$ (D) $\dfrac{1}{2\sqrt{3}}$

D-5. A uniform ladder of length 5m is placed against the wall as shown in the figure. If coefficient of friction μ is the same for both the wall and the floor then minimum value of μ for it not to slip is

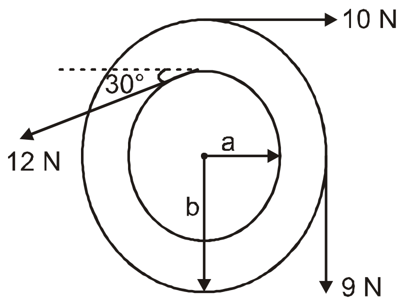


(A) $\mu = 1/2$ (B) $\mu = 1/4$ (C) $\mu = 1/3$ (D) $\mu = 1/5$

SECTION (E) : Rotation about fixed axis

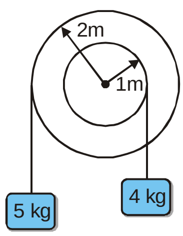
E-1. A sphere is rotating about a diameter.
(A) The particles on the surface of the sphere do not have any linear acceleration.
(B) The particles on the diameter mentioned above do not have any linear acceleration.
(C) Different particles on the surface have different angular speeds.
(D) All the particles on the surface have same linear speed.

E-2. In the figure a = 5cm and b = 20cm. If the M.I. of the wheel is 3200kg-m^2, the angular acceleration would be



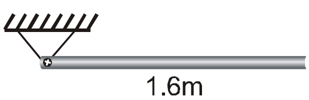
(A) 10^{-1} rad/s^2 (B) 10^{-2}rad/s^2 (C) 10^{-3} rad/s^2 (D) 10^{-4} rad/s^2

E-3. The moment of inertia of the pulley system shown in figure is 4 kg m^2. The radii of bigger and smaller pulleys are 2m and 1m, respectively. The angular acceleration of the pulley system is:



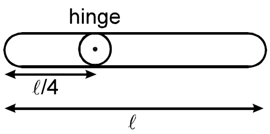
(A) 2.1 rad/s^2 (B) 4.2 rad/s^2 (C) 1.2 rad/s^2 (D) 0.6 rad/s^2.

E-4. The uniform rod of mass 20 kg and length 1.6 m is pivoted at its end and swings freely in the vertical plane. Angular acceleration of rod just after the rod is released from rest in the horizontal position as shown in figure is :



(A) $\frac{15g}{16}$ (B) $\frac{17g}{16}$ (C) $\frac{16g}{15}$ (D) $\frac{g}{15}$

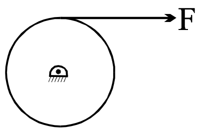
E-5. A uniform rod is hinged as shown and is released from a horizontal position. The angular velocity of the bar as it passes the vertical position is:



(A) $\sqrt{\frac{12g}{3\ell}}$ (B) $\sqrt{\frac{2g}{3\ell}}$ (C) $\sqrt{\frac{24g}{7\ell}}$ (D) $\sqrt{\frac{3g}{7\ell}}$

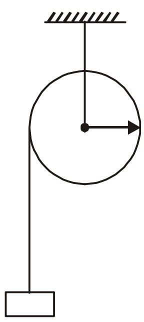
E-6. Two men support a uniform horizontal rod at its two ends. If one of them suddenly lets go, the force exerted by the rod on the other man will:
(A) remain unaffected
(B) increase
(C) decrease
(D) become unequal to the force exerted by him on the beam.

E-7. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,



(A) its angular velocity increases, but force on hinge remains constant
(B) its angular velocity remains same, but force on hinge increases
(C) its angular velocity increases and force on hinge increases
(D) its angular velocity remains same and force on hinge is constant

E-8. A solid cylinder of radius R is free to rotate about its horizontal axis. A string is wound around it and a mass m is attached to its free end. When m falls through a distance h, its speed at that instant is



(A) proportional to R (B) proportional to 1 / R
(C) proportional to 1 / R^2 (D) independent of R

SECTION (F) : Angular Momentum & its conservation

F-1. A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin
(A) is zero (B) remains constant (C) goes on increasing (D) goeson decreasing

F-2. A particle of mass 1 kg is projected at an angle θ with horizontal. Its co-ordinates at any instant are (5m, 5m) and it is having velocity components along X-axis and Y-axis as 8 m/s and 4 m/s respectively. Its angular momentum about origin is
(A) $-20\ \hat{k}$ (B) $+20\ \hat{k}$ (C) $-60\ \hat{k}$ (D) $+60\ \hat{k}$

F-3. If $\vec{\tau} \times \vec{L} = 0$ for a rigid body, where $\vec{\tau}$ = resultant torque & $\vec{L}$ = angular momentum about a point and both are non - zero. Then:
(A) $\vec{L}$ = constant (B) $|\vec{L}|$ = constant (C) $|\vec{L}|$ will increase (D) $|\vec{L}|$ may increase.

F-4. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 seconds. The magnitude of this torque is-

(A) $\frac{3A_0}{4}$ (B) A_0 (C) $\frac{4A_0}{3}$ (D) $12A_0$

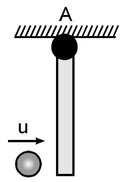
F-5. If a person sitting on a rotating stool with his hands outstretched, suddenly lowers his hands, then his

(A) Kinetic energy will decrease (B) Moment of inertia will decrease
(C) Angular momentum will increase (D) Angular velocity will remain constant

F-6. A man is standing at the centre of a rotating disc of radius R, hinged at its center. When the man starts walking from the centre towards the periphery of the disc, then

(A) Angular momentum of the system starts decreasing
(B) Kinetic energy of the system starts increasing
(C) Angular velocity of the system starts decreasing
(D) Kinetic energy first increases then decreases

F-7. In the given figure a ball strikes a rod elastically and rod is hinged smoothly at point A. Then which of the statement(s) is/are correct for the collision?

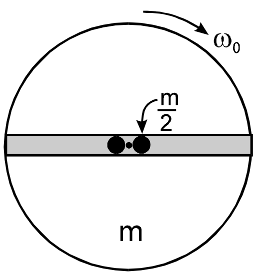


(A) linear momentum of system (ball + rod) is conserved
(B) angular momentum of system about hinged point A is conserved
(C) kinetic energy of system is conserved
(D) linear momentum of ball is conserved.

F-8. A playground merry-go-round is at rest, pivoted about a frictionless axis. A child of mass m runs along a path tangential to the rim with speed v and jumps on to the merry-go-round. If R is the radius of the merry-go-round and I is its moment of inertia, then the angular velocity of the merry-go-round and the child is -

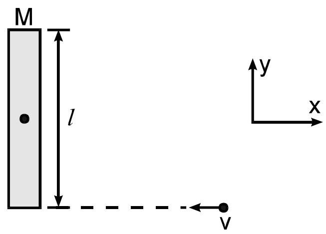
(A) $\frac{mvR}{mR^2 + I}$ (B) $\frac{mvR}{I}$ (C) $\frac{mR^2 + I}{mvR}$ (D) $\frac{I}{mvR}$

F-9. A disc of mass 'm' and radius R is free to rotate in horizontal plane about a vertical smooth fixed axis passing through its centre. There is a smooth groove along the diameter of the disc and two small balls of mass $\frac{m}{2}$ each are placed in it on either side of the centre of the disc as shown in fig. The disc is given initial angular velocity ω_0 and released. The angular speed of the disc when the balls reach the end of disc is :



(A) $\frac{\omega_0}{2}$ (B) $\frac{\omega_0}{3}$ (C) $\frac{2\omega_0}{3}$ (D) $\frac{\omega_0}{4}$

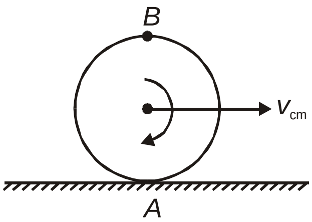
F-10. A rod of mass m and length ℓ is kept on a surface that has no friction. If a ball comes & hits the lower end with a velocity v, then the velocity of centre of mass will be :



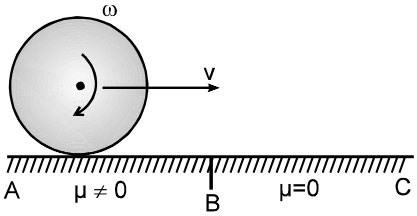
(A) zero
(B) along negative x - axis
(C) along positive x - axis
(D) cannot determine with the given data

SECTION (G) : Combined Translational & Rotation Motion

G-1. A disc is undergoing in pure rolling on a horizontal surface, then which of the statement about angular momentum of the disc is incorrect?
(A) Angular momentum about A is conserved under any situation
(B) Angular momentum about B is conserved only if surface is smooth
(C) Angular momentum about any point on the disc is conserved
(D) None of these

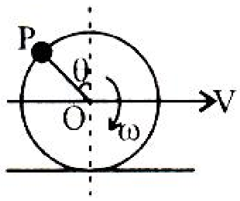


G-2. As shown in the figure, a disc of mass m is rolling without slipping a angular velocity ω. When it crosses point B disc will be in:



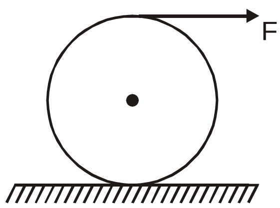
(A) translational motion only
(B) pure rolling motion
(C) rotational motion only
(D) none of these

G-3. A disc of radius R rolls on a horizontal surface with linear velocity V and angular velocity ω. There is a point P on circumference of disc t angle θ, which has a vertical velocity. Here θ is equal to (ωR > V)



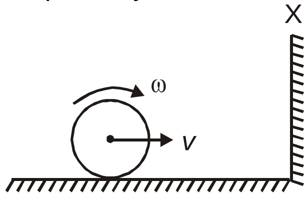
(A) $\pi + \sin^{-1}\dfrac{V}{R\omega}$
(B) $\dfrac{\pi}{2} - \sin^{-1}\dfrac{V}{R\omega}$
(C) $\pi - \cos^{-1}\dfrac{V}{R\omega}$
(D) $\pi + \cos^{-1}\dfrac{V}{R\omega}$

G-4. A solid sphere of radius R and mass M is pulled by a force F acting at the top of the sphere as shown in figure. Friction coefficient is sufficient enough to provide rolling without slipping. Work done by force F when the centre of mass moves a distance S is :



(A) FS
(B) 2FS
(C) Zero
(D) $\dfrac{3FS}{2}$

G-5. A solid cylindre is rolling without slipping with velocity of its centre of mass v and angular velocity about its centre of mass ω on a horizontal frictionless surface as shown in figure. If it collides with a frictionless vertical wall X, then after collision its velocity and angular velocity respectively become

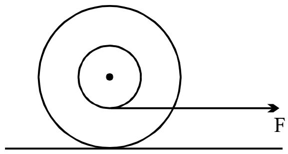


(A) $\frac{v}{2}, \frac{\omega}{2}$
(B) $-v, -\omega$
(C) $-v, \omega$
(D) $v, -\omega$

G-6. A hoop of radius r, mass m kg rolls along a horizontal floor so that its centre of mass has a speed v m/s. How much work has to be done to stop it ?

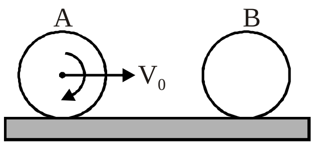
(A) $\frac{1}{2}mv^2$ (B) mv^2 (C) $2mv^2$ (D) $\frac{1}{3}mv^2$.

G-7. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards
(B) Thread winds, spool rotates clockwise and friction acts leftwards
(C) Thread winds, spool moves to the right and friction act rightwards
(D) Thread winds, spool moves to the right and friction does not come into existence.

G-8. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :



(A) 1 : 1 (B) 2 : 3 (C) 3 : 2 (D) None of these

G-9. A uniform rod AB of mass m and length l rest on a smooth horizontal surface. An impulse P is applied to the end B. The time taken by the rod to turn through a right angle is

(A) $\pi ml/12P$ (B) $\pi ml/6P$ (C) ml/6P (D) none of these

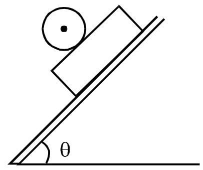
G-10. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping.
(A) the heavier sphere reaches the bottom first
(B) the bigger sphere reaches the bottom first
(C) the two spheres reach the bottom together
(D) the information given is not sufficient to tell which sphere will reach the bottom first.

G-11. A solid cylinder rolls down on inclined plane which has friction sufficient to prevent sliding. The ratio of rotational energy to total kinetic energy is

(A) 1/2 (B) 1/3 (C) 2/3 (D) 3/4

G-12. A sphere cannot roll on:
(A) a smooth horizontal surface. (B) a smooth inclined surface.
(C) a rough horizontal surface. (D) a rough inclined surface.

G-13. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank.



(A) up the plane (B) down the plane

(C) horizontally (D) Zero

G-14. A hollow sphere and a solid sphere having same mass and same radii are rolled down on a rough inclined plane

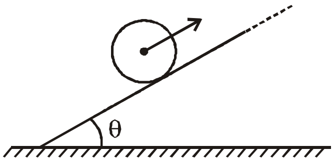
(A) the hollow sphere reaches the bottom first

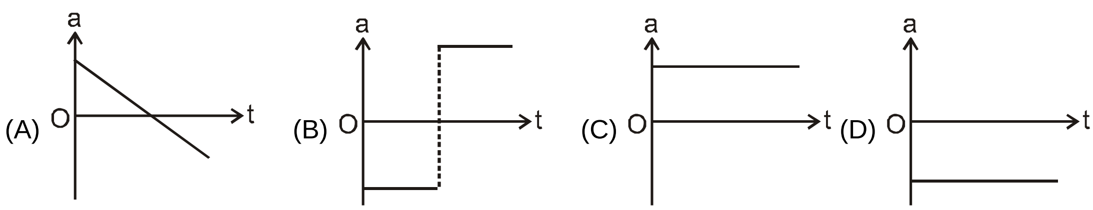
(B) the solid sphere reaches the bottom with greater speed

(C) the solid sphere reaches the bottom with greater kinetic energy

(D) the two spheres will reach the bottom with same linear momentum .

G-15. A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration 'a' of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline) ? Assume that the sphere rolling up has a positive velocity.



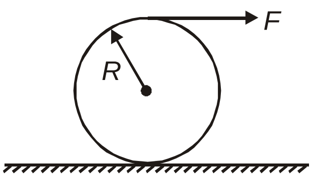


G-16. A uniform disc of mass m and radius R is rolling up a rough inclined plane which makes an angle of 30° with the horizontal. If only gravitational and frictional forces are acting on the disc, the magnitude of the frictional force acting along the inclined plane is

(A) $\frac{mg}{6}$ upwards (B) $\frac{mg}{6}$ downwards

(C) $\frac{mg}{4}$ upwards (D) $\frac{mg}{4}$ downwards

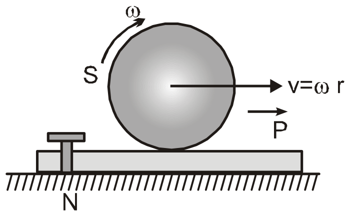
G-17. A ring of mass m and radius R is acted upon by a force F as shown in the figure, there is sufficient friction between the ring and the ground then the force of friction force necessary for pure rolling is



(A) $\frac{F}{2}$ forward (B) $\frac{F}{3}$ forward

(C) Zero (D) $\frac{F}{4}$ backward

G-18. A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin N. If N is suddenly removed:



(A) S will begin to slip on P

(B) P will begin to move backwards

(C) the speed of S will decrease and its angular velocity will increase

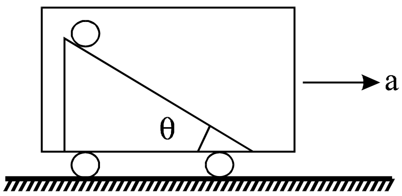
(D) there will be no change in the motion of S and P will still be at rest.

G-19. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of a smooth incline and released. Least time will be taken in reaching the bottom by

(A) the solid sphere (B) the hollow sphere

(C) the disc (D) all will take same time.

G-20. A smooth inclined plane fixed in a car accelerating on a horizontal road is shown in figure. The angle of incline θ is related to the acceleration a of the car as $a = g \tan\theta$. If the sphere is set in pure rolling on the incline

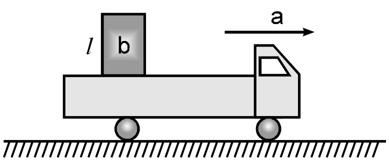


(A) it will continue pure rolling (B) it will slip up the plane

(C) its angular velocity will increase (D) its angular velocity will decrease.

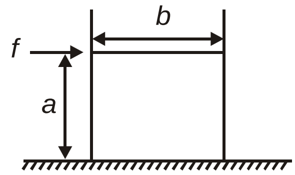
SECTION (H) : Toppling

H-1. A box of dimensions ℓ and b is kept on a truck moving with an acceleration a. If box does not slide, maximum acceleration for it to remain in equilibrium (w.r.t.truck) is :



(A) $\frac{g\ell}{b}$ (B) $\frac{gb}{\ell}$ (C) g (D) none of these

H-2. A cuboidal block of height a and width b is placed on the horizontal surface with sufficient friction then for a given force



(A) Probability of toppling is more of b > a

(B) Probability of toppling is more of a > b

(C) Probability of toppling is more of a = b

(D) Block will not topple

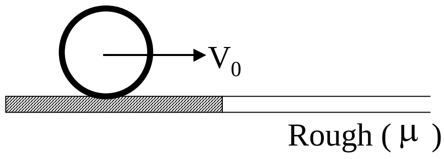
PART - II : MISLLANEOUS QUESTIONS

1. COMPREHENSION

COMPREHENSION # 1

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in fig

1. Choose the incorrect statement :



(A) The friction does negative translational work
(B) The friction does positive rotational work
(C) The net work done by friction is zero
(D) Friction force converts translational kinetic energy into rotational kinetic energy in rotational kinetic energy

2. Velocity of centre of mass of the ring when it starts rolling motion

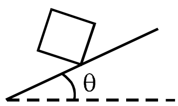
(A) $\frac{V_0}{4}$ (B) $\frac{V_0}{2}$ (C) $\frac{V_0}{3}$ (D) $\frac{2V_0}{3}$

3. Linear distance moved by the centre of mass of the ring on the rough surface till it starts rolling is

(A) $\frac{3}{8}\frac{V_0^2}{\mu g}$ (B) $\frac{2V_0^2}{8\mu g}$ (C) $\frac{V_0^2}{\mu g}$ (D) $\frac{5V_0^2}{8\mu g}$

COMPREHENSION # 2

A square frame of mass m is made of four identical uniform rods of length L each. This frame is placed on an inclined plane such that one of its diagonals is parallel to the inclined plane as shown in figure, and is released.



4. The moment of inertia of square frame about the axis of the frame is :

(A) $\frac{mL^2}{3}$ (B) $\frac{2mL^2}{3}$ (C) $\frac{4mL^2}{3}$ (D) $\frac{mL^2}{12}$

5. The frictional force acting on the frame just after the release of the frame assuming that it does not slide is :

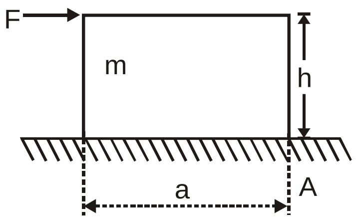
(A) $\frac{mg\sin\theta}{3}$ (B) $\frac{2mg\sin\theta}{7}$ (C) $\frac{3mg\sin\theta}{5}$ (D) $\frac{2mg\sin\theta}{5}$

6. The acceleration of the center of square frame just after the release of the frame assuming that it does not slide is :

(A) $\frac{g\sin\theta}{3}$ (B) $\frac{2g\sin\theta}{7}$ (C) $\frac{3g\sin\theta}{5}$ (D) $\frac{2g\sin\theta}{5}$

COMPREHENSION # 3

When a force F is applied on a block of mass m resting on a horizontal surface then there are two possibilities, either the block moves by translation or it moves by toppling. If the surface is smooth then the block always translates but on a rough surface it topples only when the torque of the applied force F is greater than the torque of mg about a point in contact with the ground.
When the force F is applied the body may topple about A or it may translate.



Answer the following question.

7. When the block topples about A, the normal force –
(A) passes through centre of mass
(B) is / zero
(C) shifts to the right and passes through right most edge containing A
(D) depends on μ

8. The block will move in pure translation if –

(A) $F = \frac{mga}{h}$ (B) $F < \frac{mga}{2h}$ (C) $F > \frac{mga}{2h}$ (D) None

9. The block will topple about A if –

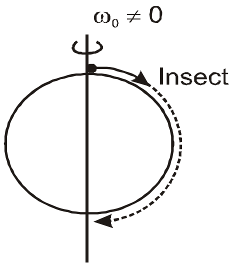
(A) $F = \frac{mga}{h}$ (B) $F < \frac{mga}{2h}$ (C) $F > \frac{mga}{2h}$ (D) None

10. If the block be a cube of edge a and μ = 0.2 then –
(A) The body will translate before topple (B) The body will topple before translate
(C) The body may translate or topple (D) None

11. If the block is a cube of edge a and μ = 0.6 then–
(A) The body will translate before topple (B) The body will topple before translate
(C) The body first translates and then topples (D) None

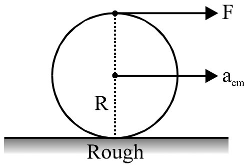
2. MATCH THE COLUMN

12. A solid sphere is freely rotating about an axis as shown in figure. An insect follows the dotted path on the circumference of sphere as shown. Match the following :
(consider sphere and insects as a system)



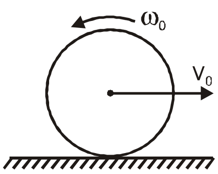
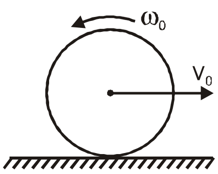
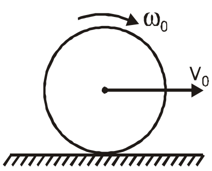
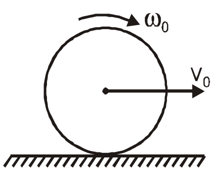
Column-I	Column-II
(A) Moment of inertia	(P) will remain conserved.
(B) Angular velocity	(Q) will first increase then decrease
(C) Angular momentum	(R) will first decrease then increase
(D) Rotational kinetic energy	(S) will continuously decrease

13. In the adjacent figure a uniform rigid body of mass m and radius R is kept at rest on a rough horizontal surface. A constant horizontal force F is applied at the top most point the body. The body starts rolling without slipping. Different shapes of bodies are given in the column I and based on this problem some physical quantities related to them are given in column II. Match the correct pair



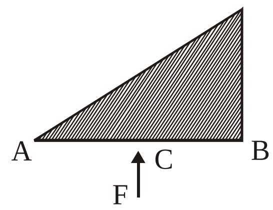
Column I	**Column II**
(A) Solid sphere	(P) Friction force is zero
(B) Ring	(Q) Magnitude of friction force is maximum
(C) Hollow sphere	(R) Acceleration of C. O. M. is 4F/3 m
(C) Disc	(S) Magnitude of friction force is F/5

14. A uniform disc of mass M and radius R lies on a fixed rough horizontal surface at time t = 0. Initial angular velocity ω_0 of each disc (magnitude and sense of rotation) and horizontal velocity v_0 of centre of mass is shown for each situation of column-I. Match each situation in column-I with the results given in column-II.

Column-I	**Column-II**
(A)  It is given that $v_0 = 2R\omega_0$	(p) The magnitude of angular speed keeps on decreasing till the disc starts rolling without slipping.
(B)  It is given that $2v_0 = R\omega_0$	(q) After the disc starts rolling without slipping, the angular velocity is nonzero and in clockwise sense
(C)  It is given that $v_0 = 2R\omega_0$	(r) After the disc starts rolling without slipping, the velocity of centre of disc is towards right
(D)  It is given that $2v_0 = R\omega_0$	(s) After the disc starts rolling without slipping, the kinetic energy of disc is less than its initial value.

3.TRUE/FALSE

15. (i) A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and **(a)** passing through A, **(b)** passing B, by the application of the same force, F, at C (midpoint of AB) as shown in the figure. The angular acceleration in both the cases will be the same.



(ii) A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega\sqrt{5}$.

(iii) **1.** Net torque on a system due to all internal forces about any point is zero. (TRUE)

2. For a rigid body undergoing fixed axis rotation, the direction of its angular acceleration and angular velocity are always same.

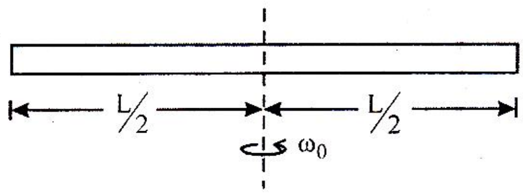
(iv) A particle may have angular momentum even though the particle is not moving in a circle.

(v) A ring is rolling without sliding on a fixed surface. The centripetal acceleration of each particle with respect to the centre of the ring is same.

(vi) A wheel is rolling without sliding on a rough surface. The friction acting on the wheel must be kinetic.

4. FILL IN THE BLANKS

16. (i) A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the midpoint of the rod (see figure). There are no external forces.



When the beads reach the ends of the rod, the angular velocity of the system is ....

(ii) A uniform disc of mass m and radius R is rolling up a rough inclined plane which makes an angle 30º with the horizontal. If the coefficient of static and kinetic friction are each to μ and the only forces acting are gravitational and frictional, then the magnitude of the frictional force acting on the disc is .... and its direction is ....(write up or down) the inclined plane.

(iii) A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly below the centre of the face, at a height a/4 above the base. The minimum value of F for which the cube begins to tilt about the edge............. (Assume that the cube does not slide).

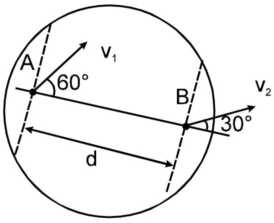
EXERCISE # 2

PART - I : MIXED OBJECTIVE

* ***Marked Questions are having more than one correct option.***

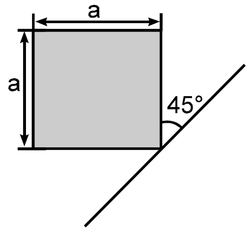
SINGLE CORRECT ANSWER TYPE

1. Two points A & B on a disc have velocities v_1 & v_2 at some moment. Their directions make angles 60° and 30° respectively with the line of separation as shown in figure. The angular velocity of disc is :



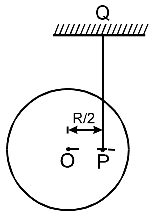
(A) $\frac{\sqrt{3}v_1}{d}$ (B) $\frac{v_2}{\sqrt{3}d}$ (C) $\frac{v_2 - v_1}{d}$ (D) $\frac{v_2}{d}$

2. The moment of inertia of a thin sheet of mass M of the given shape about the specified axis is (axis and sheet both are in same plane:)



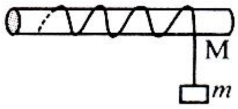
(A) $\frac{7}{12}Ma^2$ (B) $\frac{5}{12}Ma^2$ (C) $\frac{1}{3}Ma^2$ (D) $\frac{1}{12}Ma^2$

3. A uniform disc of mass M and radius R is released from the shown position. PQ is a string, OP is a horizontal line, O is the centre of the disc and distance OP is R/2. Then tension in the string just after the disc is released will be :



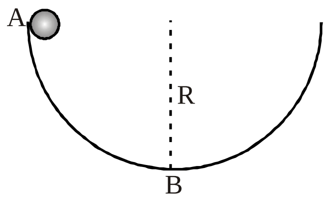
(A) $\frac{Mg}{2}$ (B) $\frac{Mg}{3}$ (C) $\frac{2Mg}{3}$ (D) none of these

4. A cord is wound round the circumference of a solid cylinder of radius R and mass M. The axis of the cylinder is horizontal. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, the angular velocity of the cylinder will be



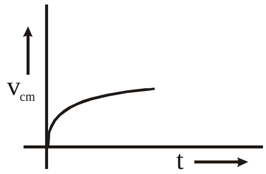
(A) $\frac{2mg}{M+2m}$ (B) $\sqrt{\frac{2gh}{R}}$ (C) $\sqrt{\frac{4mgh}{(M+2m)R^2}}$ (D) None of these

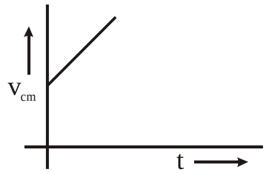
5. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.

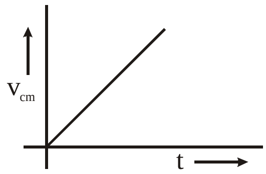


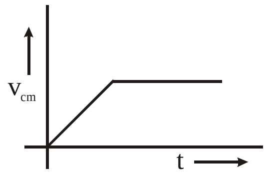
(A) $\frac{10}{7}$mg (B) $\frac{17}{7}$mg (C) $\frac{5}{7}$ mg (D) $\frac{7}{5}$mg

6. A uniform disc of radius r is rotated in clockwise sense with angular speeds ω and kept vertical on a rough surface. Which of the graph of V_{cm} - t is correct.

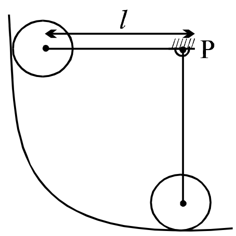
(A)


(B)


(C)


(D)


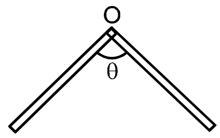
7. A sphere of mass M and radius R is attached by a light rod of length l to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. the angular momentum of the system about P when the rod becomes vertical is :



(A) $M\sqrt{\frac{10}{7}gl}\ [l+R]$ (B) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{2}{5}R\right]$

(C) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{7}{5}R\right]$ (D) none of the above

8. A thin rod of length L and mass M is bend at the middle point O as shown in figure. Consider an axis passing through the middle point O and perpendicular to the plane of the bent rod. Then moment of inertia about this axis is :

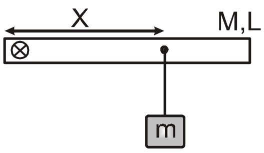


(A) (2/3) mL^2 (B) (1/3) mL^2 (C) (1/12) mL^2 (D) (1/24) mL^2

9. The moment of inertia of a door of mass m, length 2ℓ and width ℓ about its longer side is

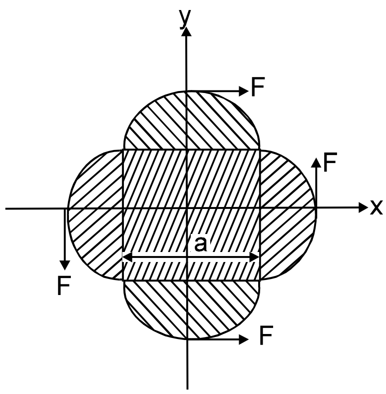
(A) $\frac{11\,m\ell^2}{24}$ (B) $\frac{5\,m\ell^2}{24}$ (C) $\frac{m\ell^2}{3}$ (D) none of these

10. A small block is hanged by a string of small length at a distance ' x ' from left end on a uniform rod of length L and mass M. The rod is in horizontal position and hinged at left end as shown in figure. Then minimum value of ' x ' ($x \neq 0$) for which initial acceleration will be independent of ' m ' mass of the block.



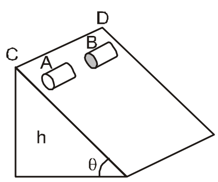
(A) L (B) $\frac{2L}{3}$ (C) $\frac{L}{3}$ (D) can't be determined

11. A planar object made up of a uniform square plate and four semicircular discs of the same thickness and material is being acted upon by four forces of equal magnitude as shown in figure. The coordinates of point of application of forces is given by



(A) (0, a) (B) (0, –a) (C) (a, 0) (D) (–a, 0)

12. Two cylinders A and B are released on a fixed rough inclined plane of angle θ with the horizontal from height 'h'. A is a hollow cylinder and 'B' is a solid cylinder. Both perform pure rolling. Find the distance between the two cylinders when one of them reaches the bottom. Assume that the axis of both cylinders is parallel to the edge CD always.

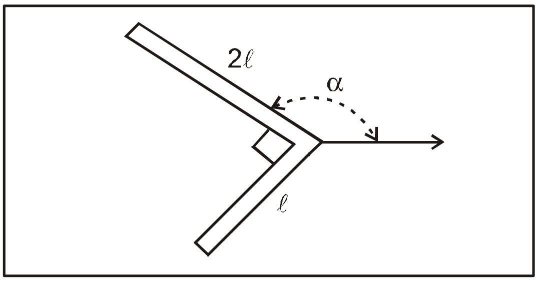


(A) $\frac{h}{2\sin\theta}$ (B) $\frac{h}{4\sin\theta}$ (C) $\frac{h}{\sin\theta}$ (D) $\frac{h}{8\sin\theta}$

13. A solid homogeneous cylinder of height h and base radius r is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = a + bt^2$. If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

(A) 2gr/bh (B) 4gr/bh (C) gr/bh (D) gr/2bh

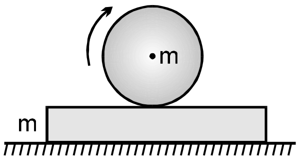
14. A straight uniform metal rod of length 3ℓ is bent through right angle as shown. The bent rod is then placed on a rough horizontal table. A light string is attached to the vertex of the right angle. The string is then pulled horizontally so that the rod translates at constant velocity. Then the angle α which the side 2ℓ makes with string is :



TOP VIEW

(A) $\pi - \tan^{-1}\frac{1}{2}$ (B) $\pi - \sin^{-1}\frac{1}{2}$ (C) $\pi - \tan^{-1}\frac{1}{4}$ (D) $\pi - \sin^{-1}\frac{1}{4}$

15. A sphere of mass 'm' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass 'm'. The co-efficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the centre of sphere relative to the plank will be:



(A) zero (B) μg (C) (7/5) μg (D) 2 μg

16. Two cylinders having radii 2R and R and moments of inertia 4I and I about their central axes are supported by axles perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. During this

(A) angular momentum of system is conserved
(B) kinetic energy is conserved
(C) neigher the angular momentum nor the kinetic energy is conserved
(D) both the angular momentum and kinetic energy are conserved.

17. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$ The x-coordinate of centre of mass is

(A) x = 2 (B) x = 0 (C) x = 1 (D) x = 3

18. If the positions of two like parallel forces are interchanged, their resultant shifts by one–fourth of the distance between them then the ratio of their magnitude is:

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 3 : 5

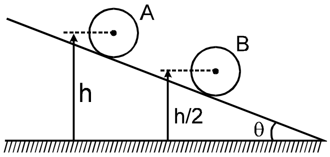
19. A string is wrapped around a cylinder of mass m and radius R. The string is pulled vertically upward to prevent the centre of mass from falling as the cylinder unwinds the string. The length of the string unwound when the cylinder has reached a speed ω will be :

(A) $\frac{R\omega}{4g}$ (B) $\frac{R^2\omega^2}{4g}$ (C) $\frac{R\omega}{8g}$ (D) $\frac{R^2\omega^2}{8g}$

20. A stick of length l and mass m lies on a frictionless horizontal surface on which it is free to move in any direction. A ball of same mass m moving with speed v_0 perpendicularly to this length collides at the end of stick. Find the angular velocity when the ball sticks to the end of the stick.

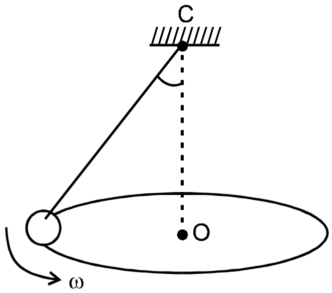
(A) $\frac{12}{7}\frac{v_0}{l}$ (B) $\frac{6v_0}{5l}$ (C) $\frac{24v_0}{5l}$ (D) $\frac{3v_0}{7l}$.

21. Two identical balls A & B of mass m each are placed on a fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two balls collides. Ball A rolls down without slipping on inclined plane & collide elastically with ball B. The kinetic energy of ball A just after the collision with ball B is :



(A) $\frac{mgh}{7}$ (B) $\frac{mgh}{2}$ (C) $\frac{2mgh}{5}$ (D) $\frac{7mgh}{5}$

22. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C, then



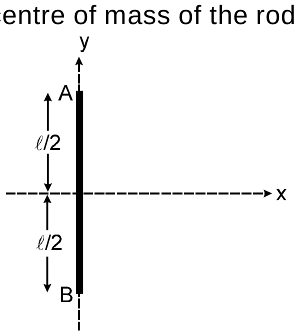
(A) $\vec{L}$ is constant (B) only direction of $\vec{L}$ is constant

(C) only magnitude of $\vec{L}$ is constant (D) none of the above.

23. A homogeneous disc of mass M and radius R rotates about vertical axis with a uniform angular velocity ω_0. If the brakes are applied with a radial force F and coefficient of friction μ, the time taken to bring the disc to rest is :

(A) $\frac{MR\omega_0}{4\mu F}$ (B) $\frac{MR\omega_0}{2\mu F}$ (C) $\frac{MR\omega_0}{\mu F}$ (D) $\frac{2MR\omega_0}{\mu F}$

24. A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along positive x-direction at end A of the rod. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$ (C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

MULTIPLE CORRECT ANSWER(S) TYPE

25. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about this point. It follows that:

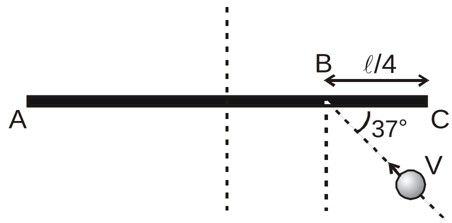
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instant of time.

(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

(C) the magnitude of $\vec{L}$ does not change with time.

(D) $\vec{L}$ does not change with time.

26. A rod AC of length ℓ and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes rod at point B making angle 37^0 with the rod. The collision is elastic. After collision :



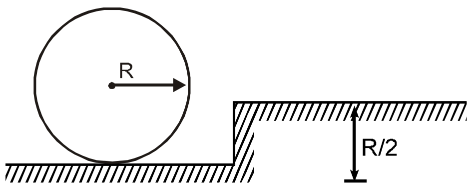
(A) The angular velocity of the rod will be $\frac{72}{55}\frac{v}{\ell}$

(B) The centre of the rod will travel a distance $\frac{\pi\ell}{3}$ in the time in which it makes half rotation

(C) Impulse of the impact force is $\frac{24mV}{55}$

(D) None of these

27. A wheel (to be considered as a ring) of mass m and radius R rolls without sliding on a horizontal surface with constant velocity v. It encounters a step of height R/2 at which it ascends without sliding.



(A) the angular velocity of the ring just after it comes in contact with the step is 3v/4R

(B) the normal reaction due to the step on the wheel just after the impact is $\frac{mg}{2}+\frac{9\,mv^2}{16\,R}$

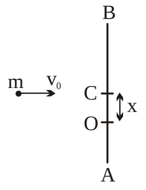
(C) the normal reaction due to the step on the wheel increases as the wheel ascends

(D) the friction will be absent during the ascent.

28. A paritcle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.
(A) Angular momentum of the particle about O is increasing .
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing .
(D) The angular velocity of the particle about O is increasing.

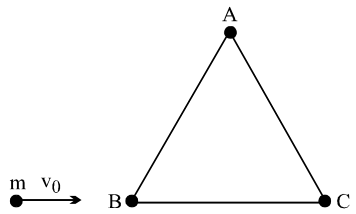
PART - II : SUBJECTIVE QUESTIONS

1. A uniform rod AB of length L and mass M is lying on a smooth table. A small particle of mass m strike the rod with a velocity v_0 at point C at a distance x from the centre O. The particle comes to rest after collision. Then find the valueof x, so that point A of the rod remains stationary just after collision.

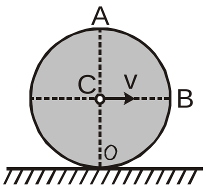


2. A force $\vec{F} = A\hat{i} + B\hat{j}$ is applied to a point whose radius vector relative to the origin of coordinates O is equal to $\vec{r} = a\hat{i} + b\hat{j}$, where a, b &, A, B are constants, and $\hat{i}$, $\hat{j}$ are the unit vectors of the x and y axes. Find the moment N and the arm ℓ of the force relative to the point O.

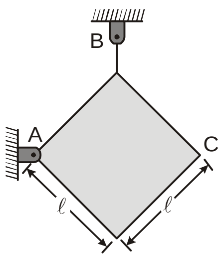
3. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact .
Calculate the time required by the triangle ABC to complete half-revolution in its subsequent motion.



4. A cylinder rolls without slipping over a horizontal plane with constant velocity. The radius of the cylinder is equal to r. Find the curvature radii of trajectories traced out by the points A and B in Fig.

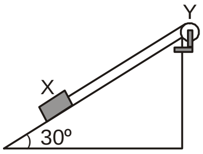


5. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that moment;



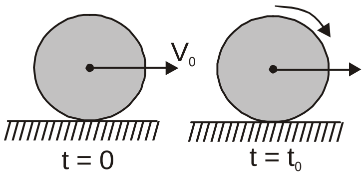
(a) The angular acceleration of the plate.
(b) The acceleration of corner C.
(c) The reaction at A.

6. A block X of mass 0.5 kg is held by a long massless string on a fixed frictionless inclined plane inclined at 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and radius 0.2 m as shown in figure. The drum is given an initial angular velocity such that block X starts moving up the plane.



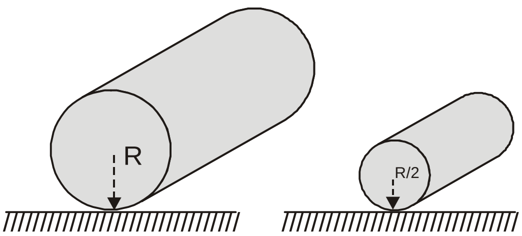
(a) Find the tension in the string during motion.
(b) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

7. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds, it acquires a purely rolling motion as shown in figure.

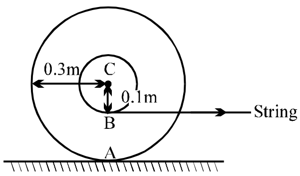


(a) Calculate the velocity of the centre of mass of the disc at t_0 .
(b) Assuming the coefficient of friction to be μ, calculate t_0 . Also calculate the work done by the frictional force as a function of time & the total work done by it over a time t much longer than t_0.

8. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.

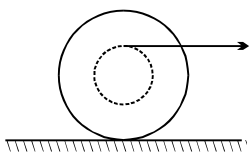


9. A wheel is made to roll without slipping, towards right, by pulling a string wrapped around a coaxial spool as shown in figure. With what velocity the string should be pulled so that the centre of the wheel moves with a velocity of 3 m/s?



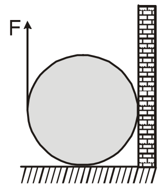
10. A disc rolls on a table. The ratio of its K.E. of rotation to the total K.E. is 1/c . Find c .

11. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. The acceleration of the point on the thread which is being pulled (assuming that the spool rolls purely on the floor) is 2K (in m/s^2) then find K .

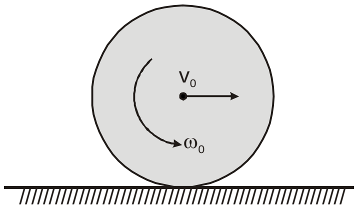


12. A wheel rotating at a speed of 600 rpm (revolutions per minute) about its axis is brought to rest by applying a constant torque for 10 seconds. Find the angular deceleration (rev/s^2) and the angular velocity (rev/s) 5 seconds after the application of the torque.

13. Figure shows a vertical force F that is applied tangentially to a uniform cylinder of weight W = 16N. The coefficient of static friction between the cylinder and all surfaces is 0.5. Find the maximum force that can be applied without causing the cylinder to rotate.



14. A sphere of mass m and radius r is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero(v_0 = 2m/s , r = 1m and the coefficient of friction μ_k = 0.05), Find;



(a) The required ω_0 .
(b) The time required for the sphere to come to rest .

15. A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the acceleration the stationary one and decelerating the moving one. if the common magnitude of the acceleration and deceleration be one revolution per second square, how long will it take before the two cylinders have equal angular speed (in sec.)?

EXERCISE # 3

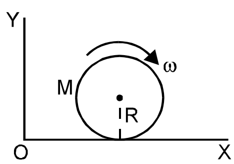
PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

* ***Marked Questions are having more than one correct option.***

1. A smooth sphere 'A' is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity 'v'. It collides elastically and head on with an identical sphere 'B' at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then **[JEE - 99' 2/200]**

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$ (C) $\omega_A = \omega$ (D) $\omega_B = \omega$

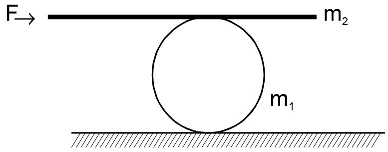
2. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is



[JEE - 99' 2/200]

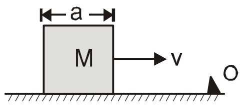
(A) (1/2)MR$^2\omega$ (B) MR$^2\omega$ (C) (3/2)MR$^2\omega$ (D) 2MR$^2\omega$

3. A man pushes a cylinder of mass m_1 with help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find : **[JEE 99' 6+4/200]**



(a) The accelerations of the plank and the centre of mass of the cylinder, and
(b) The magnitudes and directions of frictional forces at contact points.

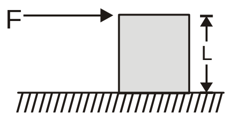
4. A cubical block of side a is moving with velocity v on a horizontal smooth plane, as shown in figure. It hits a ridge at point O. The angular speed of the block after it hits O is .



[JEE - 99' 2/200]

(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$ (C) $\frac{\sqrt{3}v}{\sqrt{2}a}$ (D) zero

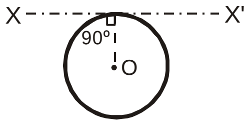
5. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



[JEE - 2000 (Scr' 1/35]

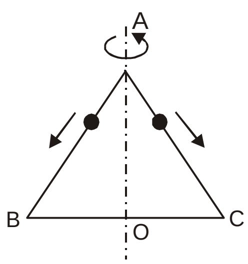
(A) Infinitesimal (B) $\frac{mg}{4}$ (C) mg/2 (D) $mg(1-\mu)$

6. A thin wire of length L and uniform linear density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **[JEE - 2000 Scr' 1/35]**



(A) $\frac{\rho L^3}{8\pi^2}$ (B) $\frac{\rho L^3}{16\pi^2}$ (C) $\frac{5\rho L^3}{16\pi^2}$ (D) $\frac{3\rho L^3}{8\pi^2}$

7. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are



[JEE - 2000 Scr.' 1/35]

(A) Angular velocity and total energy (kinetic energy and potential energy)
(B) Total angular momentum and total energy
(C) Angular velocity and moment of inertia about the axis of rotation
(D) Total angular momentum and moment of inertia about axis AO

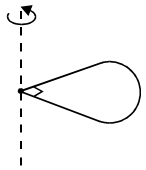
8. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits the end A of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

(a) Find the ratio m/M. **[JEE - 2000 (Mains) 10/100]**

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

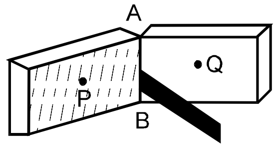
9. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M . It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertial about the axis of rotation is :



[JEE - 01 (Scr.)' 3/105]

(A) $\frac{1}{2}$ MR^2 (B) $\frac{1}{4}$ MR^2 (C) $\frac{1}{8}$ MR^2 (D) $\sqrt{2}$ MR^2

10. Two heavy metallic plates are joined together at 90º to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kg–m^2. Two rubber obstacles P and Q are fixed one on each metallic plate at a distance 0.5m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1rad/sec and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,



(a) Find the location of the centre of mass of the laminar sheet from AB

(b) At what angular velocity does the laminar sheet come back after the first impact

(c) After how many impacts, does the laminar sheet come to rest. **[JEE - 01(Mains) 8+1+1/100]**

11. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are : **[JEE - 02 (Scr.) 3/90]**

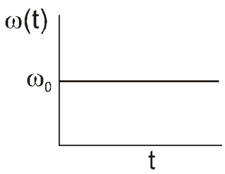
(A) Up the incline while ascending and down the incline while descending

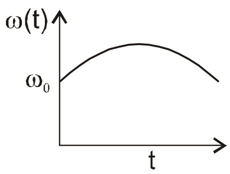
(B) Up the incline while ascending as well as descending

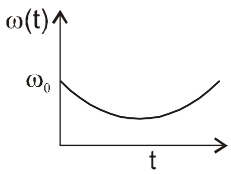
(C) Down the incline while ascending and up the incline while descending

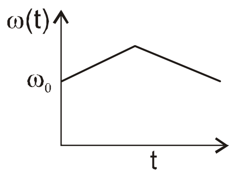
(D) Down the incline while ascending as well as descending.

12. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform . Now the platform is given an angular velocity ω_0 . When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform) the angular velocity of the platform ω(t) will vary with time t as : **[JEE - 02 (Scr.) 3/90]**

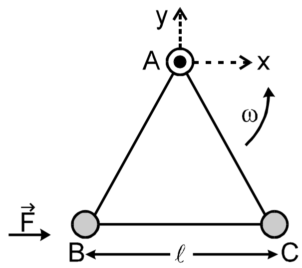
(A) 

(B) 

(C) 

(D) 

13. Three particles A, B and C each of mass m are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x - y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A as shown in figure. The body is set into rotational motion on the table about A with a constant angular velocity ω.



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

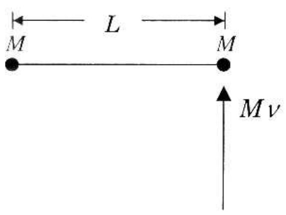
(b) At time T, when the side BC is parallel to the x – axis, a force F is applied on B along BC as shown. Obtain the x – component and the y – component of the force exerted by the hinge on the body, immediately after time T. **[JEE Mains 02, (1+4)/60]**

14. A particle is in uniform circular motion in a horizontal plane. Its angular momentum is constant when the origin is taken at :

(A) centre of the circle (B) any point on the circumference of the circle

(C) any point inside the circle (D) any point outside the circle **[JEE Sc. 2003' 3/84]**

15. Two particles, each of mass M, are connected by a rod of negligible mass and length L. The system is lying on a horizontal frictionless surface. An impulse Mv, perpendicular to the rod, is given at one end of the rod as shown in the figure. The angular velocity acquired by the rod is **[JEE Sc. 2003' 3/84]**

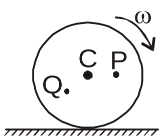


(A) $\frac{4v}{L}$ (B) $\frac{2v}{L}$ (C) $\frac{v}{L}$ (D) $\frac{v}{4L}$

16. A platform is revolving in horizontal plane about a fixed axis and a boy is sitting at centre. The initial kinetic energy of system is K. If the boy stretches his arms then moment of inertia of system becomes double. Final kinetic energy of system is : **[JEE Sc. 2004' 3/84]**

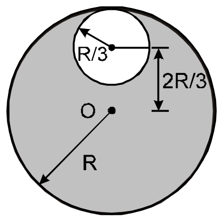
(A) K (B) $\frac{K}{2}$ (C) $\frac{K}{4}$ (D) 2K

17. A disc is moving without slipping on ground then the relation between magnitude of velocity of points P, C and Q is [distance CP = CQ] **[JEE Sc. 2004' 3/84]**



(A) Q > C > P (B) P > C > Q (C) C > Q > P (D) All will be same

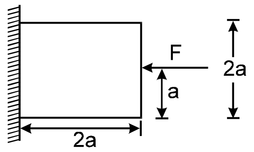
18. A disc has mass 9m. A hole of radius $\frac{R}{3}$ is cut from it as shown in the figure. The moment of inertia of the remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is : **[JEE Scr. 2005 , 3/84]**



(A) $8\ mR^2$ (B) $4\ mR^2$ (C) $\frac{40}{9}mR^2$ (D) $\frac{37}{9}mR^2$

19. A particle moves in circular path with decreasing speed. Which of the following is correct
(A) $\vec{L}$ is constant **[JEE Scr. 2005 , 3/84]**
(B) only direction of $\vec{L}$ is constant
(C) acceleration $\vec{a}$ is towards the centre
(D) it will move in a spiral and finally reach the centre

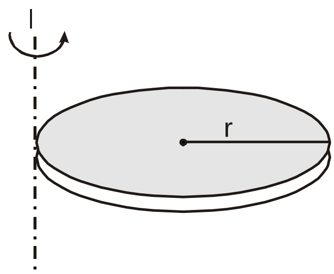
20. A block of mass m is held fixed against a wall by applying a horizontal force F. Which of the following option is incorrect. **[JEE Scr. 2005 , 3/84]**



(A) friction force = mg
(B) normal will not produce torque
(C) F will not produce torque
(D) normal reaction = F

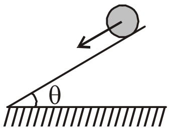
21. A rod of mass M, length L hinged at its one end is in vertical equilibrium position. A bullet of mass m, moving with velocity v strikes the lower end of the rod and gets embedded into it. Find the angular velocity of the rod just after the collision. **[JEE 2005 , 2/60]**

22. A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to : **[JEE 2006 , 3/184]**



(A) $\frac{2}{\sqrt{15}}R$ (B) $\frac{2}{\sqrt{5}}R$ (C) $\frac{3}{\sqrt{15}}R$ (D) $\frac{\sqrt{3}}{\sqrt{15}}R$

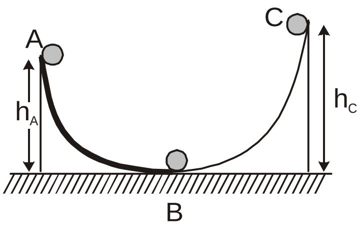
23.* A solid sphere is in pure rolling motion on an inclined surface having inclination θ.



[JEE 2006 , 5/184]

(A) frictional force acting on sphere is $f = \mu$ mg cos θ.
(B) f is dissipative force.
(C) friction will increase its angular velocity and decreases its linear velocity.
(D) If θ decreases, friction will decrease.

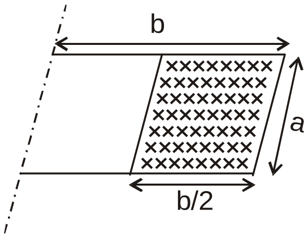
24.* A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A,B and C respectively, then :



[JEE 2006 , 5/184]

(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$
(C) $h_A = h_C$; $K_B = K_C$ (D) $h_A < h_C$; $K_B > K_C$

25. A rectangular plate of mass M of dimensions (a × b) is hinged along one edge. The plate is maintained in horizontal position by colliding a ball of mass m, per unit area, elastically 100 times per second this ball is striking on the right half shaded region of the plate as shown in figure. Find the required speed of the ball (ball is colliding in only half part of the plate as shown). (It is given M = 3 kg, m = 0.01 kg, b = 2 m, a = 1 m, g = 10 m/s^2) **[JEE 2006, 6/184]**



Paragraph for Question Nos. 26 to 28

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[JEE-2007, 12/162]**

26. The ratio x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

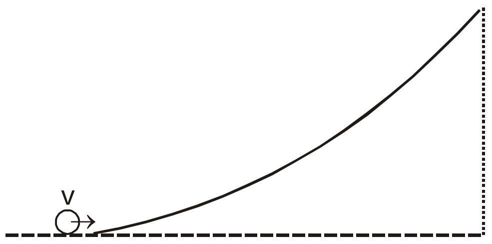
27. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

28. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

29. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is : **[JEE-2007, 3/162]**



(A) ring (B) solid sphere (C) hollow sphere (D) disc

30. **STATEMENT – 1**

If there is no external torque on a body about its centre of mass, then the velocity of the center of mass remains constant. **[JEE-2007, 3/162]**

because

STATEMENT – 2

The linear momentum of an isolated system remains constant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

31. **STATEMENT -1**

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first. **[JEE-2008, 3/163]**

and

STATEMENT -2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is a correct explanation for STATEMENT -1

(B) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT -1

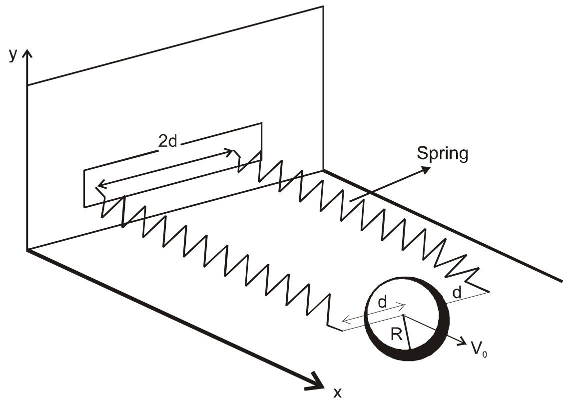
(C) STATEMENT -1 is True, STATEMENT -2 is False

(D) STATEMENT -1 is False, STATEMENT -2 is True.

Paragraph for Question Nos. 32 to 34

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. **[JEE-2008, 3×4/163]**

Figure :



32. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) –kx (B) –2kx (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

33. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

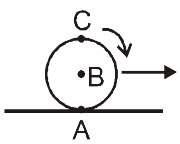
(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

34. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

35. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009, 4/160, –1]**
(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not changes in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

36*. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[JEE 2009, 4/160, –1]**



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

37. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then **[JEE 2009, 3/160, –1]**

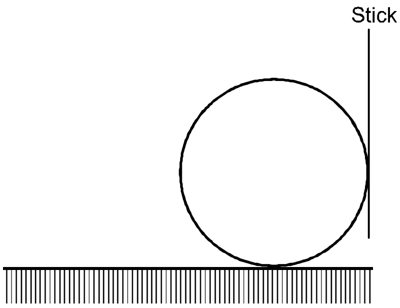
(A) at θ = 30°, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

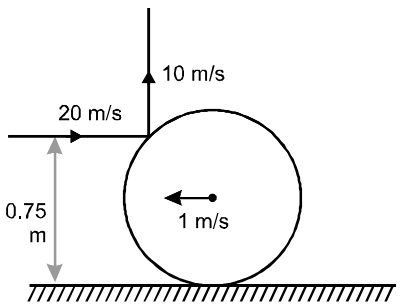
(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

38. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[IIT-JEE 2011; 4/160]**



39. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg–m^2, then N is : **[IIT-JEE 2011; 4/160]**

40. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[IIT-JEE 2011; 4/160]**



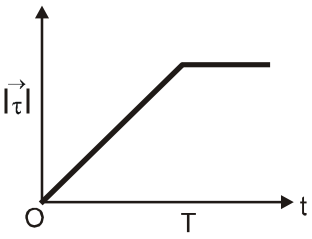
(A) the ring has pure rotation about its stationary CM.

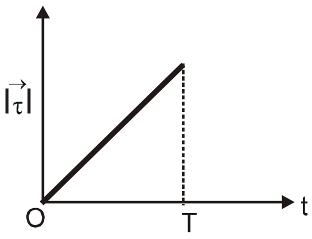
(B) the ring comes to a complete stop.

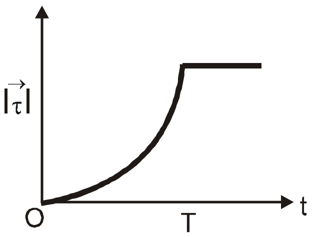
(C) friction between the ring and the ground is to the left.

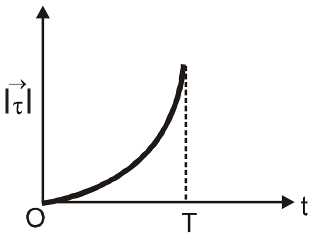
(D) there is no friction the ring and the ground.

41. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about O, as a function of time is best represented by which plot ? **[JEE 2012 (3, –1)/136]**

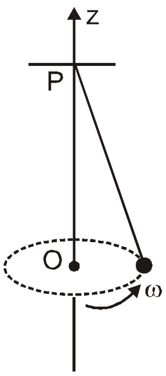
(A)


(B)


(C)


(D)


42. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion If the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system , calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then :



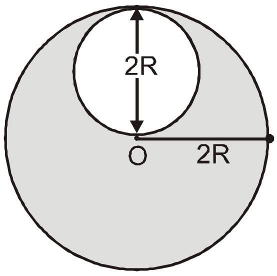
(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ vaires with time

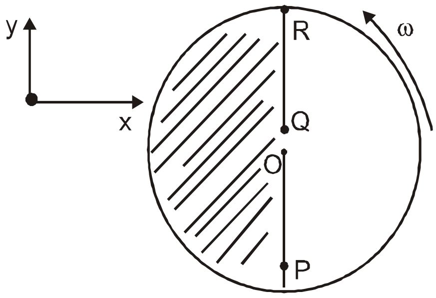
(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time **[JEE 2012 (3, –1)/136]**

43. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The mopment of inertial of this lamina aobut axes passing through O and P is I_0 and I_p, respectively. Both these axes are perpendicular to the plane of the lamina. The ration $\frac{I_p}{I_o}$ to the nearest integer is :



[JEE 2012 (4, 0)/136]

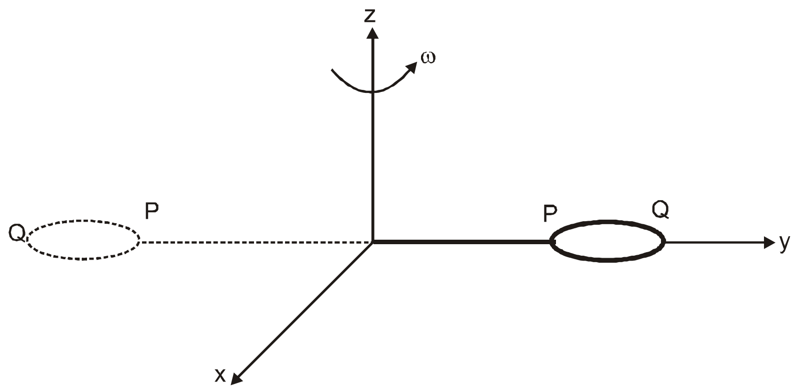
44. (A) Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velcoity of projection is in the y-z plane and is same for both pebbls with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout Then



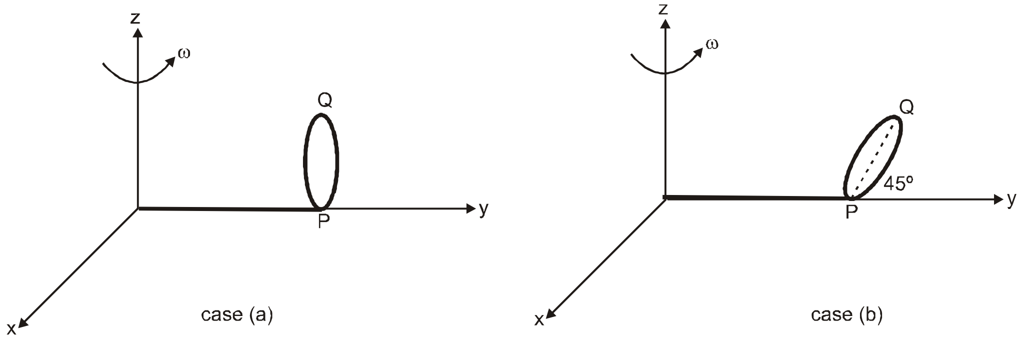
(A) P lands in the shaded region and Q in the unshaded region
(B) P lands in the unshaded region and Q in the shaded region
(C) Both P and Q land in the unshaded region
(D) Both P and Q land in the shaded region **[JEE 2012 (3, –1)/136]**

Paragraph for Question 45 and 56

The general motion of a rigid body can be considered to be a combination of (i) a motion of its center of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal firctionless plane with angular speed ω, the motion a any instant can be taken as a combination of (i) a rotation of the center of mass of the disc about the z-axis and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientaion of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure: Case (a) the disc with face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel. to x-axis, in both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



45. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct ? **[JEE 2012 (3, –1)/136]**

(A) It is $\sqrt{2}\omega$ for both the cases.

(B) It is ω for case (a) ; and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a) ; and $\sqrt{2}\omega$ for case (b).

(D) It is ω for both the cases.

46. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct ?

(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a) ; and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a) ; and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a) ; and is at 45° to the x-z plane and is normal to the plane of the disc for case (b). **[JEE 2012 (3, –1)/136]**

47. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

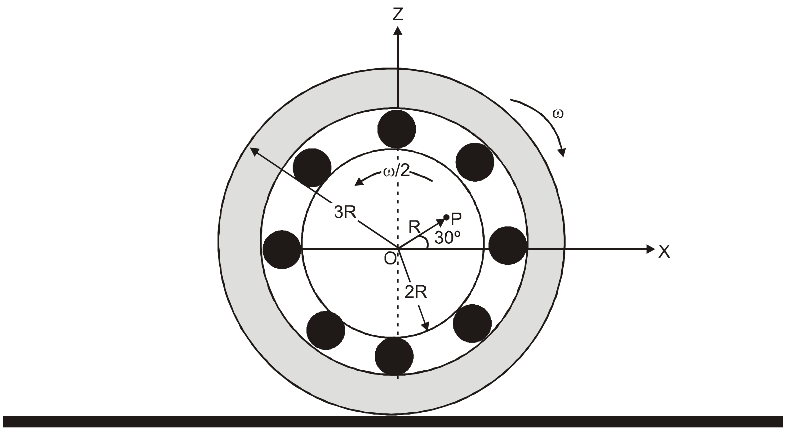
(A) Both cylinders P and Q reach the ground at the same time. **[JEE 2012 (4, 0)/136]**

(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

48. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30º with the horizontal . Then with respect to the horizontal surface.



(A) The point O has a linear velocity $3R\omega\hat{i}$.

(B) The point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) The point P has a linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) The point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right) R\omega\hat{i} + \frac{1}{4} R\omega\hat{k}$ **[JEE 2012 (4, 0)/136]**

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

* ***Marked Questions are having more than one correct option.***

1. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc ? **[AIEEE 4/300 2002]**

(1) $\left(\frac{M+m}{M}\right)\omega_1$ (2) $\left(\frac{M+m}{m}\right)\omega_1$ (3) $\left(\frac{M}{M+4m}\right)\omega_1$ (4) $\left(\frac{M}{M+2m}\right)\omega_1$

2. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. The maximum acceleration down the plane is for (no rolling) : **[AIEEE 4/300 2002]**

(1) solid sphere (2) hollow sphere (3) ring (4) all same

3. Moment of inertia of a circular wire of mass M and radius R about its diameter is : **[AIEEE 4/300 2002]**

(1) $\frac{MR^2}{2}$ (2) MR^2 (3) $2MR^2$ (4) $\frac{MR^2}{4}$

4. A thin and circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. If the another disc of same dimensions but of mass M/4 is placed gently on the first disc co-axially, then the new angular velocity of the system is : **[AIEEE 4/300 2002]**

(1) $\frac{5}{4}\omega$ (2) $\frac{2}{3}\omega$ (3) $\frac{4}{5}\omega$ (4) $\frac{3}{2}\omega$

5. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate thickness t/4. Then the relation between the moment of inertia I_X and I_Y is : **[AIEEE 4/300 2003]**

(1) $I_Y = 32I_X$ (2) $I_Y = 16I_X$ (3) $I_Y = I_X$ (4) $I_Y = 64\,I_X$

6. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is : **[AIEEE 4/300 2003]**

(1) L/4 (2) 2L (3) 4L (4) L/2

7. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then : **[AIEEE 4/300 2003]**

(1) $\vec{r}\cdot\vec{F} = 0$ and $\vec{F}\cdot\vec{\tau} \neq 0$ (2) $\vec{r}\cdot\tau \neq 0$ and $\vec{F}\cdot\vec{\tau} = 0$

(3) $\vec{r}\cdot\tau \neq 0$ and $\vec{F}\cdot\vec{\tau} \neq 0$ (4) $\vec{r}\cdot\tau = 0$ and $\vec{F}\cdot\vec{\tau} = 0$

8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ? **[AIEEE 4/300 2004]**

(1) Moment of inertia (2) Angular momentum

(3) Angular velocity (4) Rotational kinetic energy

9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that : **[AIEEE 4/300 2004]**

(1) $I_A = I_B$ (2) $I_A > I_B$ (3) $I_A < I_B$ (4) $I_A / I_B = d_A/d_B$

where d_A and d_B are their densities.

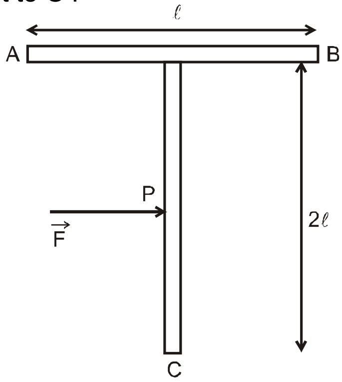
10. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is : **[AIEEE 4/300 2005]**

(1) $\frac{R_2}{R_1}$ (2) $\left(\frac{R_1}{R_2}\right)^2$ (3) 1 (4) $\frac{R_1}{R_2}$

11. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is : **[AIEEE 4/300 2005]**

(1) $\frac{1}{4}Mr^2$ (2) $\frac{2}{5}Mr^2$ (3) Mr^2 (4) $\frac{1}{2}Mr^2$

12. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C : **[AIEEE 4/300 2005]**



(1) $\frac{2}{3}\ell$ (2) $\frac{3}{2}\ell$ (3) $\frac{4}{3}\ell$ (4) ℓ

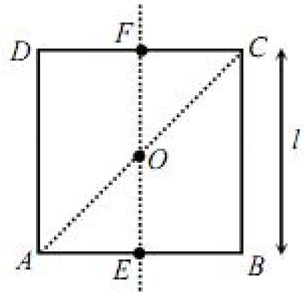
13. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' : **[AIEEE 4.5/1802006]**

(1) $\frac{\omega m}{(m+2M)}$ (2) $\frac{\omega(m+2M)}{m}$ (3) $\frac{\omega(m-2M)}{(m+2M)}$ (4) $\frac{\omega m}{(m+M)}$

14. Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia about an axis passing through A and parallel to BD is : **[AIEEE 4.5/180 2006]**

(1) $m\ell^2$ (2) $2m\ell^2$ (3) $\sqrt{3}\,m\ell^2$ (4) $3m\ell^2$

15. For the given uniform square lamina ABCD, whose centre is O, **[AIEEE 3/120 2007]**



(1) $\sqrt{2}I_{AC} = I_{EF}$ (2) $I_{AD} = 2I_{EF}$ (3) $I_{AC} = I_{EF}$ (4) $I_{AC} = \sqrt{2}I_{EF}$

16. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ ? with the horizontal. Then its acceleration is : **[AIEEE 3/120 2007]**

(1) $\frac{g\sin\theta}{1+I/MR^2}$ (2) $\frac{g\sin\theta}{1+MR^2/I}$ (3) $\frac{g\sin\theta}{1-I/MR^2}$ (4) $\frac{g\sin\theta}{1-MR^2/I}$

17. Angular momentum of the particle rotating with a central force is constant due to : **[AIEEE 3/120 2007]**

(1) constant force
(2) constant linear momentum
(3) zero torque
(4) constant torque

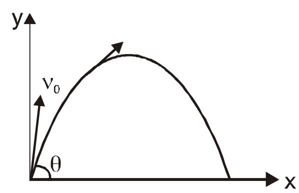
18. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is : **[AIEEE 3/105 2008]**

(1) $\frac{1}{12}ma^2$ (2) $\frac{7}{12}ma^2$ (3) $\frac{2}{3}ma^2$ (4) $\frac{5}{6}ma^2$

19. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its cenfre of mass rises to a maximum height of : **[AIEEE 4/144 2009]**

(1) $\frac{1}{6}\frac{\ell\omega}{g}$ (2) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (3) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (4) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$

20. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0\sin\theta}{g}$, the angular momentum of the particle is **[AIEEE 4/144 2010]**



(1) $-mg\,v_0\,t^2\cos\theta\,\hat{j}$
(2) $mg\,v_0\,t\cos\theta\,\hat{k}$
(3) $-\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{k}$
(4) $\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{i}$

21. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : **[AIEEE 2010, 4/ 144]**

(1) remains unchanged
(2) continuously decreases
(3) continuously increases
(4) first increases and then decreases

26. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :

(1) $\frac{3}{2}g$ (2) g (3) $\frac{2}{3}g$ (4) $\frac{g}{3}$ **[AIEEE 2010, 4/144]**

29. A pulley of radius 2m is rotated about its axis by a force F = $(20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2 , the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) less than 3
(2) more than 3 but less than 6
(3) more than 6 but less than 9
(4) more than 9 **[AIEEE 2010, 4/144]**

EXERCISE # 4

NCERT QUESTIONS

1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body ?

2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

3. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system ?

4. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?

5. (A) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value ? Assume that the turntable rotates without friction.
(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy ?

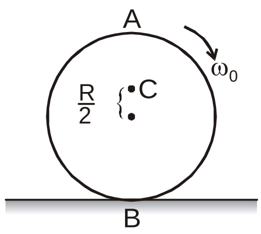
6. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping.

7. A uniform solid cylinder of mass 5 kg and radius 30 cm, and free to rotate about its axis, receives an angular impulse of 3 kg m s^{-1} initially followed by a similar impulse after every 4 s. What is the angular speed of the cylinder 30 s after the initial impulse ? The cylinder is at rest initially.

8. To maintain a rotor at a uniform angular speed or 200 rad s^{-1}, an engine needs to transmit a torque of 180 N m. What is the power required by the engine ?
(Note : uniform angular velocity in the absence of friction implies zero torque. In particle, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

9. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. (a) What is the angular speed of the two-disc system ? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy ? Take $\omega_1 \neq \omega_2$.

10. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. Figure ? Will the disc roll in the direction indicated?



11. Explain why friction is necessary to make the disc in figure roll in the direction indicated.
(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.
(b) What is the force of friction after perfect rolling begins ?

12. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10 π rad s^{-1}. Which of the two will start to roll earlier ? The co-efficient of kinetic friction is $\mu_K = 0.2$.

13. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The co-efficient of static friction $\mu_s = 0.25$.
(a) How much is the force of friction acting on the cylinder ?
(b) What is the work done against friction during rolling ?
(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly ?

14. Read each statement below carefully, and state, with reasons, if it is true or false :
(a) During rolling without slipping on a fixed inclined surface, the force of friction acts in the same direction as the direction of motion of the CM of the body (only weight and contact force act).
(b) The instantaneous speed of the point of contact during rolling without slipping on a fixed surface is zero.
(c) The instantaneous acceleration of the point of contact during rolling without slipping on a fixed surface is zero.
(d) For rolling without slipping on a fixed surface motion, total work done by friction is zero.
(e) A wheel moving down a perfectly frictionless fixed inclined plane will undergo rolling with slipping (only weight and contact force act).

ANSWERS

Exercise # 1

PART-I

A-1.	(A)	**A-2.**	(B)	**B-1.**	(A)	**B-2.**	(D)	**B-3.**	(A)	**B-4.**	(B)	**B-5.**	(B)
B-6.	(C)	**B-7.**	(A)	**B-8.**	(D)	**B-9.**	(D)	**B-10.**	(D)	**B-11.***	(BC)	**B-12.**	(C)
B-13.	(B)	**B-14.**	(D)	**C-1.**	(B)	**C-2.**	(A)	**C-3.**	(A)	**C-4.**	(C)	**C-5.**	(A)
C-6.	(C)	**D-1.**	(B)	**D-2.**	(C)	**D-3.**	(A)	**D-4.**	(D)	**D-5.**	(C)	**E-1.**	(B)
E-2.	(C)	**E-3.**	(A)	**E-4.**	(A)	**E-5.**	(C)	**E-6.**	(C)	**E-7.**	(A)	**E-8.**	(D)
F-1.	(B)	**F-2.**	(A)	**F-3.**	(D)	**F-4.**	(A)	**F-5.**	(B)	**F-6.**	(C)	**F-7.**	(BC)
F-8.	(A)	**F-9.**	(B)	**F-10.**	(B)	**G-1.**	(C)	**G-2.**	(B)	**G-3.**	(D)	**G-4.**	(B)
G-5.	(C)	**G-6.**	(B)	**G-7.**	(B)	**G-8.**	(C)	**G-9.**	(A)	**G-10.**	(C)	**G-11.**	(B)
G-12.	(B)	**G-13.**	(D)	**G-14.**	(B)	**G-15.**	(D)	**G-16.**	(A)	**G-17.**	(C)	**G-18.**	(D)
G-19.	(D)	**G-20.**	(A)	**H-1.**	(B)	**H-2.**	(B)						

PART-II

1.	(C)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(D)	**6.**	(C)	**7.**	(C)
8.	(B)	**9.**	(C)	**10.**	(A)	**11.**	(B)	**12.**	(A) – Q ; (B) – R ; (C) – P ;(D) – R				

13. (A) – (Q) ; (B) – (P) ; (C) – (S) ; (D) – (R)   **14.** (A) – q,r,s ; (B) – p,s ; (C) – q,r,s ; (D) – p,q,r,s

15. (i) F ; (ii) F ; (iii) **1.** T, **2.** F ; (iv) T ; (v) F ; (iv) F   **16.** (i) $\frac{M\omega_0}{M+6m}$; (ii) $\frac{mg}{6}$,up ; (iii) $\frac{2}{3}$ mg

Exercise # 2

PART-I

1.	(D)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(D)	**7.**	(B)
8.	(C)	**9.**	(C)	**10.**	(B)	**11.**	(B)	**12.**	(B)	**13.**	(C)	**14.**	(C)
15.	(D)	**16.**	(C)	**17.**	(D)	**18.**	(D)	**19.**	(B)	**20.**	(B)	**21.**	(A)
22.	(C)	**23.**	(B)	**24.**	(D)	**25.**	(ABC)	**26.**	(ABC)	**27.**	(AC)	**28.**	(ACD)

PART-II

1. **L/6** **2.** N = (aB – bA)k, where k is the unit vector of the z axis ℓ = |aB –bA|/ $\sqrt{A^2+B^2}$

3. $t = \dfrac{6a\pi}{\sqrt{3}\,v_0}$ **4.** $\mathbf{R_A = 4r, R_B = 2\sqrt{2}\,r}$ **5.** (a) $\dfrac{3g}{2\sqrt{2}\ell}$ (cw) (b) $\dfrac{3}{2}g\downarrow$ (c) $\dfrac{Mg}{4}\uparrow$

6. (a) 1.633 N (b) 1.224 m

7. (a) $v = \dfrac{2v_0}{3}$; $t_0 = \dfrac{v_0}{3}\mu g$ (b) $w = -\mu mg\,(v_0 t - \dfrac{3}{2}\mu g t^2)$; $-\dfrac{1}{6} m v_0^2$

8. $\upsilon = \sqrt{\dfrac{14gR}{3}}$ **9.** **2 m/s** **10.** 3 **11.** 8 **12.** 1 , 5 **13.** 6

14. (a) 5 (b) 4 **15.** 25

Exercise # 3

PART-I

1. (C) **2.** (C)

3. (a) $a_C = \dfrac{4F}{3m_1+8m_2}$; $a_P = \dfrac{8F}{3m_1+8m_2}$

(b) friction at the top of the cylinder = $3m_1F/(3m_1 + 8m_2)$ towards right,
friction at the bottom = $m_1F/(3m_1 + 8m_2)$ towards right

4. (A) **5.** (C) **6.** (D) **7.** (B) **8.** (a) $\dfrac{m}{M} = \dfrac{1}{4}$ (b) AP = $\dfrac{2L}{3}$ (c) v_p =

9. (A) **10.** **(a)** x = 0.1 m **(b)** ω = 1 rad/s **(c)** never **11.** (B) **12.** (B)

13. **(a)** $\sqrt{3}\, m\omega^2 \ell$ **(b)** $F_y = \sqrt{3}\, m\omega^2 \ell$ $F_x = -F/4$ **14.** (A) **15.** (C)

16.	(B)	**17.**	(B)	**18.**	(B)	**19.**	(B)	**20.**	(B)	**21.**	$\dfrac{3mv}{\ell(M+3m)}$		
22.	(A)	**23.***	(CD)	**24.***	(AB)	**25.**	V = 10 m/s	**26.**	(C)	**27.**	(A)		
28.	(B)	**29.**	(D)	**30.**	(D)	**31.**	(D)	**32.**	(D)	**33.**	(D)	**34.**	(C)
35.	(A)	**36.***	(BD)	**37.**	(B)	**38.**	4	**39.**	9	**40.**	(AC)	**41.**	(B)
42.	(C)	**43.**	3	**44.**	(A)	**45.**	(D)	**46.**	(A)	**47.**	(D)	**48.**	(AB)

PART-II

1.	(3)	**2.**	(4)	**3.**	(1)	**4.**	(3)	**5.**	(4)	**6.**	(1)	**7.**	(4)
8.	(2)	**9.**	(3)	**10.**	(4)	**11.**	(4)	**12.**	(3)	**13.**	(1)	**14.**	(4)
15.	(3)	**16.**	(1)	**17.**	(3)	**18.**	(3)	**19.**	(3)	**20.**	(3)	**21.**	(4)
22.	(3)	**23.**	(2)										

Exercise # 4

1. The geometrical centre of each. No, the CM may lie outside the body,k as in case of a ring, a hollow sphere, a hollow cylinder, a hollow cube etc.

2. Located on the line joining H and C1 nuclei at a distance of 1.24 Å from the H end.

3. The speed of the CM of the (trolley + child) system remains unchanged (equal to v) because no external force acts on the system. The forces involved in running on the trolley are internal to this system.

4. Kinetic Energy = 3125 J; Angular Momentum = 62.5 J s

5. (a) 100 rev/ min (use angular momentum conservation).
(b) The new kinetic energy is 2.5 times the initial kinetic energy of rotation. the child uses his internal energy to increase his rotational kinetic energy.

6. 25 s^{-2}, 10 m s^{-2}

7. 320/3 rad s^{-1} (angular impulse = change in angular momentum).

8. 36 kW

9. (a) By angular momentum conservation, the common angular speed

$$\omega = (I_1\omega_1 + I_2\omega_2) / (I_1 + I_2)$$

(b) The loss is due to energy dissipation in frictional contact which brings the two discs to a common angular speed ω. However, since frictional torques are internal to the system, angular momentum is unaltered.

10. Velocity of A = ω_0R in the same direction as the arrow; velocity of B = ω_0R in the opposite direction to the arrow; velocity of C = ω_0 R/2 in the same direction as the arrow. The disc will not roll on a frictionless plane.

11. (a) Frictional force at B opposes velocity of B. Therefore, frictinal force is in the same direction as the arrow. The sense of frictional torque is such as to oppose angular motion. ω_0 and τ are both normal to the paper, the first into the paper, and the second coming out of the paper.
(b) Frictional force decreases the velocity of the poin of contact B. Perfect rolling ensures when this velocity is zero. Once this is so, the force of friction is zero.

12. Frictional force causes the CM to accelerate from its initial zero velocity. Frictional torque cuases retardation in the initial angular speed ω_0. The equations of motion are : μ_k mg = ma and μ_k mgR = $I\alpha$, which yield v = μ_k gt/I. Rolling beings when v = Rω. For a ring, I = mR^2, and rolling begins at t = ω_0 R/2 μ_kg. For a disc, I = ½mR^2 and rolling starts at break line t = Rω_0/ 3 μ_kg. Thus, the disc begins to roll earlier than the ring, for the same R and ω_0. The actual times can be obtained for R = 10 cm, ω_0 = 10 π rad s^{-1}, μ_k = 0.2.

13. (a) 16.4 N (b) Zero (c) 37° approx

14. (a) False, (b) True, (c) False, (d) True, (e) False