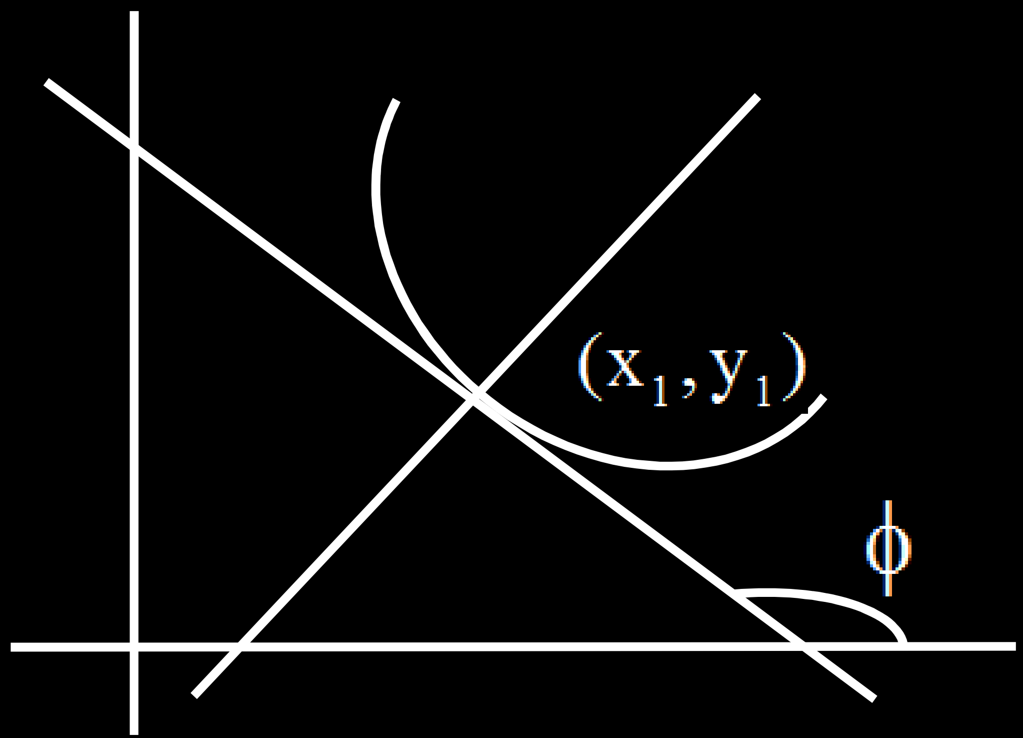
Application Of Derivative

Tangent & Normal

Equation of a tangent at P (x_1, y_1)

$$y - y_1 = \left.\frac{dy}{dx}\right|_{x_1, y_1} (x - x_1)$$



Equation of a normal at (x_1, y_1)

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{x_1, y_1}}(x - x_1)$$

* If $\left.\frac{dy}{dx}\right]_{x_1, y_1}$ exists. However in some cases $\frac{dy}{dx}$ fails to exist but still a tangent can be drawn e.g. case of vertical tangent. Also (x_1, y_1) must lie on the tangent, normal line as well as on the curve.

$$y = x^{\frac{1}{3}}$$

Q. A line is drawn touching the curve $y = \dfrac{2}{3-x}$.

Find the line if its slope/gradient is 2.

Q. Find the tangent and normal for $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$.

Q. Find tangent to $x = a \sin^3 t$ and $y = a \cos^3 t$ at $t = \pi/2$.

Vertical Tangent :

Concept : y = f(x) has a vertical tangent at the point $x = x_0$ if

$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} = \infty \text{ or } -\infty \text{ but not both}$$

Q. Which of the following cases the function f(x) has a vertical tangent at x = 0.

(i) $f(x) = x^{\frac{1}{3}}$

(ii) $f(x) = \text{sgn } x$

(iii) $f(x) = x^{\frac{2}{3}}$

(iv) $f(x) = \sqrt{|x|}$

(v) $f(x) = \begin{cases} 0 \text{ if} & x < 0 \\ 1 \text{ if} & x \geq 0 \end{cases}$

If a curve passes through the origin, then the equation of the tangent at the origin can be directly written by equating to zero the lowest degree terms appearing in the equation of the curve.

Q. $x^2 + y^2 + 2gx + 2fy = 0$ Find equation of tangent at origin

Q. $x^3 + y^3 - 3x^2y + 3xy^2 + x^2 - y^2 = 0$

Find equation of tangent at origin.

Q. Find equation of tangent at origin to $x^3 + y^2 - 3xy = 0$.

Some Common Parametric Coordinates On A Curve

Q. For $\sqrt{x} + \sqrt{y} = \sqrt{a}$ take $x = a\cos^4\theta$ & $y = \sin^4\theta$.

Q. for $y^2 = x^3$, take $x = t^2$ and $y = t^3$.

Q. $y^2 = 4ax$ $(at^2, 2at)$

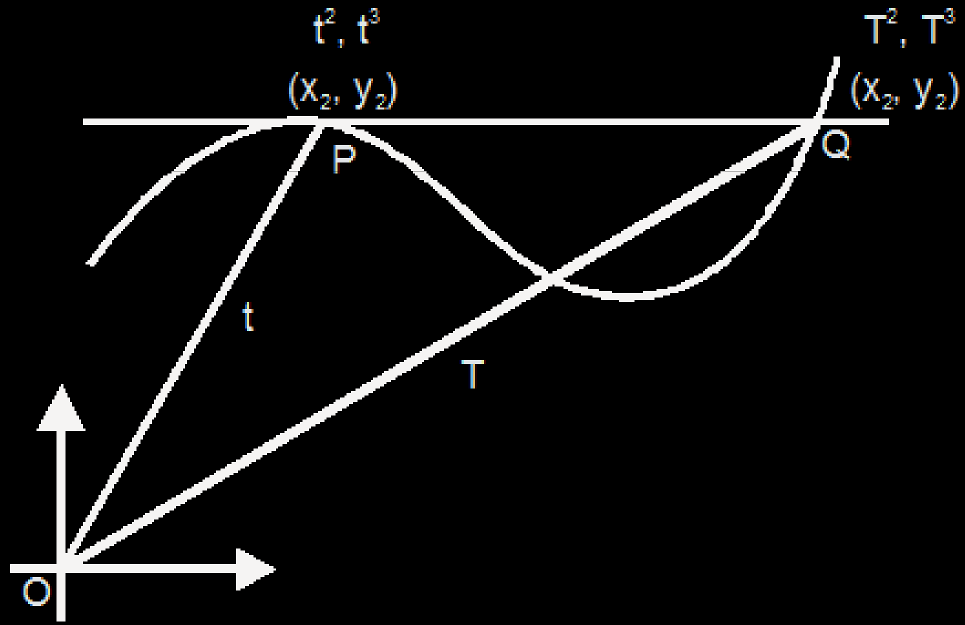
Q. $xy = c^2$

Q. for $y^2 = x^3$, take $x = t^2$ and $y = t^3$.

Note :

The tangent at P meeting the curve again at Q.

$$\Rightarrow \left.\frac{dy}{dx}\right|_{p} = \frac{y_2 - y_1}{x_2 - x_1}$$



Consider the examples $y^2 = x^3$ find $\frac{m_{OP}}{m_{OQ}}$.

Take $P(t^2, t^3)$

Q. Find the equation of a tangent and normal at $x = 0$ if they exist on the curve $y = x^{\frac{1}{3}}(1 - \cos x)$

Q. Equation of the normal to the curve $x^2 = 4y$ which passes through (1, 2).

Q. Normal to the curve $x^2 = 4y$ which passes through $(4, -1)$.

Q. Find the equation of tangent and normal to the curve $f(x)=\begin{cases} x-2 & \text{if } x<1 \\ x^2-x-1 & \text{if } x\geq 1 \end{cases}$ at $x=1$ if it exists.

Q. A curve in the plane is defined by the parametric equations $x = e^{2t} + 2e^{-t}$ and $y = e^{2t} + e^{t}$. An equation for the line tangent to the curve at the point $t = ln\ 2$ is

(A) $5x - 6y = 7$ (B) $5x - 3y = 7$

(C) $10x - 7y = 8$ (D) $3x - 2y = 3$

Q. Tangent to the curve $y = \sin^{-1}\frac{2x}{1+x^2}$ at $x = \sqrt{3}$

Q. Find the equation of the tangent to the curve $y = be^{\frac{-x}{a}}$ at the point where the curve crosses the y-axis.

Q. Prove that all points on the curve

$$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$$

at which the tangent is parallel to the x-axis lie on a parabola.

Q. Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their point of contacts lie on the curve $x^2y^2 = x^2 - y^2$.

Q. Show that the portion of the tangent to the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta$ intercepted between the coordinate axes is constant.

Q. If $y = e^x$ and $y = kx^2$ touch each other, find k.

Angle of Intersection of two Curves :

Definition : The angle of intersection of two curves at a point P is defined as the angle between the two tangents to the curve at their point of intersection.

Q. If the curves are orthogonal then

$$\left(\frac{dy_1}{dx}\right)\left(\frac{dy_2}{dx}\right) = -1$$

everywhere where ever they intersect.

Q. Find the acute angle between the curves

(i) $y = \sin x$ & $y = \cos x$

Q. If θ is the angle between $y = x^2$ and $6y = 7 - x^3$ at (a, a), Find θ.

Q. Find the angle between the curve $2y^2 = x^3$ and $y^2 = 32x$

Q. Find the condition for the two concentric ellipses $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ to intersect orthogonally.

Rate Measure

Q. If the side of an equilateral triangle increases at the rate of $\sqrt{3}$ cm/sec and area increase at the rate 12 cm^2/sec then the side of the equilateral triangle is ________.

Q. An aeroplane is flying horizontally at a height of $\frac{2}{3}$ km with a velocity of 15 km/hr. Find the rate at which it is receding from a fixed point on the ground which it passed over 2 minutes ago.

Q. The height h of a right circular cone is 20 cm and is decreasing at the rate of 4 cm/sec. At the same time, the radius r is 10 cm and is increasing at the rate of 2 cm/sec. Find the rate of change of the volume in cm^3/sec.

Q. If tangent at point P for curve. $x = 2t - t^2$ & $y = t + t^2$ passes through Pt Q (1, 1) find possible co-ordinate of P.

Q. Find shortest distance b/w line $y = x - 2$ & parabola $y = x^2 + 3x + 2$.

Q. Find Point on $3x^2 - 4y^2 = 72$ nearest to line $3x + 2y + 1 = 0$.

Length of Tangent, Normal, Subtangent And Subnormal :

Q. Find everything for hyperbola $xy = 4$ at Pt $(2, 2)$

Q. Show that for the curve $by^2 = (x + a)^3$ the square of the subtangent varies as the subnormal.

Q. Show that at any point on the hyperbola $xy = c^2$, the subtangent varies as the abscissa and the subnormal varies as the cube of the ordinate of the point of contact.

Approximation And Differentials

Q. Use differential to approximate.

Q. $\sqrt{101}$

Q. $(25)^{\frac{1}{3}}$

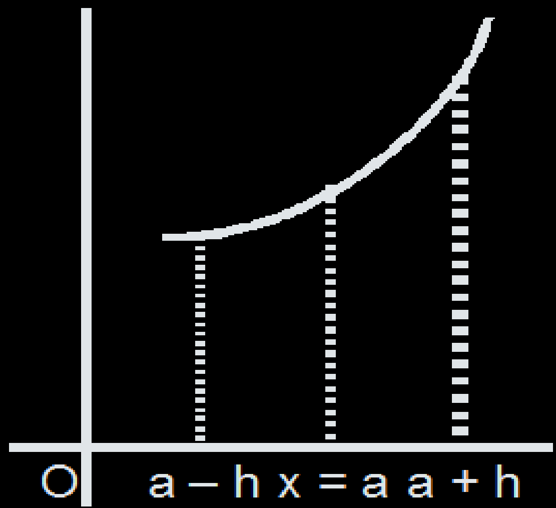
Monotonocity

Functions are said to be monotonic if they are either increasing or decreasing in their entire domain e.g. $f(x) = e^x$; $f(x) = \mathit{l}nx$ & $f(x) = 2x + 3$ are some of the examples of functions which are increasing whereas $f(x) = -x$; $f(x) = e^{-x}$ and $f(x) = \cot^{-1}(x)$ are some of the examples of the functions which are decreasing.

Q. Functions which are increasing as well as decreasing in their domain are said to be non monotonic e.g. $f(x) = \sin x$; $f(x) = ax^2 + bx + c$ and $f(x) = |x|$, however in the interval $\left[0, \frac{\pi}{2}\right]$, $f(x) = \sin x$ will be said to be increasing.

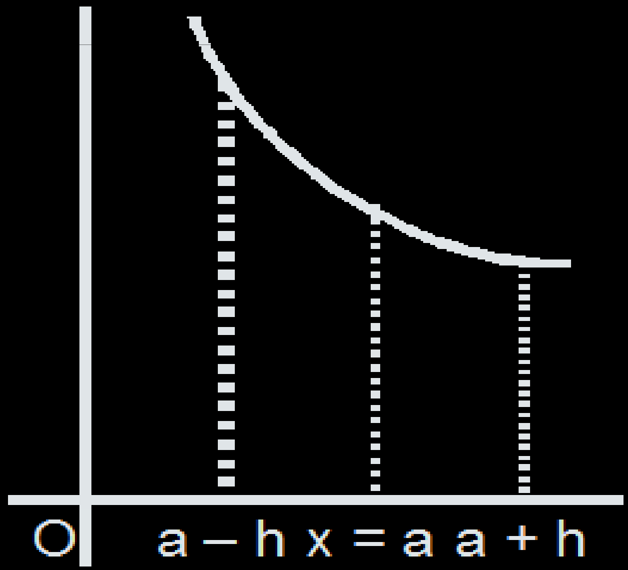
Monotonocity of A Function At A Point

A function is said to be monotonic increasing at $x = a$ if $f(x)$ satisfies $\left.\begin{array}{l} f(a+h) < f(a) \\ f(a-h) > f(a) \end{array}\right]$ for a small positive h.



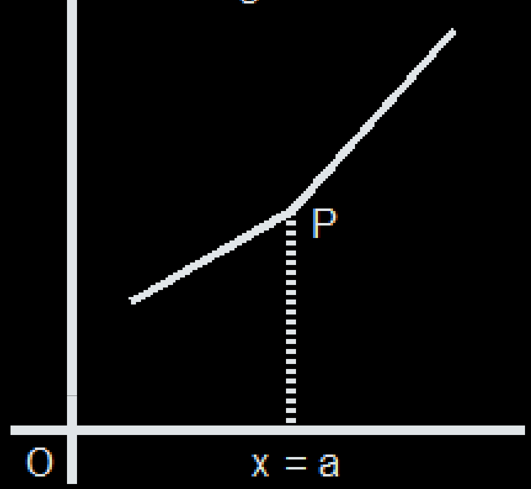
And monotonic decreasing at x = a if and

$$\left.\begin{array}{l} f(a+h) < f(a) \\ f(a-h) > f(a) \end{array}\right]$$



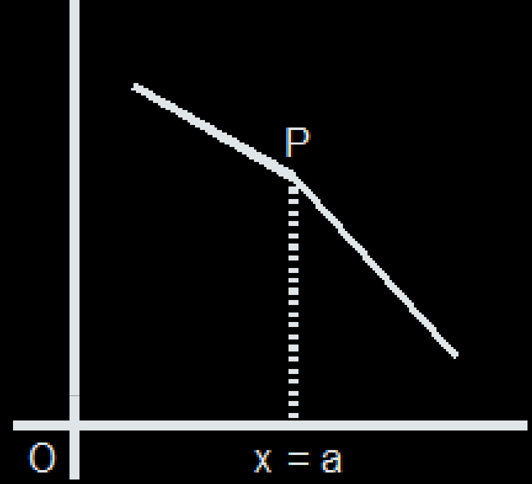
Q. It should be noted that we can talk of monotonocity of $f(x)$ at $x = a$ only if $x = a$ lies in the domain of f, without any consideration of continuity or differentiability of $f(x)$ at $x = a$.

fig. – 1



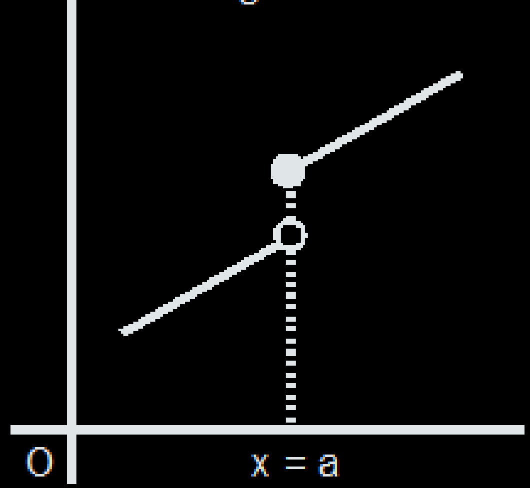
increasing
at x = a

fig. – 2



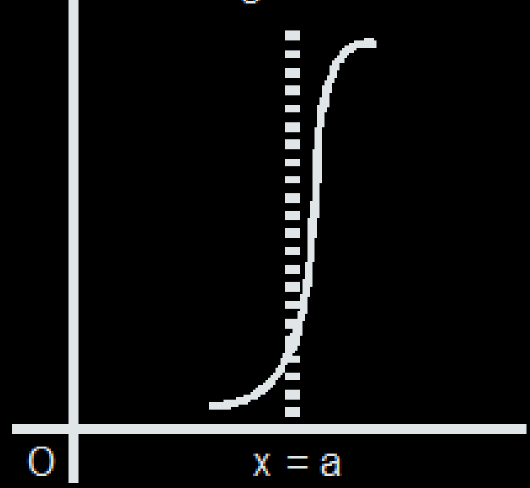
decreasing
at x = a

fig. – 3



increasing
at x = a

fig. – 4



increasing
at x = a

Monotonocity In An Interval

Non Decreasing/Non Increasing

Point of Inflection

(i) Tangent crosses the curve

(ii) $f''(x) = 0$

(iii) $f'(x)$ is extremum

If $x_1, x_2 \in$ domain of f and if

(i) $x_1 > x_2 \Leftrightarrow f(x_1) > f(x_2)$, f is strictly increasing.

(ii) $x_1 > x_2 \Leftrightarrow f(x_1) \geq f(x_2)$, f is non decreasing.

Note :

If f is increasing then nothing definite can be said about the function $f'(x)$ w.r.t. its increasing or decreasing behaviour.

Illustrations

Q. Discuss monotonic behaviour of the function $f(x) = x^2 \cdot e^{-x}$

Q. $f(x) = x + \ln(1 - 4x)$

Q. $$f(x) = \frac{x}{\ell n\, x}$$

Q. $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3$ in $[0, \pi]$
Also find maximum and minimum value of function

Q. $f(x) = ax - \sin x$

Find range of a if $f(x)$ is monotonic

Q. If the function $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ is always decreasing $\forall\ x \in R$, find $'a'$.

Q. Prove that $f(x) = \frac{2}{3}x^9 - x^6 + 2x^3 - 3x^2 + 6x - 1$ is always increasing.

Q. Prove that $f(x)=\int_{x^2}^{x^3}\frac{dt}{\ell nt}$, $(x>0)$ is always an increasing function of x.

Q. The set of integral value(s) of ‘b’ for which $f(x) = \sin 2x - 8(b + 2)\cos x - (4b^2 + 16b + 6)x$ is monotonic decreasing for $\forall\ x \in R$ and has no critical point, is

(A) $\{-10, -9, 2, 3\}$ (B) $\{-7, -8, -1, 0\}$

(C) $\{-8, 1, 5, 6\}$

(D) $\{-100, -200, 100, 200\}$

Q. Consider the function,

$f(x) = x^3 - 9x^2 + 15x + 6$ for $1 \le x \le 6$ and

$$g(x) = \begin{cases} \min. f(t) & \text{for } 1 \le t \le x, 1 \le x \le 6 \\ x - 18 & \text{for } x > 6 \end{cases}$$

then which of the following hold(s) good **?**

(A) $g(x)$ is differentiable at $x = 1$

(B) $g(x)$ is discontinuous at $x = 6$

(C) $g(x)$ is continuous and derivable at $x = 5$

(D) $g(x)$ is monotonic in $(1, 5)$

Q. Find greatest and the least values of the function $f(x) = e^{x^2 - 4x + 3}$ in $[-5, 5]$

Q. $y = \int_{\frac{5\pi}{4}}^{x} (3\sin t + 4\cos t)dt \quad \text{in} \left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$

Q. $$y = \int_{\frac{5\pi}{3}}^{x} (6\cos u - 2\sin u)\,du \quad \text{in} \left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$$

Q. $f(x) = \cos 3x - 15 \cos x + 8$ in $\left[\frac{\pi}{3}, \frac{3\pi}{2}\right]$

Q. Use the function $f(x) = x^{\frac{1}{x}}$ $(x > 0)$ to ascertain whether π^e or e^π is greater.

Q. Find minimum of x^x $(x > 0)$

Q. Find the image of interval **[–1, 3]** under the mapping specified by the function

$$f(x) = 4x^3 - 12x$$

Q. Let $f(x) = \begin{cases} -x^3 + \dfrac{(b^3 - b^2 + b - 1)}{(b^2 + 3b + 2)} & , 0 \le x < 1 \\ 2x - 3 & , 1 \le x \le 3 \end{cases}$

Find all possible real values of b such that f(x) has the smallest value at x = 1.

Establishing Inequalities

Q. Prove that $2 \sin x + \tan x \geq 3x$ for $\left(0 \leq x < \frac{\pi}{2}\right)$

Q. Find the set of values of x for which

$$\ln(1 + x) > \frac{x}{1+x}$$

Q. Prove that $\ell n\left(\frac{x+1}{x}\right)-\frac{1}{x+1}>0$ in $(-\infty,-1)\cup(0,\infty)$

Q. Find the smallest positive constant A such that $\ln x \leq Ax^2$ for all $x > 0$.

Rolle’s & Mean Value Theorem

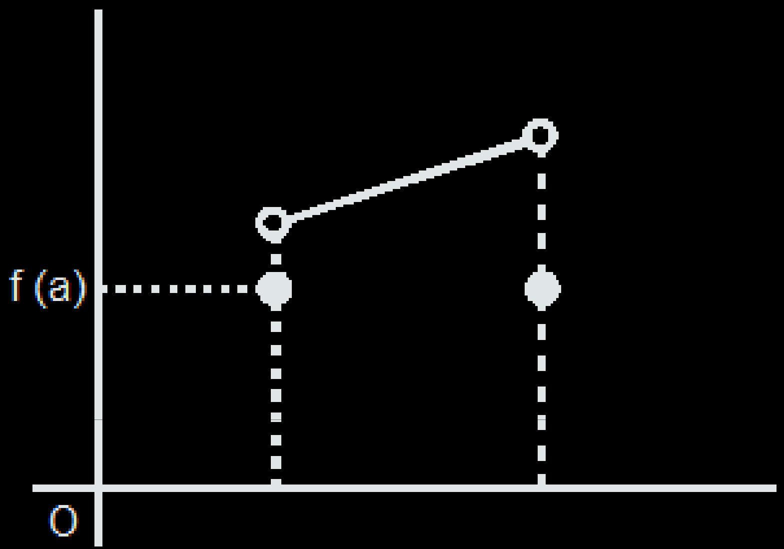
Let f(x) be a function of x subject to the following conditions :

(i) f(x) is a continuous function of x in the closed interval of $a \le x \le b$.

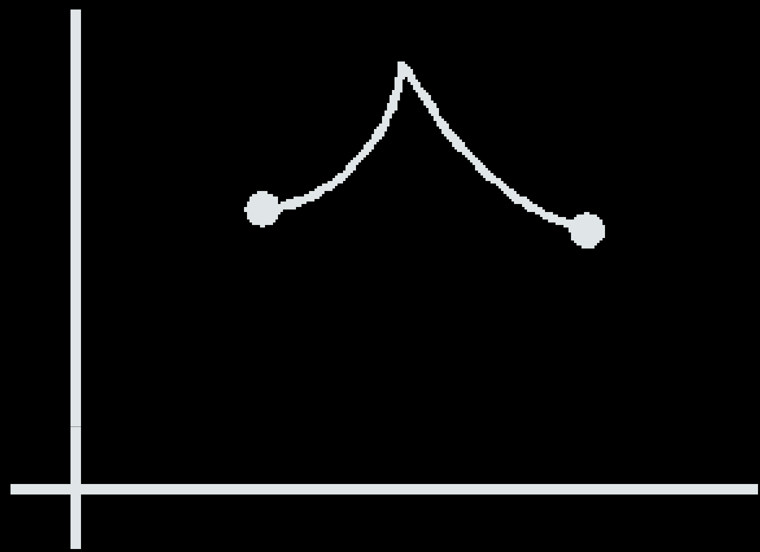
(ii) $f'(x)$ exists for every point in the open interval $a < x < b$.

(iii) $f(a) = f(b)$.

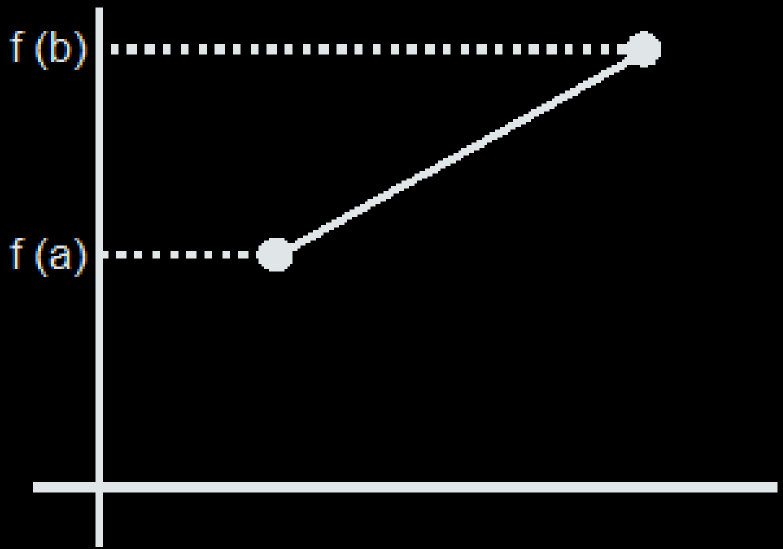
Then there exists at least one point $x = c$ such that $a < c < b$ where $f'(c) = 0$.



f is not continuous in [a, b]
and Rolles theorem failing



f is not derivable in open (a, b)



f (a) ≠ f(b)

Q. Verify Rolle's Theorem for

$f(x) = x(x + 3)e^{-x/2}$ in $[-3, 0]$

Also find c of Rolle's Theorem

Q. $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$

Q. $f(x) = x^3 - 3x^2 + 2x + 5$ in $[0, 2]$

Q. $f(x) = 1 - x^{2/3}$ in $[-1, 1]$

Q. $f(x) = |x|$ in $[-1, 1]$

Q. Let $n \in \mathbf{N}$. If the value of c prescribed in Rolle's theorem for the function $f(x) = 2x(x - 3)^n$ on $[0, 3]$ is $3/4$ then n is equal to

(A) 1 (B) 3

(C) 5 (D) 7

Q. Show that between any two roots of the equation $e^x \cos x = 1$ there exists at least one root of $e^x \sin x - 1 = 0$.

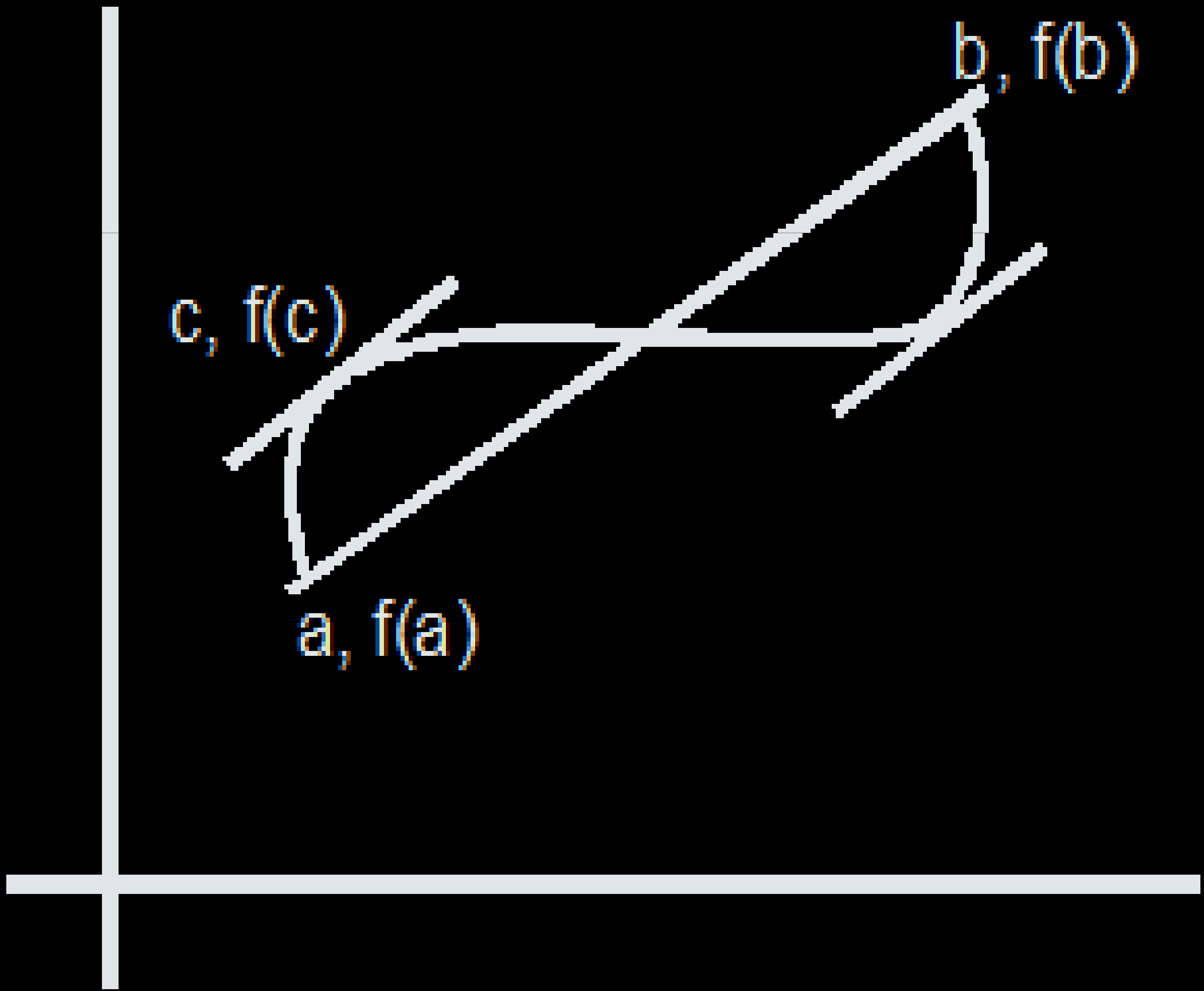
Q. Consider the function $f(x) = \begin{cases} x \sin \dfrac{\pi}{x} & \text{for } x > 0 \\ 0 & \text{for } x = 0 \end{cases}$

then the number of points in (0, 1) where the derivative $f'(x)$ vanishes, is

(A) 0 (B) 1

(C) 2 (D) infinite

LMVT THEOREM : (Lagrange's Mean Value Theorem)



b, f(b)
c, f(c)
a, f(a)

Let f(x) be a function of x subject to the following conditions :

(i) f(x) is a continuous function of x in the closed interval of $a \le x \le b$.

(ii) $f'(x)$ exists for every point in the open interval $a < x < b$.

(iii) $f(a) \neq f(b)$.

Then there exists at least on point $x = c$ such that $a < c < b$ where $f'(c) = \frac{f(b) - f(a)}{b - a}$

Geometrically, the slope of the secant line joining the curve at $x = a$ & $x = b$ is equal to the slope of the tangent line drawn to the curve at $x = c$.

Note the following :

Rolles theorem is a special case of LMVT

Q. $y = lx^2 + mx + n$ in $[a, b]$ find c of L.M.V.T.

Q. Find c of LMVT

$$f(x) = \sqrt{x-1} \text{ in } [1,3]$$

Q. $f(x) = \dfrac{x}{x+2}$ on $[1, 4]$

Q. Using LMVT prove that $|\cos a - \cos b| \le |a - b|$

Q. Find a point on the curve $f(x) = \sqrt{x-2}$ in [2, 3] when the tangent is parallel to the chord joining the end points.

Q. If $a < b$, show that a real number 'c' can be found in (a, b) such that $3c^2 = a^2 + ab + b^2$

Q. Use LMVT to prove that $\tan x > x$ for $x \in \left(0, \frac{\pi}{2}\right)$

Q. If $f(x)$ is continuous on $[0, 2]$, differentiable on $(0, 2)$, $f(0) = 2$, $f(2) = 8$, and $f'(x) \leq 3$ for all x in $(0, 2)$, then $f(1)$ has the value equal to

(A) 3 (B) 5 (C) 10

(D) There is not enough information

Q. If f(x) and g(x) are continuous on [a, b] and derivable in (a, b) then show that

$$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a)\begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix} \text{ where } a < c < b$$

Q. Prove that the equation $\frac{x^3+1}{x^2+1} = 5$ has no root in [0, 2]

Q. Number of roots of the function $f(x) = \frac{1}{(x+1)^3} - 3x + \sin x$ is

(A) 0

(B) 1

(C) 2

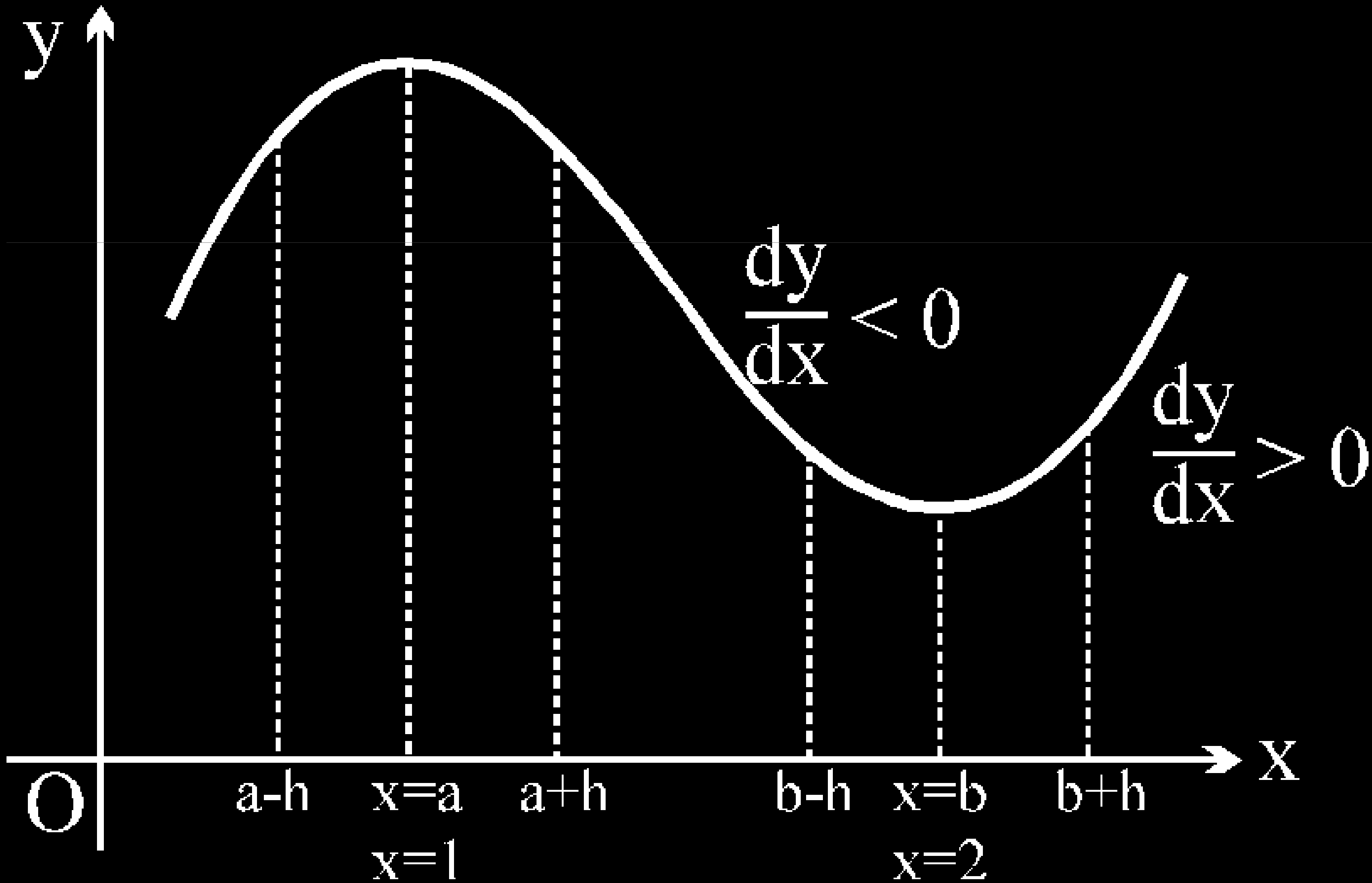
(D) more than 2

MAXIMA AND MINIMA

A function f(x) is said to have a maximum at x = a if f (a) is greater than every other value assumed by f (x) in the immediate neighbourhood of x = a. Symbolically

$$\left.\begin{array}{l} f(a) > f(a+h) \\ f(a) > f(a-h) \end{array}\right] \Rightarrow x = a \text{ gives maxima}$$

for a sufficiently small positive h.


y
$\frac{dy}{dx} < 0$
$\frac{dy}{dx} > 0$
x
O
a-h
x=a
x=1
a+h
b-h
x=b
x=2
b+h

Similarly, a function f (x) is said to have a minimum value at x = b if f (b) is least than every other value assumed by f (x) in the immediate neighbourhood at x = b. Symbolically if

$$\left.\begin{array}{l} f(b) < f(b+h) \\ f(b) < f(b-h) \end{array}\right] \Rightarrow x = b$$

gives minima for a sufficiently small positive h.

(i) the maximum & minimum values of a function are also known as local/relative maxima or local/relative minima as these are the greatest & least values of the function relative to some neighbourhood of the point in question.

(ii) the term 'extremum' or (extremal) or 'turning value' is used both for maximum or a minimum value.

(iii) a maximum (minimum) value of a function may not be the greatest (least) value in a finite interval.

(iv) a function can have several maximum & minimum values & a minimum value may even be greater than a maximum value.

(v) maximum & minimum values of a continuous function occur alternately & between two consecutive maximum values there is a minimum value & vice versa.

Use Of Second Order Derivative In Ascertaining The Maxima Or Minima

$$\text{at local maxima, } \frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0$$

at local minima, $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

Hence if

(a) $f(a)$ is a maximum value of the function f then $f'(a) = 0$ & $f''(a) < 0$.

(b) $f(b)$ is a minimum value of the function f, if $f'(b) = 0$ & $f''(b) > 0$.

However, if $f''(c) = 0$ then the test fails. In this case f can still have a maxima or minima or point of inflection (neither maxima nor minima). In this case revert back to the first order derivative check for ascertaining the maxima or minima.

Q. Prove that :

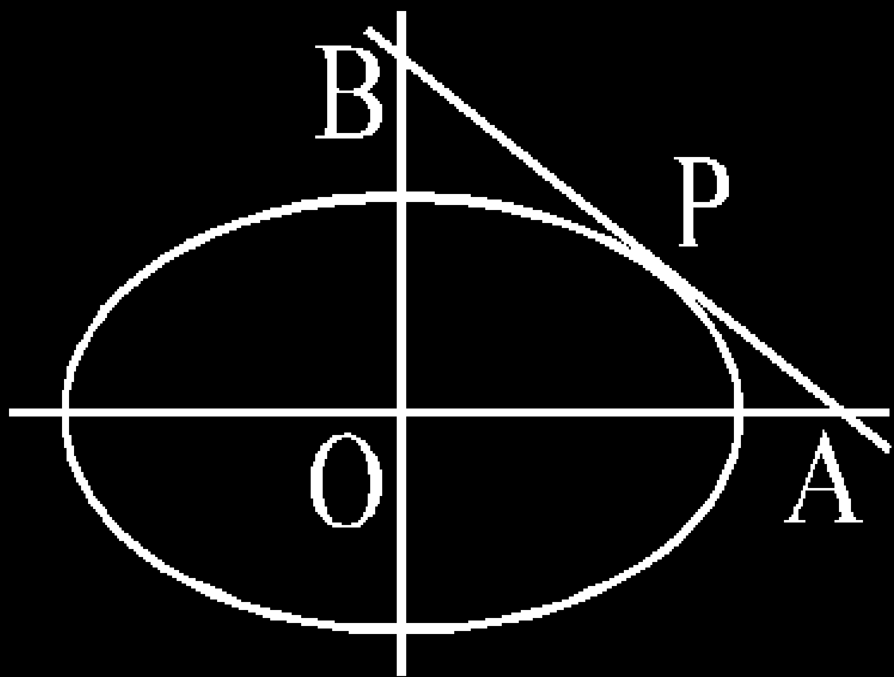
For a given slant height volume of conical tent is maximum if $\theta = \tan^{-1}\sqrt{2}$

θ is semi vertical Angle

Q. A wire of length 20 cm is cut into two pieces. One piece converted into a circle and the other into a square. Where the wire is to be cut from so that the sum total of the areas of two plane figures is **(a)** minimum **(b)** maximum.

Q. A point P is given on the circumference of a circle of radius r. Chord QR is parallel to the tangent at P. Determine the maximum possible area of the triangle PQR.

Q. Find the coordinates of the point P on the curve $\frac{x^2}{8}+\frac{y^2}{18}=1$ in the 1^{st} quadrant so that the area of the triangle formed by the tangent at P and the coordinate axes is minimum.



Q. Find the equation of a line through (1, 8) cutting the positive semi axes at A and B if

(i) the area of ΔOAB is minimum

(ii) its intercept between the coordinate axes is minimum.

(iii) sum of its intercept on the coordinates axes is minimum.

Q. Find the altitude of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height 'h'.

Q. Find the altitude of the right cone of maximum volume that can be inscribed in a sphere of radius R.

Q. A straight line l passes through the points (3, 0) and (0, 4). The point A lies on the parabola $y = 2x - x^2$. Find the distance p from point A to the straight line and indicate the coordinates of the point A (x_0, y_0) on the parabola for which the distance from the parabola to the straight line is the least.

Useful Formulae Of Mensuration To Remember

1. Volume of a cuboid = lbh.
2. Surface area of a cuboid = $2(lb + bh + hl)$.
3. Volume of a prism = area of the base x height.
4. Lateral surface of a prism = perimeter of the base x height.
5. Total surface of a prism = lateral surface + 2 area of the base
   (Note that lateral surfaces of a prism are all rectangles).

6. Volume of a pyramid $= \frac{1}{3}$ (area of the base) × (height).

7. Curved surface of a pyramid $= \frac{1}{2}$(perimeter of the base) x slant height.

   (Note that slant surfaces of a pyramid are triangles).

8. Volume of a cone $= \pi r^2 h$.

9. Curved surface of a cylinder $= 2 \pi rh$.

10. Total surface of a cylinder $= 2 \pi rh + 2 \pi r^2$.

11. Volume of a sphere $= \pi r^3$.

12. Surface area of a sphere $= 4 \pi r^2$.

13. Area of a circular sector $= \frac{1}{2} r^2 q$, when q is in radians.

Significance Of The Sign Of 2nd Order Derivative And Points Of Inflection

The sign of the 2^{nd} order derivative determines the concavity of the curve. Such points such as C & E on the graph where the concavity of the curve changes are called the points of inflection.

(i) $\frac{d^2y}{dx^2} > 0 \Rightarrow$ **concave upwards**

(ii) $\frac{d^2y}{dx^2} < 0 \Rightarrow$ **concave downwards**

At the point of inflection we find that

$$\frac{d^2y}{dx^2} = \mathbf{0} \ \& \ \frac{d^2y}{dx^2} \ \textbf{changes sign}$$

Inflection points can also occur if $\frac{d^2y}{dx^2}$ fails to exist. For example, consider the graph of the function defined as,

$$f(x) = \begin{cases} x^{3/5} & \text{for } x \in (-\infty, 1) \\ 2 - x^2 & \text{for } x \in (1, \infty) \end{cases}$$

Q. Prove that $\sin x + 2x \geq \dfrac{3x\cdot(x+1)}{\pi} \quad \forall\, x \in \left[0, \dfrac{\pi}{2}\right]$

Different Graphs of The Cubic

$y = ax^3 + bx^2 + cx + d$

Q. If the cubic $y = x^3 + px + q$ has 3 distinct real roots then prove that $4p^3 + 27q^2 < 0$.

Q. For a cubic

$$f(x) = \frac{ax^3}{3} + (a+2)x^2 + (a-1)x + 2. \ (a > 0)$$

Find the value of 'a' for which it has

(1) + ve point of maximum

(2) – ve point of minimum

(3) + ve point of minimum

(4) – ve point of maximum

(5) – ve point of inflection

(6) + ve point of inflection

Q. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2

and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$

Then the value of p(2) is

Q. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set

$$A = \{x \mid x^2 + 20 \le 9x\}$$ is

Q.1 Let $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$

Then at $x = 0$, ' f ' has :

(A) a local maximum (B) no local maximum

(C) a local minimum (D) no extremum.

[JEE 2000 Screening, 1 out of 35]

Q.2 Find the area of the right angled triangle of least area that can be drawn so as to circumscribe a rectangle of sides 'a' and 'b', the right angle of the triangle coinciding with one of the angles of the rectangle.

[REE 2001 Mains, 5 out of 100]

Q.3 (a) Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(A) $[0, 1]$

(B) $\left(0, \frac{1}{2}\right]$

(C) $\left[\frac{1}{2}, 1\right]$

(D) $(0, 1]$

Q.3 (b) The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)$. .... $(\cos\alpha_n)$, under the restrictions

$$O \le \alpha_1, \alpha_2, \ldots\ldots\ldots\ldots, \alpha_n \le \frac{\pi}{2}$$

and $\cot\alpha_1 \cdot \cot\alpha_2 \ldots\ldots\ldots \cot\alpha_n = 1$ is

(A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) 1

[JEE 2001 Screening, 1 + 1 out of 35]

Q.4 If a_1, a_2 ,....... , a_n are positive real numbers whose product is a fixed number e, the minimum value of $a_1+a_2+a_3+.....+a_{n-1}+2a_n$ is

(A) $n(2e)^{1/n}$ (B) $(n+1)e^{1/n}$

(C) $2ne^{1/n}$ (D) $(n+1)(2e)^{1/n}$

[JEE 2002 Screening]

Q.5 (a) Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$, is minimum.

Q.5 (b) For a circle $x^2 + y^2 = r^2$, find the value of 'r' for which the area enclosed by the tangents drawn from the point P(6, 8) to the circle and the chord of contact is maximum.

[JEE 2003, Mains, 2+2 out of 60]

Q.6 Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$.

Then f

(A) is bounded

(B) has a local maxima

(C) has a local minima

(D) is strictly increasing [JEE 2004 (Scr.)]

Q.7 If $P(x)$ be a polynomial of degree 3 satisfying $P(-1) = 10$, $P(1) = -6$ and $P(x)$ has maximum at $x = -1$ and $P'(x)$ has minima at $x = 1$. Find the distance between the local maximum and local minimum of the curve.

[JEE 2005 (Mains), 4 out of 60]

Q.8 (a) If f (x) is cubic polynomial which has local maximum at x = – 1. If f(2) = 18, f(1) = – 1 and f '(x) has local maxima at x = 0, then

(A) the distance between (–1, 2) and (a, f (a)), where x = a is the point of local minima $2\sqrt{5}$.

(B) f (x) is increasing for x $\in (1, 2\sqrt{5}]$

(C) f (x) has local minima at x = 1

(D) the value of f(0) = 5

Q.8 (b)

$$f(x) = \begin{cases} e^x & 0 \le x \le 1 \\ 2 - e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$$

and $g(x) = \int_0^x f(t)\,dt$, $x \in [1, 3]$

then g(x) has

(A) local maxima at $x = 1 + \ln 2$ and local minima at $x = e$

(B) local maxima at $x = 1$ and local minima at $x = 2$

(C) no local maxima

(D) no local minima [JEE 2006, 5 marks each]

Q.8 (c) If f (x) is twice differentiable function such that f(a)=0, f(b)=2, f(c)= –1, f(d)=2, f(e)=0, where a < b < c < d < e, then find the minimum number of zeros of

$g(x) = \left(f'(x)\right)^2 + f(x).f''(x)$ in the interval [a, e].

[JEE 2006, 6]

Q.9 (a) The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases} \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

Q.9 (b) **Comprehension:**

Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \quad 0 < a < 2$$

(i) Which of the following is true?

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$

(B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$

(C) $f'(1) f'(-1) = (2 - a)^2$

(D) $f'(1) f'(-1) = -(2 + a)^2$

Q.9 (ii) Which of the following is true?

(A) $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$

(B) $f(x)$ is increasing on $(-1, 1)$ and has a local maximum at $x = 1$

(C) $f(x)$ is increasing on $(-1, 1)$ but has neither a local maximum and nor a local minimum at $x = 1$.

(D) $f(x)$ is decreasing on $(-1, 1)$ but has neither a local maximum and nor a local minimum at $x = 1$.