# BINOMIAL

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# *KEY CONCEPTS*

## BINOMIAL EXPONENTIAL & LOGARITHMIC SERIES

**1. BINOMIAL THEOREM :**

The formula by which any positive integral power of a binomial expression can be expanded in the form of a series is known as **Binomial Theorem** .

If $x, y \in R$ and $n \in N$, then ;

$$(x + y)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} y + {}^nC_2\, x^{n-2}y^2 + ..... + {}^nC_r\, x^{n-r}y^r + ..... + {}^nC_n y^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r}\, y^r.$$

This theorem can be proved by Induction .

**OBSERVATIONS :**

**(i)** The number of terms in the expansion is $(n + 1)$ i.e. one or more than the index .

**(ii)** The sum of the indices of $x$ & $y$ in each term is $n$ .

**(iii)** The binomial coefficients of the terms ${}^nC_0, {}^nC_1$ .... **equidistant** from the beginning and the end are equal.

**2. IMPORTANT TERMS IN THE BINOMIAL EXPANSION ARE :**

**(i)** General term  **(ii)** Middle term

**(iii)** Term independent of x &  **(iv)** Numerically greatest term

**(i)** The general term or the $(r + 1)^{th}$ term in the expansion of $(x + y)^n$ is given by;
$T_{r+1} = {}^nC_r\, x^{n-r} \cdot y^r$

**(ii)** The middle term(s) is the expansion of $(x + y)^n$ is (are) :

**(a)** If n is even, there is only one middle term which is given by ;

$$T_{(n+2)/2} = {}^nC_{n/2} \cdot x^{n/2} \cdot y^{n/2}$$

**(b)** If n is odd, there are two middle terms which are :

$$T_{(n+1)/2} \quad \& \quad T_{[(n+1)/2]+1}$$

**(iii)** Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero.

**(iv)** To find the Numerically greatest term is the expansion of $(1 + x)^n$, $n \in N$ find $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{{}^nC_{r-1} x^{r-1}} = \frac{n-r+1}{r} x$ . Put the absolute value of x & find the value of r Consistent with the inequality $\frac{T_{r+1}}{T_r} > 1$.

Note that the Numerically greatest term in the expansion of $(1 - x)^n$, $x > 0$, $n \in N$ is the same as the greatest term in $(1 + x)^n$ .

**3.** If $\left(\sqrt{A} + B\right)^n = I + f$, where I & n are positive integers, n being odd and $0 < f < 1$, then $(I + f) \cdot f = K^n$ where $A - B^2 = K > 0$ & $\sqrt{A} - B < 1$.
If n is an even integer, then $(I + f)(1 - f) = K^n$.

**4. BINOMIAL COEFFICIENTS :**

**(i)** $C_0 + C_1 + C_2 + ....... + C_n = 2^n$

**(ii)** $C_0 + C_2 + C_4 + ....... = C_1 + C_3 + C_5 + ....... = 2^{n-1}$

**(iii)** $C_0^2 + C_1^2 + C_2^2 + .... + C_n^2 = {}^{2n}C_n = \frac{(2n)!}{n!\, n!}$

**(iv)** $C_0.C_r + C_1.C_{r+1} + C_2.C_{r+2} + ... + C_{n-r}.C_n = \frac{(2n)!}{(n+r)(n-r)!}$

**REMEMBER :**

**(i)** $(2n)! = 2^n . n! [1. 3. 5 ...... (2n - 1)]$

## 5. BINOMIAL THEOREM FOR NEGATIVE OR FRACTIONAL INDICES :

If $n \in Q$, then $(1+x)^n = 1 + n x + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + ......\infty$ Provided $|x| < 1$.

**Note :**

**(i)** When the index n is a positive integer the number of terms in the expansion of $(1+x)^n$ is finite i.e. $(n+1)$ & the coefficient of successive terms are :
$^nC_0, {}^nC_1, {}^nC_2, {}^nC_3 ..... {}^nC_n$

**(ii)** When the index is other than a positive integer such as negative integer or fraction, the number of terms in the expansion of $(1+x)^n$ is infinite and the symbol $^nC_r$ cannot be used to denote the Coefficient of the general term.

**(iii)** Following expansion should be remembered ($|x| < 1$).
**(a)** $(1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - .... \infty$ **(b)** $(1 - x)^{-1} = 1 + x + x^2 + x^3 + x^4 + .... \infty$
**(c)** $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + .... \infty$ **(d)** $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + ..... \infty$

**(iv)** The expansions in ascending powers of x are only valid if x is 'small'. If x is large i.e. $|x| > 1$ then we may find it convinient to expand in powers of $\frac{1}{x}$, which then will be small.

## 6. APPROXIMATIONS :

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1.2} x^2 + \frac{n(n-1)(n-2)}{1.2.3} x^3 .....$

If $x < 1$, the terms of the above expansion go on decreasing and if x be very small, a stage may be reached when we may neglect the terms containing higher powers of x in the expansion. Thus, if x be so small that its squares and higher powers may be neglected then $(1+x)^n = 1 + nx$, approximately. This is an approximate value of $(1+x)^n$.

## 7. EXPONENTIAL SERIES :

**(i)** $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + .......\infty$ ; where x may be any real or complex & $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

**(ii)** $a^x = 1 + \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a + \frac{x^3}{3!} \ln^3 a + .......\infty$ where $a > 0$

**Note :**

**(a)** $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + .......\infty$

**(b)** e is an irrational number lying between 2.7 & 2.8. Its value correct upto 10 places of decimal is 2.7182818284.

**(c)** $e + e^{-1} = 2\left(1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + .......\infty\right)$

**(d)** $e - e^{-1} = 2\left(1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + .......\infty\right)$

**(e)** Logarithms to the base 'e' are known as the Napierian system, so named after Napier, their inventor. They are also called **Natural Logarithm**.

## 8. LOGARITHMIC SERIES :

**(i)** $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots.\infty$ where $-1 < x \le 1$

**(ii)** $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots.\infty$ where $-1 \le x < 1$

**(iii)** $\ln\frac{(1+x)}{(1-x)} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\infty\right)$ $|x| < 1$

**REMEMBER :** **(a)** $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ \infty = \ln 2$ **(b)** $e^{\ln x} = x$

**(c)** $\ln 2 = 0.693$ **(d)** $\ln 10 = 2.303$

## ***EXERCISE–I***

Q.1 Find the coefficients : (i) $x^7$ in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ (ii) $x^{-7}$ in $\left(ax - \frac{1}{bx^2}\right)^{11}$

(iii) Find the relation between a & b, so that these coefficients are equal.

Q.2 If the coefficients of the $r^{th}$, $(r+1)^{th}$ & $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in AP, find r.

Q.3 Find the term independent of x in the expansion of (a) $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{2x^2}\right]^{10}$ (b) $\left[\frac{1}{2}x^{1/3} + x^{-1/5}\right]^8$

Q.4 If the coefficients of $2^{nd}$, $3^{rd}$ & $4^{th}$ terms in the expansion of $(1+x)^{2n}$ are in AP, show that $2n^2 - 9n + 7 = 0$.

Q.5 Given that $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, find the values of :
(i) $a_0 + a_1 + a_2 + \ldots + a_{2n}$ ; (ii) $a_0 - a_1 + a_2 - a_3 \ldots + a_{2n}$ ; (iii) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2$

Q.6 If a, b, c & d are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, $n \in N$, prove that $\frac{a}{a+b} + \frac{c}{c+d} = \frac{2b}{b+c}$.

Q.7 Find the value of x for which the fourth term in the expansion, $\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}} + \frac{1}{5^{\log_5\sqrt[3]{2^{x-1}+7}}}\right)^8$ is 336.

Q.8 Prove that : ${}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots + {}^{r}C_r = {}^{n}C_{r+1}$.

Q.9 (a) Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

(b) Show that ${}^{2n-2}C_{n-2} + 2.{}^{2n-2}C_{n-1} + {}^{2n-2}C_n > \frac{4n}{n+1}$ , $n \in N$ , $n > 2$

Q.10 In the expansion of $\left(1 + x + \frac{7}{x}\right)^{11}$ find the term not containing x.

Q.11 Show that coefficient of $x^5$ in the expansion of $(1+x^2)^5 . (1+x)^4$ is 60.

Q.12 Find the coefficient of $x^4$ in the expansion of :
(i) $(1 + x + x^2 + x^3)^{11}$ (ii) $(2 - x + 3x^2)^6$

Q.13 Find numerically the greatest term in the expansion of :

(i) $(2+3x)^9$ when $x = \frac{3}{2}$ (ii) $(3-5x)^{15}$ when $x = \frac{1}{5}$

Q.14 Given $s_n = 1 + q + q^2 + ..... + q^n$ & $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + .... + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$, prove that ${}^{n+1}C_1 + {}^{n+1}C_2 . s_1 + {}^{n+1}C_3 . s_2 + .... + {}^{n+1}C_{n+1} . s_n = 2^n . S_n$.

Q.15 Prove that the ratio of the coefficient of $x^{10}$ in $(1-x^2)^{10}$ & the term independent of x in $\left(x - \frac{2}{x}\right)^{10}$ is 1 : 32 .

Q.16 Let $(1+x^2)^2 . (1+x)^n = \sum_{K=0}^{n+4} a_K . x^K$. If $a_1, a_2$ & $a_3$ are in AP, find n.

Q.17 If ${}^nJ_r = \frac{(1-x^n)(1-x^{n-1})(1-x^{n-2}).................(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3).................(1-x^r)}$, prove that ${}^nJ_{n-r} = {}^nJ_r$.

Q.18 Prove that $\sum_{K=0}^{n} {}^nC_K \sin Kx . \cos(n-K)x = 2^{n-1} \sin nx$ .

Q.19 The expressions $1+x$, $1+x+x^2$, $1+x+x^2+x^3$,............ $1+x+x^2+.......... + x^n$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 + ................$, then,

(a) how many terms are there in the product.

(b) show that the coefficients of the terms in the product, equidistant from the beginning and end are equal.

(c) show that the sum of the odd coefficients = the sum of the even coefficients = $\frac{(n+1)!}{2}$

Q.20 Find the coeff. of

(a) $x^6$ in the expansion of $(ax^2 + bx + c)^9$ .

(b) $x^2 y^3 z^4$ in the expansion of $(ax - by + cz)^9$ .

(c) $a^2 b^3 c^4 d$ in the expansion of $(a - b - c + d)^{10}$.

Q.21 If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ & $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

Q.22 Find the coefficient of $x^r$ in the expression of :

$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + ..... + (x+2)^{n-1}$

Q.23(a) Find the index n of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient $(n \in N)$.

(b) For which positive values of x is the fourth term in the expansion of $(5+3x)^{10}$ is the greatest.

Q.24 Prove that $\frac{(72)!}{(36!)^2} - 1$ is divisible by 73.

Q.25 Find the number of divisors of the number

$$N = {}^{2000}C_1 + 2 \cdot {}^{2000}C_2 + 3 \cdot {}^{2000}C_3 + ...... + 2000 \cdot {}^{2000}C_{2000}.$$

Q.26 Show that the integral part in each of the following is odd. $n \in N$

(A) $\left(5 + 2\sqrt{6}\right)^n$ (B) $\left(8 + 3\sqrt{7}\right)^n$ (C) $\left(6 + \sqrt{35}\right)^n$

Q.27 Show that the integral part in each of the following is even. $n \in N$

(A) $\left(3\sqrt{3}+5\right)^{2n+1}$ (B) $\left(5\sqrt{5}+11\right)^{2n+1}$

Q.28 If $\left(7+4\sqrt{3}\right)^n = p+\beta$ where n & p are positive integers and $\beta$ is a proper fraction show that $(1-\beta)(p+\beta) = 1$.

Q.29 Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$.

Q.30 Let I denotes the integral part & F the proper fractional part of $\left(3+\sqrt{5}\right)^n$ where $n \in N$ and if $\rho$ denotes the rational part and $\sigma$ the irrational part of the same, show that

$$\rho = \frac{1}{2}(I+1) \text{ and } \sigma = \frac{1}{2}(I+2F-1).$$

## ***EXERCISE–II***

### (On combinatorial coefficients)

If $C_0, C_1, C_2, \ldots, C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n$, $n \in N$, then prove the following :

Q.1 $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{(2n)!}{n!\,n!}$ **(This result is to be remembered)**

Q.2 $C_0 C_1 + C_1 C_2 + C_2 C_3 + \ldots + C_{n-1} C_n = \frac{(2n)!}{(n+1)!\,(n-1)!}$

Q.3 $C_1 + 2C_2 + 3C_3 + \ldots + n \cdot C_n = n \cdot 2^{n-1}$

Q.4 $C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = (n+2)2^{n-1}$

Q.5 $C_0 + 3C_1 + 5C_2 + \ldots + (2n+1)C_n = (n+1)\,2^n$

Q.6 $(C_0+C_1)(C_1+C_2)(C_2+C_3) \ldots (C_{n-1}+C_n) = \frac{C_0 \cdot C_1 \cdot C_2 \ldots C_{n-1}(n+1)^n}{n!}$

Q.7 If $P_n$ denotes the product of all the coefficients in the expansion of $(1+x)^n$, $n \in N$, show that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$.

Q.8 $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots + \frac{n \cdot C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

Q.9 $2 \cdot C_o + \frac{2^2 \cdot C_1}{2} + \frac{2^3 \cdot C_2}{3} + \frac{2^4 \cdot C_3}{4} + \ldots \frac{2^{n+1} \cdot C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

Q.10 $C_o C_r + C_1 C_{r+1} + C_2 C_{r+2} + \ldots + C_{n-r} C_n = \frac{2n!}{(n-r)!\,(n+r)!}$

Q.11 $C_o - \frac{C_1}{2} + \frac{C_2}{3} - \ldots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

Q.12 $1 \cdot C_o^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \ldots + (2n+1)\,C_n^2 = \frac{(n+1)\,(2n)!}{n!\,n!}$

Q.13 If $a_0, a_1, a_2, \ldots$ be the coefficients in the expansion of $(1+x+x^2)^n$ in ascending powers of x, then prove that : (i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - \ldots = 0$ .

(ii) $a_0 a_2 - a_1 a_3 + a_2 a_4 - \ldots + a_{2n-2}\, a_{2n} = a_{n+1}$ or $a_{n-1}$.

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$ ; where $E_1 = a_0 + a_3 + a_6 + \ldots$ ; $E_2 = a_1 + a_4 + a_7 + \ldots$ & $E_3 = a_2 + a_5 + a_8 + \ldots$

Q.14 If $(1+x)^n = C_0 + C_1x + C_2x^2 + .... + C_n x^n$, then show that the sum of the products of the $C_i$'s taken two at a time, represented by $\underset{0 \le i < j \le n}{\Sigma\Sigma} C_i C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$ .

Q.15 If $(1 + x + x^2 + ... + x^p)^n = a_0 + a_1x + a_2x^2 + ... + a_{np} . x^{np}$ , then find the value of : $a_1 + 2a_2 + 3a_3 + .... + np . a_{np}$

Q.16 $1^2. C_0 + 2^2. C_1 + 3^2. C_2 + 4^2. C_3 + .... + (n+1)^2 C_n = 2^{n-2} (n+1) (n+4)$ .

Q.17 $C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \frac{C_3}{4}x^3 + ....... + \frac{C_n}{n+1} . x^n = \frac{(1+x)^{n+1} - 1}{(n+1)x}$

Q.18 $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + ........ = \frac{2^n}{n+1}$

Q.19 $\frac{C_0}{2} + \frac{C_1}{3} + \frac{C_2}{4} + \frac{C_3}{5} + ........ + \frac{C_n}{n+2} = \frac{1+n.2^{n+1}}{(n+1)(n+2)}$

Q.20 Prove that , $({}^{2n}C_1)^2 + 2 . ({}^{2n}C_2)^2 + 3 . ({}^{2n}C_3)^2 + ... + 2n . ({}^{2n}C_{2n})^2 = \frac{(4n-1)!}{[(2n-1)!]^2}$

## ***EXERCISE–III***

Q.1 If in the expansion of $(1 + x)^m (1 - x)^n$, the co-efficients of x and $x^2$ are 3 and – 6 respectively, then m is : [JEE '99, 2 (Out of 200)]

(A) 6 (B) 9 (C) 12 (D) 24

Q.2 For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

Q.3 For any positive integers m , n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$ . Prove that

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + ........ + \binom{m}{m} = \binom{n+1}{m+1}$$

Hence or otherwise prove that ,

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + ........ + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2} .$$

[ JEE '2000 (Mains), 6 ]

Q.4 Find the largest co-efficient in the expansion of $(1 + x)^n$, given that the sum of co-efficients of the terms in its expansion is 4096 . [REE '2000 (Mains)]

Q.5 In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5th and 6th terms is zero. Then $\frac{a}{b}$ equals

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

[ JEE '2001 (Screening), 3]

Q.6 Find the coeffcient of $x^{49}$ in the polynomial [ REE '2001 (Mains) , 3 ]

$$\left(x-\frac{C_1}{C_0}\right)\left(x-2^2\cdot\frac{C_2}{C_1}\right)\left(x-3^2\cdot\frac{C_3}{C_2}\right)\ldots\ldots\ldots\left(x-50^2\cdot\frac{C_{50}}{C_{49}}\right)$$ where $C_r = {}^{50}C_r$.

Q.7 The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q}=0$ if $P<q$) is maximum when m is

(A) 5 (B) 10 (C) 15 (D) 20

[ JEE '2002 (Screening) , 3 ]

Q.8(a) Coefficient of $t^{24}$ in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is

(A) ${}^{12}C_6 + 2$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) none

[JEE 2003, Screening 3 out of 60]

(b) Prove that : $2^K.\binom{n}{0}\binom{n}{K} - 2^{K-1}\binom{n}{1}\binom{n-1}{K-1} + 2^{K-2}\binom{n}{2}\binom{n-2}{K-2}\ldots\ldots(-1)^K\binom{n}{K}\binom{n-K}{0} = \binom{n}{K}$.

[JEE 2003, Mains-2 out of 60]

Q.9 ${}^{n-1}C_r = (K^2-3).{}^nC_{r+1}$, if $K \in$

(A) $[-\sqrt{3}, \sqrt{3}]$ (B) $(-\infty, -2)$ (C) $(2, \infty)$ (D) $(\sqrt{3}, 2]$

[JEE 2004 (Screening)]

Q.10 The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12}\ldots\ldots + \binom{30}{20}\binom{30}{30}$ is, where $\binom{n}{r} = {}^nC_r$.

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$ (C) $\binom{60}{30}$ (D) $\binom{31}{10}$

[JEE 2005 (Screening)]

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# ANSWER KEY

## *EXERCISE–I*

**Q.1** **(i)** ${}^{11}C_5\frac{a^6}{b^5}$ **(ii)** ${}^{11}C_6\frac{a^5}{b^6}$ **(iii)** ab = 1 **Q.2** r = 5 or 9 **Q.3** (a) $\frac{5}{12}$ (b) $T_6$ **=7**

**Q.5** **(i)** $3^n$ **(ii)** 1, **(iii)** $a_n$ **Q.7** x = 0 or 1

**Q.9** **(a)** $101^{50}$ (Prove that $101^{50} - 99^{50} = 100^{50}$ + some +ive qty) **Q.10** $1 + \sum_{k=1}^{5} {}^{11}C_{2k} . {}^{2k}C_k\ 7^k$

**Q.12** **(i)** 990 **(ii)** 3660 **Q.13** **(i)** $T_7 = \frac{7.3^{13}}{2}$ **(ii)** 455 x $3^{12}$ **Q.16** n = 2 or 3 or 4

**Q.19 (a)** $\frac{n^2+n+2}{2}$ **Q.20** **(a)** $84b^6c^3 + 630ab^4c^4 + 756a^2b^2c^5 + 84a^3c^6$**; (b)** $-1260 . a^2b^3c^4$ **; (c)** $-12600$

**Q.22** ${}^nC_r(3^{n-r} - 2^{n-r})$ **Q.23** (a) n = 12 (b) $\frac{5}{8} < x < \frac{20}{21}$ **Q.25** 8016 **Q.29** 500

## *EXERCISE–II*

**Q15** Differentiate the given expn. & put x = 1 to get the result $\frac{np}{2}(p+1)^n$

**Q.18** Consider $\frac{1}{2}[(1+x)^n + (1-x)^n] = C_0 + C_2x^2 + C_4x^4 + \ldots$.. Integrate between 0 & 1.

**Q.19** Multiply both sides by x the expn. $(1+x)^n$. Integrate both sides between 0 & 1.

## *EXERCISE–III*

**Q.1** C **Q.2** D **Q.4** ${}^{12}C_6$ **Q.5** B

**Q.6** – 22100 **Q.7** C **Q.8** **(a)** A **Q.9** D **Q.10** A