CIRCLE

MANOJ CHAUHAN SIR(IIT-DELHI)
EX. SR. FACULTY (BANSAL CLASSES)

KEY CONCEPTS (CIRCLE)

STANDARD RESULTS :

1. EQUATION OF A CIRCLE IN VARIOUS FORM:

(a) The circle with centre (h, k) & radius 'r' has the equation;
$(x - h)^2 + (y - k)^2 = r^2$.

(b) The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre as:
$(-g, -f)$ & radius $= \sqrt{g^2 + f^2 - c}$.

Remember that every second degree equation in x & y in which coefficient of x^2 = coefficient of y^2 & there is no xy term always represents a circle.

If $g^2 + f^2 - c > 0 \Rightarrow$ real circle.
$g^2 + f^2 - c = 0 \Rightarrow$ point circle.
$g^2 + f^2 - c < 0 \Rightarrow$ imaginary circle.

Note that the general equation of a circle contains three arbitrary constants, g, f & c which corresponds to the fact that a unique circle passes through three non collinear points.

(c) The equation of circle with (x_1, y_1) & (x_2, y_2) as its diameter is :
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

Note that this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

2. INTERCEPTS MADE BY A CIRCLE ON THE AXES :

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co-ordinate axes are $2\sqrt{g^2 - c}$ & $2\sqrt{f^2 - c}$ respectively.

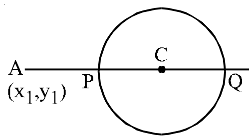
NOTE :

If $g^2 - c > 0 \Rightarrow$ circle cuts the x axis at two distinct points.
If $g^2 = c \Rightarrow$ circle touches the x-axis.
If $g^2 < c \Rightarrow$ circle lies completely above or below the x-axis.

3. POSITION OF A POINT w.r.t. A CIRCLE :

The point (x_1, y_1) is inside, on or outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.
according as $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \Leftrightarrow 0$.

Note : The greatest & the least distance of a point A from a circle with centre C & radius r is AC + r & AC – r respectively.



4. LINE & A CIRCLE :

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then :

(i) $p > r \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.
(ii) $p = r \Leftrightarrow$ the line touches the circle.
(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.
(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

5. PARAMETRIC EQUATIONS OF A CIRCLE :

The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are :
$x = h + r\cos\theta$; $y = k + r\sin\theta$; $-\pi < \theta \le \pi$ where (h, k) is the centre,
r is the radius & θ is a parameter.

Note that equation of a straight line joining two point α & β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}.$$

6. TANGENT & NORMAL :

(a) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is, $x x_1 + y y_1 = a^2$. Hence equation of a tangent at $(a\cos\alpha, a\sin\alpha)$ is ; $x\cos\alpha + y\sin\alpha = a$. The point of intersection of the tangents at the points $P(\alpha)$ and $Q(\beta)$ is $\dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$.

(b) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

(c) $y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$ and the point of contact is $\left(-\dfrac{a^2m}{c}, \dfrac{a^2}{c}\right)$.

(d) If a line is normal/orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1).$$

7. A FAMILY OF CIRCLES :

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is : $S_1 + K S_2 = 0$ $(K \neq -1)$.

(b) The equation of the family of circles passing through the point of intersection of a circle $S = 0$ & a line $L = 0$ is given by $S + KL = 0$.

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form :

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ where K is a parameter.}$$

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.
In case the line through (x_1, y_1) is parallel to y - axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(x - x_1) = 0$.
Also if line is parallel to x - axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1) = 0$.

(e) Equation of circle circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ & $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co-efficient of $xy = 0$ & co-efficient of x^2 = co-efficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ is $L_1L_3 + \lambda L_2L_4 = 0$ provided co-efficient of x^2 = co-efficient of y^2 and co-efficient of $xy = 0$.

8. LENGTH OF A TANGENT AND POWER OF A POINT :

The length of a tangent from an external point (x_1, y_1) to the circle
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.
Square of length of the tangent from the point P is also called **THE POWER OF POINT** w.r.t. a circle. Power of a point remains constant w.r.t. a circle.
Note that : power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

9. DIRECTOR CIRCLE:

The locus of the point of intersection of two perpendicular tangents is called the **DIRECTOR CIRCLE** of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

10. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT :

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\frac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplication can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note that : the shortest chord of a circle passing through a point 'M' inside the circle, is one chord whose middle point is M.

11. CHORD OF CONTACT :

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is :
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$

REMEMBER :

(a) Chord of contact exists only if the point 'P' is not inside.

(b) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2+L^2}}$.

(c) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2+L^2}$

Where R is the radius of the circle & L is the length of the tangent from (x_1, y_1) on $S = 0$.

(d) Angle between the pair of tangents from $(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$

where R = radius ; L = length of tangent.

(e) Equation of the circle circumscribing the triangle PT_1T_2 is :
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0.$

(f) The joint equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c.$

12. POLE & POLAR :

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q & R is called the **POLAR OF THE POINT P** ; also P is called the **POLE OF THE POLAR**.

(ii) The equation to the polar of a point $P(x_1, y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $\mathbf{xx_1 + yy_1 = a^2}$, & if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$. Note that if the point (x_1, y_1) be on the circle then the chord of contact, tangent & polar will be represented by the same equation.

(iii) Pole of a given line $Ax + By + C = 0$ w.r.t. any circle $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$.

(iv) If the polar of a point P pass through a point Q, then the polar of Q passes through P.

(v) Two lines L_1 & L_2 are conjugate of each other if Pole of L_1 lies on L_2 & vice versa Similarly two points P & Q are said to be conjugate of each other if the polar of P passes through Q & vice-versa.

13. COMMON TANGENTS TO TWO CIRCLES :

(i) Where the two circles neither intersect nor touch each other, there are FOUR common tangents, two of them are transverse & the others are direct common tangents.

(ii) When they intersect there are two common tangents, both of them being direct.

(iii) When they touch each other :

(a) **EXTERNALLY :** there are three common tangents, two direct and one is the tangent at the point of contact.

(b) **INTERNALLY :** only one common tangent possible at their point of contact.

(iv) Length of an external common tangent & internal common tangent to the two circles is given by:

$$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} \quad \& \quad L_{int} = \sqrt{d^2 - (r_1 + r_2)^2} .$$

Where d = distance between the centres of the two circles. r_1 & r_2 are the radii of the two circles.

(v) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.

Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

14. RADICAL AXIS & RADICAL CENTRE :

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given ;

$S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

NOTE THAT :

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

(e) Radical axis bisects a common tangent between the two circles.

(f) The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles.

(g) A system of circles, every two which have the same radical axis, is called a coaxal system.

(h) Pairs of circles which do not have radical axis are concentric.

15. ORTHOGONALITY OF TWO CIRCLES :

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is : $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

Note :

(a) Locus of the centre of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.

(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.

EXERCISE–I

Q.1 Determine the nature of the quadrilateral formed by four lines $3x + 4y - 5 = 0$; $4x - 3y - 5 = 0$; $3x + 4y + 5 = 0$ and $4x - 3y + 5 = 0$. Find the equation of the circle inscribed and circumscribing this quadrilateral.

Q.2 A circle S = 0 is drawn with its centre at (–1, 1) so as to touch the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ externally. Find the intercept made by the circle S = 0 on the coordinate axes.

Q.3 The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = l^2 + m^2$.

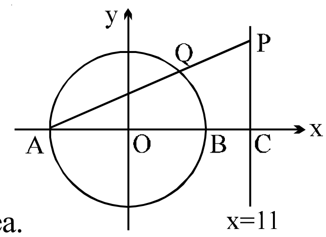
Q.4 One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A & B are the points (–3, 4) & (5,4) respectively, then find the area of the rectangle.

Q.5 Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 & L_2 are equal, then find the equation(s) which represent L_1.

Q.6 A circle passes through the points (–1, 1), (0, 6) and (5, 5). Find the points on the circle the tangents at which are parallel to the straight line joining origin to the centre.

Q.7 Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.

Q.8 In the given figure, the circle $x^2 + y^2 = 25$ intersects the x-axis at the point A and B. The line x = 11 intersects the x-axis at the point C. Point P moves along the line x = 11 above the x-axis and AP intersects the circle at Q. Find



(i) The coordinates of the point P if the triangle AQB has the maximum area.

(ii) The coordinates of the point P if Q is the middle point of AP.

(iii) The coordinates of P if the area of the triangle AQB is $(1/4)^{th}$ of the area of the triangle APC.

Q.9 A circle is drawn with its centre on the line $x + y = 2$ to touch the line $4x - 3y + 4 = 0$ and pass through the point (0, 1). Find its equation.

Q.10 A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and moved along a tangent to the circle passing through the point D (3, – 3). Find the following.

(i) Equation of the tangents at A and B.

(ii) Coordinates of the points A and B.

(iii) Angle ADB and the maximum and minimum distances of the point D from the circle.

(iv) Area of quadrilateral ADBC and the ΔDAB.

(v) Equation of the circle circumscribing the ΔDAB and also the intercepts made by this circle on the coordinate axes.

Q.11 Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

Q.12 Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal length on it.

Q.13 Find the locus of the middle points of portions of the tangents to the circle $x^2 + y^2 = a^2$ terminated by the coordinate axes.

Q.14 Tangents are drawn to the concentric circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ at right angle to one another. Show that the locus of their point of intersection is a 3^{rd} concentric circle. Find its radius.

Q.15 Find the equation to the circle which is such that the length of the tangents to it from the points (1, 0), (2, 0) and (3, 2) are 1, $\sqrt{7}$, $\sqrt{2}$ respectively.

Q.16 Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and centres (–3, 0), (–1, 0), (1, 0) and (3, 0) respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x.

Q.17 If the variable line $3x - 4y + k = 0$ lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circle, then the range of k is (a, b) where a, b $\in$ I. Find the value of (b – a).

Q.18 Obtain the equations of the straight lines passing through the point A(2, 0) & making 45° angle with the tangent at A to the circle $(x + 2)^2 + (y - 3)^2 = 25$. Find the equations of the circles each of radius 3 whose centres are on these straight lines at a distance of $5\sqrt{2}$ from A.

Q.19 A variable circle passes through the point A (a, b) & touches the x-axis; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

Q.20 Find the locus of the mid point of all chords of the circle $x^2 + y^2 - 2x - 2y = 0$ such that the pair of lines joining (0, 0) & the point of intersection of the chords with the circles make equal angle with axis of x.

Q.21 A circle with center in the first quadrant is tangent to y = x + 10, y = x – 6, and the y-axis. Let (h, k) be the center of the circle. If the value of $(h + k) = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of a + b.

Q.22 A circle C is tangent to the x and y axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope – 1. If the points A and B are on the y-axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ sq. units then find the radius of the circle.

Q.23 Circles C_1 and C_2 are externally tangent and they are both internally tangent to the circle C_3. The radii of C_1 and C_2 are 4 and 10, respectively and the centres of the three circles are collinear. A chord of C_3 is also a common internal tangent of C_1 and C_2. Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where *m*, *n* and *p* are positive integers, *m* and *p* are relatively prime and *n* is not divisible by the square of any prime, find the value of $(m + n + p)$.

Q.24 Find the equation of the circle passing through the three points (4, 7), (5, 6) and (1, 8). Also find the coordinates of the point of intersection of the tangents to the circle at the points where it is cut by the straight line $5x + y + 17 = 0$.

Q.25 The line $2x - 3y + 1 = 0$ is tangent to a circle S = 0 at (1, 1). If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S.

Q.26 Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.

Q.27 Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point (–1, – 1).

Q.28 Given that a right angled trapezium has an inscribed circle. Prove that the length of the right angled leg is the Harmonic mean of the lengths of bases.

Q.29 Let **K** denotes the square of the diameter of the circle whose diameter is the common chord of the two circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$

and **W** denotes the sum of the abscissa and ordinates of a point P where all variable chords of the curve $y^2 = 8x$ subtending right angles at the origin, are concurrent.

and **H** denotes the square of the length of the tangent from the point (3, 0) on the circle $2x^2 + 2y^2 + 5y - 16 = 0$.

Find the value of KWH.

Q.30 Let $S_1 = 0$ and $S_2 = 0$ be two circles intersecting at P (6, 4) and both are tangent to x-axis and line y = mx (where m > 0). If product of radii of the circles $S_1 = 0$ and $S_2 = 0$ is $\frac{52}{3}$, then find the value of m.

EXERCISE–II

Q.1 Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \left(d^2/2\right) = 0$.

Q.2 A rhombus ABCD has sides of length 10. A circle with centre 'A' passes through C (the opposite vertex) likewise, a circle with centre B passes through D. If the two circles are tangent to each other, find the area of the rhombus.

Q.3 Let A, B, C be real numbers such that
(i) (sin A, cos B) lies on a unit circle centred at origin.
(ii) tan C and cot C are defined.
If the minimum value of $(\tan C - \sin A)^2 + (\cot C - \cos B)^2$ is $a + b\sqrt{2}$ where $a, b \in I$, find the value of $a^3 + b^3$.

Q.4 An isosceles right angled triangle whose sides are 1, 1, $\sqrt{2}$ lies entirely in the first quadrant with the ends of the hypotenuse on the coordinate axes. If it slides prove that the locus of its centroid is $(3x - y)^2 + (x - 3y)^2 = \frac{32}{9}$.

Q.5 Real number x, y satisfies $x^2 + y^2 = 1$. If the maximum and minimum value of the expression $z = \frac{4 - y}{7 - x}$ are M and m respectively, then find the value $(2M + 6m)$.

Q.6 The radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = 3/4$ or $f = 2$.

Q.7 Find the equation of the circle through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12=0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ & cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Q.8 The centre of the circle S = 0 lie on the line $2x - 2y + 9 = 0$ & S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points & find their coordinates.

Q.9(a) Find the equation of a circle passing through the origin if the line pair, $xy - 3x + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8=0$ then find the value of k.

(b) Find the equation of the circle which cuts the circle $x^2 + y^2 - 14x - 8y + 64 = 0$ and the coordinate axes orthogonally.

Q.10 Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9=0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Interpret the locus.

Q.11 Find the equation of a circle which touches the line x + y = 5 at the point (–2, 7) and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally.

Q.12 Find the equation of the circle passing through the point (–6 , 0) if the power of the point (1, 1) w.r.t. the circle is 5 and it cuts the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ orthogonally.

Q.13 Consider a family of circles passing through two fixed points A (3, 7) & B(6, 5). The the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.

Q.14 Find the equation of circle passing through (1, 1) belonging to the system of co–axal circles that are tangent at (2, 2) to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2 + y^2 = 4$.

Q.15 The circle C : $x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through two fixed points for every real number k. Find
(i) the coordinates of these two points. **(ii)** the minimum value of the radius of a circle C.

Q.16 Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ & $x^2 + y^2 + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

Q.17 The circles, which cut the family of circles passing through the fixed points $A \equiv (2, 1)$ and $B \equiv (4, 3)$ orthogonally, pass through two fixed points (x_1, y_1) and (x_2, y_2), which may be real or imaginary. Find the value of $\left(x_1^3 + x_2^3 + y_1^3 + y_2^3\right)$.

Q.18 Find the equation of a circle which touches the lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and is contained in the given circle.

Q.19 Find the equation of the circle which passes through the origin, meets the x-axis orthogonally & cuts the circle $x^2 + y^2 = a^2$ at an angle of 45°.

Q.20 Consider two circles C_1 of radius 'a' and C_2 of radius 'b' (b > a) both lying in the first quadrant and touching the coordinate axes. In each of the conditions listed in **column-I**, the ratio of b/a is given in **column-II.**

	Column-I		**Column-II**
(A)	C_1 and C_2 touch each other	(P)	$2+\sqrt{2}$
(B)	C_1 and C_2 are orthogonal	(Q)	3
(C)	C_1 and C_2 intersect so that the common chord is longest	(R)	$2+\sqrt{3}$
(D)	C_2 passes through the centre of C_1	(S)	$3+2\sqrt{2}$
		(T)	$3-2\sqrt{2}$

EXERCISE–III

Q.1 (a) The triangle PQR is inscribed in the circle, $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) & (– 4, 3) respectively, then $\angle QPR$ is equal to
(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$

(b) If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ & $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then 'k' is :
(A) 2 or – 3/2 (B) – 2 or – 3/2 (C) 2 or 3/2 (D) – 2 or 3/2
[JEE '2000 (Screening), 1+1]

Q.2 (a) Extremities of a diagonal of a rectangle are (0, 0) & (4, 3). Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal.

(b) Find the point on the straight line, $y = 2x + 11$ which is nearest to the circle, $16(x^2 + y^2) + 32x - 8y - 50 = 0$.

(c) A circle of radius 2 units rolls on the outerside of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles is inclined at an angle of 60° with x-axis. [REE '2000 (Mains) 3 + 3 + 5]

Q.3 (a) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals

(A) $\sqrt{PQ \cdot RS}$ (B) $\dfrac{PQ + RS}{2}$ (C) $\dfrac{2PQ \cdot RS}{PQ + RS}$ (D) $\sqrt{\dfrac{(PQ)^2 + (RS)^2}{2}}$
[JEE '2001 (Screening) 1 out of 35]

(b) Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA.
[JEE '2001 (Mains) 5 out of 100]

Q.4 (a) Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. [REE '2001 (Mains) 3 out of 100]

(b) Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ.

[REE '2001 (Mains) 5 out of 100]

Q.5 (a) If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

(b) If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

[JEE '2002 (Scr)3 + 3 out of 270]

Q.6 The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$ [JEE '2004 (Scr)]

Q.7 Line $2x + 3y + 1 = 0$ is a tangent to a circle at (1, -1). This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points (0, –1) and (– 2, 3). Find equation of circle.

[JEE '2004, 4 out of 60]

Q.8 A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

[JEE '2005 (Scr)]

Q.9(a) Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(A) 3 (B) 2 (C) 3/2 (D) 1

(b) Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement-1: The tangents are mutually perpendicular.

because

Statement-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. [JEE 2007, 3+3]

Q.10(a) Consider the two curves

$C_1 : y^2 = 4x$; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

(b) Consider, $L_1 : 2x + 3y + p - 3 = 0$; $L_2 : 2x + 3y + p + 3 = 0$,
where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

STATEMENT-1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

STATEMENT-2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

(c) **Comprehension (3 questions together):**

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}\,x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}\,x,\ y = 0$

[JEE 2008, 3+3 + 4 + 4 + 4]

Q.11(a) Tangents drawn from the point P(l, 8) to the circle

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$
(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

(b) The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is [JEE 2009, 3 + 4]

ANSWER SHEET

EXERCISE–I

Q.1 square of side 2; $x^2 + y^2 = 1$; $x^2 + y^2 = 2$ **Q.2** zero, zero
Q.4 32 sq. unit **Q.5** $x - y = 0$; $x + 7y = 0$ **Q.6** (5, 1) & (–1, 5)
Q.7 $4x - 3y - 25 = 0$ **OR** $3x + 4y - 25 = 0$
Q.8 **(i)** (11, 16), **(ii)** (11, 8), **(iii)** (11, 12)
Q.9 $x^2 + y^2 - 2x - 2y + 1 = 0$ **OR** $x^2 + y^2 - 42x + 38y - 39 = 0$
Q.10 (i) $3x - 4y = 21$; $4x + 3y = 3$; (ii) A(0, 1) and B (–1, – 6); (iii) 90°, $5(\sqrt{2} \pm 1)$ units
(iv) 25 sq. units, 12.5 sq. units; (v) $x^2 + y^2 + x + 5y - 6$, x intercept 5; y intercept 7
Q.11 $x^2 + y^2 - 2x - 2y = 0$ **Q.12** $2x - 2y - 3 = 0$ **Q.13** $a^2(x^2 + y^2) = 4x^2y^2$
Q.14 $x^2 + y^2 = a^2 + b^2$; $r = \sqrt{a^2 + b^2}$ **Q.15** $2(x^2 + y^2) + 6x - 17y - 6 = 0$ **Q.16** 63 **Q.17** 6
Q.18 $x - 7y = 2$, $7x + y = 14$; $(x - 1)^2 + (y - 7)^2 = 3^2$; $(x - 3)^2 + (y + 7)^2 = 3^2$;
$(x - 9)^2 + (y - 1)^2 = 3^2$; $(x + 5)^2 + (y + 1)^2 = 3^2$
Q.20 $x + y = 2$ **Q.21** 10 **Q.22** r = 15 **Q.23** 19
Q.24 (– 4, 2), $x^2 + y^2 - 2x - 6y - 15 = 0$ **Q.25** $x^2 + y^2 - 6x + 4y=0$ **OR** $x^2 + y^2 + 2x - 8y + 4=0$
Q.26 $x^2 + y^2 + x - 6y + 3 = 0$ **Q.27** $5x^2 + 5y^2 - 8x - 14y - 32 = 0$ **Q.29** 64 **Q.30** $\sqrt{3}$

EXERCISE–II

Q.2 75 sq. unit **Q.3** 19 Q.5 4 **Q.7** $x^2 + y^2 + 16x + 14y - 12 = 0$
Q.8 (– 4, 4) ; (– 1/2, 1/2) **Q.9** (a) $x^2 + y^2 + 4x - 6y = 0$; k = 1; (b) $x^2 + y^2 = 64$
Q.10 $9x - 10y + 7 = 0$; radical axis **Q.11** $x^2 + y^2 + 7x - 11y + 38 = 0$ **Q.12** $x^2 + y^2 + 6x - 3y = 0$
Q.13 $\left(2, \frac{23}{3}\right)$ **Q.14** $x^2 + y^2 - 3x - 3y + 4 = 0$ **Q.15** (1, 0) & (1/2,1/2); $r = \frac{1}{2\sqrt{2}}$
Q.16 $4x^2 + 4y^2 + 6x + 10y - 1 = 0$ **Q.17** 40 **Q.18** $x^2 + y^2 - 12x - 12y + 64 = 0$
Q.19 $x^2 + y^2 \pm a\sqrt{2}\,x = 0$ **Q.20** (A) S; (B) R ; (C) Q ; (D) P

EXERCISE–III

Q.1 **(a)** C **(b)** A
Q.2 **(a)** $6x - 8y + 25 = 0$ & $6x - 8y - 25 = 0$; **(b)** (–9/2 , 2)
(c) $x^2 + y^2 + 4x - 12 = 0$, T_1: $\sqrt{3}x - y + 2\sqrt{3} + 4 = 0$, T_2: $\sqrt{3}x - y + 2\sqrt{3} - 4 = 0$ (D.C.T.)
T_3: $x + \sqrt{3}\,y - 2 = 0$, T_4: $x + \sqrt{3}\,y + 6 = 0$ (T.C.T.)

Q.3 **(a)** A; **(b)** OA = $3(3 + \sqrt{10})$ **Q.4** **(a)** $x^2 + y^2 + 14x - 6y + 6 = 0$; **(b)** $2px + 2qy = r$
Q.5 **(a)** C; **(b)** A **Q.6** C
Q.7 $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **Q.8** D **Q.9** (a) B; (b) A
Q.10 **(a)** B; **(b)** C; **(c)** (i) D, (ii) A, (iii) D **Q.11** (a) B; (b) 8