Definite Integration
Summation/Area Under Curve

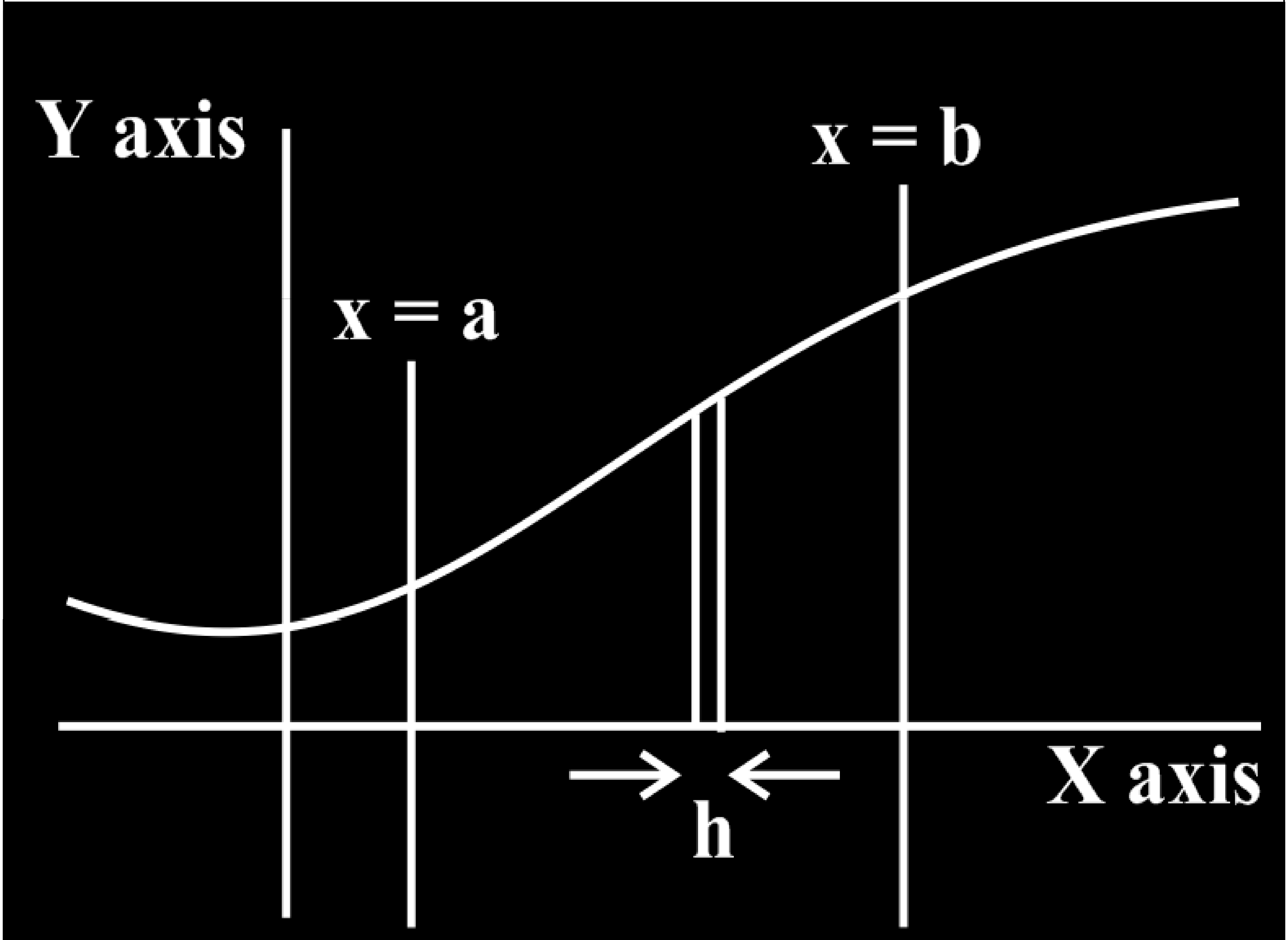
Definition :

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

is called definite integral of f between limits a & b

where $\frac{d}{dx}\left(F(x)\right) = f(x)$

Note : f (x) is bounded & continuous in [a, b]


Y axis
x = b
x = a
X axis
h

Dividing into n Vertical stripes each of width h

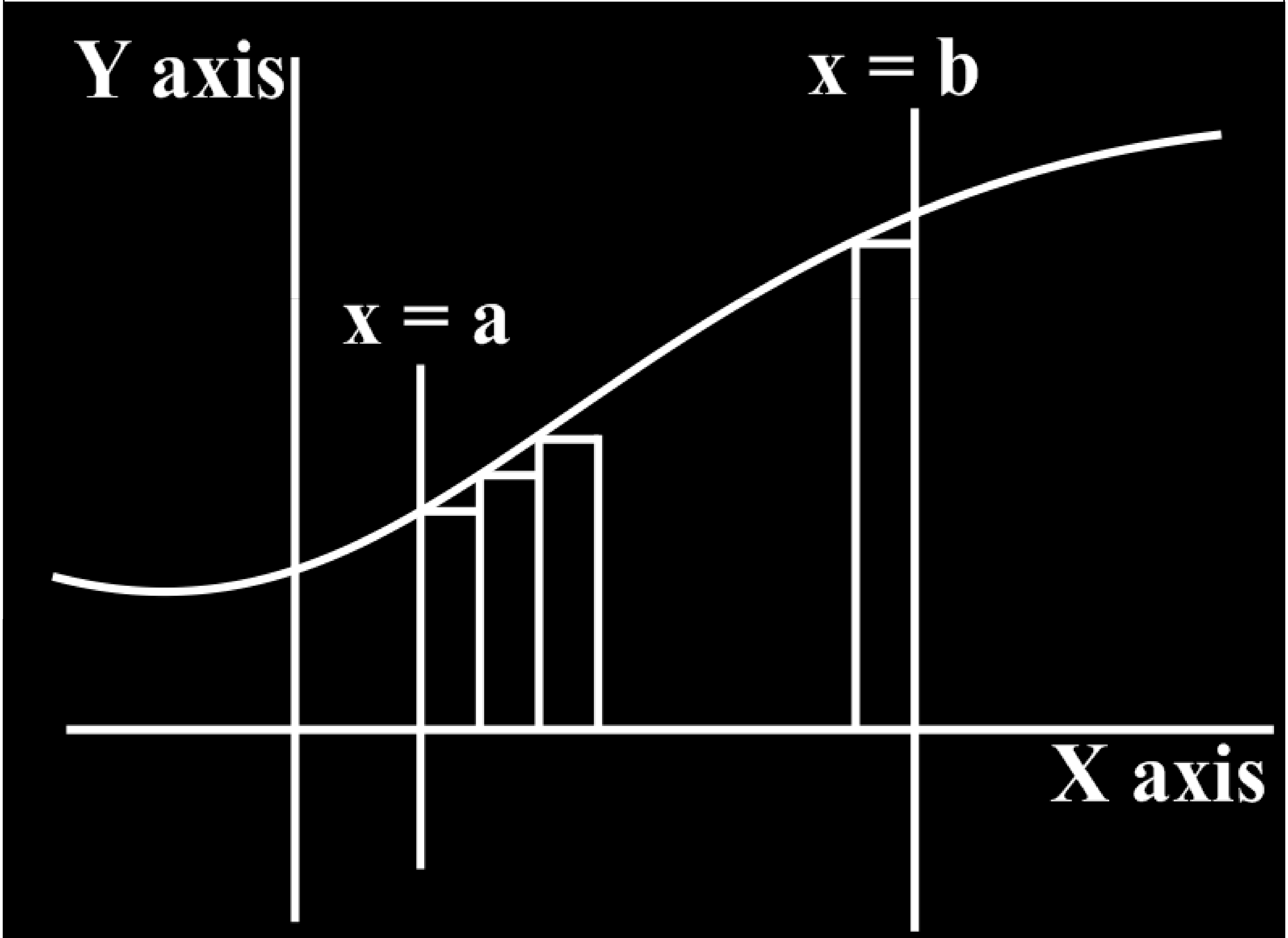
$h + h + h \ldots\ldots$ n times $= b - a$

$nh = b - a$

As $n \to \infty \qquad h \to 0$

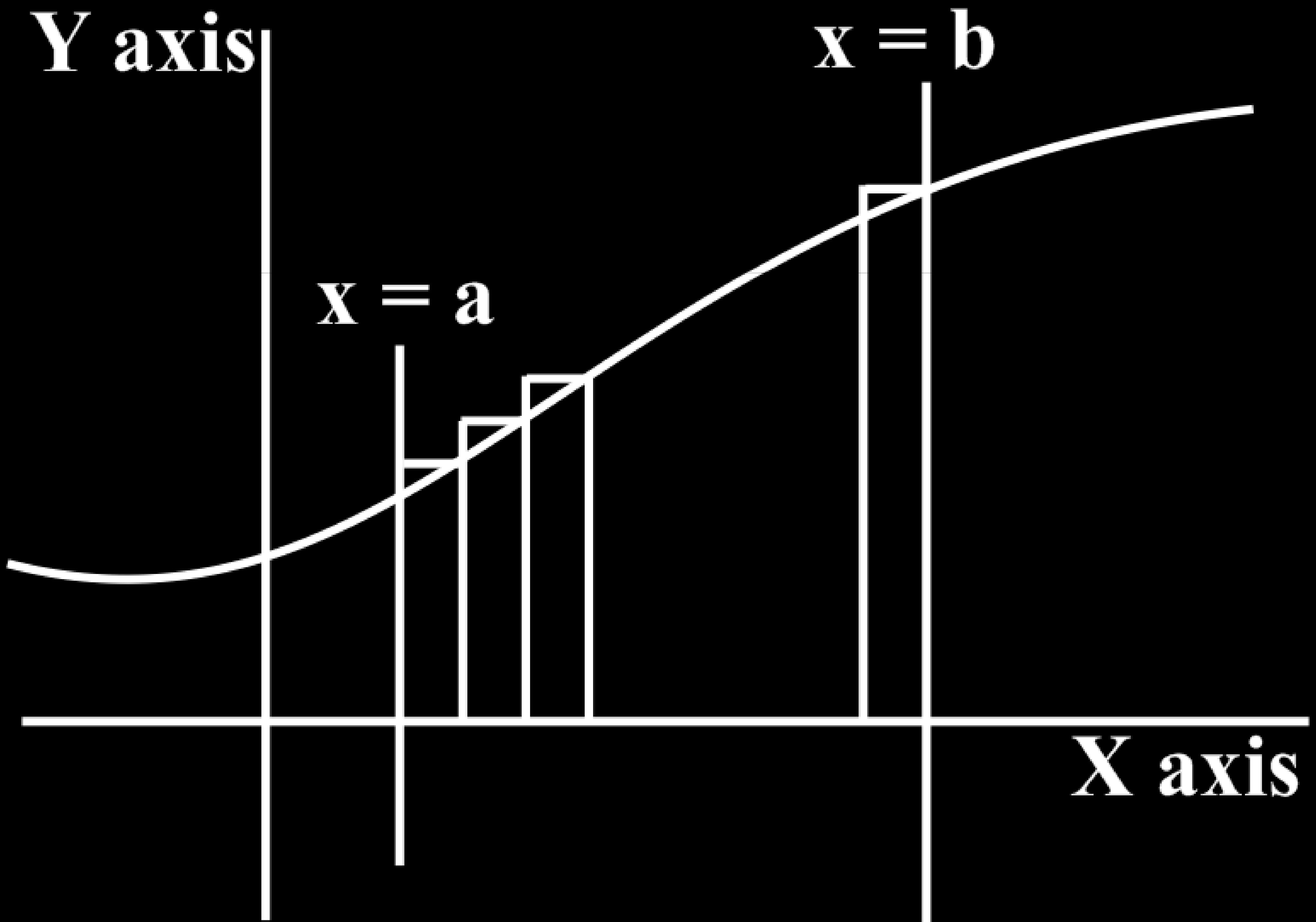
Area can be calculated by 2 ways
First Method

$$S_n = \lim_{\substack{h \to 0 \\ n \to \infty}} h\left(f(a) + f(a+h) + \ldots\ldots\ldots f(a+(n-1)h)\right)$$


Y axis
x = b
x = a
X axis

Second Method

$$s_n = \lim_{\substack{h \to 0 \\ n \to \infty}} h\left(f(a+h) + f(a+2h) + \ldots\ldots\ldots f(a+nh)\right)$$


Y axis
x = b
x = a
X axis

S_n < Required Area < s_n

Examples

Q. By 1st Principle $\int_0^1 e^x\,dx$

Q. $$\int_{0}^{2} x\,dx$$

Note

1. If $\int_a^b f(x)\,dx = 0$,

then the equation f (x) = 0 has atleast one root in (a, b) provided f is continuous in (a, b).

Note that the converse is not true.

2. Area below x-axis is Negative

3. $$\lim_{n\to\infty}\left(\int_a^b f_n(x)\,dx\right)=\int_a^b\left(\lim_{n\to\infty} f_n(x)\right)dx$$

Q. $$\lim_{n\to\infty} \int_{-1}^{1} \left(1+\frac{t}{n}\right)^{n} dt$$

4. $$\int_{a}^{b} f\,dx = \int_{a}^{c^-} f\,dx + \int_{c^+}^{b} f\,dx$$

Q. $\int_{1}^{3} [x]\, dx$

Q. $$\int_0^{\pi/2} \sin x \, dx = \int_0^{\pi/2} \cos x \, dx = 1$$

Q. $$\int_0^{\pi/2} \sin^2 x\, dx = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$$

Q. $\int_0^{\pi/2} \sin^3 x \, dx = \int_0^{\pi/2} \cos^3 x \, dx = \frac{2}{3}$

Q. $\int_0^{\pi/2} \sin^4 x \, dx = \int_0^{\pi/2} \cos^4 x \, dx = \frac{3\pi}{16}$

Q. If g(x) is the inverse of f(x) and f(x) has domain $x \in [a, b]$ where $f(a) = c$ and $f(b) = d$ then the value of

$$\int_a^b f(x)\,dx + \int_c^d g(y)\,dy = (bd - ac)$$

Q. $\int_{0}^{1} e^{x}\, dx + \int_{1}^{e} \ln x\, dx$

Q. $f:[0,1]\rightarrow[e,e^{\sqrt{e}}]$

$$I=\int_0^1 e^{\sqrt{e^x}}dx+2\int_e^{e^{\sqrt{e}}}\ln(\ln x)\,dx$$

Q. $$\int_{3}^{8} \frac{\sin\sqrt{x+1}}{\sqrt{x+1}}\,dx$$

Q. $$\int_{0}^{\pi/4} \cos 2x \sqrt{4 - \sin 2x}\, dx$$

Q. $\int_{0}^{1} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Q. $\int_{\alpha}^{\beta} \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} \; (\beta > \alpha)$

Q. $$\int_0^{1/2} \frac{dx}{\left(1-2x^2\right)\sqrt{1-x^2}}$$

Q. $\int_{0}^{1} x \, \ell n(1+2x)\, dx$

Q. $$\int_{0}^{\ln 2} \frac{e^x}{1+e^x}\,dx$$

Q. $\int_{0}^{\pi/2} \frac{dx}{4+5\sin x}$

(A) $\frac{1}{3}\ln 2$ (B) $\frac{4}{3}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $2\ln\frac{3}{2}$

Q. The value of integral

$$\int_0^{2008}\left(3x^2-8028x+(2007)^2+\frac{1}{2008}\right)dx$$ equals

(A) $(2008)^2$ (B) $(2009)^2$

(C) 2009 (D) 1

Q. $\int_{1}^{e} (x+1)e^{x}\ln x \, dx$

Q. $$\int_{2}^{4} \frac{\sqrt{x^2 - 4}}{x^4} \, dx$$

Q. $\int_{-1}^{1} x^2 d(\ln x)$

Q. $$\int_0^{\pi/16}\left(\frac{\sin x+\sin 2x+\sin 3x+\ldots\ldots\ldots\ldots+\sin 7x}{\cos x+\cos 2x+\cos 3x+\ldots\ldots\ldots\ldots+\cos 7x}\right)dx$$

Q. $$\int_0^1 xe^{-x}\,dx$$

Q. $\displaystyle\int_{\frac{1}{\sqrt{3}}}^{1} \cot^{-1}x \, dx$

Q. $\int_{0}^{\infty} x^{n} e^{-x} \, dx$

Q. Assume that f'' is continuous and that $f(1)=3$, $f'(1) = 2$ and

$\int_0^1 f(x)\,dx = 5$ Find the value of $\int_0^1 x^2 f''(x)\,dx$

Q. $$\int_0^{2\pi} [(1+x)\cos x + (1-x)\sin x]\,dx$$

Q. Let $I = \int_0^{\pi/2} \frac{\cos x}{a\cos x + b\sin x} dx$ and $J = \int_0^{\pi/2} \frac{\sin x}{a\cos x + b\sin x} dx$

where a > 0 and b > 0. Compute the values of I and J.

Assignment - 1

Q. Let $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{\sqrt{x(1-x)}}\,dx$

Q. $\int_0^{\ell n 2} x\,e^{-x}\,dx$

Q. $\int_1^e \left(\frac{1}{\sqrt{x\,\ln x}}+\sqrt{\frac{\ln x}{x}}\right)dx$

Q. Given $f'(x)=\frac{\cos x}{x}$, $f\left(\frac{\pi}{2}\right)=a$, $f\left(\frac{3\pi}{2}\right)=b$.

Find the value of the definite integral $\int_{\pi/2}^{3\pi/2} f(x)\,dx$

Q. $\displaystyle\int_{-1}^{1} \frac{x\,dx}{\sqrt{5-4x}}$

Q. $\displaystyle\int_{2}^{e} \left(\frac{1}{\ell n\,x} - \frac{1}{\ell n^{2}\,x}\right) dx$

Q. $\displaystyle\int_{0}^{\pi/4} \frac{\sin 2x}{\sin^{4} x + \cos^{4} x}\, dx$

Q. $\displaystyle\int_{0}^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

Q. $\displaystyle\int_{0}^{\pi/4} \frac{\sin^{2} x \,.\, \cos^{2} x}{\left(\sin^{3} x + \cos^{3} x\right)^{2}}\, dx$

Q. $\displaystyle\int_{1/3}^{3} \frac{\sin^{-1}\dfrac{x}{\sqrt{1+x^{2}}}}{x}\, dx$

Q. $\displaystyle\int_{2}^{3} \frac{dx}{\sqrt{(x-1)\,(5-x)}}$

Q. $\displaystyle\int_{3/2}^{2} \left(\frac{x-1}{3-x}\right)^{1/2} dx$

Q. $\int_{0}^{\pi/4} x \cos x \cos 3x \, dx$

Q. $\int_{0}^{\pi/2} \frac{dx}{5+4\sin x}$

Q. $\int_{2}^{3} \frac{dx}{(x-1)\sqrt{x^2-2x}}$

Q. $\int_{0}^{\pi/2} \frac{dx}{1+\cos\theta \cdot \cos x} \quad \theta \in (0, \pi)$

Q. $\int_{0}^{\pi/4} \cos 2x \sqrt{1-\sin 2x} \, dx$

Q. $\int_{0}^{\frac{\ln 3}{2}} \frac{e^x+1}{e^{2x}+1} dx$

Q. $\int_{0}^{3} \sqrt{\frac{x}{3-x}} \, dx$

Q. $\int_{0}^{1/2} \frac{dx}{\left(1-2x^2\right)\sqrt{1-x^2}}$

Q. $\int_{1}^{2} \frac{dx}{x\left(x^{4}+1\right)}$

Q. $\int_{0}^{\pi/2} \sin\phi \cos\phi \sqrt{\left(a^{2}\sin^{2}\phi + b^{2}\cos^{2}\phi\right)}\, d\phi \quad a \neq b \ (a > 0,\ b > 0)$

Q. $\int_{0}^{3\pi/4} \left((1+x)\sin x + (1-x)\cos x\right) dx$

Q. $\int_{\pi/2}^{\pi} x^{\sin x}\left(1 + x\cos x \cdot \ln x + \sin x\right) dx$

Q. $\int_{0}^{1} x\,(\tan^{-1} x)^{2}\, dx$

Q. Suppose that f, f' and f'' are continuous on $[0, ln2]$ and that $f(0) = 0$, $f'(0) = 3$, $f(ln 2) = 6$, $f'(ln 2) = 4$ and

$$\int_0^{ln 2} e^{-2x} \cdot f(x)\, dx = 3$$

Find the value of $\int_0^{ln 2} e^{-2x} \cdot f''(x)\, dx$

Q. $\displaystyle\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$ where $-\pi < \alpha < \pi$

Q. $\int_{a}^{b} \frac{dx}{\sqrt{1+x^2}}$ where $a=\frac{e-e^{-1}}{2}$ & $b=\frac{e^2-e^{-2}}{2}$

Q. $\int_{0^+}^{1} \frac{x^x(x^{2x}+1)(\ln x+1)}{x^{4x}+1}dx$

Q. $\int_{0}^{1} x^5 \sqrt{\frac{1+x^2}{1-x^2}}\, dx$

Q. Suppose that the function f, g, f ' and g' are continuous over [0,1], $g(x) \neq 0$ for $x \in [0, 1]$, $f(0) = 0$, $g(0) = \pi$, $f(1) = \frac{2009}{2}$ and $g(1) = 1$. Find the value of the definite integral,

$$\int_0^1 \frac{f(x)\cdot g'(x)\left\{g^2(x)-1\right\}+f'(x)\cdot g(x)\left\{g^2(x)+1\right\}}{g^2(x)}dx$$

Q. $\int_0^{\pi/4} \frac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\, d\theta$

Q. $\int_0^{\pi} \theta \sin^2\theta \cos\theta \; d\theta$

Q. $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2}\, dx$

Q. $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\, dx$

Q. Let $A=\int_{3/4}^{4/3}\frac{2x^2+x+1}{x^3+x^2+x+1}\,dx$

then find the value of e^A..

Q. $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}}\,dx$ Q. $\int_{-1}^{1}\left(\frac{d}{dx}\left(\frac{1}{1+e^{1/x}}\right)\right)dx$

Q. $\int_1^e \frac{dx}{\ln(x^x e^x)}$

Q. $\int_0^{\pi}\left[\cos^2\left(\frac{3\pi}{8}-\frac{x}{4}\right)-\cos^2\left(\frac{11\pi}{8}+\frac{x}{4}\right)\right]dx$

Q. If $f(\pi) = 2$ & $\int_0^{\pi} (f(x) + f''(x)) \sin x \, dx = 5$, then find $f(0)$

Q. $\int_a^b \frac{|x|}{x} dx$

Q. $\int_{\ell n2}^{\ell n3} f(x)dx$, where $f(x) = e^{-x} + 2e^{-2x} + 3e^{-3x} + .. \infty$

Q. $\int_0^{\pi/2} \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}} \frac{\operatorname{cosec} x}{\sqrt{1 + 2\operatorname{cosec} x}} dx$

Q. $\int_0^1 x f''(x) \, dx$, where $f(x) = \cos(\tan^{-1} x)$

Q. (a) If g (x) is the inverse f (x) and f (x) has domain x∈ [1, 5], where f(1)=2 and f(5)=10 then find the value of

$$\int_1^5 f(x)\,dx + \int_2^{10} g(y)\,dy$$

(b) Suppose f is continuous, f (0) = 0, f (1) = 1, f' (x) > 0 and $\int_0^1 f(x)\,dx = \frac{1}{3}$. Find the value of the definite integral $\int_0^1 f^{-1}(y)\,dy$

ANSWER KEY

Q.1 $\frac{\pi^2}{4}$ Q.2 $\frac{1}{2}\ell n\left(\frac{e}{2}\right)$ Q.3 $2\sqrt{e}$ Q.4 $2-\frac{\pi}{2}(a-3b)$ Q.5 $\frac{1}{6}$ Q.6 $e-\frac{2}{\ell n 2}$

Q.7 $\frac{\pi}{4}$ Q.8 $ln\frac{4}{3}$ Q.9 $\frac{1}{6}$ Q.10 $\frac{\pi ln 3}{2}$ Q.11 $\frac{\pi}{6}$ Q.12 $\frac{\sqrt{3}}{2}-1+\frac{\pi}{6}$

Q.13 $\frac{\pi-3}{16}$ Q.14 $\frac{2}{3}\tan^{-1}\frac{1}{3}$ Q.15 $\frac{\pi}{3}$ Q.16 $\frac{\theta}{\sin\theta}$

Q.17 $\frac{1}{2}\left(\frac{\pi}{6}+ln3-ln2\right)$ Q.18 $\frac{1}{3}$ Q.19 $\frac{3\pi}{2}$ Q.20 $\frac{1}{2}\ell n\left(2+\sqrt{3}\right)$ Q.21 $\frac{1}{4}ln\frac{32}{17}$

Q.22 $\frac{1}{3}\frac{a^3-b^3}{a^2-b^2}$ Q.23 (a) $2\left(\sqrt{2}+1\right)$; (b) $\left(\pi-\frac{\pi^2}{4}\right)$ Q.24 $\frac{\pi}{4}\left(\frac{\pi}{4}-1\right)+\frac{1}{2}\ell n 2$ Q.25 13

Q.26 $\frac{\alpha}{2\sin\alpha}$ if $\alpha \neq 0$; $\frac{1}{2}$ if $\alpha = 0$ Q.27 1 Q.28 0 Q.29 $\frac{3\pi+8}{24}$

Q.30 2009 Q.31 $\frac{1}{20}\ln 3$ Q.32 $-\frac{4}{9}$ Q.33 $\frac{1}{2}$ Q.34 $\frac{\pi}{2}$

Q.35 $\frac{16}{9}$ Q.36 $\frac{\pi}{2}$ Q.37 $\frac{2}{1+e}$ Q.38 $\ln 2$ Q.39 $\sqrt{2}$ Q.40 3

Q.41 $|b| - |a|$ Q.42 $\frac{1}{2}$ Q.43 $\pi/3$ Q.44 $1-\frac{3}{2\sqrt{2}}$ Q.45 (a) 48 (b) 2/3

Properties

P – 1

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

P – 2

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dt$$

P – 3

$$\int_a^b f(x) = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Provided f has a piece wise continuity

Examples

Q. $$\int_0^{3/2} x[x^2]\,dx$$

Q. $$\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}$$

Q. $\int_{0}^{3} |5x-9|\,dx$

Q. $\int_{-e}^{-1/e} \left| \ln|x| \right| dx$

(A) $2-\frac{2}{e}$

(B) 2e

(C) $-\frac{2}{e}$

(D) $\frac{2}{e}$

Q. $\int_{-1}^{3}\left[x+\frac{1}{2}\right]dx$

Q. $\int_{0}^{2\pi} \sqrt{1 - \sin 2x}\, dx$

Q. $\int_{0}^{2\pi} |1 + 2\cos x|\,dx$

(A) $\frac{2\pi}{3} + 2\sqrt{3}$

(B) $\frac{2\pi}{3} + 3 + 3\sqrt{3}$

(C) $\frac{2\pi}{3} + 4\sqrt{3}$

(D) $2\pi/3$

Q. $\int_0^2 [x^2 - x + 1]\, dx$

(A) 1

(B) $\frac{3-\sqrt{5}}{2}$

(C) $\frac{5-\sqrt{5}}{2}$

(D) $\frac{9-\sqrt{5}}{2}$

P - 4

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_{0}^{a} (x)\,dx & \text{if } f(x) \text{ is even} \end{cases}$$

Examples

Q. $$\int_{-1/2}^{1/2} \sec x \ln \frac{1-x}{1+x} dx$$

Q. $$\int_{-1/2}^{1/2}\left([x]+\ln\left(\frac{1+x}{1-x}\right)\right)dx$$

Q. $$\int_{-2}^{2} \left| 1 - x^2 \right| dx$$

Q. $\int_{-\pi/4}^{\pi/4} f(x)\,dx$ where $f(x) = \dfrac{x^7 - 3x^5 + 3x^3 - x + 1}{\cos^2 x}$

Q. $$\int_{-1}^{3/2} |x \sin \pi x| \, dx$$

Q. $$\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$$

(A) π

(B) 2π

(C) 3π

(D) $5\pi/2$

P – 5 (King Rule)

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \text{ or } \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

1. King laga ke add kar diya
2. Most time Denominator remains slightly change or unchange
3. x In numerator

Examples

Q. $$\int_{0}^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$$

Q. $$\int_{-\pi/2}^{\pi/2}\left(\frac{1}{(2007)^{x}+1}\right)\cdot\frac{\sin^{2008}x}{\sin^{2008}x+\cos^{2008}x}\,dx$$

Q. $$\int_{\pi/6}^{\pi/3} \sin 2x \ln(\tan x)\, dx$$

Q. $$\int_{50}^{100} \frac{\ln x}{\ln x + \ln(150 - x)} dx$$

Q. $$\int_{\pi/8}^{3\pi/8} \ln\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$$

Q. $\int_{0}^{\pi/4} \ln(1+\tan x)\,dx$

Q. $\int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+e^x} dx$

Q. $$\int_{0}^{\pi} \frac{dx}{1+2^{\tan x}}$$

Q. $\int_{0}^{1} \cot^{-1}(1 - x + x^2)\,dx$

Q. $$\int_{2}^{3} \frac{x^2 dx}{2x^2 - 10x + 25}$$

Q. $$\int_{\pi/4}^{3\pi/4} \frac{x\sin x}{1+\sin x}\,dx$$

Q. $\int_{0}^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Q. $$\int_{0}^{\pi/4} \frac{x\,dx}{1+\cos 2x+\sin 2x}$$

Q. $\int_{0}^{1} \ln\left(\frac{1}{x} - 1\right) dx$

Q. $$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Q. **Prove that**

$$I = \int_0^{\infty} \frac{\ln x \, dx}{ax^2 + bx + a} = 0$$

Examples

Q. **Prove that :**

$$\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$$

Q. $$\int_{a/2}^{\sqrt{3}a/2} \frac{dx}{x+\sqrt{a^2-x^2}} \, (a>0)$$

Q. $$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$$

Q. $\int_{0}^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

Q. $$\int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cos x}\,dx$$

Q. $\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Q. $\int_{0}^{\pi} \frac{\sin 8x}{\sin x} dx$

Q. $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

Q. $\int_{0}^{\pi/2n} \frac{dx}{1+\tan^{n}(nx)}$

Q. $\int_{0}^{1} \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

Q. $\int_{0}^{\pi/2} \sqrt{\sin 2\theta}\, \sin\theta \, d\theta$

Q. $\int_{-\pi}^{\pi}(\cos px - \sin qx)^2\, dx, p, q \in I$

Q. $$\int_0^{\pi/2} \frac{\sin 8x \, . \ln(\cot x)}{\cos 2x} dx$$

Q. $\int_{-2}^{2}\left(x^{3}f(x)+x.f''(x)+2\right)dx$

Where f(x) is an even differentiable function.

Q. $$I = \int_{\pi/2}^{3\pi/2} [2\sin x]\, dx$$

(A) $-\pi$

(B) 0

(C) $-\frac{\pi}{2}$

(D) $\frac{\pi}{2}$

Q. $\int_{1/2}^{2} \frac{\ln x}{1+x^2}\, dx$

Q. $$\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}\,dx$$

Q. $$\int_0^{2a} f(x)dx = \begin{cases} 0 & \text{If } f(2a-x) = -f(x) \\ 2\int_0^{a} f(x)dx & \text{If } f(2a-x) = f(x) \end{cases}$$

Q. $$I = \int_0^{2\pi} \sin^4 x \, dx$$

Q. $I = \int_{0}^{2\pi} cox^5 x \, dx$

Q. $$I = \int_0^{\pi} \frac{dx}{1 + 2\sin^2 x}$$

Q. $$\int_0^{\pi} \frac{\sin x}{\sin 4x}$$

Q. $\int_{0}^{\pi} \sin^3 x \cos^3 x \, dx$

Q. $\int_0^{\frac{\pi}{2}} \ell n(\sin x)dx = \int_0^{\frac{\pi}{2}} \ell n(\cos x)dx = \int_0^{\frac{\pi}{2}} \ell n(\sin 2x)dx$

are equal to $\left(-\frac{\pi}{2}\ln 2\right)$

Q. $\int_0^1 \ell n \sin\left(\frac{\pi x}{2}\right) dx$

Q. $\int_{0}^{\pi} x \ell n(\sin x)dx$

Q. $\int_{0}^{\pi} \ell n(1-\cos x)dx$

Q. $$\int_0^1 \frac{\sin^{-1} x}{x}$$

Q. $\int_{0}^{\frac{\pi}{2}} (2\cos^2 x)\ell n(\sin 2x)dx$

Q. $\int_{0}^{\pi} x\left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$

Q. $\int_0^{\pi} x\left(\sin^2\left(\cos^2 x\right)\cos\left(\sin^2 x\right)\right)dx$

Q. $$\int_0^{2\pi} \frac{x(\sin x)^{2n}}{(\sin x)^{2n} + (\cos x)^{2n}}, n \in N$$

Q. $\int_{0}^{2\pi} x\sin^4 x\cos^6 x \, dx$

P - 7

$$\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx \text{ where } f(T+x) = f(x)\ n \in I$$

Examples

Q. $$\int_0^{2n\pi} \left(|\sin x| - \left[\left| \frac{\sin x}{2} \right| \right] \right) dx$$

[.] Denotes greatest integer function.

Q. $$\int_{0}^{1000} e^{x-[x]}\,dx$$

Q. $$\int_{0}^{200\pi} \sqrt{1+\cos x}\, dx$$

Q. $$\int_{0}^{2000\pi} \frac{dx}{1+e^{\sin x}}$$

Q. $\int_{0}^{n\pi+v} |\cos x|\,dx$ where $\frac{\pi}{2} < v < \pi$ & $n \in N$

Derivatives Of Antiderivatives (Leibnitz Rule)

If f is continuous then

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\left(h(x)\right).h'(x) - f\left(g(x)\right).g'(x)$$

(Integral of a continuous function is always differentiable)

Examples

Q. $\mathbf{f(x)} = \int_{x^2}^{x^3} \mathbf{t\,dt}$, $f'(2) = $ **?**

Q. $g(x)=\int_{0}^{\cos x} t^2\,dt$, $g'\left(\pi/4\right)=?$

Q. $g(x) = \int_{x}^{x^2} \cos t\,dt$, $g'(0) = ?$

$G(x) = \int_{2}^{x^2} \frac{dt}{1+\sqrt{t}} \; (x > 0)$. Find $G'(9)$.

Q. $f(x) = \int_{3}^{e^{3x}} \frac{t}{\ln t}\,dt$, $f'(\ln 2) = ?$

Q. If $x = \int_{1}^{t^2} z \ln z \, dz$ and $y = \int_{t^2}^{1} z^2 \ln z \, dz$ find $\frac{dy}{dx}$

(A) $-t^2$ (B) $-2t^2$ (C) 1 (D) $-1/t^2$

Q. $x = \int_0^y \frac{dt}{\sqrt{1+4t^2}}$, If $\frac{d^2y}{dx^2} = ky$, find k.

(A) 2 (B) 4 (C) – 8 (D) –4

Q. $$\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2 \, dt}{x \sin x}$$

Q. $$\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

Prove that above is constant function of x.

Q. $f(x) = \int_{e^{2x}}^{e^{3x}} \frac{t}{\ln t}\, dt \quad x > 0.$

Find derivative of f(x) w.r.t. *l*n x when x = *l*n 2

Q. $f(x) = \int_0^x \frac{\sin^2 \frac{t}{2}}{t} dt$ then find $\underset{x \to 0}{\text{Limit}} \frac{f'(x)}{x}$

Q. If $y = \int_0^{z^2} \frac{dx}{1+x^3}$ find $\frac{d^2y}{dz^2}$ at $z = 1$

(A) –2 (B) –4 (C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$

Q. Let f(x) is a derivable function satisfying

$$f(x) = \int_0^x e^t \sin(x - t)\,dt \text{ and } g(x) = f''(x) - f(x).$$

Find the range of g(x).

Q. $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4+1}\, dt$

Q. **Evaluate** $\lim_{x \to \infty} x \int_0^x \left(e^{t^2 - x^2}\right) dt$

Q. $\int_0^x f(t)\,dt = x\cos(\pi x)$, for $x > 0$, $f(4)$ is equal to

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q. $\lim_{x \to 0} \frac{1}{x} \int_0^x (1 - \tan 2t)^{1/t} \, dt$

Q. Finding function by Leibnitz

$$f(x) = 1 + \int_0^x f(t)\,dt$$

Q. Let $f(x)$ be a continuous function such that $f(x) > 0$ for all $x \geq 0$ and

$$(f(x))^{101} = 1 + \int_0^x f(t)\,dt.$$

The value of $(f(101))^{100}$ is

(A) 100 (B) 101 (C) $\frac{101}{100}$ (D) $(101)^{\frac{1}{100}}$

Q. $f^2(x) = \int_0^x \frac{f(t).\sin t\, dt}{2+\cos t}$ $\quad f(x) \neq 0$

Find f(x)

DEFINITE INTEGRAL AS A LIMIT OF SUM

Working Rule :

Step 1

Replace $\frac{1}{n} \rightarrow dx$

$$\sum \rightarrow \int$$

$$\frac{r}{n} \rightarrow x$$

Examples

Q. $$\underset{n\to\infty}{\text{Limit}} \frac{n^2}{\left(n^2+1\right)^{3/2}}+\frac{n^2}{\left(n^2+2^2\right)^{3/2}}+\ldots\ldots+\frac{n^2}{\left[n^2+(n-1)^2\right]^{3/2}}$$

Q. $\underset{n\to\infty}{\text{Limit}} \frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots+\frac{1}{4n}$

Q. $\underset{n \to \infty}{\text{Limit}} \dfrac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots\ldots + 2\sqrt{n}}{n\sqrt{n}}$

Q. $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+\ldots\ldots+\frac{3}{5n}\right]$

(A) $\tan^{-1}2+\frac{1}{2}\ln 5$

(B) $\tan^{-1}2+\frac{1}{2}\ln 2$

(C) $\tan^{-1}2+\frac{1}{2}\ln 3$

(D) $\tan^{-1}2+\frac{1}{2}\ln 4$

$$\underset{n\to\infty}{\text{Limit}}\ \frac{n}{(n+1)\sqrt{(2n+1)}}+\frac{n}{(n+2)\sqrt{2(2n+2)}}+\frac{n}{(n+3)\sqrt{3(2n+3)}}$$

……… up to n terms

Q. $$\lim_{n\to\infty}\left(\tan^{-1}\frac{1}{n}\right)\left(\sum_{k=1}^{n}\frac{1}{1+\tan(k/n)}\right)$$

has the value equal to

(A) $\frac{1+\ln(\cos 1)}{2}$ (B) $\frac{1+\ln(\sin 1)}{2}$

(C) $\frac{1-\ln(\sin 1+\cos 1)}{2}$ (D) $1 + \ln(\sin1 + \cos1)$

Q. $$\lim_{n\to\infty} \frac{\left[(n+1)(n+2).....(n+n)\right]^{1/n}}{n}$$

Q. $$\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)^{\frac{2}{n^2}}\cdot\left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}}\cdot\left(1+\frac{3^2}{n^2}\right)^{\frac{6}{n^2}}\cdots\cdots\left(1+\frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}\right]$$

Q. $\lim_{n\to\infty} \left({}^{2n}C_n\right)^{1/n}$

(A) 4

(B) 4/e

(C) $4/e^2$

(D) 2/e

ESTIMATION OF DEFINITE INTEGRAL AND GENERAL INEQUALITIES

For a monotonic increasing function in (a, b)

$$(b-a)f(a) < \int_a^b f(x)\,dx < (b-a)f(b)$$

For a monotonic decreasing function in (a, b)

$$f(b).\ (b-a) < \int_a^b f(x)\,dx < (b-a)f(a)$$

For a non monotonic function in (a, b)

$$f(c).\ (b-a) < \int_a^b f(x)\,dx < (b-a)\,f(b)$$

Function is maximum at x=b and minimum at x=c

In addition to this note that

$$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$$

equality holds when f(x) lies completely above the x-axis

Examples

Q. $$\frac{\pi}{128} < \int_{\pi/4}^{\pi/2} (\sin x)^{10}\, dx < \frac{\pi}{4}$$

Q. $1 < \int_{0}^{\pi/2} \frac{\sin x}{x} dx < \frac{\pi}{2}$

Q. $$\frac{e-1}{3} < \int_{1}^{e} \frac{dx}{2+\ln x} < \frac{e-1}{2}$$

Q. $$\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4 - x^2 + x^5}} < \frac{\pi}{6}$$

Q. $1 \le \int_0^{\pi/2} \sqrt{1-\sin^3 x}\, dx \le \frac{\pi}{2}$

Walli's Theorem & Reduction Formula

$$\int_0^{\pi/2} \sin^n x \cos^m x \, dx = \frac{[(n-1)(n-3)....1 \text{ or } 2][(m-1)(m-3)....1 \text{ or } 2]}{(m+n)(m+n-2).....1 \text{ or } 2} K$$

(m, n are non-negative integer)

$$\text{where } K = \begin{cases} \dfrac{\pi}{2} & \text{if } m, n \text{ both are even} \\ 1 & \text{otherwise} \end{cases}$$

Example

Q. $\int_0^{2\pi} x \sin^6 x \cos^4 x \, dx$

SOME INTEGRALS WHICH CAN NOT BE FOUND IN TERMS OF KNOWN ELEMENTRY FUNCTIONS

Q. $\int \frac{\sin x}{x} dx$

Q. $\int \frac{\cos x}{x} dx$

Q. $\int \sqrt{\sin x}\, dx$

Q. $\int \sin x^2 dx$

Q. $\int \cos x^2 dx$

Q. $\int x \tan x\, dx$

Q. $\int e^{-x^2} dx$

Q. $\int e^{x^2} dx$

Q. $\int \frac{x^3}{1+x^5} dx$

Q. $\int \left(1+x^2\right)^{1/3} dx$

Q. $\int \frac{dx}{\ln x}$

Q. $\int \sqrt{1+k^2 \sin^2 x}\, dx \quad k \in R$

DIFFERENTIATION AND INTEGRATING SERIES

Find the sum of series

Q. $\dfrac{x^2}{1.2} - \dfrac{x^3}{2.3} + \dfrac{x^4}{3.4} - \ldots\ldots + (-1)^{n+1}\dfrac{x^{n+1}}{n(n+1)} + \ldots\ldots \quad |x|<1$

Q. If | x | < 1 then find the sum of the series

$$\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+\ldots\ldots\infty$$

then prove that $f(x)=\frac{1}{x}-\cot x$