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DETERMINANT

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KEY CONCEPTS
DETERMINANT

1. The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called the determinant of order two.

Its value is given by : $D = a_1 b_2 - a_2 b_1$

2. The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order three.

Its value can be found as : $D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ **OR**

$D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ .... and so on .

In this manner we can expand a determinant in 6 ways using elements of ;
R_1, R_2, R_3 or C_1, C_2, C_3 .

3. Following examples of short hand writing large expressions are :

(i) The lines :
$a_1x + b_1y + c_1 = 0$........ (1)
$a_2x + b_2y + c_2 = 0$........ (2)
$a_3x + b_3y + c_3 = 0$........ (3)

are concurrent if , $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Condition for the consistency of three simultaneous linear equations in 2 variables.

(ii) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

(iii) Area of a triangle whose vertices are (x_r, y_r) ; $r = 1, 2, 3$ is :

$D = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ If $D = 0$ then the three points are collinear.

(iv) Equation of a straight line passsing through (x_1, y_1) & (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

4. MINORS :

The minor of a given element of a determinant is the determinant of the elements which remain after deleting the row & the column in which the given element stands. For example, the minor of a_1 in (Key Concept 2) is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ & the minor of b_2 is $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$.

Hence a determinant of order two will have "4 minors" & a determinant of order three will have "9 minors".

5. COFACTOR :

If M_{ij} represents the minor of some typical element then the cofactor is defined as :
$C_{ij} = (-1)^{i+j} . M_{ij}$; Where i & j denotes the row & column in which the particular element lies. Note that the value of a determinant of order three in terms of 'Minor' & 'Cofactor' can be written as :
$D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ **OR** $D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ & so on ....

6. PROPERTIES OF DETERMINANTS :

P– 1 : The value of a determinant remains unaltered, if the rows & columns are inter changed.

e.g. if $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$

D & D′ are transpose of each other . If D′ = – D then it is **Skew symmetric** determinant but $D' = D \Rightarrow 2D = 0 \Rightarrow D = 0 \Rightarrow$ Skew symmetric determinant of third order has the value zero.

P– 2: If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only. e.g.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ & $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $D' = -D$.

P– 3: If a determinant has any two rows (or columns) identical, then its value is zero.

e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then it can be verified that $D = 0$.

P– 4: If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

e.g. If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $D' = KD$

P–5: If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants. e.g.

$$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P– 6: The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row

(or column). e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3+na_2 & b_3+nb_2 & c_3+nc_2 \end{vmatrix}. \text{ Then } D' = D .$$

Note : that while applying this property **ATLEAST ONE ROW (OR COLUMN)** must remain unchanged .

P– 7: If by putting $x = a$ the value of a determinant vanishes then $(x - a)$ is a factor of the determinant.

7. MULTIPLICATION OF TWO DETERMINANTS :

(i) $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \text{x} \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1 l_1 + b_1 l_2 & a_1 m_1 + b_1 m_2 \\ a_2 l_1 + b_2 l_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}$

Similarly two determinants of order three are multiplied.

(ii) If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$ then, $D^2 = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ where A_i, B_i, C_i are cofactors

PROOF : Consider $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix}$

Note : $a_1A_2 + b_1B_2 + c_1C_2 = 0$ etc.

therefore, $D \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^3 \Rightarrow \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^2$ OR $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = D^2$

8. SYSTEM OF LINEAR EQUATION (IN TWO VARIABLES) :

(i) Consistent Equations : Definite & unique solution. [intersecting lines]

(ii) Inconsistent Equation : No solution . [Parallel line]

(iii) Dependent equation : Infinite solutions . [Identical lines]

Let $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ then :

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given equations are inconsistent

& $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ Given equations are dependent

9. CRAMER'S RULE : [Simultaneous Equations Involving Three Unknowns]

Let ,$a_1x + b_1y + c_1z = d_1$ ...(I) ; $a_2x + b_2y + c_2z = d_2$ ...(II) ; $a_3x + b_3y + c_3z = d_3$ ...(III)

Then, $x = \frac{D_1}{D}$, $Y = \frac{D_2}{D}$, $Z = \frac{D_3}{D}$.

Where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$; $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$; $D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ & $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

NOTE :

(a) If $D \neq 0$ and alteast one of $D_1, D_2, D_3 \neq 0$, then the given system of equations are consistent and have unique non trivial solution.

(b) If $D \neq 0$ & $D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have trivial solution only.

(c) If $D = D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have infinite solutions.

In case $\left.\begin{matrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{matrix}\right\}$ represents these parallel planes then also

$D = D_1 = D_2 = D_3 = 0$ but the system is inconsistent.

(d) If $D = 0$ but atleast one of D_1, D_2, D_3 is not zero then the equations are inconsistent and have no solution.

10. If x , y , z are not all zero , the condition for $a_1x + b_1y + c_1z = 0$; $a_2x + b_2y + c_2z = 0$ & $a_3x + b_3y + c_3z = 0$ to be consistent in x , y , z is that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Remember that if a given system of linear equations have **Only Zero** Solution for all its variables then the given equations are said to have **TRIVIAL SOLUTION**.

EXERCISE–I

Q.1 (a) Prove that the value of the determinant $\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix}$ is real.

(b) On which one of the parameter out of a, p, d or x, the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$ does not depend.

Q.2 Without expanding as far as possible, prove that

(a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$ (b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = [(x-y)(y-z)(z-x)(x+y+z)]$

Q.3 If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = 0$ and x, y, z are all different then, prove that $xyz = -1$.

Q.4 Using properties of determinants or otherwise evaluate $\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$.

Q.5 Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

Q.6 If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$ then prove that $D' = 2D$.

Q.7 Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

Q.8 Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$.

Q.9 Show that the value of the determinant $\begin{vmatrix} \tan(A+P) & \tan(B+P) & \tan(C+P) \\ \tan(A+Q) & \tan(B+Q) & \tan(C+Q) \\ \tan(A+R) & \tan(B+R) & \tan(C+R) \end{vmatrix}$ vanishes for all values of

A, B, C, P, Q & R where $A+B+C+P+Q+R=0$

Q.10 Prove that $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

Q.11 For a fixed positive integer n, if D= $\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

Q.12 Solve for x

(a) $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0.$ (b) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$

Q.13 If $a+b+c=0$, solve for x : $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0.$

Q.14 If $a^2+b^2+c^2=1$ then show that the value of the determinant

$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & ba(1-\cos\theta) & ca(1-\cos\theta) \\ ab(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & cb(1-\cos\theta) \\ ac(1-\cos\theta) & bc(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$ simplifies to $\cos^2\theta$.

Q.15 If $p+q+r=0$, prove that $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.

Q.16 If a, b, c are all different & $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then prove that, $abc(ab+bc+ca) = a+b+c$.

Q.17 Show that, $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2 and find the other factor.

Q.18 Prove that : $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$.

Q.19 In a Δ ABC, determine condition under which $\begin{vmatrix} \cot\frac{A}{2} & \cot\frac{B}{2} & \cot\frac{C}{2} \\ \tan\frac{B}{2}+\tan\frac{C}{2} & \tan\frac{C}{2}+\tan\frac{A}{2} & \tan\frac{A}{2}+\tan\frac{B}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$

Q.20 Prove that : $\begin{vmatrix} (a-p)^2 & (a-q)^2 & (a-r)^2 \\ (b-p)^2 & (b-q)^2 & (b-r)^2 \\ (c-p)^2 & (c-q)^2 & (c-r)^2 \end{vmatrix} = \begin{vmatrix} (1+ap)^2 & (1+aq)^2 & (1+ar)^2 \\ (1+bp)^2 & (1+bq)^2 & (1+br)^2 \\ (1+cp)^2 & (1+cq)^2 & (1+cr)^2 \end{vmatrix}$

Q.21 Prove that $\begin{vmatrix}(a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2\\(a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2\\(a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2\end{vmatrix} = 2(a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$

Q.22 Prove that $\begin{vmatrix}2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta\\ \alpha+\beta+\gamma+\delta & 2(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)\\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) & 2\alpha\beta\gamma\delta\end{vmatrix} = 0.$

Q.23 If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = d$
and $ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1 = ax_1x_2 + by_1y_2 + cz_1z_2 = f,$

then prove that $\begin{vmatrix}x_1 & y_1 & z_1\\x_2 & y_2 & z_2\\x_3 & y_3 & z_3\end{vmatrix} = (d-f)\left[\dfrac{d+2f}{abc}\right]^{1/2}$ $(a, b, c \neq 0)$

Q.24 If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix}S_0 & S_1 & S_2\\S_1 & S_2 & S_3\\S_2 & S_3 & S_4\end{vmatrix} = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$.

Q.25 If $u = ax^2 + 2bxy + cy^2$, $u' = a'x^2 + 2b'xy + c'y^2$. Prove that

$$\begin{vmatrix}y^2 & -xy & x^2\\a & b & c\\a' & b' & c'\end{vmatrix} = \begin{vmatrix}ax+by & bx+cy\\a'x+b'y & b'x+c'y\end{vmatrix} = -\frac{1}{y}\begin{vmatrix}u & u'\\ax+by & a'x+b'y\end{vmatrix}.$$

EXERCISE–II

Q.1 Solve the following using Cramer's rule and state whether consistent or not.

(a) $\begin{aligned}x+y+z-6&=0\\2x+y-z-1&=0\\x+y-2z+3&=0\end{aligned}$ (b) $\begin{aligned}x+2y+z&=1\\3x+y+z&=6\\x+2y&=0\end{aligned}$ (c) $\begin{aligned}7x-7y+5z&=3\\3x+y+5z&=7\\2x+3y+5z&=5\end{aligned}$

Q.2 For what value of K do the following system of equations possess a non trivial (i.e. not all zero) solution over the set of rationals Q?
$x + Ky + 3z = 0$, $3x + Ky - 2z = 0$, $2x + 3y - 4z = 0$.
For that value of K, find all the solutions of the system.

Q.3 The system of equations
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$
has no solution. Find α.

Q.4 If the equations $a(y+z) = x$, $b(z+x) = y$, $c(x+y) = z$ have nontrivial solutions, then find the value of $\dfrac{1}{1+a} + \dfrac{1}{1+b} + \dfrac{1}{1+c}$.

Q.5 Given $x = cy + bz$; $y = az + cx$; $z = bx + ay$ where x, y, z are not all zero, prove that $a^2 + b^2 + c^2 + 2abc = 1$.

Q.6 Given $a = \dfrac{x}{y-z}$; $b = \dfrac{y}{z-x}$; $c = \dfrac{z}{x-y}$ where x, y, z are not all zero, prove that: $1 + ab + bc + ca = 0$.

Q.7 If $\sin q \neq \cos q$ and x, y, z satisfy the equations

$$x\cos p - y\sin p + z = \cos q + 1$$
$$x\sin p + y\cos p + z = 1 - \sin q$$
$$x\cos(p+q) - y\sin(p+q) + z = 2$$

then find the value of $x^2 + y^2 + z^2$.

Q.8 Investigate for what values of λ, μ the simultaneous equations $x+y+z=6$; $x+2y+3z=10$ & $x+2y+\lambda z=\mu$ have; (a) A unique solution. (b) An infinite number of solutions. (c) No solution.

Q.9 For what values of p, the equations : $x+y+z=1$; $x+2y+4z=p$ & $x+4y+10z=p^2$ have a solution? Solve them completely in each case.

Q.10 Solve the equations : $Kx+2y-2z=1$, $4x+2Ky-z=2$, $6x+6y+Kz=3$ considering specially the case when $K=2$.

Q.11 Let a, b, c, d are distinct numbers to be chosen from the set $\{1, 2, 3, 4, 5\}$. If the least possible positive solution for x to the system of equations $\left.\begin{matrix} ax+by=1 \\ cx+dy=2 \end{matrix}\right\}$ can be expressed in the form $\frac{p}{q}$ where *p* and *q* are relatively prime, then find the value of (p + q).

Q.12 If $bc+qr = ca+rp = ab+pq = -1$ show that $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$.

Q.13 If the following system of equations $(a-t)x + by + cz = 0$, $bx + (c-t)y + az = 0$ and $cx+ay+(b-t)z=0$ has non–trivial solutions for different values of t, then show that we can express product of these values of t in the form of determinant.

Q.14 Show that the system of equations

$$3x - y + 4z = 3\ ,\ x + 2y - 3z = -2 \text{ and } 6x + 5y + \lambda z = -3$$

has atleast one solution for any real number λ. Find the set of solutions of $\lambda = -5$.

Q.15 Solve the system of equations ; $\left.\begin{matrix} z+ay+a^2x+a^3=0 \\ z+by+b^2x+b^3=0 \\ z+cy+c^2x+c^3=0 \end{matrix}\right]$

EXERCISE–III

Q.1 If the system of equations $x - Ky - z = 0$, $Kx - y - z = 0$ and $x + y - z = 0$ has a non zero solution, then the possible values of K are

(A) –1, 2 (B) 1, 2 (C) 0, 1 (D) –1, 1

[JEE 2000 (Screening)]

Q.2 Prove that for all values of θ, $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$

[JEE 2000 (Mains), 3 out of 100]

Q.3 Find the real values of r for which the following system of linear equations has a non-trivial solution. Also find the non-trivial solutions :

$$2rx - 2y + 3z = 0$$
$$x + ry + 2z = 0$$
$$2x + rz = 0$$

[REE 2000 (Mains) , 3 out of 100]

Q.4 Solve for x the equation

$$\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$$

[REE 2001 (Mains) , 3 out of 100]

Q.5 Test the consistency and solve them when consistent, the following system of equations for all values of λ

$$x + y + z = 1$$
$$x + 3y - 2z = \lambda$$
$$3x + (\lambda + 2)y - 3z = 2\lambda + 1$$

[REE 2001 (Mains) , 5 out of 100]

Q.6 Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$$

represents a straight line. [JEE 2001 (Mains) , 6 out of 100]

Q.7 The number of values of k for which the system of equations

$$(k + 1)x + 8y = 4k$$
$$kx + (k + 3)y = 3k - 1$$

has infinitely many solutions is

(A) 0 (B) 1 (C) 2 (D) inifinite

[JEE 2002 (Screening), 3]

Q.8 The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is

(A) 3 (B) –3 (C) 2 (D) –2

[JEE 2004 (Screening)]

Q.9(a) Consider three points P = $\left(-\sin(\beta-\alpha), -\cos\beta\right)$, Q = $\left(\cos(\beta-\alpha), \sin\beta\right)$ and R = $\left(\cos(\beta-\alpha+\theta), \sin(\beta-\theta)\right)$, where $0 < \alpha, \beta, \theta < \pi/4$

(A) P lies on the line segment RQ (B) Q lies on the line segment PR
(C) R lies on the line segment QP (D) P, Q, R are non collinear

(b) Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1$$

STATEMENT-1 : The system of equations has no solution for $k \neq 3$.

and

STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True [JEE 2008, 3 + 3]

ANSWER KEY
DETERMINANT
EXERCISE–I

Q.1 (b) p

Q.4 -1 Q.11 $(ab' - a'b)(bc' - b'c)(ca' - c'a)$ Q.12 (a) $x = -1$ or $x = -2$; (b) $x = 4$

Q.13 $x = 0$ or $x = \pm\sqrt{\frac{3}{2}\left(a^2 + b^2 + c^2\right)}$ Q.17 $\lambda^2 (a^2 + b^2 + c^2 + \lambda)$

Q.19 Triangle ABC is isosceles.

EXERCISE–II

Q.1 (a) $x = 1$, $y = 2$, $z = 3$; consistent (b) $x = 2$, $y = -1$, $z = 1$; consistent
(c) inconsistent

Q.2 $K = \frac{33}{2}$, $x : y : z = -\frac{15}{2} : 1 : -3$ Q.3 -2 Q.4 2 Q.7 2

Q.8 (a) $\lambda \neq 3$ (b) $\lambda = 3, \mu = 10$ (c) $\lambda = 3, \mu \neq 10$

Q.9 $x = 1 + 2K$, $y = -3K$, $z = K$, when $p = 1$; $x = 2K$, $y = 1 - 3K$, $z = K$ when $p = 2$; where $K \in R$

Q.10 If $K \neq 2$, $\frac{x}{2(K+6)} = \frac{y}{2K+3} = \frac{z}{6(K-2)} = \frac{1}{2\left(K^2+2K+15\right)}$,

If $K = 2$, then $x = \lambda$, $y = \frac{1-2\lambda}{2}$ and $z = 0$ where $\lambda \in R$

Q.11 19 Q.13 $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Q.14 If $\lambda \neq -5$ then $x = \frac{4}{7}$; $y = -\frac{9}{7}$ and $z = 0$;

If $\lambda = 5$ then $x = \frac{4-5K}{7}$; $y = \frac{13K-9}{7}$ and $z = K$ where $K \in R$

Q.15 $x = -(a + b + c)$, $y = ab + bc + ca$, $z = -abc$

EXERCISE–III

Q.1 D   Q.3 $r = 2$; $x = k$; $y = \frac{k}{2}$; $z = -k$ where $k \in R - \{0\}$

Q.4 $x = n\pi,\ n \in I$

Q.5 If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,
$y = \frac{1}{2}(3K + 4)$ and $x = -\frac{1}{2}(5K + 2)$ where $K \in R$
If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.

Q.7 B   Q.8 D   Q.9 (a) D; (b) A