FUNCTION

MC Sir

Function

1. Cartesian Product, Functions (Introduction), Domain, Co-Domain & Range
2. Types of functions and their Domain, Range & Graphs
3. Examples on Domain, Range and Graphs
4. Identical Functions, one one, many one, onto, Into Injective, Surjective, Bijective
5. Examples

Function

MC Sir

6. P & C based problems

7. Functional Equation

8. Composite function, Homogeneous function, Bounded Function

9. Implicit, Explicit, Odd, Even Function

10. Inverse of a Function

11. Periodicity

MC Sir

Function

No. of Questions				
2008	2009	2010	2011	2012
--	--	1	2	4

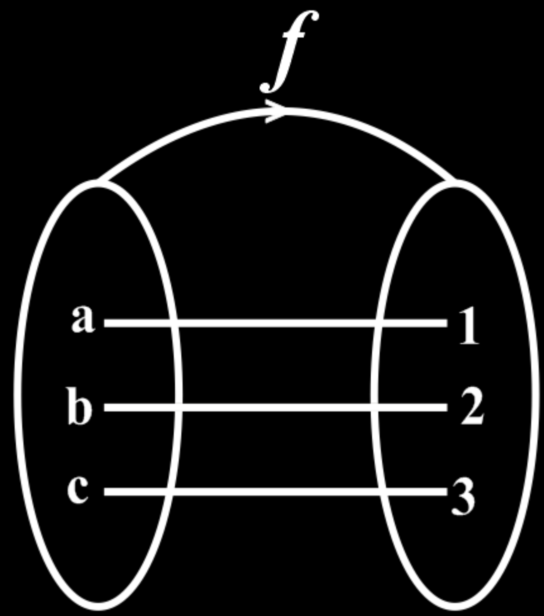
General Definition

Definition - 1

Let A and B be two sets and let there exist a rule or manner or correspondence '*f* ' which associates to each element of A, a unique element in B. Then *f* is called a <u>function</u> or <u>mapping</u> from A to B. It is denoted by the symbol

$$f : A \rightarrow B \text{ or } A \xrightarrow{f} B$$

which reads '*f*' is a function from A to B' or 'f maps A to B, If an element a $\in$ A is associated with an element b $\in$ B then b is called ‘the *f* image of a’ or ‘image of a under f’ or ‘the value of the function *f* at a’. Also a is called the pre-image of b or argument of under the function *f*.



Definition - 2

A relation R from a set A to a set B is called a function if

(i) Each element of A is associated with some element of B.

(ii) each element of A has unique image in B.

Definition - 3

Every function from A → B satisfies the following conditions.

(i) $f \subset A \times B$

(ii) $\forall a \in A \Rightarrow (a, f(a)) \in f$ *and*

(iii) $(a,b) \in f$ & $(a,b) \in f \Rightarrow \mathrm{b} = \mathrm{c}.$

Domain, Co-Domain & Range of A Function

Domain of $D_f = \{a \mid a \in A, (a, f(a)) \in f\}$

Range of $R_f = \{f(a) \mid a \in A, f(a) \in B\}$

Note

(1) It should be noted that range is a subset of co-domain.

(2) If only the rule of function is given then the domain of the function is the set of those real numbers where function is defined.

Q. Find domain of

$$\sqrt{x}\ ,\ \frac{1}{x}\ ,\ \frac{1}{x-1}\ ,\ \frac{1}{x}+\sqrt{x}$$

(3) Continuous function

If graph of a function can be drawn without taking up ruler then function is continuous.

Example : $f(x)$ **=** sinx, cosx, x

(4) For a continuous function, the interval from minimum to maximum value of a function gives the range

For Example : $f(x) = \sin x, \cos x$

Let f and g be function with domain D_1 and D_2 then the function f + g, f – g, fg, f/g are defined as

(i) $(f + g)(x) = f(x) + g(x)$; Domain $D_1 \cap D_2$

Let f and g be function with domain D_1 and D_2 then the function f + g, f – g, fg, f/g are defined as

(ii) $(f - g)(x) = f(x) - g(x)$; Domain $D_1 \cap D_2$

Let f and g be function with domain D_1 and D_2 then the function f + g, f – g, fg, f/g are defined as

(iii) $(f\,g)(x) = f(x)\,.\,g(x)$; Domain $D_1 \cap D_2$

Let f and g be function with domain D_1 and D_2 then the function f + g, f – g, fg, f/g are defined as

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$;

Domain = $D_1 \cap D_2 - \{x \mid g(x) \neq 0\}$

Example

Q. $f(x) = x^3 + 2x^2$ and $g(x) = 3x^2 - 1$.

Find Domain of $f \pm g$, $f\,g$ and f / g

Q. $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x-1}$

Domain of $f \pm g$, $f.g$ and f / g

Q. Find Domain of

$\sqrt{x^2+1}$, $f(x)=\dfrac{2x^4-x^2+1}{x^2-4}$

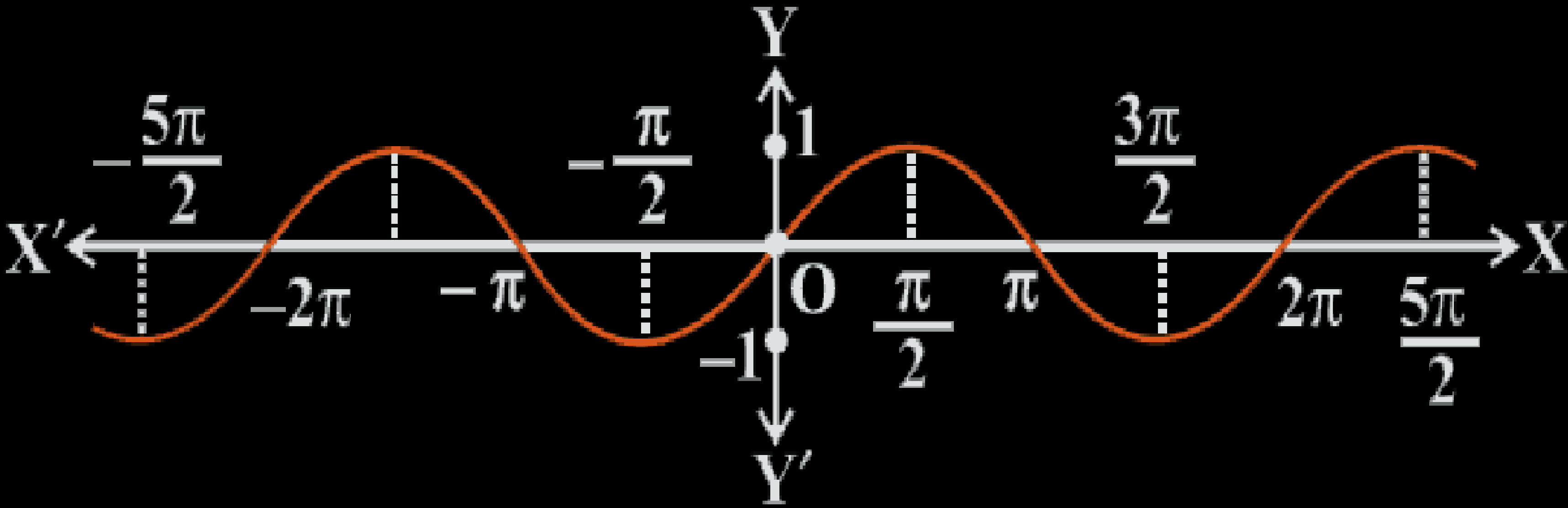
Types of Functions

Types of Functions

Trigonometric Function

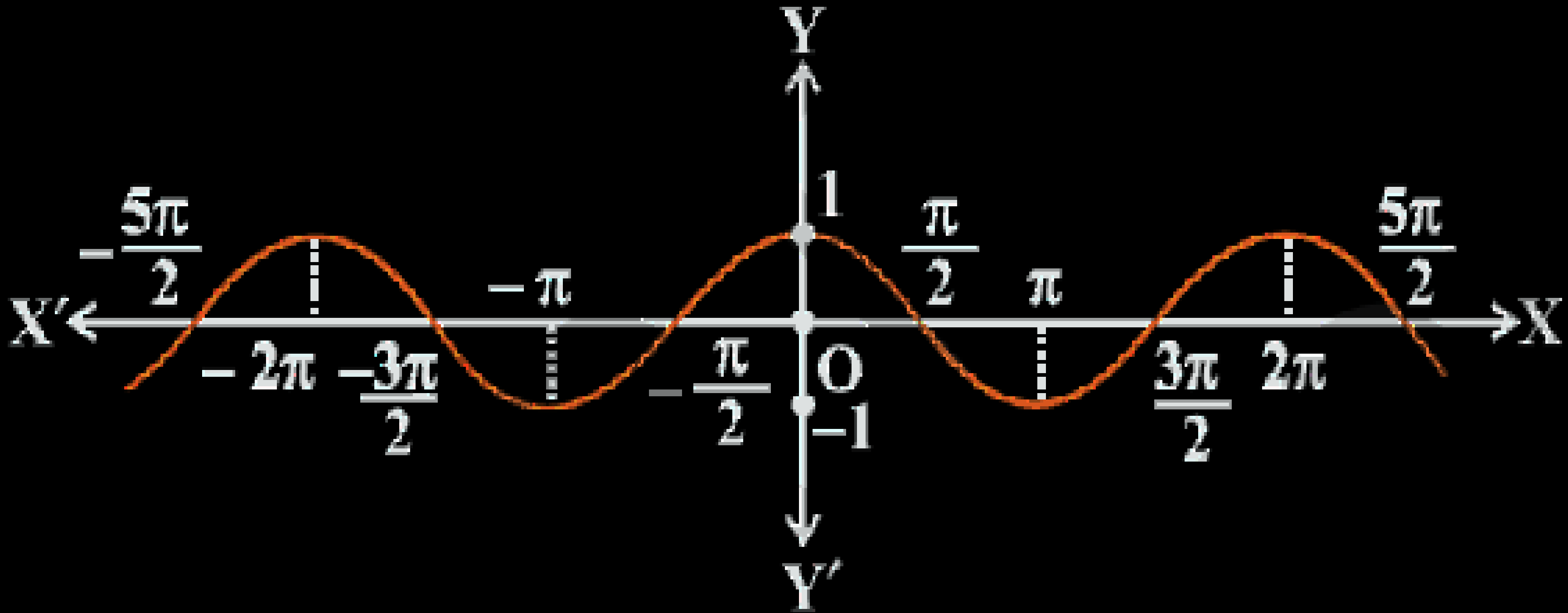
Graph of Trigonometric Function

$y = f(x) = \sin x$



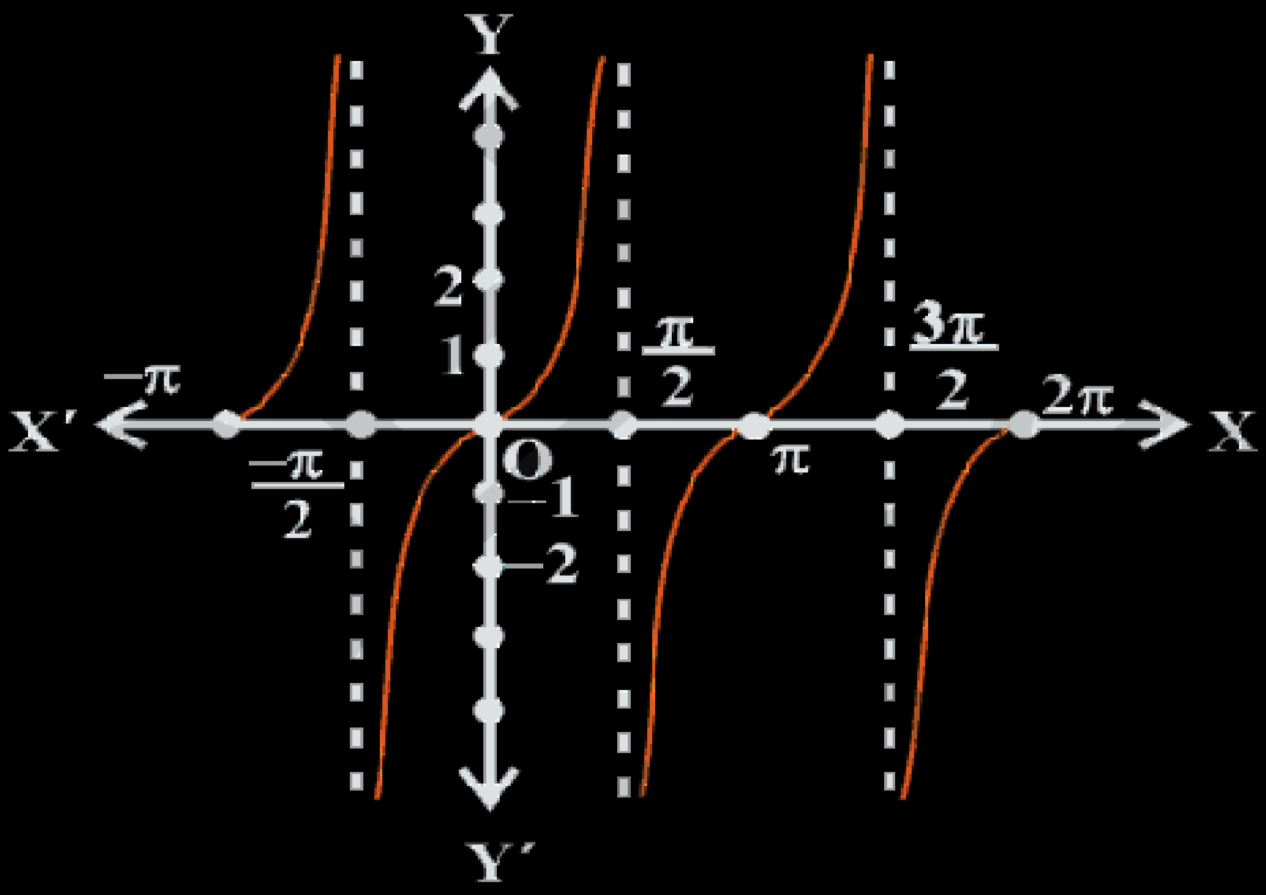
$y = \sin x$

$y = f(x) = \cos x$



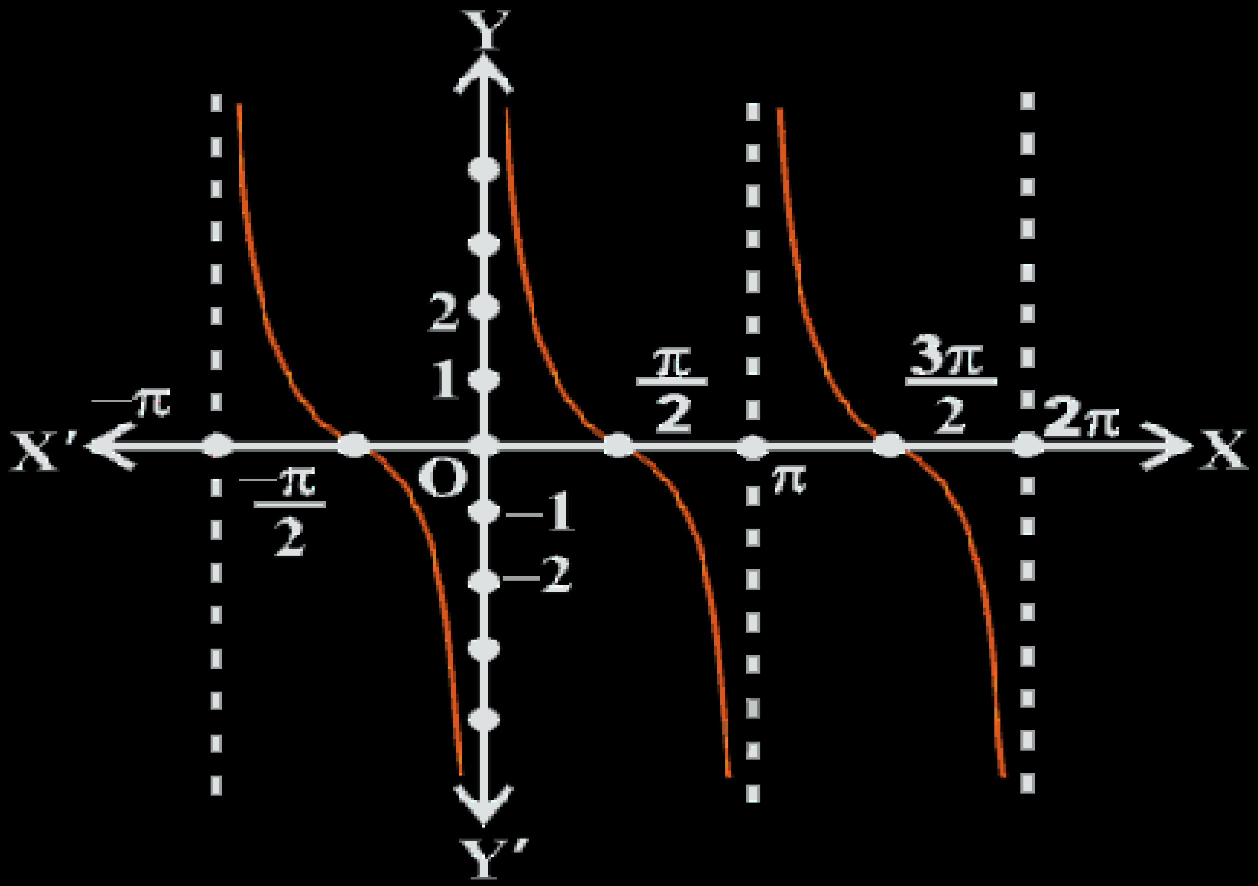
$y = \cos x$

$y = f(x) = \tan x$



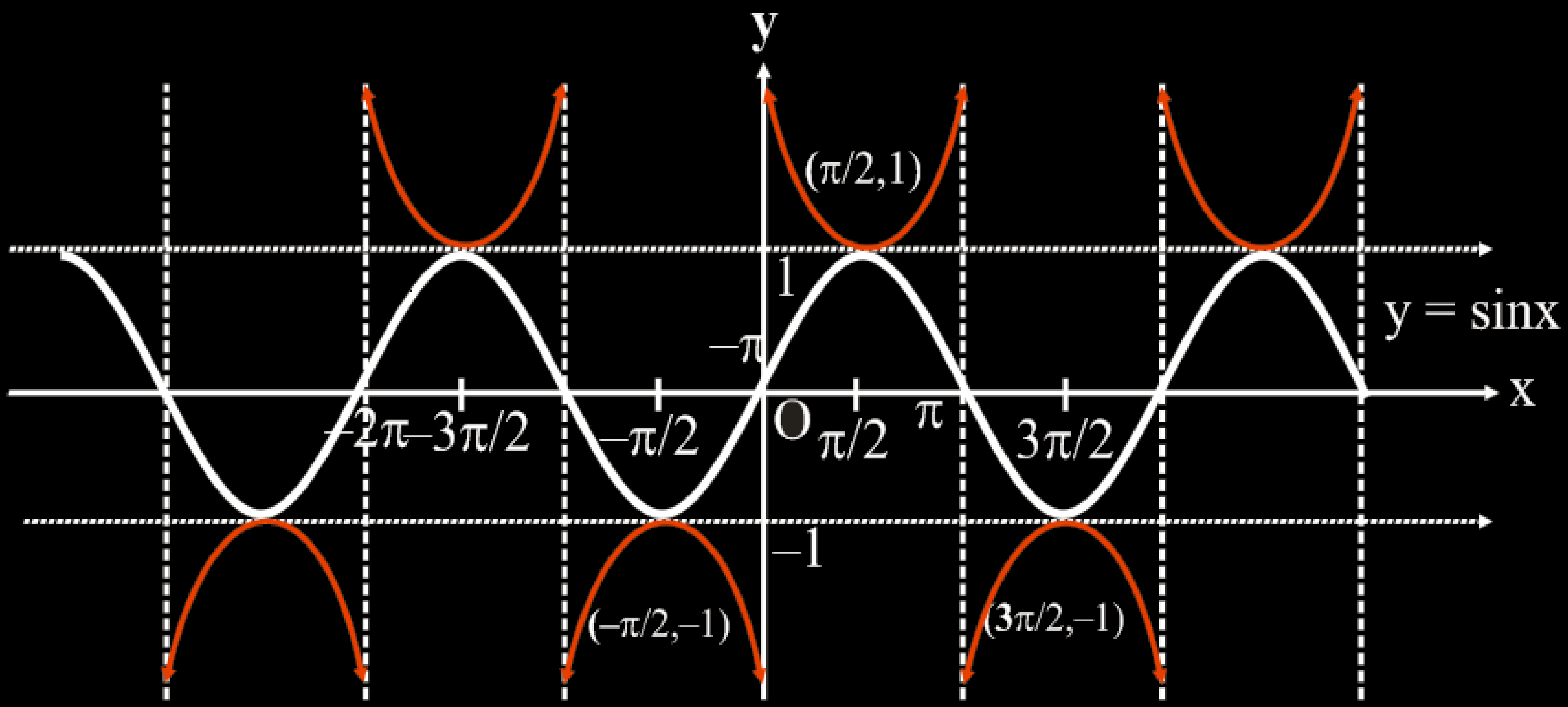
$y = \tan x$

$y = f(x) = \cot x$

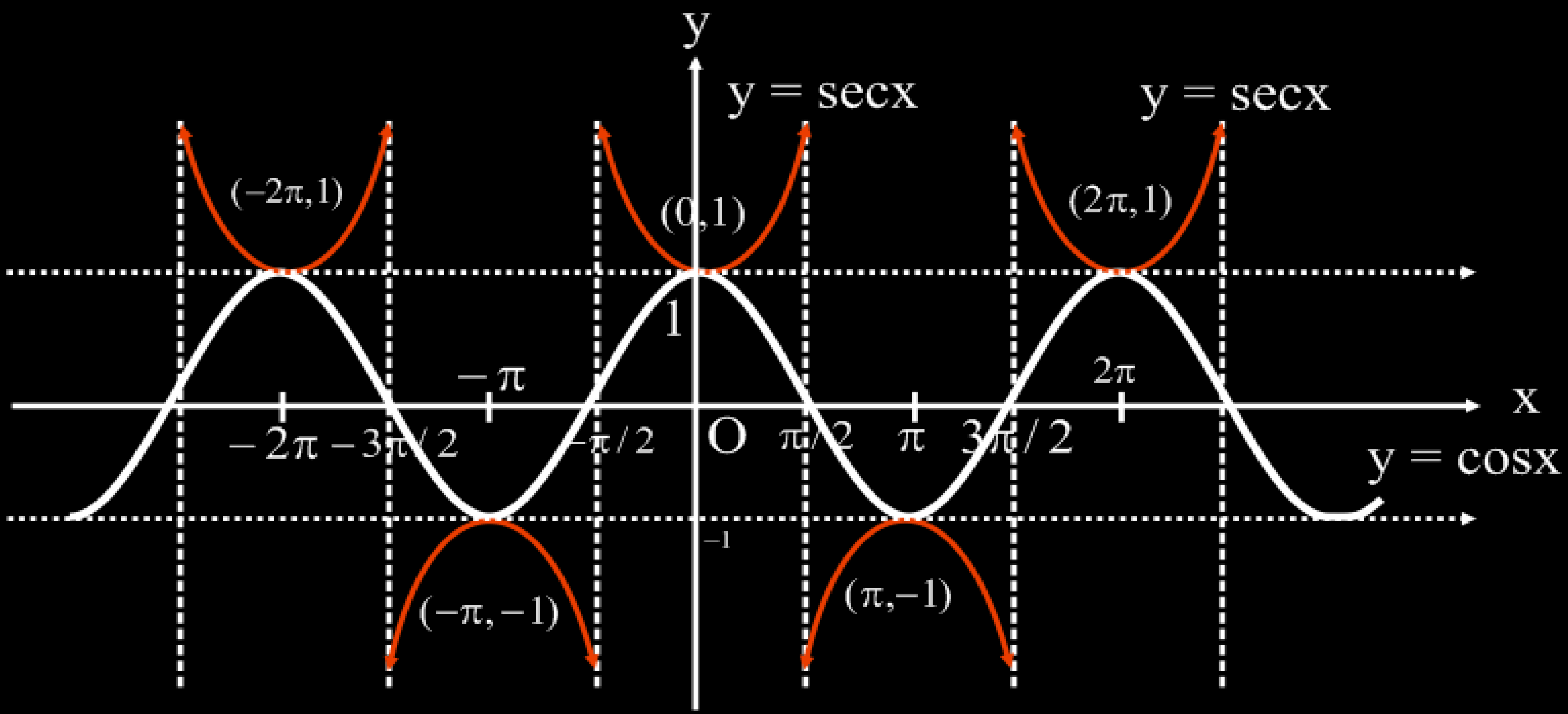


$y = \cot x$

y = f(x) = cosec x



$y = f(x) = \sec x$



Range of sin x, cos x ∈ [– 1, 1]

Range of sec x, cosec x $\in (-\infty, -1] \cup [1, \infty)$

Range of tan x, cot x $\in (-\infty, \infty)$

Examples

Q. *Find range of y*

$y = \sin(2x)$

Q. *Find range of y*

$$y = \sin(x^2)$$

Q. *Find range of y*

$y = \sin\sqrt{x}$

Q. *Find range of y*

$$y = \cos^2 x$$

Q. *Find range of y*

$$y = \cos^2 x - \sin^2 x$$

Q. ***Find range of y***

$$y = \cos^4\frac{x}{2} - \sin^4\frac{x}{2}$$

Q. *Find range of y*

$$y = \cos^4 x - \sin^4 x$$

Q. *Find range of y*

$$y = (\sin x + 2)^2 + 1$$

Q. *Find range of y*

$$y = 4\tan x \cos x$$

Polynomial Function

For Example : $f(x) = ax^2 + bx + c,$

$f(x) = ax^3 + bx^2 + cx + d$

Note

(1) A polynomial function is always continuous.

(2) A polynomial of degree one is called linear function.

i.e. $f(x) = ax + b, a \neq 0$

(3) There are two polynomial functions, satisfying the relation ;

$f(x).f(1/x) = f(x) + f(1/x)$. They are :

(i) $f(x) = x^n + 1$ & (ii) $f(x) = 1 – x^n$,

where n is a positive integer.

Q. A polynomial function satisfies the following condition

$$f(x)\ \ f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

and f(4) = 65. Find f(x) and f(3)

(4) A polynomial of degree one with no constant term is called an odd linear function.

i.e. $f(x) = ax, a \neq 0$

(5) A polynomial of degree odd has its range $(-\infty, \infty)$ but a polynomial of degree even has a range which is always subset of R.

Algebraic Function

A function *f* is called an algebraic function if it can be constructed using algebraic operations such as addition, substraction, multiplication, division and talking roots started with polynomials.

Example

$$f(x) = \sqrt{x^2 + 1}; \quad g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2) \times \sqrt[3]{x + 1}$$

Note

All polynomial are algebraic but not the converse. Function which are not algebraic are known as Transcendental function.

Fractional \ Rational Function

A rational function is a function of the form.

$$y = f(x) = \frac{g(x)}{h(x)},$$

where g (x) & h (x) are polynomial & h (x) $\neq$ 0. The domain of f(x) is set of real x such that h(x) $\neq$ 0

Example

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}; \quad D = \{x \mid x \neq \pm 2\}$$

Exponential Function

A function $f(x) = a^x$ $(a > 0, a \neq 1, x \in R)$ is called an exponential function.

Graph of a^x base a > 1

Graph of a^x base $0 < a < 1$

Logarithmic Function

$y = \log_a x,\ x > 0,\ a > 0,\ a \neq 1$

Graph of $y = \log_a x$ base $a > 1$

Graph of $y = \log_a x$ base $0 < a < 1$

Graph of *l*nx & e^x together

Q. Find Domain of $y = \sqrt{\ln x}$

Q. Find Domain of $\frac{1}{\ln x}$

Q. Find Domain of $\sqrt{4x - x^2}$

Q. Find Domain of $\log_x (2x-1)(x-3)$

Q. Find Domain of $e^{1/x}$

Q. Find Domain of $\log_{10}(\log_{10}(1 + x^3))$

Absolute Value Function/ Modulus Function

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Domain, Range & Graph of $| x |$

Q. Find Domain, Range & Graph of $y = | x - 1 |$

Q. Find Domain, Range & Graph of $y = |x| + 1$

Q. Find Domain, Range & Graph of $y = |x + 1|$

Q. Find Domain, Range & Graph of $y = |x - 1| + 1$

Q. Find Domain and Range of $f(x) = \dfrac{1}{|x|}$

Signum Function

A function y = f(x) = Sgn (x) is defined as follows :

$$y = f(x) = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ 1 & \text{for} \quad x < 0 \end{cases}$$

Domain Range & Graph of Sgn (x)

Q. Find Domain and Range of $y = \text{Sgn}\,(x^2 - 1)$

Q. Find Domain and Range of $y = \text{Sgn}(x^2 + 1)$

Q. Find Domain and Range of

$y = \text{Sgn}\,(x^2 + x + 1)$

Q. Find Domain and Range of

$\log_{10} \operatorname{sgn}(x)$

Q. Find Domain and Range of $\operatorname{sgn}(\ell n(x^2 - x + 2))$

Q. Find Domain and Range of sgn (cosx)

Q. Find Domain and Range of $\operatorname{sgn}(\cos(\sin x))$

Greatest Integer Or Step Up Function

The function $y = f(x) = [x]$ is called the greatest integer function where $[x]$ denotes the greatest integer less than or equal to x.

Domain, Range, Graph of [x]

Q. Find Domain of $f(x) = \frac{1}{[x]}$

Properties of [x]

(1) $[x] \le x < [x] + 1$ and

$x - 1 < [x] \le x$, $0 \le x - [x] < 1$

(2) $[x + m] = [x] + m$ if m is an integer.

(3) $[x] + [-x] = \begin{cases} 0 & \text{if x is an integer} \\ -1 & \text{Otherwise} \end{cases}$

Q. Find $D_f = \sqrt{([x]-1)} + \sqrt{4-[x]}$

Q. Find Domain of $\log_{10}\left(\sqrt{3}-\sqrt{7-x^2}\right)$

Q. Find Domain of $\frac{\sqrt{x-2}}{\sqrt{x-3}}$

Q. Find Domain of $\sqrt{\dfrac{\sqrt{x-2}}{x-3}}$

Q. Find Domain of $\sqrt{(x-5)\,l\,n^2(x-3)}$

Q. Find Domain of $\sqrt{\left(x^2-3x-10\right)\ln^2(x-3)}$

Q. Find Domain of $\sqrt{\left(e^{x}-1\right)(x-2)(x-3)}$

Q. Find Domain of $\sqrt{(e^x)(10^x - 1)(x-2)}$

Q. Find Domain of $\dfrac{\sqrt{\cos x - ½}}{\sqrt{(x)(1-x)}}$

Fractional Part Function {x}

It is defined as :

$g(x) = \{x\} = x - [x]$.

Domain, Range, Graph of {x}

Q. Domain & Range of $y = \dfrac{1}{\{x\}}$

Q. Domain & Range of $f(x) = \log_{10}\{x\}$

Q. Domain & Range of $y = [\sin\{x\}]$

Properties of $\{x\}$

Q. $\{x + n\} = \{x\} \quad n \in I$

Properties of {x}

Q. $\{x\} + \{-x\} = \begin{cases} 0 & x \in I \\ 1 & x \notin I \end{cases}$

Properties of {x}

Q. $[\{x\}] = 0$

Properties of $\{x\}$

Q. $\{[x]\} = 0$

Examples of Domain

Q. $$f(x) = \frac{1}{\sqrt{|x| - x}}$$

Q. $$f(x)=\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{1-x}{1+x}}$$

Q. $g(x) = \dfrac{1}{\sqrt{x - |x|}}$

Q. $f(x) = \left(\log_{\frac{x-2}{x+3}} 2\right) + \sqrt{9 - x^2}$

Q. $$f(x)=\frac{1}{[x]}+\log_{1-\{x\}}(x^2-3x+10)+\frac{1}{\sqrt{2-|x|}}+\frac{1}{\sqrt{\sec(\sin x)}}$$

Q. Find value of $\left[\frac{3}{4}\right]+\left[\frac{3}{4}+\frac{1}{100}\right]+\left[\frac{3}{4}+\frac{2}{100}\right]+....\left[\frac{3}{4}+\frac{99}{100}\right]$

Q. Find Domain of $^{x-10}C_{20-x}$

Q. Find Domain & Range of $f(x)=\dfrac{3x+|x|}{x}$

Q. Find Range of $f(x) = 4\tan x \cos x$

Q. Find Range of $f(x) = \cos^4 \frac{x}{5} - \sin^4 \frac{x}{5}$

Q. Find Range of $f(x) = \sin\sqrt{x}$

Q. Find Range of $f(x) = \cos(2\sin x)$

Q. Find Range of $f(x) = 3 - 2^x$

Q. Find Range of $f(x) = \sin(\log_2 x)$

Q. Find Range of $y = \frac{\tan\left(\pi[x-\pi]\right)}{x^2 - 3x + 4}$

Q. Find Range of $f(x) = \cos 2x - \sin 2x$

Q. Find Range of $f(x) = \cot^2\left(x - \frac{\pi}{4}\right)$

Q. Find Range of $\log_{10}(x^2 - 2x + 2)$

Q. Find Range of $\log_{\frac{1}{10}}\left(x^2+2x+2\right)$

Q. Find Range of $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

Q. Find Range of $f(x) = x + \dfrac{1}{x}$

Q. Find Range of $\dfrac{x-[x]}{1-[x]+x}$

Q. Find Range of $y = \tan\left(\{x\}\dfrac{\pi}{4}\right)$

Q. Find Range of $y = \tan\left(\frac{\pi}{4}\ \mathrm{Sgn}\left(x^2 - 1\right)\right)$

Q. Solve the equation $2[x] = x + \{x\}$

Q. Range of $y = | x^2 - x - 6 |$

Q. Range of $y = \dfrac{1}{x^2 + x + 1}$

Q. Range of $y = |\sin x| + |\cos x|$

Q. Domain of $\sqrt{x^{14}-x^{11}+x^{6}-x^{3}+x^{2}+1}$

Q. Domain of $f(x)=\sqrt{3^{x-1}+5^{x-1}+7^{x-1}-83}$

Range of Linear

$$y = ax + b \quad ;a \neq 0$$

$$y \in R$$

Example

Q. $y = f(x) = x + 1$

Range of $\frac{1}{\text{Linear}}$

$$y = \frac{1}{ax + b}$$

$$\mathbf{y \in R - \{0\}}$$

Range of $\frac{\text{Linear}}{\text{Linear}}$

$$y = \frac{ax+b}{cx+d} \qquad \mathbf{y \in R - \left\{ \frac{a}{c} \right\}}$$

Example

Q. $y = \frac{2x+3}{x+1}$, *Find range of y*

Example

Q. $y = \dfrac{1}{3x-1}$, *Find range of y*

Example

Q. $y = \dfrac{x(x-1)}{x-1}$, *Find range of y*

Example

Q. $$y=\frac{(x-1)(x-2)(x-3)}{(x-2)(x-3)}\ ,\quad \textit{Find range of } y$$

Range of

$$\frac{\text{Linear}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Linear}}$$

- Assume y

Range of $\frac{\text{Linear}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Linear}}$

- Assume y
- Check for common roots in numerator & denominator

Range of $\frac{\text{Linear}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Linear}}$

- Assume y
- Check for common roots in numerator & denominator
- Form Quadratic Equation

Range of $\frac{\text{Linear}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Linear}}$

- Assume y
- Check for common roots in numerator & denominator
- Form Quadratic Equation
- Apply $D \geq 0$ (since x is real)

Range of

$$\frac{\text{Linear}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Quadratic}}, \frac{\text{Quadratic}}{\text{Linear}}$$

- Assume y
- Check for common roots in numerator & denominator
- Form Quadratic Equation
- Apply D ≥ 0 (since x is real)
- Solve inequality in y and hence the range

Note

Always check for coefficient of x^2 not equal to zero

Example

Find range of following

Q. $$\frac{x^2 + 2x - 11}{2(x-3)}$$

Example

Find range of following

Q. $$\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

Example

Find range of following

Q. $$\frac{(x+1)(x-2)}{x(x+3)}$$

Example

Find range of following

Q.

$$\frac{x^2+2x-2}{x^2+2x+1}$$

Example

Find range of following

Q. $$\frac{x^2+14x+9}{x^2+2x+3}$$

Example

Find range of following

Q. $$\frac{x^2-5x+6}{x^2-4x+3}$$

Q. Range of $f(x) = \dfrac{1}{\ln x}$

Q. Range of $f(x) = \dfrac{1}{\sin^4 x + \cos^4 x}$

Q. Range of $f(x) = \log_2\left(\dfrac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

Q. Range of $y = [\sin x]$

Q. Range of $y = 2 - [\sin x] - [\sin x]^2$

Equal or Identical function

Two functions f & g are said to be equal if :

(i) The domain of f = the domain of g.

(ii) The range of f = the range of g and

(iii) f(x) = g(x), for every x belonging to their common domain

Q. Check if function are identical or not

Q.1 $y = \ln x^2$; $y = 2\ln x$

Q. $y = \operatorname{cosec} x \quad ; \quad y = \frac{1}{\sin x}$

Q. $f(x) = \tan x$; $g(x) = \dfrac{1}{\cot x}$

Q. $f(x) = \sec x$; $g(x) = \dfrac{1}{\cos x}$

Q. $f(x) = \cot^2 x . \cos^2 x$; $g(x) = \cot^2 x - \cos^2 x$

Q. $f(x) = \text{Sgn}\,(x^2 + 1)$; $g(x) = \sin^2 x + \cos^2 x$

Q. $f(x) = \tan^2 x . \sin^2 x$; $g(x) = \tan^2 x - \sin^2 x$

Q. $f(x) = \sec^2 x - \tan^2 x$; $g(x) = 1$

Q. $f(x) = \log_x e$; $g(x) = \dfrac{1}{\log_e x}$

Q. $f(x) = \log_e x$; $g(x) = \dfrac{1}{\log_x e}$

Q. $f(x) = \sqrt{x^2 - 1}$; $g(x) = \sqrt{x - 1} . \sqrt{x + 1}$

Q. $f(x)=\sqrt{1-x^2}$; $g(x)=\sqrt{1-x}\,.\sqrt{1+x}$

Q. $f(x) = e^{\ln e^{x}}$; $g(x) = e^{x}$

Q. $f(x)=\sqrt{\dfrac{1-\cos 2x}{2}}$; $g(x)=\sin x$

Q. $f(x) = \log(x+2) + \log(x-3) - g(x) = \log(x^2-x-6)$

Q. $f(x) – x| x |$; $g(x) = x^2 sgn\ x$

Q. $f(x) = \dfrac{1}{1+\dfrac{1}{x}}$; $g(x) = \dfrac{x}{1+x}$

Q. $f(x) = [\{x\}]$; $g(x) = \{[x]\}$

Classification of Function

One-one Function (Injective Mapping)

A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if “different elements of A have different f images in B.”

$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

Example

$R \to R$ $f(x) = x + 1$, $f(x) = e^x$, $f(x) = x^3$

Method – 1 (Graph)

If a line parallel to x-axis cuts the graph of the function atleast at two points, then f is many-one. (not one-one)

Q. Find whether function is one-one or not

$y = [x], \{x\}, \frac{1}{x}, \frac{1}{x^2}$ sinx, cosx, tanx

$x, x^2, x^3, |x|, Sgn(x), e^x$

Method – 2 by $\frac{dy}{dx}$

Meaning of $\frac{dy}{dx}$

(1) Local maxima

(2) Local Minima

$$\text{If } \frac{dy}{dx} \geq 0 \quad \forall x \in D_f \Rightarrow \text{function is one} - \text{one}$$

$$\text{Or } \frac{dy}{dx} \leq 0 \quad \forall x \in D_f \Rightarrow \text{function is one} - \text{one}$$

Find weather function is one-one or not

Q. $y = 2x + \sin x$

Find weather function is one-one or not

Q. $y = e^x + x$

Find weather function is one-one or not

Q. $y = e^{-x} - x$

Find weather function is one-one or not

Q. $y = (x - 1)(x - 2)(x - 3)$

Find weather function is one-one or not

Q. $y = x^3$

Find weather function is one-one or not

Q. $y = \dfrac{x^4+1}{x^2+1}$

Many One – Function
(Not Injective)

A function $f : A \to B$ is said to be a many one function if two or more elements of A have the same f image in B.

$x_1, x_2 \in A$, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$

Example : $R \to R$ $f(x) = [x]$; $f(x) = |x|$;

$f(x) = ax^2 + bx + c$; $f(x) = \sin x$

Note

If a function is one-one, it cannot be many-one and vice versa.

One-one + Many-one = Total number of mappings.

Onto function (Surjective mapping) (Co-Domain = Range)

If the function $f : A \to B$ is such that each element in B (co-domain) is the f image of atleast one element in A, then we say that f is a function of A ‘onto’ B.

Example

$f : R \rightarrow R$ $f(x) = 2x + 1;$

$f : R \rightarrow R^+$ $f(x) = e^x;$

$f : R^+ \rightarrow R$ $f(x) = \ln x$

Into Function

Range ≠ Co-Domain

If $f : A \rightarrow B$ is such that there exists atleast one element in co-domain which is not the image of any element in domain, then f(x) is into.

Example

$f : R \to R$ $f(x) = \{x\}$,

$[x]$,

$\text{sgn}\, x$,

$f(x) = ax^2 + bx + c$

Example

$f : R \rightarrow R \quad f(x) = [x]$,

$| x |$,

sgn x,

$ax^2 + bx + c$

Note

If a function is onto, it cannot be into and vice versa.

A polynomial of degree even define from R → R will always be into & a polynomial of degree odd defined from R → R will always be onto.

A function can be one of these four types :

(a) one-one onto (injective & surjective)

(b) One-one into (injective & surjective)

(c) Many-one onto (surjective but not injective)

(d) Many-one into (neither surjective nor injective)

If f is both injective & surjective, then it is called a **Bijective** mapping. The bijective function are also named as invertible.

Example on Classification

Q. Classify as one-one onto, one-one into, many-one onto or many-one into.

$f : R \to R$

$f(x) = e^x + e^{-x}$

Q. $f : R \to R$

$f(x) = \sqrt{1+x^2}$

Q. $f : R \rightarrow R$

$f(x) = x^3$

Q. $f : R \rightarrow R$

$f(x) = |x|\, \text{Sgn}(x)$

Q. $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\rightarrow[-1,1]$ $\quad f(x)=\sin x$

Q. $f : [-1,1] \to [-1, 1]$ $f(x) = \sin 2x$

Q. The function $f : [2, \infty) \rightarrow Y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if :

(A) $Y = R$

(B) $Y = [1, \infty)$

(C) $Y = [4, \infty)$

(D) $[5, \infty)$

Q. $f : R \rightarrow R$

$x^3 - 2x^2 + 5x + 3$

Q. $f : R \rightarrow R$

$f(x) = 2x^3 - 6x^2 - 18x + 17$

Q. If the function $f(x) = x^2 + bx + 3$ is not injective for values of x in the interval $0 \leq x \leq 1$ then b lies in

(A) $(-\infty, \infty)$

(B) $(-2, \infty)$

(C) $(-2, 0)$

(D) $(-\infty, 2)$

Q. $f : R \rightarrow R$ $f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$

Q. $f : R \to R$ is defined as

$$f(x) = \begin{cases} x^2 + 2mx - 1 & \text{for } x \le 0 \\ mx - 1 & \text{for } x > 0 \end{cases}$$

If $f(x)$ is one-one then m must lies in the interval

(A) $(-\infty, 0)$
(B) $(-\infty, 0]$
(C) $(0, \infty)$
(D) $[0, \infty]$

Q. Let a function f defined from R → R as :

$$f(x) = \begin{cases} x + m & \text{for } x < 1 \\ 2mx - 1 & \text{for } x > 1 \end{cases}$$

If the function is surjective on R then m must lie in the interval.

(A) (0,2] (B) (–∞, 0] (C) (–∞, 0) (D) (0, ∞)

Permutation Based Problems

$$f : A \rightarrow B$$

Case – I

When both the sets A and B contain an equal number of elements

Case – I

When both the sets A and B contain an equal number of elements

(i) Total number of functions

Case – I

When both the sets A and B contain an equal number of elements

(ii) Number of functions one-one

Case – I

When both the sets A and B contain an equal number of elements

(iii) Number of functions many-one

Case – I

When both the sets A and B contain an equal number of elements

(iv) Number of onto function

Case – I

When both the sets A and B contain an equal number of elements

(v) Number of into function

Case – II

When number of elements in A(domain) is more than B

Case – II

When number of elements in A(domain) is more than B

(i) Total number of functions

Case – II

When number of elements in A(domain) is more than B

(ii) One-one (injective)

Case – II

When number of elements in A(domain) is more than B

(iii) Many-one

Case – II

When number of elements in A(domain) is more than B

(iv) Number of onto function

Case – II

When number of elements in A(domain) is more than B

(v) Number of into function

Case – III

Number of elements in codomain (B) is more than A

Case – III

Number of elements in codomain (B) is more than A

(i) Total functions

Case – III

Number of elements in codomain (B) is more than A

(ii) Number of injective mapping

Case – III

Number of elements in codomain (B) is more than A

(iii) Number of many-one

Case – III

Number of elements in codomain (B) is more than A

(iv) Number of onto function

Case – III

Number of elements in codomain (B) is more than A

(v) Number of into function

Functional Equations

Q. For $x \in R$, the function $f(x)$ satisfies $2f(x) + f(1 - x) = x$ then the value of $f(4)$?

Functional Equations

Q. If $2\,f(x^2) + 3f(1/x^2) = x$ $(x \neq 0)$ then $f(x^2)$ is :

(A) $\dfrac{1-x^4}{-5x^2}$

(B) $\dfrac{1-x^2}{5x}$

(C) $\dfrac{5x^2}{1-x^4}$

(D) $-\dfrac{2x^4+x^2-3}{5x^2}$

Functional Equations

Q. Let $f(x)$ and $g(x)$ be functions which take integers as arguments.

Let $f(x+y) = f(x)+g(y)+8$ for all integer x and y. Let $f(x) = x$ for all negative integers x, and let $g(8) = 17$. The value of $f(0)$ is

(A) 17 (B) 9 (C) 25 (D) -17

Functional Equations

Q. Let $f(x) = ax^7 + bx^3 + cx - 5$, where a, b and c are constants. If $f(-7) = 7$, then $f(7)$ equals

(A) –17 (B) –7 (C) 14 (D) 21

Functional Equations

Q. The function $f : R \rightarrow R$ satisfies the condition $m\,f(x-1) + n\,f(-x) = 2|x| + 1$. If $f(-2) = 5$ and $f(1) = 1$, then $(m + n)$ equals

(A) 4/3 (B) 3 (C) 4 (D) 6

Functional Equations

Q. If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2 \ (x \neq 0)$ find $f(2)$

Functional Equations

Q. Let f be a real valued function of real and positive argument such that

$$f(x) + 3x\, f\left(\frac{1}{x}\right) = 2(x + 1)$$ for all real $x > 0$. The value of $f(10099)$ is

(A) 550 (B) 505 (C) 5050 (D) 10010

Functional Equations

Q. Let $f : R \rightarrow R$ be a function such that

$$\left(f\left(x^3+1\right)\right)^{\sqrt{x}} = 5, \quad \forall x \in (0, \infty)$$

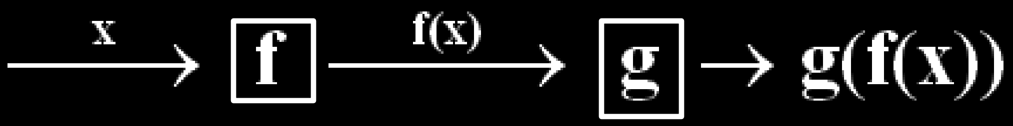
$$\left(f\left(\frac{27+y^3}{y^3}\right)\right)^{\sqrt{\frac{27}{y}}}$$ for $y \in (0, \infty)$ is equal to

(A) 5 (B) 5^2 (C) 5^3 (D) 5^6

Composite of Uniformly & Non-Uniformly Defined Function

Let f : A → B & g : B → C be two functions. Then the function gof : A → C defined by (gof) (x) = g(f(x)) ∀ x ∈ A is called the composite of the two functions

f & g. Diagrammatically



Note

(i) The image of every $x \in A$ under the function gof is the g-image of the f-image of x.

(ii) gof is defined only if $\forall\ x \in A$, $f(x)$ is an element of the domain of g so that we can take its g-image. The product gof of two functions f & g, the range of f must be a subset of the domain of g.

(iii) gof in general is not equal to fog.

Example

Q.1 If $f(x) = \log_{100x}\left(\frac{2\log_{10}x + 2}{-x}\right)$ and $g(x) = \{x\}$.

If the function (fog)(x) exists then find the range of g(x).

Q.2 $f(x) = Sgn\,(x)$ and $g(x) = 1 + x - [x]$

then $f[g(x)] = ?$, $g[f(x)] = ?$

Properties of Composite Functions

Properties of Composite Functions

(i) The composite of functions is not commutative i.e. gof ≠ fog.

Properties of Composite Functions

(i) The composite of functions is not commutative i.e. gof ≠ fog.

(ii) The composite of functions is associative i.e. if f, g, h are three functions such that fo (gof) & (fog) oh are defined, then fo (goh) oh.

Properties of Composite Functions

(i) The composite of functions is not commutative i.e. gof $\neq$ fog.

(ii) The composite of functions is associative i.e. if f, g, h are three functions such that fo (gof) & (fog) oh are defined, then fo (goh) oh.

(iii) The composite of two bijections is a bijection i.e. if f and g are two bijections such that gof is defined, then gof is also a bijection.

Examples on Composite Function

Q. If $f(x) = x^2$ and $g(x) = x - 7$

Find : (i) gof

Examples on Composite Function

Q. If $f(x) = x^2$ and $g(x) = x - 7$

Find : (ii) fog

Examples on Composite Function

Q. If $f(x) = x^2$ and $g(x) = x – 7$

Find : (iii) gog

Examples on Composite Function

Q. If $f(x) = x^2$ and $g(x) = x - 7$

Find : (iv) fof

Examples on Composite Function

Q. $f : \mathrm{R} \rightarrow \mathrm{R}$ be the function defined by

$f(x) = ax^2 - \sqrt{2}$

for some positive a. If (fof) $\sqrt{2} = -\sqrt{2}$ then the value of 'a' is

(A) $\sqrt{2}$ (B) 2

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Examples on Composite Function

Q. Let $f(x) = \sqrt{x}; g(x) = \sqrt{2 - x}$, find the domain of

(A) fog

Examples on Composite Function

Q. Let $f(x) = \sqrt{x}; g(x) = \sqrt{2 - x}$, find the domain of
(B) gof

Examples on Composite Function

Q. Let $f(x) = \sqrt{x}; g(x) = \sqrt{2-x}$, find the domain of

(C) fof

Examples on Composite Function

Q. Let $f(x) = \sqrt{x}; g(x) = \sqrt{2 - x}$, find the domain of

(D) gog

Examples on Composite Function

Q. Prove that : $f(x) = x^2$

$$\underbrace{fof\ of \ldots\ldots fof}_{n\ times} = x^{2^n}$$

Examples on Composite Function

Q. Suppose that $f(x) = x^x$ and $g(x) = x^{2x}$. Which one of the following represents the composite function f [g (x)], is

(A) $x^{x^{2x+1}}$

(B) $x^{2x^{2x}}$

(C) $x^{2x^{x+1}}$

(D) $x^{2x^{2x+1}}$

Examples on Composite Function

Q. $f(x) = \dfrac{2x - 7}{x + 3}$, find g such that g(f(x)) = x. for all x in domain of f.

Composite of Non Uniformly Defined Function

Examples

Q. $$f(x) = \begin{cases} 1 + x & \text{if } 0 \le x \le 2 \\ 3 - x & \text{if } 2 < x \le 3 \end{cases}$$ find fof

Q. $f(x) = \begin{cases} 1 - x & \text{if } 0 \le x \le 1 \\ x + 2 & \text{if } 1 < x < 2 \\ 4 - x & \text{if } 2 \le x \le 4 \end{cases}$ find (fof) (x)

Q. $f(x) = \begin{cases} 1-x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$ and $g(x) = \begin{cases} -x & \text{if } x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$

find (fog) (x) and (gof) (x)

Homogeneous Function

A function is said to be homogeneous with respect to any set of variable when each of its terms is of the same degree with respect to those variables.

$f(tx,ty) = t^n.f(x,y)$ then $f(x,y)$ is homogeneous function of degree n.

Example

Q. Find if f(x) is homogeneous or not

$$\mathbf{f(x, y) = \frac{x}{y} \; \mathit{l}n\left(\frac{y}{x}\right)}$$

Q. Find if f(x) is homogeneous or not

$$f(x, y) = x + y \cos\left(\frac{y}{x}\right)$$

Q. Find if f(x) is homogeneous or not

$$f(x, y) = x + y \quad \sin x$$

Q. Find if f(x) is homogeneous or not

$$f(x, y) = \sqrt{x^2 + y^2} + x$$

Bounded Function

(i) A function is said to be bounded if $| f(x) | \leq M$, where M is a finite quantity.

Bounded Function

(ii) If graph of function can be bounded between 2 horizontal lines.

Bounded Function

(iii) If Range of function contains ∞ or $-\infty \Rightarrow$ function is not bounded.

Example

Find if f(x) is bounded or not

Q. sgn (x)

Find if $f(x)$ is bounded or not

Q. $\sin(x)$

Find if f(x) is bounded or not

Q. tan (x)

Find if f(x) is bounded or not

Q. [x]

Find if $f(x)$ is bounded or not

Q. $\{x\}$

Find if f(x) is bounded or not

Q. $|x|$

Find if $f(x)$ is bounded or not

Q. $\cos(x)$

Find if f(x) is bounded or not

Q. $\ln(x)$

Find if f(x) is bounded or not

Q. e^x

Q. $f(x) = 2^{\frac{1}{x-1}}$ on $(0,1)$

Q. $f(x) = 2^{\frac{1}{x-1}}$ on $(1,2)$

Implicit & Explicit Function

A function defined by an equation not solved for the dependent variable is called an **IMPLICIT FUNCTION**. For eg. The equation $x^3 + y^3 = 1$ defines y as an implicit function. If y has been expressed in terms of x alone then it is called an **EXPLICIT FUNCTION**.

Example

Q. $x = 2y - y^2$. Find domain of implicit form

Odd and Even Function

A function f(x) defined on the symmetric interval (–a, a)

If f(–x) = f (x) for all x in the domain of 'f' then f is said to be an even function.

If f(–x) = –f(x) for all x in the domain of 'f' then f is said to be an odd function.

Note : A function may neither be odd nor even.

Example

Identify given below functions as odd, even or none

Q. $f(x) = x^2$

Q. $f(x) = x^4 + x^5 + x^3 + 3x$

Q. $f(x) = \cos x$

Q. $f(x) = \tan x$

Q. $f(x) = 2x^3 - x + 1$

Q. $f(x) = \ln \dfrac{1-x}{1+x}$

Q. $f(x) = x\tan^2 x + x^3 - 3x$

Q. $f(x) = x^3 + x^2$

Note

(a) $f(x) - f(-x) = 0 \Rightarrow f(x)$ is even &

$f(x) + f(-x) = 0 \Rightarrow f(x)$ is odd.

Note

(b) Inverse of an even function is not defined.

Note

(c) Every even function is symmetric about the X–axis & every odd function is symmetric about the origin.

Note

(d) Every function can be expressed as the sum of an even & an odd function.

Note

(e) The only function which is defined on the entire number line & is even and odd at the same time is $f(x) = 0$

Note

(f) If f and g both are even or both are odd then the function f.g will be even but if any one of them is odd then f.g will be odd.

Example

Q. Express e^x as sum of an odd & even function

Q. Identify whether the function given below is Odd/Even/NONE

(1) $y = \ln \frac{1-x}{1+x}$

Q. Identify whether the function given below is Odd/Even/NONE

(2) $y = x \sin^2 x - x^3$

Q. Identify whether the function given below is Odd/Even/NONE

(3) $y = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$

Q. Identify whether the function given below is Odd/Even/NONE

(4) $y = \dfrac{1+2^x}{1-2^x}$

Q. Identify whether the function given below is Odd/Even/NONE

(5) $y = \ln\left(x + \sqrt{1 + x^2}\right)$

Inverse of a Function

Let $f : A \rightarrow B$ be a one-one & onto function, then their exists a unique function $g : B \rightarrow A$ such that $f(x) = y \Leftrightarrow g(y) = x, \forall x \in A$ & $y \in B$. Then g is said to be inverse of f.

Thus $g = f^{-1} : B \rightarrow A \{(f(x), x) \mid (x, f(x)) \in f\}$.

Example

Compute the inverse of the following bijective.

Q. If $f : R \rightarrow R^{+}$, $f(x) = 10^{x+1}$

Compute the inverse of the following bijective.

Q. If $f : (-2, \infty) \to R$, $f(x) = 1 + \ln(x + 2)$

Compute the inverse of the following bijective.

Q. If $f : R \rightarrow (0,1)$, $f(x) = \dfrac{2^x}{1+2^x}$

Properties of Inverse Function

(1) The inverse of a bijection is unique.

Properties of Inverse Function

(2) If $f : A \rightarrow B$ is a bijection & $g : B \rightarrow A$ is the inverse of f, then $fog = I_B$ and $gof = I_A$, where I_A & I_B are identity functions on the sets A & B respectively.

Properties of Inverse Function

(3) The inverse of a bijection is also a bijection.

Properties of Inverse Function

(4) If f & g are two bijections $f : A \rightarrow B$. $g : B \rightarrow C$ then the inverse of gof exists and $(gof)^{-1} = f^{-1} o g^{-1}$.

Note

Function are not generally one-one onto. To make them invertible we redefine them in such a manner that they become one-one & onto in certain part of domain

Example

Q. A function

$$f:\left[\frac{3}{2},\infty\right)\rightarrow\left[\frac{7}{4},\infty\right)$$

$f(x) = x^2 - 3x + 4$. Solve the equation $f(x) = f^{-1}(x)$

Q. If $f : R \to R$ $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ is an invertible mapping find range of a.

Q. $f : [0, \infty) \rightarrow [1, \infty)$

$$f(x) = \frac{e^{x} + e^{-x}}{2} \quad \text{find } f^{-1}(x)$$

Q. If $f : [0,\infty) \rightarrow [1, \infty)$

$$f(x) = \frac{e^{x/2} - e^{-x/2}}{2}; \text{ Find } f^{-1}(x)$$

Periodic Function

A function f(x) is called periodic if there exists a positive number T ($T > 0$) called the period of the function such that $f(x + T) = f(x)$, for all values of x within the domain of x.

Note

(a) $f(T) = f(0) = f(-T)$, where 'T' is the period.

Note

(a) f (T) = f (0) = f (–T), where 'T' is the period.

(b) Inverse of a periodic function does not exist.

Example

Q. If f is periodic and $T = 1$, $f(2) = 5$, $f(9/4) = 2$

Find $f(3)$, $f(9)$, $f(-3)$, $f(\frac{1}{4})$

Q. If $f(x) = x \; \forall \; x \in [0, 2]$ and $f(x)$ is even and $T = 4$, find $f(5)$, $f(7.1)$, $f(-1)$, $f(-7)$.
Also draw graph of function.

Note

1. Every constant function is always periodic, with no fundamental period

Note

2. If f (x) has a period T & g (x) also has a period T then it does not mean that f(x) + g (x) must have a period T. e.g. $f(x) = |\sin x| + |\cos x|$.

Note

3. If f(x) has a period p, then

$\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has a period p.

Example

Q. Period of $\sin(2x + 7)$

Q. Period of cos $(2\pi x)$

Q. Period of $|\sin x|$

Q. Find period of $| \sin (3x + 7) | + | \cos (3x + 7) |$

Q. Find period of $f(x) = \cos(\sin x)$

Q. Find period of $f(x) = \sin(\cos x)$

Q. Find period of $y = x - [x]$

Q. Find period of $f(x) = \{3x\}$

Q. Find period of $f(x) = \sin^4 x + \cos^4 x$

Q. Find period of

$$f(x) = \cos\frac{2x}{3} - \sin\frac{4x}{5}$$

Q. Find period of

$$f(x) = \sin x + \cos \sqrt{2}\, x$$

Q Show that :f(x) = cos $\sqrt{x}$; x sin x and sin x + {x} are aperiodic.

Q. Find a if :

$f(x) = (a + 3)x + 5a,\ x \in R$ is periodic.

Q. $\{x\}+\left\{\frac{x}{2}\right\}+\left\{\frac{x}{3}\right\}=f$. Find period of f

Q. If $f(x) = \{x\} + \{2x\} + \{3x\}$, Find period of $f(x)$

Q. If $f(x) = f(x + 2) + f(x - 2)\ \forall\ x \in D_f$, find period of $f(x)$

GENERAL

Note

If x, y are independent variables, then :

(1) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k\,ln\,x$ or $f(x) = 0$

Note

If x, y are independent variables, then :

(2) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n, n \in R$

Note

If x, y are independent variables, then :

(3) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$.

Note

If x, y are independent variables, then :

(4) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$,

where k is a constant.

Q. Which of the following functions is periodic ?

(a) $f(x)=x-[x]$ where $[x]$ denotes the greatest integer less than or equal to the real number x

(b) $f(x) = \sin \frac{1}{x}$ for $x \neq 0$, $f(0) = 0$

(c) $f(x) = x \cos x$

(d) None of these [IIT-JEE 1982]

Q. For real x, the function $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values provided.

(a) $a > b > c$

(b) $a < b < c$

(c) $a > c < b$

(d) $a \leq c \leq b$

[IIT-JEE 1984]

Q. If $g\{f(x)\} = |\sin x|$ and $f\{g(x)\} = \left(\sin\sqrt{x}\right)^2$, then

(a) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

(b) $f(x) = \sin x$, $g(x) = |x|$

(c) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$

(d) f and g cannot be determined [IIT-JEE 1988]

Q. If $f(x) = 3x - 5$, then $f^{-1}(x)$

(a) is given by $\frac{1}{3x-5}$

(b) is given by $\frac{x+5}{3}$

(c) does not exist because f is not one-one

(d) does not exist because f is not onto

[IIT-JEE 1998]

Q. If the function $f : [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$

(b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

(d) not defined

[IIT-JEE 1999]

Q. The domain of definition of the function y (x) is given by the equation $2^x + 2^y = 2$, is

(a) $0 < x \leq 1$

(b) $0 \leq x \leq 1$

(c) $-\infty < x \leq 0$

(d) $-\infty < x < 1$

[IIT-JEE 2000]

Q. For all $x \in (0, 1)$

(a) $e^x < 1 + x$ (b) $\log_e (1 + x) < x$

(c) $\sin x > x$ (d) $\log_e x > x$

[IIT-JEE 2000]

Q. Let $f : (0, \infty) \rightarrow R$ and $F(x) = \int_0^x f(t)\,dt$.

If $F(x^2) = x^2(1 + x)$, then f (4) equals

(a) 5/4 (b) 7 (c) 4 (d) 2

[IIT-JEE 2000]

Q. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 1, & x = 0, \\ 1, & x > 0 \end{cases}$

then for all x, f [g (x)] is equal to

(a) x (b) 1

(c) f(x) (d) g(x)

[IIT-JEE 2001]

Q. If f : [1, ∞] → [2, ∞) is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals.

(a) $\frac{x+\sqrt{x^2-4}}{2}$

(b) $\frac{x}{1+x^2}$

(c) $\frac{x-\sqrt{x^2-4}}{2}$

(d) $1+\sqrt{x^2-4}$

[IIT-JEE 2001]

Q. The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

(a) R/{–1, –2)

(b) (–2, ∞)

(c) R/{–1, –2, –3}

(d) (–3, ∞) / {–1, –2}

[IIT-JEE 2001]

Q. Let $f(x) = (1 + b^2) x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(a) $[0, 1]$

(b) $\left[0, \frac{1}{2}\right]$

(c) $\left[\frac{1}{2}, 1\right]$

(d) $(0, 1]$ [IIT-JEE 2001]

Q. Let E = {1, 2, 3, 4} and F = {1, 2}, Then, the number of onto functions from E to F is

(a) 14 (b) 16 (c) 12 (d) 8

[IIT-JEE 2001]

Q. Let $f(x) = \frac{\alpha x}{x+1}, x \neq -1$. Then, for what value of α is $f[f(x)] = x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 1 (d) –1 [IIT-JEE 2001]

Q. Suppose, $f(x) = (x + 1)^2$ for $x \geq -1$. If g (x) is the function whose graph is reflection of the graph of f(x) with respect to the line y = x, then g(x) equals.

(a) $-\sqrt{x} - 1, x \geq 0$

(b) $\frac{1}{(x+1)^2}, x > -1$

(c) $\sqrt{x+1}, x \geq -1$

(d) $\sqrt{x} - 1, x \geq 0$

[IIT-JEE 2002]

Q. Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then, f is

(a) one-to-one and onto

(b) one-to-one but not onto

(c) onto but not one-to-one

(d) neither one-to-one nor onto [IIT-JEE 2002]

Q. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = \dfrac{x}{1+x}$, then f is

(a) one-one and onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto [IIT-JEE 2003]

Q. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}; x \in R$ is

(a) $(1, \infty)$

(b) $(1, 11/7)$

(c) $(1, 7/3]$

(d) $(1, 7/5)$ [IIT-JEE 2003]

Q. Domain of definition of the function

$f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ for real valued x, is

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$

(b) $\left[-\frac{1}{2},\frac{1}{2}\right]$

(c) $\left(-\frac{1}{2},\frac{1}{9}\right)$

(d) $\left[-\frac{1}{4},\frac{1}{4}\right]$

[IIT-JEE 2003]

Q. If $f(x)=\sin x+\cos x$, $g(x)=x^2-1$, then $g\{f(x)\}$ is invertible in the domain.

(a) $\left[0, \frac{\pi}{2}\right]$

(b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(d) $[0, \pi]$ [IIT-JEE 2004]

Q. $$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$$

Then, f–g is

(a) one-one and into

(b) neither one-one nor onto

(c) many one and onto

(d) one-one and onto [IIT-JEE 2005]

Q. If X and Y are two non-empty sets where $f : X \to Y$, is function is defined such that

$f(c) = \{f(x) : x \in C\}$ for $C \subseteq X$ and

$f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$,

for an $A \subseteq Y$ and $B \subseteq Y$, then

(a) $f^{-1}\{f(A)\} = A$

(b) $f^{-1}\{f(A)\} = A$ only if $f(X) = Y$

(c) $f\{f^{-1}(B)\} = B$ only if $B \subseteq f(x)$

(d) $f\{f^{-1}(B)\} = B$ [IIT-JEE 2005]

Q. Let S={1,2,3,4}. The total number of unordered pairs of disjoint subsets of S is equal to

(a) 25 (b) 34 (c) 42 (d) 41

[IIT-JEE 2010]

Q. Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x \in R$. Then, the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is

(a) $\pm\sqrt{n\pi}, n \in \{0,1,2,...\}$

(b) $\pm\sqrt{n\pi}, n \in \{1,2,...\}$

(c) $\frac{\pi}{2} + 2n\pi, n \in \{...,-2,-1,0,1,2,...\}$

(d) $2n\pi, n \in \{....,-2, -1, 0, 1, 2,...\}$

[IIT-JEE 2011]

Multiple Choice Questions

Q. If $y = f(x) = \dfrac{x+2}{x-1}$, then

(a) $x = f(y)$

(b) $f(1) = 3$

(c) y increases with x for $x < 1$

(d) f is a rational function of x [IIT-JEE 1984]

Q. If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive. Then S contains

(a) $\left(-\infty,-\frac{3}{2}\right)$

(b) $\left(-\frac{3}{2},-\frac{1}{4}\right)$

(c) $\left(-\frac{1}{4},\frac{1}{2}\right)$

(d) $\left(\frac{1}{2},3\right)$ [IIT-JEE 1986]

Q. Let g(x) be a function defined on [–1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and [x, g(x)] is $\sqrt{3}/4$ then the function g(x) is

(a) $g(x) = \pm\sqrt{1-x^2}$ (b) $g(x) = \sqrt{1-x^2}$

(c) $g(x) = -\sqrt{1-x^2}$ (d) $g(x) = \sqrt{1+x^2}$

[IIT-JEE 1989]

Q. If $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function, then

(a) $f\left(\frac{\pi}{2}\right) = -1$

(b) $f(\pi) = 1$

(c) $f(-\pi) = 0$

(d) $f\left(\frac{\pi}{4}\right) = 1$

[IIT-JEE 1991]

Q. Let $f : (0, 1) \to R$ be defined by $f(x) = \frac{b - x}{1 - bx}$, where b is a constant such that $0 < b < 1$. Then,

(a) f is not invertible on (0, 1)

(b) $f \neq f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(c) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(d) f^{-1} is differentiable on (0, 1) [IIT-JEE 2011]

Fill in the Blank Question

Q. The values of $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval.....

[IIT-JEE 1983]

Q. The domain of the function $f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is given by..... [IIT-JEE 1984]

Q. If $f(x) = \sin \log\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then the domain of f(x) is....

[IIT-JEE 1985]

Q. There are exactly two distinct linear functions, ... and …… which map {–1, 1} onto {0, 2}.

[IIT-JEE 1989]

Q. If f is an even function defined on the interval (–5, 5), then four real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are..........

[IIT-JEE 1996]

Q. If $f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cos\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $(gof)(x) = \ldots.$ [IIT-JEE 1996]

Q. Let $y = \sqrt{\dfrac{(x+1)(x-3)}{(x-2)}}$

Find all the real values of x for which y takes real values. [IIT-JEE 1980]

Q. Given $\mathbf{A} = \left\{\mathbf{x} : \frac{\pi}{6} \le \mathbf{x} \le \frac{\pi}{3}\right\}$ and f(x) = cos x–x (1+x) find f(A). [IIT-JEE 1980]

Q. Let f be a one-one function with domain {x,y,z} and range {1, 2, 3}. It is given that exactly one of the following statements is true and the remaining two are false

$f(x) = 1$, $f(y) \neq 1$, $f(z) \neq 2$ determine $f^{-1}(1)$.

[IIT-JEE 1982]

Q. Find the natural number a for which

$$\sum_{k=1}^{n} f(a+k) = 16(2^n - 1).$$

where the function f satisfies the relation

f(x+y) = f(x) f(y) for all natural numbers x, y and further f(1) = 2. [IIT-JEE 1992]

Q. A function $f : IR \to IR$, where IR, is the set of real numbers, is defined by

$$f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$$

Find the interval of values of α for which is onto. Is the functions one-to-one for $\alpha = 3$? Justify your answer. [IIT-JEE 1996]

Q. Find the range of values of t for which

$$2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1},\ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

[IIT-JEE 2005]