Integration
Introduction

Differential of y = f(x)

Q. Differential of x^4, $\sin(x)$, $\tan(x)$, $ln(x)$.

Q. If $\frac{d}{dx}(F(x)+C) = f(x) \Rightarrow \int f(x)dx = F(x) + C$

Q. Antiderivative of a periodic function need not be periodic function.

Eg. $f(x) = \cos x + 1$ is periodic but $\int (\cos x + 1)\, dx = \sin x + x + C$ is aperiodic.

(a) $$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1$$

(b) $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$n \neq -1,\ n \in R$$

Q. $\int e^{\ln\sqrt{x}}\,dx$

Q. $\int e^{-\ln x^2} dx$

Q. $\int \ln\left(\frac{1}{e^x}\right)dx$

Q. $\int \frac{dx}{2\sqrt{x}}$

Q. $\int e^{\ln 2+\ln x}dx$

Q. $\int e^{m \ln x} dx$

Q. $\int 2^{\ln x}\,dx$

(a) $\int \frac{1}{x} dx = \ln|x| + C$

(b) $\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + C$

Q. $$\int \frac{\sqrt{x^4 + x^{-4} + 2}}{x^3}\,dx$$

Q. $\int \frac{dx}{3-2x}$

Q. $$\int \frac{x\,dx}{a+bx}$$

Q. $\int \frac{x\,dx}{x^2 + 2x + 1}$

Q. $\int \frac{dx}{\sqrt{2x-3}+\sqrt{2x+3}}$

Q. $\int \frac{(1+x)^3}{x} dx$

Q. $\int \frac{1}{(x)(x+1)} dx$

Q. $\int \frac{1}{(x+2)(x-3)} dx$

Q. $\int \frac{1}{(x+n)(x-n)} dx$

Q. $\int \frac{x}{(x+2)(x+3)} dx$

(a) $\int e^{x}dx = e^{x} + C$ $\quad$ $\int e^{ax+b}dx = \frac{e^{ax}+b}{a} + C$

(b) $\int a^{x}dx = \frac{a^{x}}{\ln a}$, $\quad$ $\int a^{px+q}dx = \frac{a^{px+q}}{p\ln a}$ $a > 0$

Q. $$\int \frac{2^{x+1}-5^{x-1}}{10^x}\,dx$$

Q. $\int a^x . e^x dx$

Q. $\int (2^x + 3^x)^2 dx$

Q. $$\int \frac{e^{3x} + e^{5x}}{e^{x} + e^{-x}} dx$$

Q. $\int a^{mx} . b^{nx} dx$

Q. $\int \sin x\, dx = \int -\cos x + C$

$$\int \sin(ax+b)d = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$$

Q. $\int \sqrt{1+\sin x}$

Q. $\int \cos 2x \cos 3x$

Q. $\int \sin^4 x \, dx$

Q. $\int \frac{\cos x - \cos 2x}{1 - \cos x} dx$

Q. $\int \sin 2x \cos^2 x \, dx$

Q. $\int \sin^2 x\,dx$

Q. $\int \cos^3 x\,dx$

Q. $\int \cos^4 x \, dx$

Q. $$\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}$$

Q. $$\int \frac{\cos^3 x + \sin^3 x}{\cos x + \sin x} dx$$

Q. $\int \sin x^\circ dx$

Q.

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$$

$$\int \operatorname{cosec}^2(ax+b) = -\frac{1}{a}\cot(ax+b) + C$$

Q. $\int \frac{dx}{1+\cos x}$

Q. $$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx$$

Q. $\int \frac{1-\cos x}{1+\cos x} dx$

Q. $\int \cot^2 x \, dx$

Q. $\int \sec^2 x \operatorname{cosec}^2 x \, dx$

Q. $\int \left(\cot^2 x \cos^2 x\right) dx$

Q. $\int \tan^2 x \sin^2 x dx$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

Q. $$\int \frac{a\sin^3 x + b\cos^3 x}{\sin^2 x \cos^2 x}$$

Q. $$\int \frac{\operatorname{cosec} x + \tan^2 x + \sin^2 x}{\sin x}\, dx$$

Q. $$\int \frac{dx}{1-\sin 3x}$$

Q. $\int \frac{\sin x}{\cos^2 x}(1 - 3\sin^3 x)dx$

$$\int \frac{dx}{1+x^2} = \tan^{-1}x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

Q. $$\int \frac{x^2+\cos^2 x}{x^2+1} \operatorname{cosec}^2 x \, dx$$

Q. $$\int \frac{x^2}{1+x^2}dx$$

Q. $\int \frac{x^4}{1+x^2}dx$

Q. $\int \frac{dx}{\sqrt{9-4x^2}}$

Q. $$\int \frac{dx}{(2x-7)\sqrt{(x-3)(x-4)}}$$

Q. $\int \frac{dx}{(x^2-4x+4)(x^2-4x+5)}$

Q. find $f(x)$ if

$f'(x^2) = 1/x$ for $x > 0$, $f(1) = 1$

Q. find $f(x)$ if

$f'(\sin^2 x) = \cos^2 x$ for all x, $f(1) = 1$

Q. find f(x) if

$f'(\sin x) = \cos^2 x$ for all x, $f(1) = 1$

Assignment on Elementary Integration

Q. $\int 2^x . e^x dx$

Q. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx$

Q. $\int \frac{1-\tan^2 x}{1+\tan^2 x}\,dx$

Q. $\int \frac{1+\tan^2 x}{1+\cot^2 x}\,dx$

Q. $\int \frac{e^{5\ell nx}e^{4\ell nx}}{e^{3\ell nx}-e^{2\ell nx}}dx$

Q. $\int (e^{a \ell n x} + e^{x \ell n a}) dx (a > 0)$

Q. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

Q. $$\int \frac{1+2x^2}{x^2(1+x^2)}dx$$

Q. $\int 4\cos\frac{x}{2}.\cos^2 x.\sin\frac{21}{2}x\,dx$

Q. $\displaystyle\int \frac{\cos x - \sin x}{\cos x + \sin x}(2 + 2\sin 2x)dx$

Q. $\int (3\sin x\cos^2 x - \sin^3 x)dx$

Q. $\int \frac{(1+x)^2}{x(1+x^2)} dx$

Q. $\int \cos x^{\circ} dx$

Q. $\int \frac{(1+x)^2}{x(1+x^2)} dx$

Q. $\int \frac{x}{2x+1} dx$

Q. $\int \frac{\sec 2x - 1}{\sec 2x + 1} dx$

Q. $\int \frac{2x-1}{x-2}dx$

$$\int \frac{e^{2x}-1}{e^{x}}dx$$

Q. $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}} dx (\cos x+\sin x>0)$

Q. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$

Q. $\int \frac{x^6 - 1}{x^2 + 1} dx$

Q. $$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

Q. $\int \frac{x^4 + x^2 + 1}{2(1+x^2)} dx$

Q. $\int \sqrt{1-\sin 2x\,dx}$

Q. $$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x . \cos^2 x} dx$$

Q. $$\int \frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}\,dx$$

Q. $$\int\left[\sin^2\left(\frac{9\pi}{8}+\frac{x}{4}\right)-\sin^2\left(\frac{7x}{8}+\frac{x}{4}\right)\right]dx$$

Q. $$\int \frac{\cos 4x + 1}{\cot x - \tan x}\,dx$$

Q. A function g defined for all positive real number satisfies $g'(x^2) = x^3$ for all $x > 0$ and $g(1) = 1$. Compute $g(4)$.

Q. $$\int\left[\sin\alpha\sin(x-\alpha)+\sin^2\left(\frac{x}{2}-\alpha\right)\right]dx$$

Q. $$\int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x} dx$$

Q. $$\int \left[\frac{\cot^2 2x - 1}{2\cot 2x} - \cos 8x \cot 4x \right] dx$$

Q. $\int \frac{\cos^4 x - \sin^4 x}{\sqrt{1+\cos 4x}} dx (\cos 2x > 0)$

Q. $$\int \frac{2x^3+3x^2+4x+5}{2x+1}\,dx$$

Q. $$\int \frac{\left(x^2+\sin^2 x\right)\sec^2 x}{1+x^2}\,dx$$

Q. $$\int \frac{dx}{\sqrt{9-16x^2}}$$

Q. $$\int \frac{dx}{25 + 4x^2}$$

Q. $$\int \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)dx$$

Q. $\int \frac{dx}{1+\sin x}$

Q. $\int \frac{\cos 8x - \cos 7x}{1 + 2\cos 5x}\,dx$

Q. $\displaystyle\int \frac{2+3x^2}{x^2(1+x^2)}\,dx$

Q. $$\int \frac{\sin 2x - \sin 2k}{\sin x - \sin k + \cos x - \cos k} dx$$

Q. $$\int \frac{x^2+3}{x^6(x^2+1)}\,dx$$

Q. $\int \sin x \cos x \cos 2x \cos 4x \, dx$

Q. $\int x^x \ell n(ex)dx$

Methods of integration

1. By substitution
2. Integration by parts
3. Partial fraction
4. Misc. (kuturputur)

Substitution

$$\int [f(x)]^n f'(x)dx$$ Or

$$\int \frac{f'(x)}{[f(x)]^n}dx$$ Or

$$\int \frac{f'(x)}{f(x)}dx$$ Or

Start with y = f(x)

Illustrations

Q. $\int \tan x \sec^2 x \, dx$

Q. $\int \sqrt{\tan x}\, \sec^2 x\, dx$

Q. $\int \sqrt{5+\tan x}\,\sec^2 x\,dx$

Q. $\int \frac{\sqrt{\tan x}}{\sin 2x} dx$

Q. $\int (\tan x - x)(\sec^2 x - 1)dx$

Q. $\int (\tan^3 x - x \tan^2 x)dx$

Q. $$\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx$$

Q. $$\int \tan\sqrt{x}\,\frac{\sec^2\sqrt{x}}{2\sqrt{x}}\,dx$$

Q. $$\int \frac{2x}{\sqrt{1-x^2}}\,dx$$

Q. $$\int \frac{2x - \sqrt{\sin^{-1} x}}{\sqrt{1-x^2}} dx$$

Q. $$\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx$$

Q. $\int e^{x} \sin\left(e^{x}\right) dx$

Q. $$\int \frac{x^7}{1+x^{16}}\,dx$$

Q. $$\int \frac{x^2 \tan^{-1} x^3 dx}{1+x^6}$$

Q. $\int \ell n^2(\sec x)\tan x\,dx$

Q. $$\int \frac{\sin 2x\,dx}{1+2\sin^2 x}$$

Q. $\int \frac{\sin 2x \, dx}{1+3\cos^2 x}$

Q. $$\int \frac{\sin 2x\,dx}{a\sin^2 x + b\cos^2 x}$$

Q. $$\int \frac{\tan x \, dx}{a + b \tan^2 x}$$

Q. $$\int \frac{1}{x^{1/2}+x^{1/3}}\,dx$$

Q. $\int \frac{2x^5 - 3x^2}{1 + 3x^3 - x^6} dx$

Q. $\int \left(1-3x^{\frac{4}{3}}\right)^{\frac{1}{10}} x^{\frac{1}{3}}\, dx$

Q. $\int (\tan x)\, dx = \ln \sec x + c = -\ln(\cos x) + C$

Q. $\int (\cot x)\ dx = \ln(\sin x)$

Q. $\int \frac{\tan(\ln x)}{x} dx$

Q. $\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}\,dx$

Q. $\int \frac{\cos x}{\cos(x-a)}dx$

Q. $$\int \frac{x\cos x}{(x\sin x+\cos x)^2}\,dx$$

Q. $$\int \frac{\sin 2x \, . \, dx}{\sin 5x \, . \sin 3x}$$

Q. $\int (\sin x - \cos x).(\sin x + \cos x)^5\,dx$

Q. $\displaystyle\int x\sqrt{\frac{2\sin(x^2+1)-\sin 2(x^2+1)}{2\sin(x^2+1)+\sin 2(x^2+1)}}\,dx$

Q. $$\int \frac{x+e^{x}(\sin x+\cos x)+\sin x\cos x}{(x^{2}+2e^{x}\sin x-\cos^{2}x)^{2}}\,dx$$

Q. $\int \sec x \, dx = \ln(\sec x + \tan x) + C$ or $\ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$

Q. $\int \operatorname{cosec} x\, dx = \ln(\operatorname{cosec} x - \cot x)$ or $\ln \tan \frac{x}{2} + C$

Q. $\int \frac{\sin x}{\sin 2x} dx$

Q. $$\int \frac{dx}{4\cos^3 x - 3\cos x}$$

Q. $\displaystyle\int \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}\,dx$

Q. $$\int \frac{\operatorname{cosec}\left(\tan^{-1}x\right)}{1+x^{2}}\,dx$$

Q. $\int \frac{\cos 2x}{\sin x} dx$

Q. $\int \frac{dx}{\sin x \cos^2 x}$

Q. $\int \frac{dx}{\sqrt{3}\sin x + \cos x}$

Q. $$\int \frac{dx}{a\sin x + b\cos x}$$

Q. $\int \frac{dx}{\sec x + \operatorname{cosec} x}$

Q. $\int e^{x^2 + \ln x}\, dx$

Q. $\int \frac{dx}{e^{x}+1}$

Q. $\displaystyle\int \frac{e^x - 1}{e^x + 1}\,dx$

Q. $\int \frac{e^{x}(1+x)}{\sin^{2}(xe^{x})}dx$

Q. $\int \left(27e^{9x}+e^{12x}\right)^{1/3} dx$

Q. $\int \tan(10x).\tan(7x).\tan(3x)\,dx$

General Substitution

$$\sqrt{a^2 - x^2} \;;\; x = a\sin\theta$$

$$\sqrt{a^2 + x^2} \;;\; x = a\tan\theta$$

$$\sqrt{x^2 - a^2} \;;\; x = a\sec\theta$$

$$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}} \;;\; x^2 = a^2\cos 2\theta$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ell n\left(x+\sqrt{x^2+a^2}\right) \& \int \frac{dx}{\sqrt{x^2-a^2}} = \ell n\left(x+\sqrt{x^2-a^2}\right)$$

Example

Q. $$\int \frac{\sin 2x}{\sqrt{9-\sin^4 x}}\,dx$$

Q. $\displaystyle\int \frac{e^{x}\,dx}{\sqrt{e^{2x}-1}}$

Q. $\int \frac{dx}{4x^2+4x+5}$

Q. $$\int \frac{x\,dx}{x^4+x^2+1}$$

Q. $\int \frac{ax+b}{px^2+qx+r}dx$ or $\int \frac{ax+b}{\sqrt{px^2+qx+r}}dx$

write $ax+b = A\frac{d}{dx}\left(px^2+qx+r\right)+B$

Q. $$\int \frac{(2x+3)}{(x^2+2x+2)}dx$$

Q. $\int \frac{8x+5}{\sqrt{4x^2+4x+2}}\,dx$

Q. $\int \frac{4x+3}{3x^2+3x+1}dx$

Trigonometric Functions

$$\int \sin^m x \cos^n x \, dx \qquad m, n \in N$$

(1) If one odd other even substitute term of even power

(2) Both odd substitute small higher power T-function.

(3) Both even use T-identities to manipulate

(4) If m & n are rational numbers & $\left(\frac{m+n-2}{2}\right)$ is negative integer or m & n is negative even integer then, substitute $\left.\begin{array}{r}\tan x = t\\ \text{or} \cot x = t\end{array}\right\}$ Also create derivatives

Examples

Q. $\int \sin^5 x \cos^6 x \, dx$

Q. $\int \sin^3 x \cos^5 x \, dx$

Q. $\int \frac{dx}{\sin x \cos^3 x}$

Q. $\int \sin^2 x \cos^2 x \, dx$

Integration By Parts

Rules :–

(i) Choose 2nd Function which is easily integrable

(ii) Choose 1st & 2nd functions such that after by parts Complexity of 2nd term reduces as compared to original integration

(iii) Note sometimes 1 is taken as a function

Examples

Q. $\int x e^{x} dx$

Q. $\int x \sin x \, dx$

Q. $\int x \ell n x \, dx$

Q. $\int x \sec^2 x \, dx$

Q. $\int \ell n\, x\, dx$

Q. $\int x \tan^{-1} x \, dx$

Q. $\int \sin^{-1} x \, dx$

Q. $\int x^2 \ell n\, x\, dx$

Q. $\int x \cos x \sin^2 x \, dx$

Q. $\int \theta \tan\theta \sec^2\theta \, d\theta$

Q. $\int e^{x} \sin x \, dx$

Q. $\int e^{x} \cos x \, dx$

Q. $\int \text{cosec}^2 x \, \ell n(\sec x)\, dx$

Q. $\displaystyle\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}dx$

Q. $\int \cos^{-1}\frac{1}{x}\,dx$

Q. $\int \sec^3 x \, dx$

Q. $\int x^2 e^{3x} dx$

Q. $\int x^3 \ell n^2 x \, dx$

Q. $\int \sin x \cdot \ln(\sec x + \tan x)\,dx$

Q. $\int \frac{\cos^{-1}x}{x^3}dx$

Q. $\int \sin(\ell n\, x)$

Q. $\int \ell n(1+x)^{1+x}\, dx$

Q. $\int \ell n\left(x+\sqrt{x^2+a^2}\right)dx$

Q. $\int \frac{x\,dx}{1+\sin x}$

Q. $\int \ell nx \cdot \frac{1}{(x+1)^2} dx$

Q. $\int x \cos x \cos 2x \, dx$

Q. $\int e^{x}(1+x)\ell n(xe^{x})dx$

Q. $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Q. $\int x\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{2a-x}{a}}\right)dx$

Two Classic Integrands

(a) $\int e^{x}\left(f(x)+f'(x)\right)dx = e^{x} f(x)+c$

(b) $\int \left(f(x) + xf'(x) \right) dx = xf(x) + c$

Examples

Q. $\int e^{x}(x+1)$

Q. $\int e^{x}(\cos x+\sin x)\,dx$

Q. $\int e^{x}(\cos x-\sin x)\,dx$

Q. $\int e^x\left(\tan x+\sec^2 x\right)dx$

Q. $\int e^{x}(\sec x)(1+\tan x)\,dx$

Q. $\int e^{x}\left(\ln x+\frac{1}{x}\right)dx$

Q. $\int e^{x}\left(\frac{x}{(1+x)^{2}}\right)dx$

Q. $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx$

Q. $\int \left(\tan x + x\sec^2 x\right) dx$

Q. $$\int \frac{x+\sin x}{1+\cos x}\,dx$$

Q. $\int e^x \left(\frac{x^2+5x+7}{(x+3)^2} \right) dx$

Q. $\int(\ell nx+1)\,dx$

Q. $\int\left(\ell n \sec x + x\tan x\right)dx$

Q. $\int e^{x}\left(\frac{x^{2}+1}{(x+1)^{2}}\right)dx$

Q. $\int \frac{x - \sin x}{1 - \cos x} dx$

Q. $\int e^x\left(\frac{1}{\left(1+x^2\right)^{3/2}}+\frac{x}{\sqrt{1+x^2}}\right)dx$

Q. $\int \frac{x e^x}{(1+x)^2} dx$

Q. $\int e^{x}\left[\ell n(\sec x+\tan x)+\sec x\right]dx$

Q. $\int [\sin(\ell n\, x) + \cos(\ell n\, x)\, dx$

Q. $$\int \frac{e^{\tan^{-1}x}\left(1+x+x^2\right)}{1+x^2}\,dx$$

Q. $\int e^x \frac{1 \pm \sin x}{1 \pm \cos x} dx$

Q. $\int \frac{\ell n x}{(1+\ell n x)^2} dx$

Q. $\int \frac{x^2 e^x}{(x+2)^2} dx$

Q. $\int e^{x}(\tan x - \ell n \cos x)\,dx$

Q. $\int \frac{e^{x}(x-1)}{(x+1)^{3}}\,dx$

Q. $\int \frac{dx}{a^2 e^x + b^2 e^{-x}}$

Q. $\int\left(\frac{x-1}{1+x^2}\right)^2 dx$

Q. $\int \frac{\tan\alpha - \tan x}{\tan\alpha + \tan x}\,dx$

Q. If the primitive of the function $\mathbf{f(x)} = \dfrac{x^{2009}}{\left(1+x^2\right)^{1006}}$ w.r.t. x is equal to $\dfrac{1}{n}\left(\dfrac{x^2}{1+x^2}\right)^m + C$ then find (m + n) (where m, n $\in$ N)

Q. $\int \frac{\sqrt{1+\sin 2x}}{(1+\cos 2x)e^{-x}} dx$

Partial Fraction

Case I :

If $\dfrac{P_1(x)}{P_2(x)}$ $P_1(x), P_2(x)$ are polynomials

If degree of $P_1 \geq P_2$ Divide & Move to case (2)

Case II :

Degree of $P_1(x)$ < Degree of $P_2(x)$

(a) P_2 is linear in x

For Example : $\int \frac{x^2+2x+4}{(x-1)(x-2)(x+1)}$

Let $\frac{x^2+2x+4}{(x-1)(x-2)(x+1)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$

$A = \frac{-7}{2}$, B = 4, $C = \frac{1}{2}$

(b) Denominator is linear factor of x

(i) $$\frac{x^2+2x+4}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

(b) Denominator is quadratic in x (Not Factorized)

(ii) $$\frac{x^2+2x+4}{(x-1)\left(x^2+x+1\right)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

Important Concepts

$$\int \frac{1}{\text{Linear}} = \log$$

Q. $\int \frac{1}{\left(x^2+1\right)^2} dx$

Q. $$\int \frac{x^3 dx}{x^4 + 3x^2 + 2}$$

Q. $\int \frac{dx}{x^3+1}$

Q. $\int \frac{dx}{\sin 2x - 2\sin x}$

Q. $\int \frac{\sin x}{\sin 4x} dx$

Q. $\int \frac{1 + x\cos x}{x(1 - x^2 e^{2\sin x})} dx$

Miscellaneous
Kuturputur

Q. $\int \frac{dx}{x(x^2+1)}$

Q. $\int \frac{dx}{x(x^n + 1)}$

Q. $$\int \frac{x^7}{(1-x^2)^5}\,dx$$

Q. $\displaystyle\int \frac{x\,dx}{(1-x^4)^{3/2}}$

Q. $\int \frac{dx}{x^4(x^3+1)^2}$

Q. $\displaystyle\int \frac{x^{2}-x^{-2}}{x\sqrt{x^{2}+x^{-2}+1}}\,dx$

Q. $\int \frac{dx}{x^2\left(x+\sqrt{1+x^2}\right)}$

Q. $$\int \frac{\left(ax^2 - b\right) dx}{x\sqrt{c^2x^2 - \left(ax^2 + b\right)^2}}$$

Q. $\int \frac{x(\cos\alpha+1)\,dx}{\left(x^2+2x\cos\alpha+1\right)^{3/2}}$

Q. $\int \frac{(x-1)\,dx}{x^2\sqrt{2x^2-2x+1}}$

Q. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Q. $$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)+C$$

Q. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2 - a^2}\right) + C$

Integration By Differentiation

Q. $\int e^{x}\left(x^{3}+2x+1\right)dx$

Q. $\int e^{x}\left(2\sin x+\cos x\right)dx$

Trigonometric Functions

Type 1 :

$$\int \frac{dx}{a+b\sin^2 x} \Big/ \int \frac{dx}{a+b\cos^2 x} \Big/ \int \frac{dx}{a\sin^2 x+b\cos^2 x+c\sin x\cos x}$$

Type 2 :

$$\int \frac{dx}{a+b\sin x} \Big/ \int \frac{dx}{a+b\cos x} \Big/ \int \frac{dx}{a+b\sin x+c\cos x}$$

Examples

Q. $\int \frac{dx}{4-5\sin^2 x}$

Q. $\int \frac{dx}{(3\sin x - 4\cos x)^2}$

Q. $\int \frac{dx}{3+\cos^2 x}$

Q. $\int \frac{dx}{5+4\sin x}$

Q. $\int \frac{dx}{4+5\cos x}$

Q. $\int \frac{dx}{3+2\sin x+\cos x}$

Q. $\int \frac{dx}{\cos x(5+3\cos x)}$

Q. $\int \frac{dx}{3+5\cos x}$

Q. $\int \frac{dx}{4+5\sin x}$

Q. $\int \frac{dx}{1-\cos x+\sin x}$

Q. $\int \frac{\cos x}{5 - 3\cos x} dx$

Q. $\int \frac{\sin 2x}{(a+b\cos x)^2}\,dx$

Q. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

Q. $\displaystyle\int \frac{dx}{\sin^4 x + \cos^4 x}$

Type 3 :

$$\int \frac{a\sin x + b\cos x + c}{\ell \sin x + m\cos x + n}\,dx$$

Working Rule :

$$N_r = A\left(D_r\right) + B\frac{d}{dx}\left(D_r\right) + C$$

Compute A, B, C

Q. $\int \frac{6+3\sin x+14\cos x}{3+4\sin x+5\cos x}$

Q. $$\int \frac{11\cos x - 16\sin x}{2\cos x + 5\sin x}$$

Examples

Q. $$\int \frac{1}{1+\tan x}\,dx$$

Q. $\int \frac{1}{1+\cot x}\,dx$

Q. $\int \frac{1}{1+\sec x}\,dx$

Q. $\int \frac{1}{1+\text{cosec}x}dx$

Q. $\int \frac{\cos x + \sin x}{2 - \sin 2x} dx$

Q. $\int \frac{\sin x}{e^{x} - \sin x - \cos x} dx$

Q. $$\int \frac{x^2+1}{x^4+7x^2+1}\,dx$$

Q. $\int \frac{x^2-1}{x^4-23x^2+1}dx$

Q. $\int \frac{x^2}{x^4+1} dx$

Q. $\int \frac{(x+1)^2}{x^4+x^2+1}\,dx$

Q. $\int \frac{x^{17}}{1+x^{24}}\,dx$

Q. $\int \sqrt{\tan\theta}\, d\theta$

Q. $\int \sqrt{\cot\theta}\, d\theta$

Q. $\int \left(\sqrt{\cot\theta} \pm \sqrt{\tan\theta}\right) d\theta$

Q. $\int \frac{\cos x}{\sqrt{8-\sin 2x}}dx$

Q. $\int \frac{dx}{\operatorname{cosec} x + \cos x}$

Q. $\int \frac{\cos x}{10 + \sin 2x} dx$

Q. $\int \sqrt{\dfrac{x-\alpha}{\beta-x}}\,dx$ or $\int \sqrt{(x-\alpha)(\beta-x)}\,dx$

Put $x=\alpha\cos^2\theta$

$$\int \sqrt{\frac{x-\alpha}{x-\beta}}\,dx \quad \text{or} \quad \int \sqrt{(x-\alpha)(x-\beta)}$$

$$x=\alpha\sec^2\theta-\beta\tan^2\theta$$

nth Integration

Q. $\int (\tan x)^n \, dx$

Q. $\int (\cot x)^n \, dx$

Q. $\int (\sin x)^n \, dx$

Q. $\int (\cos x)^n \, dx$

Q. $\int (\sec x)^n \, dx$

Integration of Irrational Algebraic Function

$$\int \frac{1}{L_1 \sqrt{L_2}} dx$$

Working Rule :

Put $L_2 = t^2$

$$\int \frac{1}{Q\sqrt{L}}\,dx$$

Working Rule :

Put $L_1 = t^2$

$$\int \frac{1}{L\sqrt{Q}}\,dx$$

Working Rule :

Put $L = {}^{1}/_{t}$

$$\int \frac{1}{Q_1 \sqrt{Q_2}}\, dx$$

Case 1 :	**For**	Q_1	$D > 0$
Case 2 :	**For**	Q_1	$D = 0$
Case 3 :	**For**	Q_1	$D < 0$

$$\int \frac{1}{\sqrt{Q}}\,dx$$

Q. $\int \frac{1}{(2x+2)\sqrt{2x+1}} dx$

Q. $\int \frac{dx}{(x+1)\sqrt{x^2+2x+2}}$

Q. $$\int \frac{dx}{\left(x^2+2x+2\right)\sqrt{x+1}}$$

Q. $\displaystyle\int \frac{dx}{(x+1)(x+2)\sqrt{x^2+2x+2}}$

Q. $$\int \frac{dx}{\left(x^2+2\right)\sqrt{x^2+1}}$$