Limit

General Introduction

Notion of limit

- Limit of what ?
- Why evaluate limit ?
- How to evaluate limit ?

Left Hand Limit &
Right Hand Limit

$\underset{x \to a}{Limit}$ f(x) is said to exist at x = a if

$\underset{h \to 0}{Limit}$ f(a + h) = $\underset{h \to 0}{Limit}$ f(a–h)

= finite (disregards whether f is defined at x = a)

$f(a^+) = f(a^-)$ = finite

h is a small positive quantity

Note f(x) = [x] and {x} has no limit at all integers

Q. Find $\underset{x \to 0}{Lim}$ sgn(x)

Q. $f(x) = \frac{|x|}{x}$ limit at x = 0 = ?

Q. $\lim\limits_{x \to 0} \tan^{-1} \dfrac{1}{x}$

Q. $\displaystyle \lim_{x \to 0} \frac{1}{\ln|x|}$

Q. $\lim\limits_{x \to 0} \cot^{-1} x^2 = \ ?$

Q. $f(x) = [x] + \sqrt{\{x\}}$ Lim at $x = 0$ = ?

Q. $\lim_{x \to 1} x \, \mathrm{sgn}(x-1)$

Concept of One Sided Limit

Q. $\displaystyle Lim_{x \to 1} \frac{x}{[x]}$ $\qquad$ $\displaystyle \lim_{x \to 1} \frac{[x]}{x}$

Q. $f(x) = [\sin x]$ at $x = \frac{\pi}{2}$

Q. $g(x) = [\cos x]$ at $x = \frac{\pi}{2}$

Q. Consider : $f(x)=\begin{cases} 2x-3 & x\ge 2 \\ 4-x^2 & x<2 \end{cases}$ $\lim\limits_{x\to 2} f(x)=?$

Q. Consider the function: $f(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } x < -1 \\ 2, & \text{if } -1 \le x < 1 \\ 3, & \text{if } x = 1 \\ x+1, & \text{if } 1 < x \le 2 \\ \dfrac{-1}{(x-2)^2}, & \text{if } x > 2 \end{cases}$

Sketch the graph of f.

Determine the following limits.

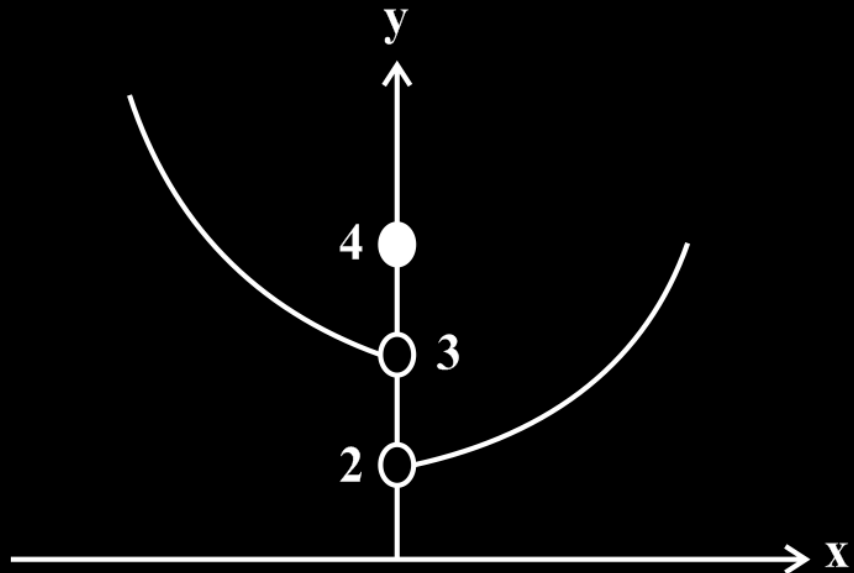
(a) $\lim_{x \to -1^+} f(x)$ (b) $\lim_{x \to -1^-} f(x)$ (c) $\lim_{x \to -1} f(x)$ (d) $\lim_{x \to 1^+} f(x)$

(e) $\lim_{x \to 1^-} f(x)$ (f) $\lim_{x \to 1} f(x)$ (g) $\lim_{x \to 2^+} f(x)$ (h) $\lim_{x \to 2^-} f(x)$

(i) $\lim_{x \to 2} f(x)$ (j) $\lim_{x \to -3} f(x)$ (k) $\lim_{x \to 5} f(x)$ (*l*) $\lim_{x \to 1.5} f(x)$

Q. Refer the figure,

The value of λ for which $2\left(\lim_{x \to 0} f(x^3 - x^2)\right) = \lambda\left(\lim_{x \to 0} f(2x^4 - x^5)\right)$ is



(A) $\frac{4}{3}$ (B) 2

(C) 3 (D) 5

Q. Let $g(x) = (x - 2)^2, x < 2$

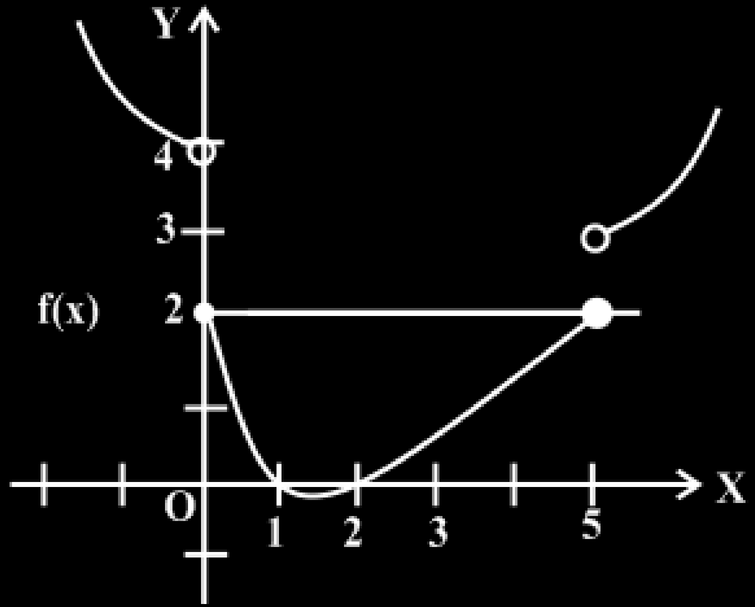
$= 7 - x, x \geq 2$

also the graph of f(x) is given then which of the following limits are non existent

(A) $\lim\limits_{x \to 2} f(g(x))$

(B) $\lim\limits_{x \to 0} g(f(x))$

(C) $\lim\limits_{x \to 5} g(f(x))$



Five Fundamental Theorems

If $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} g(x) = M$ where L and M are finite quantities then

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(5) Constant Multiple rule : $\lim\limits_{x\to c} K.f(x) = KL$

Important Notes

If $\lim\limits_{x \to c} f(x)$ and $\lim\limits_{x \to c} g(x)$ both exists then limit of

$f(x) \pm g(x)$; $f(x) \cdot g(x)$ and $\dfrac{f(x)}{g(x)}$ at x = c must exists.

If $\lim_{x \to c} f(x)$ exist but $\lim_{x \to c} g(x)$ does not exist then

(a) $\lim_{x \to c} (f \pm g)$ can not exist.

(b) Nothing definite can be said about the product or quotient.

Example

Q. $f(x) = x$; $g(x) = [x]$ (where f exist and g does not at $x \in I$)

(i) $\lim_{x \to 0} x[x]$ exists and $\lim_{x \to 1} x[x]$ does not exists

(ii) $\displaystyle \operatorname*{Lim}_{x\to 0} \frac{x}{\operatorname{sgn} x}$ exists and $\displaystyle \operatorname*{Lim}_{x\to 2} \frac{x}{[x]}$ does not exists

(iii) $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ both do not exist then nothing can be said about $(f \pm g), f \cdot g$ or $\frac{f}{g}$.

(iii) $f(x)=\frac{1}{\sin x}$ and $g(x)=\frac{1}{\tan x}$ at $x=0$

(iv) $f(x) = [x]$ and $g(x) = \{x\}$ then $(f + g)x$ exist

(v) $f(x) = \text{sgn } x$ and $g(x) = [x]$ then $\lim_{x \to 0} (\text{sgn}x+[x])$ does not exist

(v) $f(x) = [x]$ and $g(x) = \{x\}$; then $\lim_{x\to 0} [x]\cdot\{x\}$ does not exist but $\lim_{x\to 1} [x]\cdot\{x\}$ exist

(vi) $f(x) = e^{[x]}$; $g(x)$ then $\lim_{x \to 0} e^{[x]} \cdot e^{(x)}$ exists.

Various strategies to evaluate limit

List of formulaes

(i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(ii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(iii) $a^4 + a^2b^2 + b^4 = (a^2 - ab + b^2)(a^2 + ab + b^2)$

(iv) $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$

(v) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\rightarrow$ $a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$ or $a = b = c$

(vi) Binomial expansion for any index n

$$(1+x)^n = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...\infty$$

Sum of first n natural numbers

Sum of squares of first n natural numbers

Sum of cube of first n natural numbers

Sum of A.P.

Sum of G.P.

7 Indeterminant forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, 1^{\infty}, \infty^{0}, 0^{0}$$

Meaning of Indeterminant forms

Strategy to solve limits

(i) Factorisation

(ii) Rationalisation

(iii) Double rationalisation

(iv) Use of binomial theorem

(v) Algebric identities

(vi) Involving law of love

Q. $$\lim_{x \to \pi/4} \frac{\tan^3 x - 1}{\tan^2 x - 1}$$

Q. $$\lim_{x\to\pi/4}\frac{\cot^3 x-1}{\cot^3 x+\cot x-2}$$

Q. $\lim_{x \to 0} \frac{x}{\sqrt{5-x} - \sqrt{5+x}}$

Q. $$\lim_{x\to 9} \frac{3-\sqrt{x}}{4-\sqrt{2x-2}}$$

Q. $\lim_{x\to 2} \frac{(2^x - 4)(2^x + 2)}{(2^{x/2} - 2)}$

Q. $$\lim_{x \to \infty} \frac{3 + 2x + x^2}{x^2 - x + 4}$$

Q. $\lim_{n\to\infty} \frac{3+n+n^2}{n^3+n^2+n+7}$

Q. $\lim_{n\to\infty} \frac{(n)(n+1)(n+2)}{(n-1)(n+7)(n+3)}$

Q. $\lim_{n\to\infty} \frac{n^2+1}{n}$

Q. $\lim_{x\to\infty} \frac{x+1}{2x^2+x-1}$

Q. $\lim_{n\to\infty} \frac{(n+3)(n+9)}{(n-2)(n-1)}$

Q. $\lim_{n\to\infty} \frac{(n+1)^4-(n-1)^4}{(n+1)^4+(n-1)^4}$

Q. $\lim_{n\to\infty} \frac{1}{n^{2/3}}$

Q. $\lim_{x\to\infty} \frac{x^2+2x-3}{x+1}$

Q. $\lim_{n\to\infty} \frac{1}{n^{7/99}}$

Q. $$\lim_{n\to\infty}\frac{\sqrt{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt[4]{n^6+6n^5+2}-\sqrt[5]{n^7+3n^3+1}}$$

Q. $\lim_{n\to\infty} \frac{\left(3(n+1)\right)!}{(n+1)^3(3n)!}$ equal

Q. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+1}}{3x-6}$

Q. $$\lim_{n\to\infty}\prod_{r=2}^{n}\left(\frac{r}{r-1}\right)\prod_{r=2}^{n}\frac{r+3}{r+4}$$

Q. $\lim_{x \to \pm\infty} \left(\sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x - 3} \right)$

Q. $\lim_{x\to\frac{\pi}{2}} \tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$

Q. $\lim\limits_{x \to -2} \left(\dfrac{1}{x+2} - \dfrac{12}{x^3+8} \right)$

Q. $\lim_{x\to\infty}\left(\sqrt{4x^2+x}-\sqrt{\frac{4x^3}{x+2}}\right)$

Q. Let $f(x)=\dfrac{x^2-9x+20}{x-[x]}$ and $g(x)=\dfrac{x|x-3|}{(x^2-x-6)|x|}$

then which of the following holds good ?

(A) $\lim\limits_{x\to5} f(x)=1$

(B) $\lim\limits_{x\to3} g(x)=\dfrac{1}{7}$

(C) $\lim\limits_{x\to0^+} g(x)=\dfrac{-1}{2}$

(D) $\lim\limits_{x\to0^-} g(x)=\dfrac{1}{2}$

Q. $$\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n^3}$$

Assignment – 1
G.N. Berman

Q. $\lim_{n\to\infty} \frac{n+1}{n}$

Q. $\lim_{n\to\infty} \frac{1000n^3+3n^2}{0.001n^4-100n^3+1}$

Q. $\lim_{n\to\infty} \frac{(n+1)^2}{2n^2}$

Q. $\lim_{n\to\infty} \frac{(n+1)^4-(n-1)^4}{(n+1)^4+(n-1)^4}$

Q. $\lim_{n\to\infty} \frac{(n+1)^3+(n-1)^3}{(n+1)^2+(n-1)^2}$

Q. $\lim_{n\to\infty} \frac{(2n+1)^4-(n-1)^4}{(2n+1)^4+(n-1)^4}$

Q. $\lim_{n\to\infty} \frac{n^3-100n^2+1}{100n^2+15n}$

Q. $\lim_{n\to\infty} \frac{\sqrt[3]{n^3+2n-1}}{n+2}$

Q. $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+n}}{n+1}$

Q. $\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$

Q. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt[4]{n^6+6n^5+2}-\sqrt[5]{n^7+3n^3+1}}$

Q. $\lim\limits_{n\to\infty}\dfrac{\sqrt[4]{n^5+2}-\sqrt[3]{n^2+1}}{\sqrt[5]{n^4+2}-\sqrt{n^3+1}}$

Q. $\lim\limits_{n\to\infty}\dfrac{n!}{(n+1)!-n!}$

Q. $\lim\limits_{n\to\infty}\dfrac{(n+2)!+(n+1)!}{(n+3)!}$

Q. $\lim\limits_{n\to\infty}\dfrac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

Q. $$\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\dots\dots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\dots\dots+\frac{1}{3^3}}$$

Q. $$\lim_{n\to\infty}\frac{1}{n^2}\left(1+2+3+\dots\dots+n\right)$$

Q. $$\lim_{n\to\infty}\left(\frac{1+2+3+\dots\dots+n}{n+2}-\frac{n}{2}\right)$$

Q. $$\lim_{n\to\infty}\left(\frac{1-2+3-4+\dots\dots-2n}{\sqrt{n^2+1}}\right)$$

Q. $$\lim_{n\to\infty}\left(\frac{1}{1.2}+\frac{1}{2.3}+.....+\frac{1}{(n-1)n}\right)$$

Q. $$\lim_{n\to\infty}\left(\frac{1}{1.3}+\frac{1}{3.5}+......+\frac{1}{(2n-1)(2n+1)}\right)$$

L′ Opital Rule

Q. $$\lim_{x\to 1}\frac{x^n-1}{x^m-1}\,(m, n \in N)$$

Q. If $\lim\limits_{x\to 2}\left(\dfrac{x^{n+1}-2^{n+1}}{x-2}\right)=80$ and $n \in N$, find n.

Q. $\lim_{x \to 0} \dfrac{(\cos x)^{1/3} - (\cos x)^{1/2}}{\sin^2 x}$ equals

Q. $\lim_{x \to \infty} \sqrt[3]{x^3 + 3x^2} - \sqrt{x^2 - 2x}$

Q. $\lim_{x \to \infty} \left((x+1)(x+2)(x+3)\right)^{\frac{1}{3}} - x$

Q. $\text{find } a, b, c \text{ if } \lim\limits_{\substack{x \to 0 \\ x \to \infty}} \dfrac{ax^2 + bx + c}{x} = 0$

Q. $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

Q. **find a & b** $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=0$

Q. **find a & b** $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=2$

Q. find a & b $\lim_{x \to \infty}\left(\frac{x^2+1}{x+1} - ax - b\right) = \infty$

Sandwich/Squeeze play Theorem

Q. $[x] \leq x < [x] + 1$

$x - 1 < [x] \leq x$

Q. If $4x - 9 \le x^2 - 4x + 7 \ \forall \ x \ge 0$ find $\lim_{x \to 4} f(x)$

Q. $2x \le g(x) \le x^4 - x^2 + 2$ for all x then $\lim\limits_{x \to 1} g(x)$

Q. $$\lim_{x\to\infty}\frac{x^2(2+\sin^2 x)}{x+100}$$

Q. $$\lim_{x \to -\infty} \frac{5x^2 + \sin 3x}{x^2 + 10}$$

Q. The value of the limit $\lim_{x\to 0}\frac{x}{a}\left[\frac{b}{x}\right](a \neq 0)$ (where [ ] denotes the greatest integer function) is equal to

(A) a (B) b (C) $\frac{b}{a}$ (D) $1-\frac{b}{a}$

Q. Evaluate: $\lim_{x \to \infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+2}+\frac{n}{n^2+3}+\ldots\ldots+\frac{n}{n^2+n}\right)$

Assignment – 2
G.N. Berman

Q. $\lim\limits_{x \to 1} \dfrac{x}{1-x}$

Q. $\lim\limits_{x \to 1} \dfrac{(x-1)\sqrt{2-x}}{x^2-1}$

Q. $\lim\limits_{x \to \sqrt{3}} \dfrac{x^2-3}{x^4+x^2+1}$

Q. $\lim\limits_{x \to \frac{1}{2}} \dfrac{8x^3-1}{6x^2-5x+1}$

Q. $\lim\limits_{x \to 1} \dfrac{x^2-2x+1}{x^3-x}$

Q. $\lim\limits_{x \to 1} \dfrac{x^3+x-2}{x^3-x^2-x+1}$

Q. $\lim\limits_{x \to -2} \dfrac{x^3+3x^2+2x}{x^2-x-6}$

Q. $\lim\limits_{x \to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right)$

Q. $\lim_{x\to 2}\left[\frac{1}{x(x-2)^2}-\frac{1}{x^2-3x+2}\right]$

Q. $\lim_{x\to 1}\left[\frac{x+2}{x^2-5x+4}+\frac{x-4}{3\left(x^2-3x+2\right)}\right]$

Q. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}\left(m \text{ and } n \text{ integers}\right)$

Q. $\lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$

Q. $\lim_{x \to \infty} = \frac{x^4 - 5x}{x^2 - 3x + 1}$

Q. $\lim_{x \to \infty} \frac{x^2 - 1}{2x^2 + 1}$

Q. $\lim_{x \to \infty} \frac{1 + x - 3x^3}{1 + x^2 + 3x^3}$

Q. $\lim_{x \to \infty} \left(\frac{x^3}{x^2 + 1} - x \right)$

Q. $\lim_{x \to \infty} \left[\frac{3x^2}{2x - 1} - \frac{(2x - 1)(3x^2 + x + 2)}{4x^2} \right]$

Q. $\lim_{x \to \infty} \frac{(x + 1)^{10} + (x + 2)^{10} + \ldots\ldots + (x + 100)^{10}}{x^{10} + 10^{10}}$

Q. $$\lim_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)$$

Q. $$\lim_{x\to+\infty}\frac{\sqrt[3]{x^4+3}-\sqrt[5]{x^4+4}}{\sqrt[3]{x^7+1}}$$

Q. $$\lim_{x\to+\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$$

Q. $$\lim_{x\to0}\frac{\sqrt{1+x^2}-1}{x}$$

Q. $$\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$$

Q. $$\lim_{x\to0}\frac{\sqrt{1-x}-1}{x^2}$$

Q. $$\lim_{x\to+\infty}\frac{\sqrt[6]{x^7+3}+\sqrt[4]{2x^3-1}}{\sqrt[6]{x^8+x^7+1}-x}$$

Q. $$\lim_{x\to0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$$

Q. $\lim\limits_{x\to 5}\dfrac{\sqrt{x-1}-2}{x-5}$

Q. $\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

Q. $\lim\limits_{x\to 1}\dfrac{x^2-\sqrt{x}}{\sqrt{x}-1}$

Q. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$

Q. $\lim\limits_{x\to a}\dfrac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}\left(a>b\right)$

Q. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

Q. $\lim\limits_{x\to 1}\dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}\left(m \text{ and } n \text{ integers}\right)$

Q. $\lim\limits_{x \to \infty}\left(\sqrt{x+a}-\sqrt{x}\right)$

Q. $\lim\limits_{x \to \infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$

Q. $\lim\limits_{x \to \pm\infty}\left(\sqrt{x^2+1}-x\right)^{1}$

Q. $\lim\limits_{x \to \pm\infty} x\left(\sqrt{x^2+1}-x\right)$

Q. $\lim\limits_{x \to \pm\infty}\left(\sqrt{(x+a)(x+b)}-x\right)$

Q. $\lim\limits_{x \to \pm\infty}\left(\sqrt{x^2-1x-1}-\sqrt{x^2-7x+3}\right)$

Limits of Trigonometric Functions

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 = \lim_{x\to 0}\frac{x}{\sin x}$$

$$= \lim_{x\to 0} x \operatorname{cosec} x = \lim_{x\to 0}\frac{\sin^{-1} x}{x} = \lim_{x\to 0}\frac{x}{\sin^{-1} x}$$

Q. $\left[\lim_{x\to 0} \frac{\sin x}{x}\right]$

(where [ ] denotes the greatest integer function)

Q. $$\lim_{x\to 0}\left[\frac{\tan x}{x}\right]$$

Q. $\lim_{x \to \infty} x \sin \frac{1}{x}$

Q. $$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

Q. $$\lim_{x\to 0}\frac{1-\cos x^3}{x^6}$$

Q. $$\lim_{x \to 0} \frac{1 - \cos x}{\sin x}$$

Q. $\lim_{x \to 0} \frac{\sin 2x}{5x}$

Q. $\lim_{x \to 0} \sin 8x \cot 3x$

Q. $\lim_{x \to 0} \frac{1-\cos 5x}{3x^2}$

Q. $$\lim_{x \to 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3}$$

Q. $$\lim_{x\to 1}\frac{x^3+x^2-2}{\sin(x-1)}$$

Q. $$\lim_{x \to 1}(1-x)\tan\frac{\pi x}{2}$$

Q. $$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$$

Q. $$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{\sin^4 x}$$

Q. $\lim_{x\to 0} \dfrac{x^2 \cos \frac{1}{x}}{\sin x}$

Q. $$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2\frac{x}{2}}$$

Q. If $\lim_{x \to 0} x^2 \cos(\pi \sec^2 x) \operatorname{cosec}(\pi \sec^2 x) = \frac{1}{a\pi} (a \in N)$, then a is equal to

(A) 1 (B) 2 (C) 3 (D) 4

Q. $\lim_{x \to \pi/2} \frac{\tan x}{\tan 3x}$

Q. $$\lim_{x \to \pi/2} \frac{\tan x}{\tan 5x}$$

Q. $$\lim_{x\to 0}\frac{\tan(a+2x)-2\tan(a+x)+\tan a}{x^2}$$

Q. $$\lim_{x \to 1} \frac{\sin \pi x}{x - 1}$$

Q. $$\lim_{x \to 0} \frac{x^2}{\sin(\pi \sec^2 x)}$$

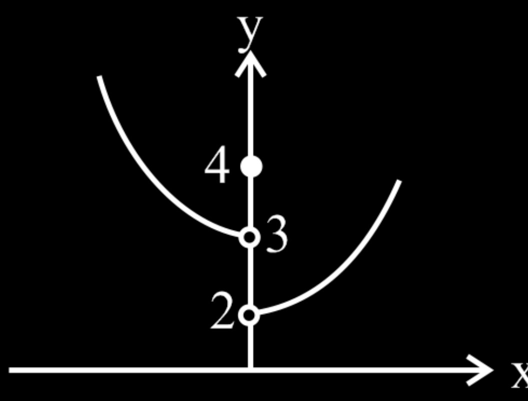
Q. $$\lim_{x \to -\infty} \frac{x^4 \sin(1/x) + x^2}{1 + |x|^3}$$

Q. $$\lim_{x\to 0} \frac{\cos\left(\frac{\pi}{2\cos x}\right)}{\sin(\sin x^2)}$$

Q. $$\lim_{x \to -0^+} \frac{\cos^{-1}(1-x)}{\sqrt{x}}$$

Q. Refer the figure, the value of $\lim_{x\to 0^+} \frac{f(-x)x^2}{\left\{\frac{1-\cos x}{[f(x)]}\right\}}$

where [.] denote greatest integer function and { . } denote fraction part function.



(A) 6 (B) 12 (C) 18 (D) 24

Q. Find the values of a, b & c so that

$$\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x.\sin x} = 2$$

Q. The value of

$$\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\ \ldots\ldots\ldots\ldots\infty \text{ is}$$

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{2}{\pi}$ (C) $\dfrac{3}{\pi}$ (D) $\dfrac{4}{\pi}$

Q. If $\lim\limits_{x\to1}\dfrac{\pi/4-\tan^{-1}x}{x^n-x}$ exists and has the value equal to $\left(-\dfrac{1}{8}\right)$, then find n.

Assignment – 3
G.N. Berman

Q. $$\lim_{x \to 0} \frac{\sin 3x}{x}$$

Q. $$\lim_{x \to 0} \frac{\tan kx}{x}$$

Q. $$\lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$$

Q. $$\lim_{x \to 0} \frac{\tan 2x}{\sin 5x}$$

Q. $$\lim_{\alpha \to 0} \frac{\sin\left(\alpha^{n}\right)}{\sin\left(\alpha\right)^{m}} \left(m \text{ and } n \text{ positive integers}\right)$$

Q. $$\lim_{x \to 0} \frac{2\arcsin x}{3x}$$

Q. $$\lim_{x \to 0} \frac{2x - \arcsin x}{2x + \arctan x}$$

Q. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

Q. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$

Q. $\lim_{\alpha\to 0}\frac{\tan\alpha}{\sqrt[3]{(1-\cos\alpha)^2}}$

Q. $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

Q. $\lim_{\alpha\to 0}\frac{\tan\alpha-\sin\alpha}{\alpha^3}$

Q. $\lim_{\alpha\to 0}\frac{(1-\cos\alpha)^2}{\tan^3\alpha-\sin^3\alpha}$

Q. $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$

Q. $\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)}$

Q. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\sqrt[3]{(1-\sin x)^2}}$

Q. $\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$

Q. $\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$

Q. $\lim_{\alpha \to \pi} \frac{\sin \alpha}{1 - \frac{\alpha^2}{\pi^2}}$

Q. $\lim_{z \to 1} (1-z) \tan \frac{\pi z}{2}$

Q. $\lim_{y \to a} \left(\sin \frac{y-a}{2} . \tan \frac{\pi y}{2a} \right)$

Q. $\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x}$

Q. $\lim_{x \to \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x}$

Q. $$\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)}$$

Q. $$\lim_{x \to \frac{\pi}{2}} \left(2x\tan x - \frac{\pi}{\cos x}\right)$$

Q. $$\lim_{x \to 0} \frac{\cos(a+x) - \cos(a-x)}{x}$$

Q. $$\lim_{x \to 0} \frac{\cos \alpha x - \cos \beta x}{x^2}$$

Q. $$\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{\tan(a+x) - \tan(a-x)}$$

Q. $$\lim_{\alpha \to \beta} \frac{\sin^2\alpha - \sin^2\beta}{\alpha^2 - \beta^2}$$

Q. $$\lim_{h \to 0} \frac{\sin(a+2h) - 2\sin(a+h) + \sin a}{h^2}$$

Q. $$\lim_{h\to 0}\frac{\tan(a+2h)-2\tan(a+h)+\tan a}{h^2}$$

Q. $$\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$$

Q. $$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$$

Q. $$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2\frac{x}{2}}$$

Q. $$\lim_{x\to 0}\frac{1-(\cos x)\sqrt{\cos 2x}}{x^2}$$

Q. $$\lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\arccos x}}{\sqrt{x+1}}$$

Exponential Functions :

$$\lim_{x\to 0}\frac{a^x - 1}{x} = \ln a \ (a > 0)$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1 \quad \text{and} \quad \lim_{x\to\infty} x\left(e^{1/x}-1\right)=1$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

Q. $\lim_{x \to 0} \dfrac{e^{4x}-1}{\tan x}$

Q. $$\lim_{x \to 0} \frac{e^{\tan x} - e^{x}}{\tan x - x}$$

Q. $$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$$

Q. $\lim_{h\to 0}\frac{a^{x+h}+a^{x-h}-2a^{x}}{h^{2}}, a>0$

Q. $$\lim_{x\to\infty} \frac{e^{1/x^2}-1}{2\arctan x^2-\pi}$$

Q. $$\lim_{x \to 0} \frac{\cos 7x - \cos 9x}{\cos x - \cos 5x}$$

1^{∞} Indeterminant form :

$$\lim_{x \to 0}(1+x)^{\frac{1}{x}} = e = \lim_{x \to \infty}\left(1+\frac{1}{x}\right)^{x}$$

$$\lim_{\substack{h\to 0\\ n\to\infty}} (\cosh)^n \to 0, \qquad \lim_{\substack{h\to 0\\ n\to\infty}} (\sec h)^n \to \infty$$

Examples

Q. $$\lim_{x\to 0}\frac{\ln(1+x)}{3^x-1}$$

Q. $$\lim_{x \to 0}(1+2x)^{5/x}$$

Q. $$\lim_{x \to e} \frac{\ln x - 1}{x - e}$$

Q. $\lim_{x \to 1} (1-x) \log_x 2$

Q. Let a, b be constants such that $\lim\limits_{x \to 1} \frac{x^2 + ax + b}{(\ln(2-x))^2}$ exist and have the value equal to *l*. Find the value of (a + b + *l*)

Generalised Formula For 1^∞

Let $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) \to \infty$

then $\lim_{x \to a} \left(f(x) \right)^{g(x)} = e^{\lim_{x \to a} g(x)[f(x) - 1]}$

Examples

Q. $$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$$

Q. $\lim\limits_{x \to 0} \left(\frac{1+x}{2+x} \right)^{\frac{1-\sqrt{x}}{1-x}}$

Q. $$\lim_{x \to \infty}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$$

Q. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{x-1}}$

Q. $\lim_{x \to \infty} \left(\frac{x+2}{2x-1} \right)^{x^2}$

Q. $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$

Q. $\lim\limits_{x \to \infty} \left(\dfrac{x+6}{x+1} \right)^{x+4}$

Q. $\lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{\frac{1}{x}}$

Q. $\lim_{x \to 0} \left(\frac{1+\tan x}{1+\sin x} \right)^{\operatorname{cosec} x}$

Q. $\lim\limits_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^{x}$

Q. $\lim_{x \to 0} \left(\frac{5}{2+\sqrt{9+x}} \right)^{\operatorname{cosec} x}$

(A) $e^{-1/6}$ (B) $e^{-1/5}$ (C) $e^{-1/25}$ (D) $e^{-1/30}$

Q. $\lim_{x \to 0} (\cos x)^{\cot^2 x}$

Q. $\lim\limits_{n \to \infty} \cos^n \dfrac{x}{\sqrt{n}}$

Q. $\displaystyle\lim_{x\to 0}\left(\cos mx\right)^{\frac{n}{x^2}}, \; m, n \in N$

Q. If $\underset{n \to 0}{\text{Lim}} \dfrac{10^x - 2^x - 5^x + 1}{\ln(\sec x)}$ is equal to $(\ln k)(\ln w)$, find least value of $(k + w)$.

Q. $\lim_{x\to\infty}\left(\frac{2\tan^{-1}x}{\pi}\right)^x$ equals e^L then L is equal to

(A) $\frac{2}{\pi}$ (B) $-\frac{2}{\pi}$ (C) $-\frac{\pi}{2}$ (D) 1

Q. $\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n$ $a > 0, b > 0, n \in N$

(A) $a^{1/b}$ (B) $b^{1/a}$ (C) a^b (D) b^a

Q. $\lim\limits_{x\to\infty}\left(\dfrac{1^{1/x}+2^{1/x}+3^{1/x}+\ldots\ldots+n^{1/x}}{n}\right)^{nx}$ $n \in N$

Q. $\lim\limits_{x \to \infty} \left(\dfrac{1^{1/x} + 2^{1/x} + 3^{1/x} + \ldots\ldots + 100^{1/x}}{100} \right)^{100x}$

Q. $\lim\limits_{x \to \infty} x^2 \sin\left(\ln\sqrt{\cos\frac{\pi}{x}}\right)$

Limit of Functions Having Built in Limit With Them

Examples

Q. $$f(x) = \lim_{n\to\infty} \frac{\cos\pi x - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}},\ \text{find} \lim_{x\to 1} f(x)$$

Q. $f(x) = \lim_{n\to\infty} \dfrac{\tan \pi x^2 + (x+1)^n \sin x}{x^2 + (x+1)^n}$, find $\lim_{x\to 0} f(x)$

Miscellaneous Types of Examples

Q. The natural number n, for which

$$\lim_{x\to 0} \frac{\left(27^x - 9^x - 3^x + 1\right)\left(1-\cos x\right)}{x^n}, n \in N$$

is a finite non zero number, is equal to

(A) 1 (B) 2 (C) 3 (D) 4

Q. $$\lim_{x \to 2} \frac{\{x\}+1}{\left[\frac{x}{2}\right]},$$

where { } denotes fractional part function and [ ] denotes greatest integer function.

Q. $\lim\limits_{x \to 1} \dfrac{\{x\}+1}{[2x]}$

where { } denotes fractional part function and [ ] denotes greatest integer function.

Q. $\lim\limits_{x \to 0} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}$

Q. $$\lim_{x \to \frac{x}{2}} \frac{e^{\tan x} - 1}{e^{\tan x} + 1}$$

Q. $\lim\limits_{x\to 0} \dfrac{a^x - b^x}{x\sqrt{1-x^2}}$

Q. $$\lim_{x \to 1} \frac{\sin^{-1} x}{\tan \frac{\pi x}{2}}$$

Q. $\lim\limits_{x \to 0} \dfrac{\cos x + 4\tan x}{2 - x - 2x^4}$

Q. $$\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{\sin^4\left(3\sqrt{x}\right)}$$

Q. $\lim_{x \to \infty} x^2 \left(1 - \cos \frac{1}{x}\right)$

Q. $\lim\limits_{x \to \infty} 5^{\frac{2x}{x+3}}$

Q. $\lim_{x \to 0} \frac{1}{x} \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

Q. $\lim\limits_{x\to 0}\dfrac{\tan x^3}{x}$

Q. $\lim\limits_{x \to 0} \frac{\ln \sec x}{x^2}$

Q. $$\lim_{x\to 0}\frac{3\sin x - x^2 + x^3}{\tan x + 2\sin^2 x + 5x^4}$$

Q. $\lim_{n \to \infty} \frac{5^{n+1} + 3^n - 2^{2n}}{5^n + 2^n + 3^{n+3}}$

Q. $\lim\limits_{x \to \infty} \frac{x}{[x]}$

Q. $$\lim_{x\to 0}\frac{1-\cos x+2\sin x-\sin^3 x-x^2+3x^4}{\tan^3 x-6\sin^2 x+x-5x^3}$$

Q. $$\lim_{x \to \frac{\pi}{4}^{+}} \left(\tan\left(\frac{\pi}{12} + x \right) \right)^{\tan 2x}$$

Q. $\lim\limits_{x \to 0} (\sin x)^{\frac{1}{x^2}}$

Q. If $\lim_{x\to 2}\frac{f(x)-5}{x-2}=3$ then Find $\lim_{x\to 2} f(x)$

Q. If $\lim\limits_{x\to 0} \dfrac{f(x)}{x^2} = 2$ then (a) $\lim\limits_{x\to 0} f(x)$ and (b) $\lim\limits_{x\to 0} \dfrac{f(x)}{x}$

Q. If $\lim\limits_{x \to 4} \dfrac{f(x) - 5}{x - 2} = 1$ then $\lim\limits_{x \to 4} f(x)$

Q. If $\lim\limits_{n\to\infty} \frac{1}{3+x^{n}} = \frac{1}{3}$ find range of x

Q. $\lim\limits_{x\to\infty}\left(\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\right)$

Q. $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

Limit Using Expansion of Function

Q. $\displaystyle\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

Q. $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

Q. $\lim\limits_{x \to 0} \dfrac{e^{x} - e^{-x} - 2x}{x^{3}}$

Q. $\lim_{x \to 0} \frac{x - \tan x}{x^2}$

Examples

Q. $$\lim_{x\to 0} \frac{e^{x^3}-1-x^3}{\sin^6 2x}$$

Q. $\lim\limits_{x \to \infty} x - x^2 \, ln\left(1 + \frac{1}{x}\right)$

Q. $$\lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x}$$

Q. $\lim\limits_{x\to 0} \dfrac{1}{\left(\sin^{-1}x\right)^2} - \dfrac{1}{x^2}$

Q. $$\lim_{x\to 0}\frac{\ln(5+x)-\ln(5-x)}{x}$$

Q. $\lim_{x \to 0} \frac{\sin x - x^2 - \{x\}.\{-x\}}{x\cos x - x^2 - \{x\}.\{-x\}}$

(A) $-\frac{1}{3}$

(B) $\frac{1}{3}$

(C) 1

(D) does not exist

Q. If $\lim\limits_{x\to 0} \dfrac{A\cos x + Bx\sin x - 5}{x^4}$ exists & finite.

Find A & B and also the limit.

Q. $\lim\limits_{x\to 0} \dfrac{\tan^2 x - x^2}{x^2 \tan^2 x}$

Q. Let $f(x) = \dfrac{4 + \sin 2x + A\sin x + B\cos x}{x^2}$. If $\lim\limits_{x \to 0} f(x)$ exists and finite. Find A and B and the limit

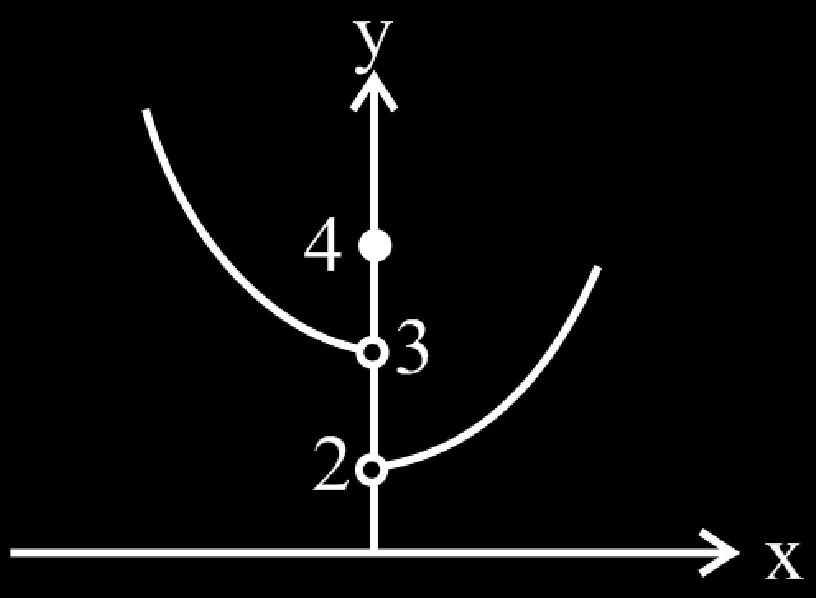
Q. Refer the figure, the value of

$$\lim_{x\to 0^-}\left(\left[3f\left(\frac{x^3-\sin^3 x}{x^4}\right)\right]-f\left(\left[\frac{\sin x^3}{x}\right]\right)\right)=$$

where [.] denote greatest integer function.

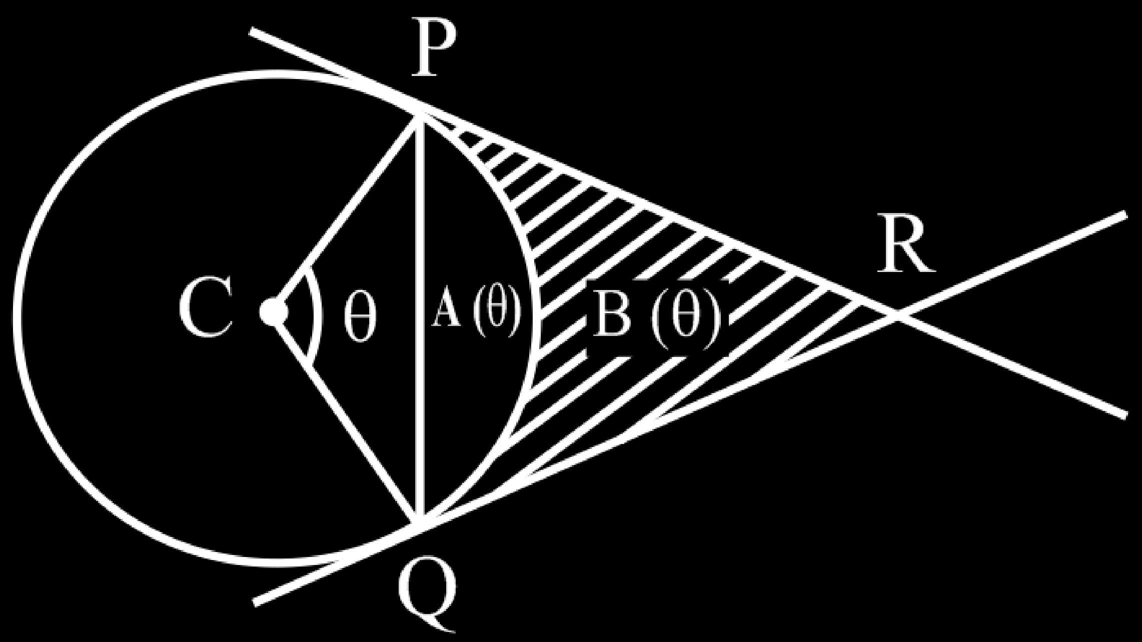
(A) 3 (B) 5

(C) 7 (D) 9



Q. An arc PQ of a circle subtends a central angle θ as shown. Let A(θ) be the area between the chord PQ and the arc PQ. Let B(θ) be the area between the tangent lines PR and QR and the arc PQ. Find

$$\lim_{\theta \to 0} \frac{A(\theta)}{B(\theta)}$$



Q. $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$ is finite and equal to L

Find a & L

Assignment – 4
G.N. Berman

Q. $\lim_{x\to\infty}\left(1-\frac{1}{t}\right)^{t}$

Q. $\lim_{x\to -1}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$

Q. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{mx}$

Q. $\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1}$

Q. $\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$

Q. $\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}$

Q. $\lim_{x\to\pm\infty}\left(\frac{x+1}{2x-1}\right)^{x}$

Q. $\lim_{x\to\pm\infty}\left(\frac{2x+1}{x-1}\right)^{x}$

Q. $$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$$

Q. $$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x^2}$$

Q. $$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

Q. $$\lim_{x\to 0}(1+\sin x)^{cosecx}$$

Q. $$\lim_{x\to 0}(1+\tan^2\sqrt{x})^{1/2x}$$

Q. $$\lim_{x\to 0}\frac{\ln(1+kx)}{x}$$

Q. $$\lim_{x\to 0}\frac{\ln(a+x)-\ln a}{x}$$

Q. $$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$$

Q. $\lim_{x\to\infty}\{x[\ln(x+a)-\ln x]\}$

Q. $\lim_{x\to e}\frac{\ln x-1}{x-e}$

Q. $\lim_{h\to 0}\frac{a^h-1}{h}$

Q. $\lim_{x\to 0}\frac{e^{2x}-1}{3x}$

Q. $\lim_{x\to 1}\frac{e^x-e}{x-1}$

Q. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$

Q. $\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x}$

Q. $\lim_{x\to 0} \frac{e^{\sin 2x} - e^{\sin x}}{x}$

Q. $\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{x}$

Q. $\lim_{x\to \infty} x(e^{1/x} - 1)$

Q. $\lim_{x\to \pm\infty} x(\sqrt{x^2 + \sqrt{x^4+1}} - x\sqrt{2})$

Q. $\lim_{x\to \pm\infty} \frac{a^x}{a^x + 1} (a > 0)$

Q. $\lim_{x\to \pm\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}} (a > 0)$

Q. $\lim_{x\to\infty} \frac{\sin x}{x}$

Q. $\lim_{x\to\infty} \frac{\arctan x}{x}$

Q. $\lim_{x\to\infty} \frac{x+\sin x}{x+\cos x}$

Q. $\lim_{x\to 1} \frac{\arcsin x}{\tan \frac{\pi x}{2}}$

Q. $\lim_{h\to 0} \frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$

Q. $\lim_{x\to\frac{\pi}{2}} \tan^2 x(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2})$

Q. $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

Q. $\lim_{x\to\infty}\left(\cos\frac{x}{2}.\cos\frac{x}{4}...\cos\frac{x}{2^n}\right)$

Q. $\lim_{x\to\infty} x^2\left(1-\cos\frac{1}{x}\right)$

Q. $\lim_{x\to\infty}(\cos\sqrt{x+1}-\cos\sqrt{x})$

Q. $\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2}-\frac{\pi}{4}\right)$

Q. $\lim_{x\to 0} x\left(\arctan\frac{x+1}{x+2}-\arctan\frac{x}{x+2}\right)$

Q. $\lim_{x\to 0}\frac{\arcsin x-\arctan x}{x^3}$

Q. $\lim_{x\to+\infty}\left(1+\frac{1}{x^n}\right)^x\ (n>0)$

Q. $\lim_{x\to 0}(\cos x)^{1/\sin x}$

Q. $\lim_{x\to 0}\frac{\ln\cos x}{x^2}$

Q. $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$

Q. $\lim\limits_{x\to 0}(\cos x+\sin x)^{1/x}$

Q. $\lim\limits_{x\to 0}(\cos x+a\sin bx)^{1/x}$