LOGARITHM

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KEY CONCEPTS (LOGARITHM)

THINGS TO REMEMBER :

1. LOGARITHM OF A NUMBER :

REMEMBER
$\log_{10}2 = 0.3010$
$\log_{10}3 = 0.4771$
$\ln 2 = 0.693$
$\ln 10 = 2.303$

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N. This number is designated as $\log_a N$.

Hence : $\boxed{\log_a N = x \Leftrightarrow a^x = N \ , \ a > 0 \ , \ a \neq 1 \ \& \ N > 0}$

If $a = 10$, then we write $\log b$ rather than $\log_{10} b$.
If $a = e$, we write $\ln b$ rather than $\log_e b$.

The existence and uniqueness of the number $\log_a N$ follows from the properties of an exponential functions.

From the definition of the logarithm of the number N to the base 'a' , we have an

identity : $\boxed{a^{\log_a N} = N \ , \ a > 0 \ , \ a \neq 1 \ \& \ N > 0}$

This is known as the **FUNDAMENTAL LOGARITHMIC IDENTITY .**

NOTE : $\log_a 1 = 0 \quad (a > 0 \ , \ a \neq 1)$
$\log_a a = 1 \quad (a > 0 \ , \ a \neq 1)$ and
$\log_{1/a} a = -1 \quad (a > 0 \ , \ a \neq 1)$

2. THE PRINCIPAL PROPERTIES OF LOGARITHMS :

Let M & N are arbitrary posiitive numbers , $a > 0, a \neq 1, b > 0, b \neq 1$ and α is any real number then ;

(i) $\log_a (M . N) = \log_a M + \log_a N$ **(ii)** $\log_a (M/N) = \log_a M - \log_a N$

(iii) $\log_a M^{\alpha} = \alpha . \log_a M$ **(iv)** $\log_b M = \dfrac{\log_a M}{\log_a b}$

NOTE : ✎ $\log_b a . \log_a b = 1 \Leftrightarrow \log_b a = 1/\log_a b$. ✎ $\log_b a . \log_c b . \log_a c = 1$

✎ $\log_y x . \log_z y . \log_a z = \log_a x$. ✎ $e^{\ln a^x} = a^x$

3.* PROPERTIES OF MONOTONOCITY OF LOGARITHM :

(i) For $a > 1$ the inequality $0 < x < y$ & $\log_a x < \log_a y$ are equivalent.

(ii) For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent.

(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$

(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$

(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$

(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

NOTE THAT :

✎ If the number & the base are on one side of the unity, then the logarithm is positive ; If the number & the base are on different sides of unity, then the logarithm is negative.

✎ The base of the logarithm 'a' must not equal unity otherwise numbers not equal to unity will not have a logarithm & any number will be the logarithm of unity.

✎ For a non negative number 'a' & $n \geq 2$, $n \in N$ $\sqrt[n]{a} = a^{1/n}$.

*** Will be covered in detail in QUADRATIC EQAUTION**

EXERCISE–I

Q.1 Let **A** denotes the value of $\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)$ + $\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$

when a = 43 and b = 57

and **B** denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$.

Find the value of $(A \cdot B)$.

Q.2(a) If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3(1.2)$ in terms of x and y.

(b) If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$.

Solve for x (Q.3 to Q.5):

Q.3 (a) If $\log_{10}(x^2 - 12x + 36) = 2$ (b) $9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3.

Q.4 Simplify: (a) $\log_{1/3}\sqrt[4]{729.\sqrt[3]{9^{-1}.27^{-4/3}}}$; (b) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

Q.5 (a) If $\log_4 \log_3 \log_2 x = 0$; (b) If $\log_e \log_5 [\sqrt{2x-2}+3] = 0$

Q.6 (a) Which is smaller? 2 or $(\log_\pi 2 + \log_2 \pi)$. (b) Prove that $\log_3 5$ and $\log_2 7$ are both irrational.

Q.7 Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that

$2(\log_a c + \log_b c) = 9\log_{ab} c$. Find the largest possible value of $\log_a b$.

Q.8 Find the square of the sum of the roots of the equation

$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$.

Q.9 Find the value of the expression $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$.

Q.10 Calculate : $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right)-6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

Q.11 Simplify: $\frac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}}3}}}{409}.\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}\right)$

Q.12 Simplify: $5^{\log_{1/5}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}}+\log_{1/2}\frac{1}{10+2\sqrt{21}}$.

Q.13 Find 'x' satisfying the equation $4^{\log_{10} x+1} - 6^{\log_{10} x} - 2.3^{\log_{10} x^2+2} = 0$.

Q.14 Given that $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \frac{2}{s^3+1}$. Write $\log_2 \frac{a^2b^5}{c^4}$ as a function of 's' (a, b, c > 0, c ≠ 1).

Q.15 Find the value of $49^{(1-\log_7 2)} + 5^{-\log_5 4}$.

Q.16 Given that $\log_2 3 = a$, $\log_3 5 = b$, $\log_7 2 = c$, express the logarithm of the number 63 to the base 140 in terms of a, b & c.

Q.17 Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$.

Q.18 Prove that $a^x - b^y = 0$ where $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$, $a > 0$, $b > 0$ & $a , b \neq 1$.

Q.19 If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. Find the value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2} \right)$.

Q.20 (a) Solve for x , $\dfrac{\log_{10}(x-3)}{\log_{10}\left(x^2-21\right)} = \dfrac{1}{2}$

(b) $\log(\log x) + \log(\log x^3 - 2) = 0$; where base of log is 10 everywhere.

(c) $\log_x 2 . \log_{2x} 2 = \log_{4x} 2$ (d) $5^{\log x} + 5\, x^{\log 5} = 3\,(a > 0)$; where base of log is a.

Q.21 If x, y > 0, $\log_y x + \log_x y = \dfrac{10}{3}$ and xy = 144, then $\dfrac{x+y}{2} = \sqrt{N}$ where N is a natural number, find the value of N.

Q.22 Solve the system of equations:

$\log_a x \ \log_a (xyz) = 48$
$\log_a y \ \log_a (xyz) = 12$, $a > 0, a \neq 1$.
$\log_a z \ \log_a (xyz) = 84$

Q.23 (a) Given : $\log_{10} 34.56 = 1.5386$, find $\log_{10} 3.456$; $\log_{10} 0.3456$ & $\log_{10} 0.003456$.

(b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.

(c) If $\log_{10} 2 = 0.3010$ & $\log_{10} 3 = 0.4771$, find the value of $\log_{10}(2.25)$.

(d) Find the antilogarithm of 0.75, if the base of the logarithm is 2401.

Q.24 If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$. Find the number of integers in :

(a) 5^{200} (b) 6^{15} & (c) the number of zeros after the decimal in 3^{-100}.

Q.25 Let '**L**' denotes the antilog of 0.4 to the base 1024.
and '**M**' denotes the number of digits in 6^{10} (Given $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$)
and '**N**' denotes the number of positive integers which have the characteristic 2, when base of the logarithm is 6.
Find the value of LMN.

<u>EXERCISE–II</u>

Note : From Q.1 to Q.7, solve the equation for x :

Q.1 $x^{\log x + 4} = 32$, where base of logarithm is 2.

Q.2 $\log_{x+1}(x^2 + x - 6)^2 = 4$

Q.3 $x + \log_{10}(1 + 2^x) = x . \log_{10} 5 + \log_{10} 6$.

Q.4 $5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$, where the base of logarithm is 10.

Q.5 $\dfrac{1 + \log_2(x-4)}{\log_{\sqrt{2}}(\sqrt{x+3} - \sqrt{x-3})} = 1$

Q.6 $\log_5 120 + (x-3) - 2 \cdot \log_5 (1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$

Q.7 $\log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log\left(\sqrt[x]{3} + 27\right).$

Q.8 If 'x' and 'y' are real numbers such that, $2\log(2y - 3x) = \log x + \log y$, find $\frac{x}{y}$.

Q.9 The real x and y satisfy $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$, find xy.

Q.10 If $a = \log_{12} 18$ & $b = \log_{24} 54$ then find the value of $ab + 5(a - b)$.

Q.11 If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$, then prove that $xyz = xy + yz + zx$.

Q.12 If $p = \log_a bc$, $q = \log_b ca$, $r = \log_c ab$, then prove that $pqr = p + q + r + 2$.

Q.13 If $\log_b a \cdot \log_c a + \log_a b \cdot \log_c b + \log_a c \cdot \log_b c = 3$ (Where a, b, c are different positive real numbers $\neq 1$), then find the value of abc.

Q.14 Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$. Find $y \in N$.

Q.15 Solve the equation $\frac{3}{2}\log_4(x+2)^2 + 3 = \log_4(4-x)^3 + \log_4(6+x)^3$.

Q.16 Find the product of the positive roots of the equation $\sqrt{(2008)}\,(x)^{\log_{2008} x} = x^2$.

Q.17 Find x satisfying the equation $\log^2\left(1 + \frac{4}{x}\right) + \log^2\left(1 - \frac{4}{x+4}\right) = 2\log^2\left(\frac{2}{x-1} - 1\right)$.

Q.18 Solve : $\log_3\left(\sqrt{x} + \left|\sqrt{x} - 1\right|\right) = \log_9\left(4\sqrt{x} - 3 + 4\left|\sqrt{x} - 1\right|\right)$

Q.19 Prove that : $2^{\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}}\right)\cdot\sqrt{\log_a b}} = \begin{cases} 2 & \text{if } b \geq a > 1 \\ 2^{\log_a b} & \text{if } 1 < b < a \end{cases}$

Q.20 Solve for x : $\log^2(4-x) + \log(4-x) \cdot \log\left(x + \frac{1}{2}\right) - 2\log^2\left(x + \frac{1}{2}\right) = 0.$

EXERCISE–III

Q.1 $\log_{3/4}\log_8(x^2+7) + \log_{1/2}\log_{1/4}(x^2+7)^{-1} = -2.$ [REE 2000, 5 out of 100]

Q.2 Number of solutions of $\log_4(x-1) = \log_2(x-3)$ is
(A) 3 (B) 1 (C) 2 (D) 0 [JEE 2001 (Screening)]

ANSWER SHEET

LOGARITHM

EXERCISE–I

Q.1 12 **Q.2** **(a)** $\frac{xy+2}{2y}$, $\frac{xy+2y-2}{2y}$; **(b)** 625 **Q.3** **(a)** x = 16 or x = – 4 **(b)** x = 5

Q.4 **(a)** – 1 **(b)** $\log_b N$ **Q.5** **(a)** 8 **(b)** x = 3 **Q.6** **(a)** 2 **Q.7** 2 **Q.8** 3721 **Q.9** 1/6

Q.10 9 **Q.11** 1 **Q.12** 6 **Q.13** $x = \frac{1}{100}$ **Q.14** $2s + 10s^2 - 3(s^3 + 1)$ **Q.15** $\frac{25}{2}$ **Q.16** $\frac{1+2ac}{2c+abc+1}$

Q.19 469 **Q.20** **(a)** x=5 **(b)** x=10 **(c)** $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$ **(d)** $x = 2^{-\log a}$ where base of log is 5.

Q.21 507 **Q.22** (a^4, a, a^7) or $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$ **Q.23** **(a)** 0.5386; $\bar{1}.5386$; $\bar{3}.5386$ **(b)** 2058 **(c)** 0.3522 **(d)** 343

Q.24 **(a)** 140 **(b)** 12 **(c)** 47 **Q.25** 23040

EXERCISE–II

Q.1 x = 2 or $\frac{1}{32}$ **Q.2** x = 1 **Q.3** x = 1 **Q.4** x = 100 **Q.5** x = 5 **Q.6** x = 1 **Q.7** $x \in \phi$

Q.8 4/9 **Q.9** $xy = 2^9$ **Q.10** 1 **Q.13** abc = 1 **Q.14** y = 6 **Q.15** x = 2 or $1 - \sqrt{33}$

Q.16 $(2008)^2$ **Q.17** $x = \sqrt{2}$ or $\sqrt{6}$ **Q.18** $[0, 1] \cup \{4\}$ **Q.20** $\left\{0, \frac{7}{4}, \frac{3+\sqrt{24}}{2}\right\}$

EXERCISE–III

Q.1 x = 3 or – 3 **Q.2** B