

# My Introduction

- At present I am Maths Faculty at ETOOS  
ACADEMY

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- Ex. Sr. Faculty of BANSAL CLASSES (KOTA)

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- **IIT- Delhi**

# **My Introduction**

- **At present I am Maths Faculty at ETOOS ACADEMY**
- **Ex. Sr. Faculty of BANSAL CLASSES (KOTA)**
- **IIT- Delhi**
- **Teaching Exp. 8 Yrs.**

# **Rank Produced by Etoos**

**AIR-24**



**SURAJ SANJAY JOG**  
**And Many Others**

# **How to Study Maths For IIT-JEE**

**(i) Write and not read maths**

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- (ii) Try to apply using formulas /tricks given**

# **How to Study Maths For IIT-JEE**

- (i) Write and not read maths**
- (ii) Try to apply using formulas /tricks given**
- (iii) Practice Practice Practice**



# How to Make Best use of the Course

**Important points / formulas are highlighted**



# **How to Make Best use of the Course**

**Important points / formulas are highlighted**



**Complete Assignment before moving to next  
lecture**

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# Course Details

Logrithms

Trigonometry Identities (Trigo – Ph1)

Quadratic Equation

Sequence & Series

Trigonometry Ph-2

(Trigonometry Equation)

# **Course Details**

**Solutions Of Triangle (Trigo Ph-3)**

**Straight Lines and Pair of Straight Lines**

**Circles**

**Permutation & Combination**



# Course Details

**Binomial Theorem**

**Functions**

**ITF**

**Limit**

**Continuity**

# **Course Details**

**Derivability**

**Method Of Derivative**

**Indefinite Integration**

**Definite Integration**

**Application Of Derivatives**

# Course Details

Vectors

3D Geometry

Determinant

Matrices

Probability

# **Course Details**

**Complex No.**

**Differential Equation**

**Area Under Curve**

**Parabola**

**Ellipse**

**Hyperbola**

# Symbols

$\neq$	(not equal)
$<$	(less than)
$>$	(greater than)
$\leq$	(less than or equal to)
$\geq$	(greater than or equal to)
$()$	(parentheses)
$\&$	(and)
$\dots$	[ellipsis (and so on)]
$\therefore$	(therefore)
$\%$	(percent)

# Symbols

$\pi$	(pi)
$\angle$	(angle)
$^{\circ}$	(degree)
$\perp$	(perpendicular)
$\parallel$	(parallel)
$\sim$	(is similar to)
$\cup$	(union)
$\cap$	(intersection)
$\in$	(is a member of)
$\notin$	(is not a member of)

# Symbols

$\subset$	(is proper subset of)
$\exists$	(there exists)
$\forall$	[(for all (universal quantifier))]
$\cong$	(is equal to or)
$\equiv$	(is equivalent)
$\wedge$	(and)
$\vee$	(or)
$\subseteq$	(is subset of)
$\supseteq$	(is super set of)
$\Leftrightarrow$	(iff or implies and is implied by)

# Symbols

$\approx$  (is approxima)

$\infty$  (infinity)

$!$  (factorial)

$\Sigma$  (sigma)

$\sqrt{\phantom{x}}$  (square root)

$\omega$  (omega)

$\Gamma$  (gamma)

$\theta$  (theta)

$\exists$  (such that)

$\Phi$  (phi)



# Symbols

$\Omega$	(omega)
$\Delta$	(delta)
$\Pi$	(pi)
$\rightarrow$	(arrow)
$\partial$	(derivative partial)
$\int$	(integral)
$\propto$	(proportional)
$\pm$	(plus or minus)
$\phi$	(empty set)
$\mathbf{R}$	(set of real number)

# Basic Maths

Revision of Class VIII, IX, X

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- Remember Tables 1-19

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- Remember Tables 1-19
- Remember Squares 1-32

# Basic Maths

Revision of Class VIII, IX, X

- Remember Tables 1-19
- Remember Squares 1-32
- Remember Cubes 1-12

# Componendo & Dividendo



$$\frac{N-D}{N+D}$$



To be applied both  
sides of the equation.

- $\left(a^x\right)^y = a^{xy}$

- $\left(a^x\right)^y = a^{xy}$

- $\frac{a^x}{a^y} = a^{x-y}$



- $\left(\mathbf{a}^x\right)^y = \mathbf{a}^{xy}$

- $\frac{\mathbf{a}^x}{\mathbf{a}^y} = \mathbf{a}^{x-y}$

- $\mathbf{a}^x \cdot \mathbf{a}^y = \mathbf{a}^{x+y}$

- $\left(a^x\right)^y = a^{xy}$

- $\frac{a^x}{a^y} = a^{x-y}$

- $a^x \cdot a^y = a^{x+y}$

- $x^a \cdot y^a = (xy)^a$

**Additive Inverse,**

**Additive Inverse,**

**Additive Identity,**

**Additive Inverse,**

**Additive Identity,**

**Multiplicative Inverse,**

**Additive Inverse,**

**Additive Identity,**

**Multiplicative Inverse,**

**Multiplicative Identity**

# Set Theory

# Classification of Sets



# Classification of Sets

- Roster or Tabular Form

# Classification of Sets

- Roster or Tabular Form
- Set – Builder Form

# Quadratic Equation

# Inequalities

# Important Algebraic Formulas

- $a^2 - b^2 = (a - b)(a + b)$

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- $(a - b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b$

# Important Algebraic Formulas

- $a^2 - b^2 = (a - b)(a + b)$
- $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
- $(a - b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(\Sigma a^2 - \Sigma ab)$



# Number Theory

# Number Theory

- Natural Number ( $\mathbb{N}$ )

# Number Theory

- Natural Number (N)
- Whole Number (W)

**Prime,**

**Prime,**

**Composite,**

**Prime,**

**Composite,**

**Twin Prime,**

**Prime,**

**Composite,**

**Twin Prime,**

**Co- Prime**

# Integers (I)



# Rational Numbers ( $\mathbb{Q}$ )

# Rational Numbers (Q)

- Converting Decimal to  $p/q$  form

# Rational Numbers (Q)

- Converting Decimal to p/q form
- Example

$$2.5 = ?$$

$$3.\overline{14} = ?$$

# **Irrational Numbers**

# Real Numbers ( $\mathbb{R}$ )

# Complex Number ( $Z$ )

N C W C I C Q C R C Z

# Exponential Form



# Exponential Form



$$(N = a^x)$$

# Exponential Form



$$(N = a^x)$$

- $a > 0$  &  $a \neq 1$

# Exponential Form



$$(N = a^x)$$

- $a > 0$  &  $a \neq 1$
- $a$  is called 'base' and  
 $x$  is called 'exponent'

# Logarithmic form

$$\log_a N = x$$

# Logarithmic form

$$\log_a N = x$$

$\log_a N$  is defined when  $N > 0$ ,  $a > 0$ ,  $a \neq 1$

# Logarithm Form

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1. Examples on value of Logarithm  
Find values :

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- $\log_{81} 27 = ?$



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- $\log_{625} 125 = ?$

# Logarithm Form

1. Examples on value of Logarithm  
Find values :

- $\log_{81} 27 = ?$

- $\log_2 (\log_2 4) = ?$

- $\log_{625} 125 = ?$

- $\log_{1/3} 9\sqrt{3} = ?$

# Fundamental Logarithm Identity



$$a^{\log_a N} = N$$

# 3 Important Deductions



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- $\log_N N = 1$

# 3 Important Deductions



- $\log_N N = 1$
- $\log_{\frac{1}{N}} N = -1$

# 3 Important Deductions



- $\log_N N = 1$
- $\log_{\frac{1}{N}} N = -1$
- $\log_a 1 = 0$



# Examples

# Examples

Find values :

- $\log_{\tan 20^\circ} \tan 70^\circ = ?$

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- $\log_{2-\sqrt{3}} (2 + \sqrt{3}) = ?$

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- $\log_{10} (0.\bar{9}) = ?$

# Examples

Find values :

- $\log_{\tan 20^\circ} \tan 70^\circ = ?$

- $\log_{2-\sqrt{3}} (2+\sqrt{3}) = ?$

- $\log_{10} (0.\bar{9}) = ?$

- $\log_5 \sqrt{5.\sqrt{5.\sqrt{5.\sqrt{5.\dots\infty}}}} = ?$

# Examples

(Integer Type)

- The value of :

$$6 + \log_{\frac{3}{2}} \left( \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}}} \right) \text{ is}$$

[JEE 2012, 4]

# Examples

Solve :

- $7^{\log_7 x} + 2x + 9 = 0$

# Examples

**Solve :**

- $7^{\log_7 x} + 2x + 9 = 0$
- $\log(\tan 5)\log(\tan 9)\log(\tan 13)\dots\dots\log(\tan 61)=?$



# Antilog/Power form

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- $\text{antilog}_8\left(\frac{2}{3}\right) = ?$

# Antilog/Power form

- $\text{antilog}_8\left(\frac{2}{3}\right) = ?$
- $\text{antilog}_{\frac{1}{100}}\left(-\frac{1}{2}\right) = ?$

# Note

**It must be noted that whenever the number and the base are on the same side of unity then logarithm of that number to that base is (+ve), however if the number and the base are located on different side of unity then logarithm of that number to that base is (-ve)**

# Examples

- $\log_{\sqrt{7}} 49 = ?$

# Examples

- $\log_{\sqrt{7}} 49 = ?$

- $\log_{10} \sqrt[3]{10} = ?$

# Examples

- $\log_{\sqrt{7}} 49 = ?$

- $\log_{10} \sqrt[3]{10} = ?$

- $\log_{\frac{1}{2}} \left( \frac{1}{8} \right) = ?$

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- $\log_2 \left( \frac{1}{32} \right) = ?$



# Examples

- $\log_{\sqrt{7}} 49 = ?$

- $\log_{10} \sqrt[3]{10} = ?$

- $\log_{\frac{1}{2}} \left( \frac{1}{8} \right) = ?$

- $\log_2 \left( \frac{1}{32} \right) = ?$

- $\log_{10} (0.001) = ?$



# Principal Properties of Log.

- $\log_a mn = \log_a m + \log_a n$



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- $\log_a mn = \log_a m + \log_a n$

- $\log_a \frac{m}{n} = \log_a m - \log_a n$

- $\log_a m^x = x \log_a m$

- $\log_{n^y} m = \frac{1}{y} \log_n m$

# Note

$\log_2 x^2 = 4$  *and*  $2\log_2 x = 4$  will not have the same solution.

# Example

1. Let  $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16}$   
 $- \log_2 12 \cdot \log_2 48 + 10$ .

# Base Change Theorem



$$\log_b a = \frac{\log_c a}{\log_c b}$$



# Base Change Theorem



$$\log_b a = \frac{\log_c a}{\log_c b}$$



$$a^{\log_b x} = x^{\log_b a}$$

# Examples

1. If  $(\log_2 3)(\log_3 4)(\log_4 5) \dots (\log_n (n+1)) = 10$ , find  $n$

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2.  $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3} = ?$

# Examples

1. If  $(\log_2 3)(\log_3 4)(\log_4 5) \dots (\log_n (n+1)) = 10$ , find  $n$

2.  $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3} = ?$

3. Prove that  $\log_2 7$  is irrational

# Examples

If  $\log_a x = b$  for permissible values of  $a$  and  $x$  then which of the following may be correct :

- (A) If  $a$  rational and  $b$  rational then  $x$  can be rational.

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If  $\log_a x = b$  for permissible values of  $a$  and  $x$  then which of the following may be correct :

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- (B) If  $a$  irrational and  $b$  rational then  $x$  can be rational.

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If  $\log_a x = b$  for permissible values of  $a$  and  $x$  then which of the following may be correct :

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- (C) If  $a$  rational and  $b$  irrational then  $x$  can be rational.

# Examples

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- (B) If  $a$  irrational and  $b$  rational then  $x$  can be rational.
- (C) If  $a$  rational and  $b$  irrational then  $x$  can be rational.
- (D) If  $a$  and  $b$  are two irrational numbers then  $x$  can be rational.



# Examples

Number of solutions of  $\log_4(x-1) = \log_2(x-3)$  is

(a) 3

(b) 1

(c) 2

(d) 0

[JEE 2001, (Screening)]

# Trichotomy

True / False

- $\log_3 5 > \log_{17} 25$

# For A Non Negative Number



$$\sqrt[n]{a} = a^{1/n}$$

# For A Non Negative Number



$$\sqrt[n]{a} = a^{1/n}$$

‘a’ &  $N \geq 2$ ,  $n \in N$

# Logarithmic Equations

- *Solve for 'x'*  $2\log_2 \frac{x-7}{x-1} + \log_2 \frac{x-1}{x+1} = 1$

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- *Solve for 'x'*  $\log_5(5^{1/x} + 125) - \log_5(6) = 1 + \frac{1}{2x}$

# Logarithmic Equations

- *Solve for 'x'*  $2\log_2 \frac{x-7}{x-1} + \log_2 \frac{x-1}{x+1} = 1$
- *Solve for 'x'*  $\log_5(5^{1/x} + 125) - \log_5(6) = 1 + \frac{1}{2x}$
- *Solve for 'x'*  $\log_5(\sqrt[3]{5} + 125) - \log_5(6) = 1 + \frac{1}{2x}$

# Examples

**Solve for 'x'**

- $5^{1+(\log_4 x)} + 5^{(\log_{1/4} x)-1} = \frac{26}{5}$



**Taking Log. Both Sides**

# Taking Log. Both Sides

- Solve for  $x$

$$3^{\log_3^2 x} + x^{\log_3 x} = 162$$

# Taking Log. Both Sides

- Solve for  $x$

$$3^{\log_3^2 x} + x^{\log_3 x} = 162$$

- Solve for  $x$

$$(x+1)^{\log_{10}(x+1)} = 100(x+1)$$

# Taking Log. Both Sides

- Let  $(x_0, y_0)$  the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$3^{\ln x} = 2^{\ln y}.$$

Then  $x_0$  is

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{2}$       (D) 6

**[JEE 2011,3]**

# Common and Natural Logarithm

# Characteristic & Mantissa

- **Standard form of a positive number**

# Examples on Characteristic & Mantissa

Using  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$ , and  $\log 7 = 0.8451$

- (1) Find the number of digits
- (A)  $6^{50}$                       (B)  $5^{25}$

# Examples on Characteristic & Mantissa

Using  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$ , and  $\log 7 = 0.8451$

(1) Find the number of digits

(A)  $6^{50}$

(B)  $5^{25}$

(2) Find the number of zeros after decimal before a significant figure start in

(A)  $\left(\frac{9}{8}\right)^{-100}$

(B)  $3^{-50}$



# Examples on Characteristic & Mantissa

Let  $\log_3 N = \alpha_1 + \beta_1$  ;  $\log_5 N = \alpha_2 + \beta_2$  ;  $\log_7 N = \alpha_3 + \beta_3$   
where  $\alpha_1, \alpha_2, \alpha_3$  are integers and  $\beta_1, \beta_2, \beta_3 \in [0, 1)$

# Examples on Characteristic & Mantissa

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where  $\alpha_1, \alpha_2, \alpha_3$  are integers and  $\beta_1, \beta_2, \beta_3 \in [0, 1)$

(i) Find the number of integral and  $\alpha_1 = 4$  and  $\alpha_2 = 2$

# Examples on Characteristic & Mantissa

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where  $\alpha_1, \alpha_2, \alpha_3$  are integers and  $\beta_1, \beta_2, \beta_3 \in [0, 1)$

- (i) Find the number of integral and  $\alpha_1 = 4$  and  $\alpha_2 = 2$
- (ii) Find the largest integral value of N if  $\alpha_1 = 5$ ,  $\alpha_2 = 3$  and  $\alpha_3 = 2$

# Examples on Characteristic & Mantissa

Let  $\log_3 N = \alpha_1 + \beta_1$  ;  $\log_5 N = \alpha_2 + \beta_2$  ;  $\log_7 N = \alpha_3 + \beta_3$   
where  $\alpha_1, \alpha_2, \alpha_3$  are integers and  $\beta_1, \beta_2, \beta_3 \in [0, 1)$

- (i) Find the number of integral and  $\alpha_1 = 4$  and  $\alpha_2 = 2$
- (ii) Find the largest integral value of N if  $\alpha_1 = 5$ ,  $\alpha_2 = 3$  and  $\alpha_3 = 2$
- (iii) Find the difference of largest and smallest integral values of N if  
 $\alpha_1 = 5$ ,  $\alpha_2 = 3$  and  $\alpha_3 = 2$

# **Modulus**

## **(Absolute Value Function)**

# Examples on Modulus

Solve for  $x$

# Examples on Modulus

Solve for  $x$

(a)  $|x - 1| + |x - 3| = 5$

# Examples on Modulus

Solve for  $x$

(a)  $|x - 1| + |x - 3| = 5$

(b)  $|x| - |x - 2| = 2$



# Examples on Modulus

Solve for  $x$

(a)  $|x - 1| + |x - 3| = 5$

(b)  $|x| - |x - 2| = 2$

(c)  $|x + 1| + |x + 2| = 2$

# Examples on Modulus

Solve for  $x$

(a)  $|x - 1| + |x - 3| = 5$

(b)  $|x| - |x - 2| = 2$

(c)  $|x + 1| + |x + 2| = 2$

(d)  $|3x - 2| + x = 11$

# Examples on Modulus

Solve for  $x$

(a)  $|x - 1| + |x - 3| = 5$

(b)  $|x| - |x - 2| = 2$

(c)  $|x + 1| + |x + 2| = 2$

(d)  $|3x - 2| + x = 11$

(e)  $|x - 2|^{10x^2 - 1} = |x - 2|^{3x}$

# More Examples on Modulus

- Least value of  $x$  satisfying

$$|x - 3| + 2|x + 1| = 4$$

# More Examples on Modulus

- Least value of  $x$  satisfying

$$|x - 3| + 2|x + 1| = 4$$

- If the sum of all solutions of the equation

$$\left(x^{\log_{10} 3}\right) - \left(3^{\log_{10} x}\right) - 2 = 0 \text{ is } \left(a^{\log_b c}\right)$$

where  $b$  and  $c$  are relatively prime and  $a, b, c \in \mathbb{N}$ .

Find the value of  $(a + b + c)$

# More Examples on Modulus

**Solve for 'x'**

- $\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4 \sqrt{(4 - x)^2}$

# More Examples on Modulus

**Solve for 'x'**

- $\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4 \sqrt{(4 - x)^2}$
- $2\log_8(2x) + \log_8(x^2 + 1 - 2x) = \frac{4}{3}$

# More Examples on Modulus

**Solve for 'x'**

- $\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4 \sqrt{(4 - x)^2}$
- $2\log_8(2x) + \log_8(x^2 + 1 - 2x) = \frac{4}{3}$
- $\frac{3}{2}\log_4(x + 2)^2 + 3 = \log_4(4 - x)^3 + \log_4(6 + x)^3.$



# More Examples on Modulus

Solve for 'x'

- $|x - 3|^{3x^2 - 10x + 3} = 1$

# More Examples on Modulus

Solve for 'x'

- $|x - 3|^{3x^2 - 10x + 3} = 1$
- $2\log_3(x - 2) + \log_3(x - 4)^2 = 0$

# More Examples on Modulus

**Solve for 'x'**

- $|x - 3|^{3x^2 - 10x + 3} = 1$
- $2\log_3(x - 2) + \log_3(x - 4)^2 = 0$
- $|x - 1|^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$

# More Examples on Modulus

**Solve for 'x'**

- $|x - 3|^{3x^2 - 10x + 3} = 1$
- $2\log_3(x - 2) + \log_3(x - 4)^2 = 0$
- $|x - 1|^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$
- $x^{(3/4)(\log_2 x)^2 + \log_2 x - (5/4)} = \sqrt{2}$

# Log. Inequalities

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- For  $a > 1$  the inequality  $0 < x < y$  &  $\log_a x < \log_a y$  are equivalent

# Log. Inequalities

- For  $a > 1$  the inequality  $0 < x < y$  &  $\log_a x < \log_a y$  are equivalent
- For  $0 < a < 1$  the inequality  $0 < x < y$  &  $\log_a x > \log_a y$  are equivalent

# **Assignment**

**Prilepko (Page No.92-93)**



# Examples

*Solve the following equations :*

- $\log_{x-1} 3 = 2$

# Examples

*Solve the following equations :*

- $\log_{x-1} 3 = 2$

- $\log_4 \left( 2 \log_3 \left( 1 + \log_2 \left( 1 + 3 \log_3 x \right) \right) \right) = \frac{1}{2}$

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- $\log_3 \left( 3^x - 8 \right) = 2 - x$

# Examples

*Solve the following equations :*

- $$\frac{\log_2(9 - 2^x)}{3 - x} = 1$$

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- $\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x$
- $\log_3(x+1) + \log_3(x+3) = 1$



# Examples

*Solve the following equations :*

- $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$

# Examples

*Solve the following equations :*

- $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$
- $\log 5 + \log(x + 10) - 1 = \log(21x - 20) - \log(2x - 1)$

# Examples

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- $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$
- $\log 5 + \log(x + 10) - 1 = \log(21x - 20) - \log(2x - 1)$
- $1 - \log 5 = \frac{1}{3} \left( \log \frac{1}{2} + \log x + \frac{1}{3} \log 5 \right)$

# Examples

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- $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$
- $\log 5 + \log(x+10) - 1 = \log(21x-20) - \log(2x-1)$
- $1 - \log 5 = \frac{1}{3} \left( \log \frac{1}{2} + \log x + \frac{1}{3} \log 5 \right)$
- $\log x - \frac{1}{2} \log \left( x - \frac{1}{2} \right) = \log \left( x + \frac{1}{2} \right) - \frac{1}{2} \log \left( x + \frac{1}{8} \right)$

# Examples

*Solve the following equations :*

- $9^{\log_3(1-2x)} = 5x^2 - 5$

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- $9^{\log_3(1-2x)} = 5x^2 - 5$

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- $x^{\frac{\log x + 5}{3}} = 10^{5+\log x}$



# Examples

*Solve the following equations :*

- $x^{\log_3 x} = 9$

# Examples

*Solve the following equations :*

- $x^{\log_3 x} = 9$
- $(\sqrt{x})^{\log_5 x - 1} = 5$

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*Solve the following equations :*

- $x^{\log_3 x} = 9$

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- $x^{\log_3 x} = 9$

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- $x^{\log x + 1} = 10^6$

- $x^{\frac{\log x + 7}{4}} = 10^{\log x + 1}$

# Examples

*Solve the following equations :*

- $x^{\log_{\sqrt{x}}(x-2)} = 9$

# Examples

*Solve the following equations :*

- $x^{\log_{\sqrt{x}}(x-2)} = 9$

- $\left(\frac{\log x}{2}\right)^{\log^2 x + \log x^2 - 2} = \log \sqrt{x}$

# Examples

*Solve the following equations :*

- $x^{\log_{\sqrt{x}}(x-2)} = 9$
- $\left(\frac{\log x}{2}\right)^{\log^2 x + \log x^2 - 2} = \log \sqrt{x}$
- $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$

# Examples

*Solve the following equations :*

- $x^{\log_{\sqrt{x}}(x-2)} = 9$
- $\left(\frac{\log x}{2}\right)^{\log^2 x + \log x^2 - 2} = \log \sqrt{x}$
- $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$
- $\log^2 x - 3 \log x = \log(x^2) - 4$



# Examples

*Solve the following equations :*

- $\log_{1/3}x - 3\sqrt{\log_{1/3}x} + 2 = 0$

# Examples

*Solve the following equations :*

- $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$
- $2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$

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- $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$
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# Examples

*Solve the following equations :*

- $\log_{1/3}x - 3\sqrt{\log_{1/3}x} + 2 = 0$
- $2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$
- $\log_2^2 x + 2\log_2 \sqrt{x} - 2 = 0$
- $(a^{\log_b x})^2 - 5x^{\log_b a} + 6 = 0$

# Examples

*Solve the following equations :*

- $\log^2(100x) + \log^2(10x) = 14 + \log\left(\frac{1}{x}\right)$

# Examples

*Solve the following equations :*

- $\log^2(100x) + \log^2(10x) = 14 + \log\left(\frac{1}{x}\right)$
- $\log_4(x+3) - \log_4(x-1) = 2 - \log_4 8$

# Examples

*Solve the following equations :*

- $\log^2(100x) + \log^2(10x) = 14 + \log\left(\frac{1}{x}\right)$
- $\log_4(x+3) - \log_4(x-1) = 2 - \log_4 8$
- $2\log_4(4-x) = 4 - \log_2(-2-x)$

**Solve Sheet**

**To Attain IIT-Level**