MOD
INTRODUCTION

Derivative by first principle

Let $y = f(x)$; $y + \Delta y = f(x + \Delta x)$

$$\therefore \quad \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(average rate of change of function)

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Above denotes the instantaneous rate of change of function and is called finding the derivative by first principle/by delta method/by ab-initio/by fundamental definition of calculus.

Q. Find equation of tangent to curve

$y = x^2$ at (3, 9)

Note that if y = f (x) then the symbols

$\frac{dy}{dx} = Dy = f'(x) = y_1$ or y' have the same meaning.

Derivative of standard functions

(1) $Dx^{n} = nx^{n-1}$, $n \in R$

(2) $D(a^{x}) = a^{x}\, ln\, a$, $a > 0$

(3) $D(e^{x}) = e^{x}$

(4) $D(ln\, x) = \frac{1}{x}$

(5) $D(\sin x) = \cos x$

(6) $D(\cos x) = -\sin x$

(7) $D(\tan x) = \sec^{2}x$

(8) $D(\cot x) - \text{cosec}^{2}x$

(9) $D(\sec x) = \sec x \tan x$

(10) $D(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(11) $D(\sin^{-1}x) = \dfrac{1}{\sqrt{1-x^2}}$

(12) $D(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$

(12) $D(\tan^{-1} x) = \dfrac{1}{1+x^2}$

(13) $D(\cot^{-1} x) = -\dfrac{1}{1+x^2}$

(14) $D(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$

(15) $D(\operatorname{cosec}^{-1} x) = -\dfrac{1}{|x|\sqrt{x^2-1}}$

→ Chain rule of derivative

→ Product rule

→ Quotient Rule

Example

Q. $e^{\sqrt{x}}$

Q. xe^{x}

Q. $x^2 \ln x$

Q. π^{x}

Q. x^{π}

Q. $$y = \frac{x}{x^2 + 1}$$

Q. $y = \cos^2 x$

Q. $y = \sin 3x$

Q. $y = \sin^{-1}x^2$

Q. $y = x^3 - 3^x$

Q. $y = 3\sin x$

Q. $\ln^2 x$

Q. $D(\tan(\tan^{-1}x))$

Q. $D\left(\cos^4\frac{x}{2}-\sin^4\frac{x}{2}\right)$

Q. $D(\cos^{-1}x + \sin^{-1}x)^n$

Q. $D\left(e^{\ell n \cot^{-1} x}\right)$

Q. $$D\left(\frac{1-\cos 2x}{\sin 2x}\right)$$

Q. $D\left(\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)\right)$

Q. $x \sin^{-1}x$

Q. $e^{x} \cdot \tan^{-1}x$

Q. If 3 functions are involved

$$D(f(x).g(x).h(x)) = f(x).g(x).h'(x) + g(x).h(x).f'(x) + h(x).f(x).g'(x)$$

$$= \frac{(fg)'(h) + (gh)'(f) + (hf)'(g)}{2}$$

Examples

Q. Let $F(x) = f(x) \cdot g(x) \cdot h(x)$. If for some $x = x_0$, $F'(x_0)$; $f'(x_0) = 4f(x_0)$; $g'(x_0) = -7g(x_0)$ and $h'(x_0) = k\,h(x_0)$ then find k.

Q. If $f(x) = (1 + x)(3 + x^2)^{1/2}(9 + x^3)^{1/3}$ then $f'(-1)$ is equal to

(A) 0

(B) $2\sqrt{2}$

(C) 4

(D) 6

Q. Let f, g and h are differentiable functions. If $f(0) = 1$; $g(0) = 2$; $h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are

$(fg)'(0) = 6$; $(g\,h)'(0) = 4$ and $(h\,f)'(0) = 5$

then compute the value of $(fgh)'(0)$.

Q. If $f(x) = 1 + x + x^2 + \ldots\ldots + x^{100}$ then $f'(1)$

Q. $$y = \frac{1 - \ell n\, x}{1 + \ell n\, x}$$

Q. $y = \dfrac{\sin^{-1} x}{\cos^{-1} x}$

Q. $$y = \frac{x^3 + 2^x}{e^x}$$

Q. $y = \dfrac{x \sin x}{1 + \tan x}$

Q. If $y = \dfrac{x^4 + x^2 + 1}{x^2 + x + 1}$ then $\dfrac{dy}{dx} = ax + b$

find a and b.

Q. If $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$, find $\left.\frac{dy}{dx}\right]_{x=\pi/4}$

Q. If $y = \dfrac{\tan^{-1} x - \cot^{-1} x}{\tan^{-1} x + \cot^{-1} x}$, find $\left.\dfrac{dy}{dx}\right]_{x=-1}$

(A) 0

(B) 1

(C) $\dfrac{2}{\pi}$

(D) –1

Q. If $y = \dfrac{x^3 + x^2 + x}{1 + x^2}$, find $\left.\dfrac{dy}{dx}\right]_{x=0}$

Q. Let g be a differentiable function of x. If $f(x) = \frac{g(x)}{x^2}$ for $x > 0$, $g(2) = 3$ and $g'(2) = -2$,

Note:

If $f'(x)$ is not defined on $x = c$ then it is wrong to conclude that $f(x)$ is not derivable at $x = c$. In such cases, LHD at $x = c$ and RHD at $x = c$.

$f(x) = x^{1/3} \sin x$ at $x = 0$

Q. $y = \sin^3 \sqrt{x}$

Q. $y = ln^3 \tan^2 (x^4)$

Q. $y = \cos^{-1}\left(\frac{ax}{b}\right)$

Q. $y = \dfrac{1}{(f(x))^n}$

Q. $y = \ln(\sec x)$

Q. $y = \sec x\left(\sqrt{\tan x}\right)$

Q. $y = \sec^2(f^3(x))$

Q. $y = \sqrt{f(x)}$

Q. Exp $(\cos^3 (\tan^{-1} x^3)^2)$

Q. $y = \cos(\ln x)$

Q. $y = f(1/x)$

Q. Suppose that f is a differentiable function such that $f(2) = 1$ and $f'(2) = 3$ and let $g(x) = f(x\,f(x))$. Find $g'(2)$

Assignment – 1
G.N. Berman

Q. (1) $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$;

(2) $y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$;

(3) $y = (\sqrt{x} + 1)\left(\dfrac{1}{\sqrt{x}} - 1\right)$;

(4) $y = \left(\dfrac{2}{\sqrt{x}} - \sqrt{3}\right)\left(4x\sqrt[3]{x} + \dfrac{\sqrt[3]{x^2}}{3x}\right)$;

(5) $y = (\sqrt[3]{x} + 2x)(1 + \sqrt[3]{x^2} + 3x)$;

(6) $y = (x^2 - 1)(x^2 - 4)(x^2 - 9)$;

(7) $y = (1 + \sqrt{x})(1 + \sqrt{2x})(1 + \sqrt{3x})$

Q. $y = \dfrac{x+1}{x-1}$

Q. $y = \dfrac{x}{x^2+1}$

Q. $s = \dfrac{3t^2+1}{t-1}$

Q. $u = \dfrac{v^3-2v}{v^2+v+1}$

Q. $y = \dfrac{ax+b}{cx+d}$

Q. $z = \dfrac{x^2+1}{3(x^2-1)} + (x^2-1)(1-x)$

Q. $u = \dfrac{\text{v}^5}{\text{v}^3 - 2}$

Q. $v = \dfrac{1 - x^3}{1 + x^3}$

Q. $y = \dfrac{2}{x^3 - 1}$

Q. $u = \dfrac{v^2 - v + 1}{a^2 - 3}$

Q. $y = \dfrac{1 - x^3}{\sqrt{\pi}}$

Q. $z = \dfrac{1}{t^2 + t + 1}$

Q. $s = \dfrac{1}{t^2 - 3t + 6}$

Q. $y = \dfrac{2x^4}{b^2 - x^2}$

Q. $y = \dfrac{x^2 + x - 1}{x^3 + 1}$

Q. $y = \dfrac{3}{(1 - x^2)(1 - 2x^3)}$

Q. $y = \dfrac{ax + bx^2}{am + bm^2}$

Q. $y = \dfrac{a^2 b^2 c^2}{(x - a)(x - b)(x - c)}$

Q. $f(x) = (x^2 + x + 1)(x^2 - x + 1)$. Find $f'(0)$ and $f'(1)$.

Q. $F(x) = (x - 1)(x - 2)(x - 3)$. Find $F'(0)$, $F'(1)$ and $F'(2)$.

Q. $F(x) = \dfrac{1}{x+2} + \dfrac{3}{x^2+1}$. Find F′ (0) and F′ (–1).

Q. (1) $(x - a)(x - b)(x - c)(x - d)$

(2) $(x^2 + 1)^4$

(3) $(1 - x)^{20}$

(4) $(1 + 2x)^{30}$

(5) $(1 - x^2)^{10}$

(6) $(5x^3 + x^2 - 4)^5$

(7) $(x^3 - x)^6$

(8) $\left(7x^2 - \dfrac{4}{x} + 6\right)^6$

(9) $s = \left(t^3 - \dfrac{1}{t^3} + 3\right)^4$

(10) $y = \left(\dfrac{x+1}{x-1} \right)^2$ (11) $y = \left(\dfrac{1+x^2}{1+x} \right)^5$

(12) $y = (2x^3 + 3x^2 + 6x + 1)^4$

Q. $y = \cos^2 x$

Q. $y = \frac{1}{4}\tan^4 x$

Q. $y = \cos x - \frac{1}{3}\cos^3 x$

Q. $y = 3\sin^2 x - \sin^3 x$

Q. $y = \frac{1}{3}\tan^3 x - \tan x + x$

Q. $y = x\sec^2 x - \tan x$

Q. $y = \sec^2 x + \mathrm{cosec}^2 x$

Q. $y = \sin 3x$

Q. $y = a\cos\frac{x}{3}$

Q. $y = 3\sin(3x + 5)$

Q. $y = \tan\frac{x+1}{2}$

Q. $y = \sqrt{1 + 2\tan x}$

Q. $y = \sin\frac{1}{x}$

Q. $y = \sin(\sin x)$

Q. $y = \cos^3 4x$

Q. $y = \sin\frac{1}{x}$

Q. $y = \sin(\sin x)$

Q. $y = \cos^3 4x$

Q. $y = \sqrt{\tan\frac{x}{2}}$

Q. $y = \sin\sqrt{1+x^2}$

Q. $y = \cot\sqrt[3]{1+x^2}$

Q. $y = \sqrt{1+\tan\left(x+\frac{1}{x}\right)}$

Q. $y = \cos^2\frac{1-\sqrt{x}}{1+\sqrt{x}}$

Q. $y = \left(1+\sin^2 x\right)^4$

Q. $y = \sin^2(\cos 3x)$

Q. $y - x \arcsin x$

Q. $y = \frac{\arcsin x}{\arccos x}$

Q. $y = (\arcsin x)^2$

Q. $y = x\arcsin x + \sqrt{1-x^2}$

Q. $y = \sin x + \cos x$

Q. $y = \dfrac{x}{1 - \cos x}$

Q. $y = \dfrac{\tan x}{x}$

Q. $p = \phi \sin \phi + \cos \phi$

Q. $z = \dfrac{\sin \alpha}{\alpha} + \dfrac{\alpha}{\sin \alpha}$

Q. $s = \dfrac{\sin t}{1 + \cos t}$

Q. $y = \dfrac{x}{\sin x + \cos x}$

Q. $y = \dfrac{x \sin x}{1 + \tan x}$

Q. $y = \dfrac{1}{\arcsin x}$

Q. $y = x \sin x \arctan x$

Q. $y = \dfrac{\arccos x}{x}$

Q. $y = \sqrt{x}\ \arctan x$

Q. $y = (\arccos x + \arcsin x)^n$

Q. $y = \operatorname{arcsec} x$

Q. $y = \dfrac{x}{1+x^2} - \arctan x$

Q. $y = \dfrac{\arcsin x}{\sqrt{1-x^2}}$

Q. $y = \dfrac{x^2}{\arctan x}$

Q. $y = \arcsin(x - 1)$

Q. $y = \arccos \dfrac{2x-1}{\sqrt{3}}$

Q. $y = \arctan x^2$

Q. $y = \arcsin \dfrac{2}{x}$

Q. $y = \arcsin(\sin x)$

Q. $y = \arctan^2 \dfrac{1}{x}$

Q. $y = \sqrt{1-(\arccos x)^2}$

Q. $y = \arcsin \sqrt{\dfrac{1-x}{1+x}}$

Q. $y = \dfrac{1}{2}\sqrt[4]{\arcsin\sqrt{x^2+2x}}$

Q. $y = \arcsin \dfrac{\sin \alpha \sin x}{1 - \cos \alpha \sin x}$

Q. $y = \arccos \dfrac{b + a \cos x}{a + b \cos x}$

Q. $y = \arctan(x - \sqrt{1 + x^2})$

Q. $y = x^2 \log_3 x$

Q. $y = ln^2 x$

Q. $y = x \log_{10} x$

Q. $y = \sqrt{ln\, x}$

Q. $y = \dfrac{x-1}{\log_2 x}$

Q. $y = x \sin x \ln x$

Q. $y = \dfrac{1}{\ln x}$

Q. $y = \dfrac{\ln x}{x^n}$

Q. $y = \dfrac{1 - \ln x}{1 + \ln x}$

Q. $y = \dfrac{\ln x}{1 + x^2}$

Q. $y = x^n \ln x$

Q. $y = \sqrt{1 + \ln^2 x}$

Q. $y = \ln (1 - 2x)$

Q. $y = \ln (x^2 - 4x)$

Q. $y = \ln \sin x$

Q. $y = \log_3 (x^2 - 1)$

Q. $y = \ln \tan x$

Q. $y = \ln \arccos 2x$

Q. $y = \ln^4 \sin x$

Q. $y = \arctan [\ln (ax+b)]$

Q. $y = (1 + \ln \sin x)^n$

Q. $y = \log_2 [\log_3 (\log_5 x)]$

Q. $y = ln \arctan \sqrt{1+x^2}$

Q. $y = 2^x$

Q. $y = 10^x$

Q. $y = \frac{1}{3^x}$

Q. $y = \frac{x}{4^x}$

Q. $y = x\ 10^x$

Q. $y = xe^x$

Q. $y = \frac{x}{e^x}$

Q. $y = \frac{x^3 + 2^x}{e^x}$

Q. $y = e^x \cos x$

Q. $y = \frac{e^x}{\sin x}$

Q. $y = \frac{\cos x}{e^x}$

Q. $y = 2^{\frac{x}{\ln x}}$

Q. $y = x^3 - 3^x$

Q. $y = \sqrt{1+e^x}$

Q. $y = (x^2 - 2x + 3)e^x$

Q. $y = \frac{1+e^x}{1-e^x}$

Q. $y = \frac{1-10^x}{1+10^x}$

Q. $y = \frac{e^x}{1+x^2}$

Q. $y = xe^{x}(\cos x + \sin x)$

Q. $y = e^{-x}$

Q. $y = 10^{2x-3}$

Q. $y = e^{\sqrt{x+1}}$

Q. $y = \sin(2^{x})$

Q. $y = 3^{\sin x}$

Q. $y = a^{\sin 3^{x}}$

Q. $y = e^{\arcsin^{2} x}$

Q. $y = 2^{3^{x}}$

Q. $y = e^{\sqrt{\ln x}}$

Q. $y = \sin(e^{x^{2}+3x-2})$

Q. $y = 10^{1-\sin^{4} 3x}$

LOGARITHMIC DIFFERENTIATION

(i) A function which is the product or quotient of a number of functions **OR**

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(i) A function which is the product or quotient of a number of functions **OR**

(ii) A function of the form $[f(x)]^{g(x)}$ where f & g are both derivable, it will be found convinient to take the logarithm of the function first & then differentiate **OR** express $= (f(x))^{g(x)} = e^{g(x).\ln(f(x))}$ and then differentiate.

Examples

Q. If $y = \sin x . \sin 2x . \sin 3x \ldots\ldots \sin nx$, find y'.

Q. If f(x) = (x + 1) (x + 2) (x + 3) ……(x + n) then $f'(0)$ is

(A) n!

(B) $\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{n}\right)$

(C) $\frac{n(n+1)}{2}$

(D) n!

Q. If $f(x) = \prod_{n=1}^{100} (x - n)^{n(101-n)}$ then find $\frac{f(101)}{f'(101)}$

Q. Find derivative of

$y = (\sin x)^{\ln x}$

Q. $y = x^{\tan x} + (\sin x)^{\cos x}$

Q. $y = (\sin x)\left(e^{\sqrt{\sin x}}\right)(\ell n\, x)\left(x^{\cos^{-1} x}\right)$

Q. $y = (x^{\ln x})(\sec x)^{3x}$

Q. If $y = (\sin x)^{\ln x} \operatorname{cosec}(e^x(a + bx))$ and $a + b = \frac{\pi}{2e}$ then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) $(\sin 1) \ln \sin (1)$

(B) 0

(C) $\ln \sin (1)$

(D) $1 + \ln (\sin 1)$

Q. If $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ then $\left.\frac{dy}{dx}\right]_{x=1}$ is

(A) 4

(B) 5/2

(C) 3

(D) not defined

Q. $y = \sqrt{x}^{\sqrt{x}} \cdot e^{x^2}$ Find $y'(1)$

Q. If $f(x) = y = \pi^2 + 2^x + x^2 + x^{1/x}$, then find the slope of the line perpendicular to the tangent on the graph of $y = f(x)$ at $x = 1$.

Assignment – 2
G.N. Berman

Q. $y = x^{x^2}$

Q. $y = x^{x^x}$

Q. $y = (\sin x)^{\cos x}$

Q. $y = (l\text{n } x)^x$

Q. $y = (x + 1)^{2/x}$

Q. $y = x^3 e^{x^2} \sin 2x$

Q. $y = x^{ln\, x}$

Q. $y = x^{1/x}$

Q. $y = x^{\sin x}$

Q. $y = \left(\dfrac{x}{1+x}\right)^x$

Parametric Differentiation

Q. In some situation curves are represented by the equations e.g. $x = \sin t$ & $y = \cos t$. If $x = f(t)$ and $y = g(t)$ then

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{g'(t)}{f'(t)}$$

Q. Find derivate of y w.r.t. x if

$x = a(\cos t + t \sin t)$ and $y = a (\sin t - t \cos t)$

Q. $x = \dfrac{3at}{1+t^3}; y = \dfrac{3at^2}{1+t^3}$

Q. $x = a\sec^2\theta$; $y = a\tan^2\theta$

Q. $x = a\sqrt{\cos 2t}\ \cos t$ and $y = a\sqrt{\cos 2t}\ \sin t$ then,

find $\left.\dfrac{dy}{dx}\right]_{t=\pi/6}$

Q. $x = \cos t + t \sin t - t^2/2 \cos t$

$y = \sin t - t \cos t - t^2/2 \sin t$

Q. $y = a \sin^3 t$

$x = a \cos^3 t$

Derivative of f(x) w.r.t. g(x)

If y = f (x) and z = g (x) then derivative of f (x) w.r.t. g(x) is given by

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz} = \frac{f'(x)}{g'(x)}$$

Q. Derivative of $(\ln x)^{\tan x}$ w.r.t. x^x.

Q. Derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\sqrt{1 - x^2}$ when $x = \frac{1}{2}$

Q. Define derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $\forall x \in R$.

Q. Differential coefficient of $e^{\sin^{-1}x}$ w.r.t. $e^{-\cos^{-1}x}$ is independent of x.

Derivative of Implicit Function

$$\phi\,(x, y) = 0$$

Q. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\ell n\, x}{(1+\ell n\, x)^2}$

Q. If $\sin y = x \sin(a + y)$ then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ Also find $\frac{dy}{dx}$ explicitly.

Q. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots\infty}}}$ find $\frac{dy}{dx}$ $(\sin x > 0)$.

Q. $y = x^{x^{x^{x^{\cdot^{\cdot^{\cdot^{\infty}}}}}}}$

Q. $y = (\ell n\, x)^{(\ell n\, x)^{(\ell n\, x)^{\cdot^{\cdot^{\cdot \infty}}}}}$

Q. $y = \cfrac{x}{1+\cfrac{x}{2+\cfrac{x}{1+\cfrac{x}{2+\dots}}}}$, prove that $y' = \frac{1}{1+y}$

Q. If $\sqrt{1-x^2}+\sqrt{1-y^2}$ = a(x–y) then prove that

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

Q. A curve is described by the relation $ln(x + y) = xe^y$. Find the tangent to the curve at (0,1)

Q. If $y^5 + xy^2 + x^3 = 4x + 3$, then find $\frac{dy}{dx}$ at (2,1)

Derivative of Inverse Function

Examples

Q. If $y = f(x) = x^3 + x^5$ and g is the inverse of f find $g'(2)$

Q. Let $f(x) = \exp(x^3 + x^2 + x)$ for any real number x, and let g be the inverse function for *f*. The value of $g'(e^3)$ is

(A) $\frac{1}{6e^3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{34e^{39}}$

(D) 1

Q. If g is the inverse of f and $f'(x) = \frac{1}{1+x^n}$,

prove that $g'(x) = 1 + \left(g(x)\right)^n$

Q. If $f(x) = x^3 + e^{x/2}$ & $g(x) = f^{-1}(x)$

Find $g'(1)$

Q. If $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all x, y

$f'(0)$ exists & $f'(0) = -1$, $f(0) = 1$ find $f(2)$.

Q. If $f\left(\frac{x+2y}{3}\right) = \frac{f(x)+2f(y)}{3}$ for all x, y

$f'(0)$ exists & $f'(0) = 1$, $f(0) = 2$ find $f(x)$.

Q. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all x, y

$f'(2) = 2$ find $f(x)$.

Q. If $f(0) = 0$, $f'(0) = 2$ then Differentiation of $y = f(f(f(f(f(x)))))$ at $x = 0$

Q. If $y = \frac{\sin x}{1+} \frac{\cos x}{1+} \frac{\sin x}{1+} \frac{\cos x}{1+} \ldots\ldots\ldots\infty$

(A) equal to 0

(B) equal to 1/2

(C) equal to 1

(D) non existent

Q. $f = |x|^{|\sin x|}$, find $f'(-\pi/4)$

nth Order Derivatives

Examples

Q. Find nth order derivative of sinx, cosx, x^n, x^{n+1}

$\frac{d^2y}{dx^2}$ is double derivative of y w.r.t. x $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

Q. Find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$ if $y = \sin t,\ x = \cos t$

Q. $\sqrt{x} + \sqrt{y} = 4 \quad \dfrac{dx}{dy} \qquad \text{at } y = 1$

Q. $y = \sqrt{x \ln x}$ $\quad y'$ at $x = e$

Q. Use the substitution $x = \tan\theta$ to show that the equation,

$$\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{y}{\left(1+x^2\right)^2} = 0 \text{ changes to } \frac{d^2y}{d\theta^2} + y = 0$$

Q. Starting with $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$. Prove that $\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$

Q. If $y^2 = 4ax$, $\dfrac{d^2y}{dx^2} \cdot \dfrac{d^2x}{dy^2} = -\dfrac{2a}{y^3}$

A homogeneous equation of degree n represents '*n*' straight lines passing through the origin.

Q. If $x^3 + 3x^2y - 6xy^2 + 2y^3 = 0$, then $\left.\frac{d^2y}{dx^2}\right]_{(1,1)} = ?$

Q. If $y = \left(\frac{1}{x}\right)^x$ then prove that $y_2(1) = 0$ i.e. $\frac{d^2y}{dx^2} = 0$

Q. If $e^{x+y} = y^2$ then prove that $y'' = \dfrac{2y}{(2-y)^3}$

Derivative of Determinants

$$\text{If } F(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ l & m & n \end{vmatrix}$$ where all functions are differentiable then

$$D'(x) = \begin{vmatrix} f' & g' & h' \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ l' & m' & n' \end{vmatrix}$$

This result may be proved by first principle and the same operation can also be done column wise.

Remainder Theorem

Note

If $(x - r)$ is a factor of the polynomial repeated m times then r is a root of the equation $f'(x) = 0$ repeated $(m - 1)$ times.

Q. If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$

then find $f'(x)$

Q. $$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ P & P^2 & P^3 \end{vmatrix}$$

P is constant , if f '' (0) = 0 find P.

Q. f, g, h are polynomial degree 2 then prove that

$$\Phi(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$$ is constant polynomial.

Q. If $y = \begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$ then $\dfrac{dy}{dx} = ?$

Q. If $f = \begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos n\pi/2 & 4 \\ \sin x & \sin n\pi/2 & 8 \end{vmatrix}$, find $\dfrac{d^n}{dx^n}\left(f(x)\right)_{x=0}$

Q. If $f = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, prove that

$$f'(x) = 3x^2 + 2x\,(a^2 + b^2 + c^2)$$

Q. If $f(x)=\begin{vmatrix}(1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3}\\(1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3}\\(1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3}\end{vmatrix}$

then find coefficient of x in the expansion of f(x).

Q. The new definition of derivative of a function is given by

$$f'(x) = \lim_{h \to 0} \frac{f^5(x+h) - f^5(x)}{h}$$

& $f(x) = x\,l\text{n}x$ find $(f'(x))_{x=e}$

Q. $x = a\cos\theta,\ y = b\sin\theta$ find $\dfrac{d^3y}{dx^3}$

L' Hospital's Rule (0/0, ∞/∞)

Q. $$\lim_{x\to 0}\frac{x\cos x-\ell n(1+x)}{x^2}$$

Q. Find a and b if $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$

Q. $\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^2 + x - \ell n(1-x)}$

Q. $$\lim_{x\to 0}\frac{1+\sin x-\cos x+\ell n(1-x)}{x\tan^2 x}$$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{6}$ (D) DNE

Q. $$\lim_{x \to 0} \frac{\log_{\sec\frac{x}{2}}(\cos x)}{\log_{\sec x}(\cos(x/2))}$$

(A) 1 (B) 16 (C) 4 (D) 2

Q. $\lim_{x\to 1} \frac{x^x - x}{x - 1 - \ell nx}$

Q. $\lim\limits_{x\to 0} \dfrac{1-\cos x\cdot\cos 2x\cdot\cos 3x}{x^2}$

Q. $\lim_{x\to 0} \frac{(\cos ax)^{1/m} - (\cos bx)^{1/n}}{x^2}$

Q. $\lim_{x \to 0^+} (\operatorname{cosec} x)^{\frac{1}{\ell n x}}$

Q. $\lim\limits_{x \to \pi/2} (\sec x)^{\cot x}$

Q. $\lim_{x \to 0^+} (\cot x)^{\frac{1}{\ell n x}}$

Q. $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$

Q. $\lim_{x \to 0} x^x$

Q. $\lim\limits_{x \to 1} \left(1 - x^2\right)^{\frac{1}{\ell n(1-x)}}$

Q. $$\lim_{x\to 4} \frac{\sqrt{2x+1}+\sqrt{x-3}-4}{\sqrt{(3x+4)}+\sqrt{5x+5}-9}$$

Q. $\lim_{x \to 0} (\tan x)^{\sin x}$

Q. $\lim_{x \to 0^+} (\sin x)^{\frac{1}{x}}$

Q. f(x) be different function & f '' (0) = 2 then

$$\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$$