Permutation & Combination

1. Fundamental Principle of Counting, Multiplication Rule
2. Factorial, Permutation & Combination, Examples
3. Properties, Circular Permutations, Clockwise and Anticlockwise Arrangements
4. Arrangement of Alike Objects
5. All Possible Selections
6. Number of Divisors of N
7. Distribution of Identical Objects (Begger's Method), Maximizing ^nC_r, Grid Problem, De-

Permutation & Combination

No. of Questions				
2008	2009	2010	2011	2012
1	1	--	--	1

PERMUTATION AND COMBINATION

PERMUTATION AND COMBINATION

To count without actually counting

5 Models

Model - 1

Example

Q.	Number of way in which one can travel from town T_1 to T_3 via town T_2 and T_4 in definite
order

Model - 2

Cinema Hall

Q. Number of ways to view a movie given number of ways are differentiated by exit and entry.

Model - 3

Tossing of a coin

Model - 4

Placing Books P,C,M,H on a shelf having 4 partition

Example

Q. Number of ways to keep 5 different books in 5 distinct partitions

Model - 5

Keeping P | C | M | E books on a shelf having 5 partition.

Example

Q.   Ways to arrange 4 people on 5 different seats.

Q. Number of ways if :
(i) 3 digit numbers using the digits 1,2,3,4,5 without repetition.

Q. Number of ways if :
   (ii)   If repetition is allowed

Q. Number of ways if :
   (iii) Numbers of 3 digit even numbers without repetition

Q.   Number of ways if :
      (iv)   3 digit odd number without repetition.

Q. 10 True / False questions. A student answered all questions no negative marking.
(i) Ways to obtain maximum marks

Q.   10 True / False questions. A student answered all questions no negative marking.
(ii)   Ways to obtain minimum marks

Q. 10 True / False questions. A student answered all questions no negative marking.
(iii) Ways to get positive marks

Q. 10 True / False questions. A student answered all questions no negative marking.

(iv) At least one answer is wrong

Q. How many 3 digit number can be formed without using 0,2,3,4,5,6. (by default repetition is allowed)

Q.   10 students compete in a swimming race. In how   many ways can they occupy the first 3 positions.

Q.   7 Flags of different colour. Number of different   signals that can be transmitted by the use 2   flags one above the other.

Q. How many 3 digit numbers more than 600 can be formed using 2, 3, 4, 6, 7

Q.   How many numbers between 3000 & 4000 can be made from 3, 4, 5, 6, 7, 8 without repetition

Q. How many numbers divisible by 5 lying between 4000 & 5000 can be formed from 4,   5, 6, 7, 8 ?

Q. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then, the number of words which have at least one letter repeated, is
(A) 69760 (B) 30240
(C) 99748 (D) None of these

[IIT-JEE 1980]

Q. How many numbers greater than thousand but not greater than 4000 can be formed with 0, 1, 2, 3, 4

(i)   repetition is not allowed

Q. How many numbers greater than thousand but not greater than 4000 can be formed with 0, 1, 2, 3, 4
(ii)   repetition is allowed

Q.   A men has 6 friend in how many he send invitation cards to them if he has 3 servant to carry cards

Factorial

0! = 1 ;

1! = 1 ;

2! = 2 ;

3! = 6 ;

4! = 24.

Note

Factorial up to 7 must be remembered

Example

Q.   Prove that : $(2n!) = 2^n.n! [1.3.5……..(2n – 1)]$.

Q.   If $(100)! = 2^m \cdot I$ where I is an odd integer then find m

Prime factorization

Example

Q.   Find the number of zeros in the end of 100!

Theorems

T - 1

Number of permutations of n distinct things taken all at a time symbolysed as :

$$^{n}P_{n} = P(n.n) = A_{n}^{n} = n!$$

T - 2

Number of permutations of n distinct things taken r at a time $0 \leq r \leq n$ as :

$$^nP_r = P(n.r) = A_r^n = \frac{n!}{(n-r)!}$$

T - 3

Number of combinations / selections of n distinct things taken r ($0 \leq r \leq n$) at a time

$$^n C_r = C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Note

$$^nC_r = {}^nC_{n-r} \; ;$$

$$^nC_0 = 1 \; ;$$

$$^nC_n = 1$$

$$^nC_x = {}^nC_y \Rightarrow x = y \;\; \text{or} \; x + y = n$$

$$^nP_r = r! \; {}^nC_r$$

$$^nC \quad + {}^nC \quad = {}^{n+1}C$$

Example

Q.   If $^{7}C_{2} + {}^{7}C_{3} + {}^{8}C_{4} + {}^{9}C_{5} + {}^{10}C_{6} + {}^{11}C_{7} = {}^{12}C_{x}$.

Find x

Q. The value of the expression

$$^{47}C_4 + \sum_{i=1}^{5} {}^{52-i}C_3$$

(A) $^{47}C_5$

(B) $^{52}C_5$

(C) $^{52}C_4$

(D) None of these

[IIT-JEE 1980]

Q.   If $^{n+2}C_n = 45$ find n

Q.   If $^nC_{90} = {}^nC_{11}$ find $^nC_2$

Q. $5\ {}^nP_3 = 24\ {}^nC_4$ find n

Q.   There are n points in a plane, no 3 of which are collinear. Find
(i)   Number of straight lines formed by joining these points.

Q.   There are n points in a plane, no 3 of which are collinear. Find
(ii)   Number of triangles.

Q. (i) Find Number of diagonals in a n polygon

Q.   (ii)   Number of diagonals of a n sided polygon are 44. Find number of sides of the polygon.

Q.   In a birthday party of 101 people, every person shakes hand with every other person. Find number of hand shakes in the party.

Q.   10 points in a plane no 3 being collinear except
     4 which are in the same line find.

Q.   10 points in a plane no 3 being collinear except 4 which are in the same line find.

(i)   Number of different lines

Q.   10 points in a plane no 3 being collinear except
     4 which are in the same line find.
     (ii)   Number of Δ

Q. 5 boys along with Ram & Shyam are to be arranged in a line.

Q.   5 boys along with Ram & Shyam are to be arranged in a line.
(i)   Ways to arrange such that Ram & Shyam are always together.

Q. 5 boys along with Ram & Shyam are to be arranged in a line.
(ii) Ways to arrange such that Ram & Shyam are never together.

Q.   6 parallel lines are cut by other 5 parallel lines. Find numbers of parallelograms.

Q. A five digits number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4, and 5, without repetition. The total number of ways this can be done, is [IIT-JEE 1989]

(A) 216   (B) 240   (C) 600   (D) 3125

Q. A train having 12 stations enroute has to be stopped at 4 stations. Number of ways the train can be stopped if no two of the stopping stations are consecutive.

Q. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. The number of possible arrangements is

(A) $^6C_3 \times {}^4C_2$ (B) $^4P_2 \times {}^4P_3$

(C) $^4C_2 + {}^4P_3$ (D) None of these

[IIT-JEE 1982]

Q. A grandfather with 4 grand children takes them to a garden without taking the same 3 children together

(i)   How many time grandfather goes.

Q.   A grandfather with 4 grand children takes them to a garden without taking the same 3 children together
(ii)   How frequently a particular child goes.

Q.   7 different colours of rainbow are to be arranged so that blue & green never come together.

Q.   An *n* digit number is a positive number with exactly *n* digits. Nine hundred distinct *n*-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of *n* for which this is possible, is   [IIT-JEE 1998]
(A) 216   (B) 240   (C) 600   (D) 3125

Q.   Number of words with 10 different consonants & 4 different vowels
(i)   If each word has 3 consonant and 2 vowels

Q.   Number of words with 10 different consonants & 4 different vowels
(ii)   If each word has 3 consonant and 2 vowels and begins with a consonant.

Q.   Number of words with 10 different consonants & 4 different vowels

(iii)   If each word has 3 consonant and 2 vowels and terminal letters are vowel

Q.   Number   of   words   with   10   different consonants & 4 different vowels
(iv)   If each word has 3 consonant and 2 vowels are not together

Q. Number of words with 10 different consonants & 4 different vowels

(v) If each word has 3 Consonants and 2 Vowels and Vowels occupy odd position

Q.   In morse telegraphy there are four arms & each arm is capable of taking 5 distinct position including position of rest. How many different signals can be transferred.

Q.   Number of ways in which 5 different books can be distributed between 3 children if each child gets at least one book.   **[IIT-JEE 2012]**

Q. Boat Problem :
    An 8 oared boat to be manned from a crew of 11 of whom 3 can only steer but can not row. 8 can row but can not steer. In how many ways the staff can be arranged if two of the men can only row on bow side.

Q.   4 boys and 4 girls are to be seated in a line. Find

(i)   Number of ways they can be seated

Q. 4 boys and 4 girls are to be seated in a line. Find
(ii) No two girls are together

Q. 4 boys and 4 girls are to be seated in a line. Find
(iii) A least one girl is separated from rest of the girls/ Not all girls are together

Q.   4 boys and 4 girls are to be seated in a line. Find
(iv)   Girls and Boys are alternate

Q.   4 boys and 4 girls are to be seated in a line. Find
(v)   Boys & Girls alternate & a particular
boy & girl are always together.

Q. 4 boys and 4 girls are to be seated in a line. Find (vi) A particular boy and girl are never adjacent to each other.

Q.   4 boys and 4 girls are to be seated in a line. Find
(vii) If they are 4 married couples then
number of ways if each couple is together.

Q.   8 straight lines & 6 circles lie in a plane. Find maximum number of their intersection points.

Q.   A cricket team comprising of A,B,C,……J, K is to be sent for batting. If 'A' wants to bat before 'J' and 'J' wants to bat after G. Then the number of batting orders if other players could go in any order.

Formation of Group

Every selection has a corresponding rejection resulting in formation of two groups / teams / sets

Number of ways to form 2 teams each containing 2 players

Number of ways to form 2 teams one containing 1 player and other 2 player

Number of ways to form 2 groups each containing three elements

Example

Q.   Number of ways in which 6 toys can be distributed equally between Kamla, Vimla, Sweety.

Q. Number of ways in which 30 soldiers can be deputed equally on 3 border post.

Q. Number of ways in which 200 people can be divided into 100 couples.

Q. Numbers of ways in which 12 different books can be tide up in 3 bundles one containing 2 and the other 2 containing 5 books.

Q. 6 different books to be distributed between Kamla, Vimla, Sweety. If each girls gets atleast one book ?

Q.   8 different computer are to be distributed in 5 schools. If each school gets at least one computer. Find number of ways to distribute ?

Q. For a game in which every pair has to play with every other pair, 6 men are available number of games which can be played ?

Q.   Number of ways in which 8 person can be seated in three different taxies each having three seats for passengers and duly numbered.
(i)   If internal arrangement matters.
(ii)   If Internal arrangement of persons inside the taxi is immaterial.

Q.   In a jeep there are 3 seats in front & 3 at back. Number of ways 6 persons of different height be seated So that every one in front is shorter than the person directly behind him.

Q.	A rack has 5 different pairs of shoes. Number of ways in which 4 shoes can be chosen from it, so that there is no complete pair.

Q. 6 newly married couples are in a Birthday Party. In how many ways 4 persons can be selected so that they form :

(i)   No couple

Q. 6 newly married couples are in a Birthday Party. In how many ways 4 persons can be selected so that they form :
(ii) Exactly 1 couple

Q.   6 newly married couples are in a Birthday Party. In how many ways 4 persons can be selected so that they form :

(iii)   At least 1 couple

Q. 6 newly married couples are in a Birthday Party. In how many ways 4 persons can be selected so that they form :

(iv) At most 1 couple

Arrangement of alike objects

Example

Q.   How many different words can be formed using all letters of the word MAHABHARAT ?

Q. 21 white & 19 black balls are arranged in a line. Balls of same colour are alike. Find no of arrangements if black balls are separated ?

Q. How many different nine digit numbers can be formed from the number 22,33,55,888 by rearranging its digits so that the odd digits occupy even position [IIT-JEE 2000]
(A) 16   (B) 36   (C) 60   (D) 180

Q. The number of arrangements of the letters of the word BANANA in which the two *N*'s do not appear adjacently, is [IIT-JEE 2002]
(A) 40 (B) 60 (C) 80 (D) 100

Q. ASSASSINATION ?
(i) Ways to arrange such that no two vowels are consecutive ?

Q. ASSASSINATION ?
(ii)   Ways to arrange no two 'S' occur together

Q.   ASSASSINATION ?
(iii)   Ways to arrange keeping position of each vowel fixed

Q.   ASSASSINATION ?
       (iv) Ways to arrange without changing relative order of vowels and consonants.

Q. Number of numbers greater than a million using digits 0, 2, 2, 3, 3, 4, 5. Of these how many are divisible by 5.

Q. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is [IIT-JEE 2007]
(A) 360   (B) 192   (C) 96   (D) 48

Q.   How many different words can be formed using all the letters of the words HONOLULU if no two alike letters are together.

Q.   How many 8 digit numbers can be formed using two 1's, two 2's, two 3's, one 4 and one 5 so that no two consecutive digits are identical.

Q.   The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is   [IIT-JEE 2009]

(A) 55   (B) 66   (C) 77   (D) 88

Q.   Number of 7 digit numbers if sum of their digits is (i) 63 (ii) 62 (iii) 61 (iv) 60.

Selection of Identical Objects

Example

Q.   Number of ways of selecting 5 letters from MISSISSIPPI ?

Q. Number of ways of selecting 5 letters from INDEPENDENCE.

Q. How many 6-letter words can be formed from INTEGRATION if each word has 3 vowels & 3 consonants ?

Q. Number of numbers greater than 1000 from the digits 1,1,2,3,4,0 taken 4 at a time ?

Circular Permutation

Ways to arrange *n* different things taken all at a time

$= \underline{|n-1}$

Ways to permute 'n' objects taken 'r' at a time

$$^{n}C_{r} \, (r-1)\,!$$

Example

Q.   Number of ways in which 7 Chinese & 7 Indians can be seated on a round table, so that no two Indians are together ?

Q. Number of ways in which 5 boys and 5 girls can be seated on a circle alternatly if a particular B_1 and G_1 are never adjacent to each other in any arrangement.

Q. Out of 10 flowers of different colours, how many different garlands can be made if each garland consists of 6 flower of different colour.

Q.   16 knights are sitting on a round table. 7 knights are required to rescue Sudip. Every consecutive knight is hostile to each other. Number of ways a team of 7 knights such that no two hostile knights team up ?

Q. 'n' people are sitting in a circle. Number of ways in which 3 people can be selected if no of two them are consecutive ?

Q. How many hexagons can be constructed by joining vertices of a quindecagon (15 sides) if none of the sides of hexagon are also the side of quindecagon ?

Total Number of selections

Selecting atleast one out of n different things

Total number of selections when 'p' alike of one kind 'q' alike of another kind & 'r' alike of another kind.

Q.   Out of 3 different Maths '4' different physics & '5' different chemistry books, how many different selections can be made so that the selection consists of

(i)   Exactly one book on each subject ?

Q.	Out of 3 different Maths '4' different physics & '5' different chemistry books, how many different selections can be made so that the selection consists of

(ii)	Atleast one book on each subject.

Q.   Out of 2 grapes, 3 mangoes & 4 apples, how many different selections of fruits can be made if each selection has

(i)   Atleast one fruit of every species.

Q. Out of 2 grapes, 3 mangoes & 4 apples, how many different selections of fruits can be made if each selection has
(ii) Atleast one fruit.

Q. Consider the number N = 75600
   (i) Number of divisors ?

Q. Consider the number N = 75600
(ii) Number of proper divisors **?**

Q. Consider the number N = 75600
(iii) Number of odd divisors **?**

Q.   Consider the number N = 75600
      (iv)   Number of even divisors ?

Q. Consider the number N = 75600
 (v) Number of divisors divisible by 5 ?

Q. Consider the number N = 75600
(vi) Number of divisors divisible by 10 ?

Q. Consider the number N = 75600

(vii) Number of divisors divisible by 12 ?

Q.   Consider the number N = 75600

(viii) Sum of all the divisors **?**

Q.   Consider the number N = 75600

(ix)   Number of ways in which N can be resolved as a product of two divisors ?

Q.   Consider the number N = 75600
      (x)   Number of ways in which N can be resolved as a product of two divisors   which are relatively prime.

Q. Number of divisors of the form $(4n + 2)$, $n \geq 0$ of the integer 240 is [IIT-JEE 1998]
(A) 4   (B) 8   (C) 10   (D) 3

Summation of Numbers

Q.   Sum of all the numbers greater than 10000 formed by the digits 1,3,5,7,9 no digit being repeated.

Method - 1

Method – 2

Method - 3

Example

Q.   Sum of all the numbers greater than 10000 formed by the digits 0,2,4,6,8 no digit being repeated.

Q.   Sum of all the numbers greater than 10000 formed by the digits 0,1,2,4,5 no digit being repeated.

Q.   Sum of all distinct four digit numbers that contain only the digits 1,2,3,4,5 each atmost once is given by

Distribution of alike objects Beggar Method

Example

Q.   Number of ways to distribute 10 identical coins to 3 beggars K/B/S each getting none, one or more **?**

Q. A shelf contains 6 separate compartments. Number of ways in which 12 indistinguishable (like) marbles can be placed in the compartments so that no compartment is empty.

Q. Number of natural solutions of
$x + y + z = 102$?

Q. Number of non-negative integral solution of the inequality

$$x + y + z + t \leq 30$$

Q.   Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates as (0, 0), (0, 102), (102, 0) is ?

Q.   Number of ways 30 marks can be allotted to 8 questions if atleast 2 marks are to be given to each question assuming that marks are to be allotted in non-negative integral values ?

Q.   There are unlimited no. of red, white, blue, green balls. They are all alike except for the colour.

(i)   Number of ways is which selection of 10 balls can be made ?

Q. There are unlimited no. of red, white, blue, green balls. They are all alike except for the colour.

(ii) Number of ways in which selection of 10 balls containing balls of all four different colours?

Q. A man has to buy 25 mangoes in 4 different varieties. Buying atleast 4 of each variety. In how many ways can he plan his purchase if mangoes of each variety are identical & available in abundance ?

Q.   Number of different throws in a throw of 'n' alike dice.

Q. Number of ways in which 2 apples, 3 oranges and 4 mangoes can be distributed between R|S| G if each child receives none, one or more number of fruits. (Fruits of the same species are alike)

Q.   A super market offers ice-cream in 10 different varieties. Ice-creams of each variety are identical. In how many ways.

(i)   4 ice-creams can be selected without any restriction.

Q. A super market offers ice-cream in 10 different varieties. Ice-creams of each variety are identical. In how many ways.

(ii) Atleast 2 ice-cream of same variety are taken.

Maximizing ^nC_r

Q.   There are '2n' letters which are either a's or b's, find maximum number of arrangement of these letters ?

Q. A man wants to give as many number of parties as he can out of his 20 friends by calling the same number of people but not the same group. How many should he call so that number parties is maximum ?

Grid Problem

Example

Q.   Number of ways an ant can reach from (1, 1) to (4, 5) via shortest path ?

Q. Number of ways to move from (0, 0) to (7, 8) via (4, 5) ?

Dearrangement

Example

Q.   Two letters are to be kept in 2 directed envelopes. Number of ways they can be placed if none of the letter goes into its own envelope ?

Q. Three letters are to be kept in 3 directed envelopes. Number of ways they can be placed if none of the letter goes into its own envelope ?

Q.   Four letters are to be kept in 4 directed envelopes. Number of ways they can be placed if none of the letter goes into its own envelope ?

Coefficient Method
(To be done in binomial)

Coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r$ $\left(\mathbf{n} \in \mathbf{N}\right)$

Example

Q.   In an exam maximum marks for each of the 3 papers are 50 each and for the 4ᵗʰ paper 100. Number of ways in which a person can secure 60% marks.

Q.   3 different dice are thrown. Number of ways in which a total of k $(9 \leq k \leq 14)$ can be had.

Q. Show that the number of different selections of 5 letters which can be made from five a's four b's three 3 c's: two d's and one e is 71.

Q.   Prove that the number of combinations of n letters out of 3n letters of which n are 'a' and n are 'b' and the rest unlike is $(n + 2) 2^{n-1}$.

Q.   A bag contains 6n tickets numbered from 0, 1, 2…. 6n – 1. In how many ways 3 tickets can be selected so that the sum of the numbers shown on them is equal to 6n.

Q.   In a shooting competition a man can score 0, 2, 3, 4 or 5 points for each shot number of ways in which he can secure 30 in 7 shot.

IIT Questions
[Fill in the Blanks]

Q.1 In a certain test, a_i, students gave wrong answer to at least i question, where i = 1, 2, ..., k. No student gave more than k wrong answers. The total number of wrong answer given is ....

[IIT-JEE 1982]

Q.2 Total number of ways in which six '+' and four '-' sings can be arranged in a line such that no two '-' sings occur together is ……

[IIT-JEE 1988]

Q.3 There are four balls of different colours and four boxes of colours, same as those of the balls. The numbers of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is….. [IIT-JEE 1988]

IIT Questions
[Analytical & Descriptive Questions]

Q.1 Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty ?

[IIT-JEE 1981]

Q.2   *m* men and *n* women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated, is   [IIT-JEE 1983]

$$\frac{m!(m+1)!}{(m-n+1)!}$$

Q.3 7 relatives of a man comprise 4 ladies and 3 gentlemen, his wife has also 7 relatives ; 3 of them are ladies and 3 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 man's relative and 3 of the wife's relatives ?

[IIT-JEE 1982]

Q.4 A box contains two white ball, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw ?

[IIT-JEE 1986]

Q.5   A student is allowed to select atmost n books from n collection of $(2n + 1)$ books. If the total number of ways in which be can select at least one books is 63, find the value of n.

Q.6   Eighteen guests have to be seated half on each side of a long table. Four particular guests desire to sit on one particular side and three other on the other side. Determine the number of ways in which the sitting arrangement can be   made.

[IIT-JEE 1991]

Q.7 A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee ? In how many of these committees
(a) the women are in majority ?
(b) the men are in majority ?
[IIT-JEE 1994]

Q.8 Using permutation or otherwise, prove that $\dfrac{n^2!}{(n!)^n}$ is an integer, where n is a positive integer.

[IIT-JEE 2004]

Q.9 Two planes P_1 and P_2 pass through origin. Two lines L_1 and L_2 also passing through origin are such that L_1 lies on P_1 but not on P_2, L_2 lies on P_2 but not on P_1, A, B, C are three points other than origin, then prove that the permutation [A′ B′ C′] of [ABC] exists. Such that

(a) A lies on L_1, B lies on P_1 not on L_1, does not lie on P_1.

(b) A′ lies on L_2, B lies on P_2 no on L_2, C does not lies on P_2.