MC SIR

CLASS : XII (ABCD) **Dpp on Probability (After 1^{st} Lecture)** ***DPP. NO.- 1***

Q.1 6 married couples are standing in a room. If 4 people are chosen at random, then the chance that exactly one married couple is among the 4 is :

(A*) $\frac{16}{33}$ (B) $\frac{8}{33}$ (C) $\frac{17}{33}$ (D) $\frac{24}{33}$

[Hint: $n(S) = {}^{12}C_4 = 55 \times 9 = 495$

$n(A) = {}^6C_1 . {}^5C_2 . 2^2 = 6 \times 10 \times 4$

$P(E) = \frac{6\times10\times4}{55\times9} = \frac{2.2.4}{11.3} = \frac{16}{33}$ Ans]

Q.2 A committee of 5 is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is :

(A) 1/2 (B) 5/9 (C*) 4/9 (D) 2/3

[Hint : $\frac{{}^7C_3 + {}^7C_5}{{}^9C_5}$]

Q.3 A quadratic equation is chosen from the set of all the quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots is :

(A*) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

Q.4 The probability that a positive two digit number selected at random has its tens digit at least three more than its unit digit is

(A*) 14/45 (B) 7/45 (C) 36/45 (D) 1/6

[Sol. $n(S) = 9 \cdot 10 = 90$

$n(A) = 1 + 2 + 3 + ... + 7 = \frac{7\cdot8}{2} = 28$;

$p = \frac{28}{90} = \frac{14}{45}$ Ans.]

x (Tens)	y(units)
1 or 2 is not possible at ten's place	
3	0
4	0,1
5	0,1,2
⋮	
9	0,1,2,3,4,5,6

Q.5 A 5 digit number is formed by using the digits 0, 1, 2, 3, 4 & 5 without repetition. The probability that the number is divisible by 6 is :

(A) 8 % (B) 17 % (C*) 18 % (D) 36 %

[Hint: Number should be divisible by 2 and 3.

$n(S) = 5 \cdot 5!$; $n(A)$: reject '0' $= 2 \cdot 4!$

reject 3, $4! + 2 \cdot 3 \cdot 3!$

Total $n(A) = 3 \cdot 4! + 6 \cdot 3! = 18 \cdot 3!$

$\therefore \quad p = \dfrac{18 \cdot 3!}{5 \cdot 5!} = 18\%$]

Q.6 A card is drawn at random from a well shuffled deck of cards. Find the probability that the card is a

(i) king or a red card (ii) club or a diamond (iii) king or a queen

(iv) king or an ace (v) spade or a club (vi) neither a heart nor a king.

[**Ans.** (i) $\frac{7}{13}$, (ii) $\frac{1}{2}$, (iii) $\frac{2}{13}$, (iv) $\frac{2}{13}$, (v) $\frac{1}{2}$, (vi) $\frac{9}{13}$]

Q.7 A bag contain 5 white, 7 black, and 4 red balls, find the chance that three balls drawn at random are all white. [Ans. 1/56]

[Hint: $n(s) = {}^{16}C_3$; $\dfrac{n(A)}{n(s)} = \dfrac{{}^5C_3}{{}^{16}C_3} = \dfrac{1}{56}$ Ans.]

Q.8 If four coins are tossed, Two events A and B are defined as

A: No two consecutive heads occur

B: At least two consecutive heads occur.

Find P(A) and P(B). State whether the events are equally likely, mutually exclusive and exhaustive.

[Ans. 1/2; 1/2]

[Hint: P(A) = P(B) = 1/2 / ME / Exh. / EL $\boxed{0\ 1}$ 2 3 4]

Q.9 Thirteen persons take their places at a round table, Find the odds against two particular persons sitting together. [Ans. 5 : 1]

Q.10 A has 3 shares in a lottery containing 3 prizes and 9 blanks, B has 2 shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success. [Ans. 952 to 715]

Q.11 There are three works, one consisting of 3 volumes, one of 4 and the other of one volume. They are placed on a shelf at random, find the chance that volumes of the same works are all together. [Ans. $\frac{3}{140}$]

[Hint: $V_1 V_2 V_3 \mid V_4 V_5 V_6 V_7 \mid V_8$; $n(s) = 81$; $\dfrac{3!3!4!}{8!} = \dfrac{36}{8 \cdot 7 \cdot 6 \cdot 5} = \dfrac{3}{140}$ Ans.]

Q.12 5 persons entered the lift cabin on the ground floor of an 8 floor building. Suppose that each of them independently and with equal probability, can leave the cabin at any other floor, starting from the first, find the probability that all 5 persons leave at different floors.

[Hint: $n(S) = 8^5$; $n(A) = {}^8C_5 \cdot 5!$]

Q.13 Consider a function $f(x)$ that has zeroes 4 and 9. Given that Mr. A randomly selects a number from the set $\{-10, -9, -8, \ldots.. 8, 9, 10\}$, what is the probability that Mr. A chooses a zero of $f(x^2)$?

[Sol. The zeroes, for the $f(x^2)$ are ± 2 and ± 3 i.e. four zeroes. [Ans. $\frac{4}{21}$]

In the set of integers from $[-10, 10]$

There are 21 elements.

Four of these are the zeroes.

Therefore, the probability is $P = \frac{4}{21}$ Ans.]

Q.14**(a)** A fair die is tossed. If the number is odd, find the probability that it is prime. [Ans. 2/3]

(b) Three fair coins are tossed. If both heads and tails appear, determine the probability that exactly one head appears. [Ans. 1/2]

[Hint: H H T (3) ; H T T (3) $\Rightarrow$ n (s) = 6]

Q.15 *n* different books $(n \geq 3)$ are put at random in a shelf. Among these books there is a particular book 'A' and a particular book B. The probability that there are exactly 'r' books between A and B is

(A) $\frac{2}{n(n-1)}$ (B*) $\frac{2(n-r-1)}{n(n-1)}$ (C) $\frac{2(n-r-2)}{n(n-1)}$ (D) $\frac{(n-r)}{n(n-1)}$

[Sol. $n \begin{cases} A, B \\ (n-2) \text{ others} \end{cases}$

r books from the remaining $(n-2)$ books can be selected in ${}^{n-2}C_r$ ways and arranged between A and B in $r!$ ways, also A and B can be interchanged in $2!$ ways.

Hence $n(E) = \boxed{{}^{n-2}C_r \cdot r! \cdot 2!}\,(n-r-1)!$; $\boxed{A\ \boxed{B_1\ B_2 \ldots\ldots. B_r}\ B}$ $(n-r-2)$ other books

$$\therefore \quad n(E) = \frac{(n-2)! \cdot 2! \cdot (n-r-1)! \cdot r!}{r!(n-r-2)!} = 2! \cdot (n-2)! \cdot (n-r-1)$$

also $n(S) = n!$

$$P(E) = \frac{2(n-2)! \cdot (n-r-1)}{n!} = \frac{2(n-r-1)}{n(n-1)}$$ **Ans.**]

Q.16 A coin is biased so that heads is three times as likely to appear as tails. Find P(H) and P(T). If such a coin is tossed twice find the probability that head occurs at least once.

[Hint: P (T) = p; P (H) = 3p; p = 1/4; $1 - \frac{1}{4^2} = \frac{15}{16}$] [**Ans.** 3/4, 1/4; 15/16]

Q.17 Nine number 1, 2, 3, ..., 9 are put into a 3 × 3 array so that each number occur exactly once. Find the probability that the sum of the numbers in atleast one horizontal row is greater than 21. [Ans. 1/7]

[Sol. There are four subsets of {1, 2, 3, ..., 9} that adds to greater than 21.

i.e. 24{7, 8, 9}, {6, 9, 8}23, {5, 8, 9}22, {6, 7, 9}22

The number of 3 × 3 array having 7, 8, 9 as a row is 3(3!)(6!)

This is true for each of the four sets.

Hence the number of 3 × 3 array having a row that sums > 21 is (4)(3)(3!)(6!)

Also total ways = 9!

$\therefore$ Probability $= \frac{(4)(3)(3!)(6!)}{9!} = \frac{1}{7}$ **Ans.**

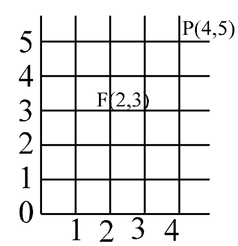
Note that exactly one row can contain elements whose sum is greater than 21.]

Q.18 Mr. A lives at origin on the cartesian plane and has his office at (4, 5). His friend lives at (2, 3) on the same plane. Mr. A can go to his office travelling one block at a time either in the + y or + x direction. If all possible paths are equally likely then the probability that Mr. A passed his friends house is

(A) 1/2 (B*) 10/21 (C) 1/4 (D) 11/21

[Sol. $n(S) = \frac{9!}{4! \cdot 5!} = 126$ **[12th, 18-10-2008]**

n(A) = 0 to F and F to P



$= \frac{5!}{2! \cdot 3!} \cdot \frac{4!}{2! \cdot 2!} = 10 \cdot 6 = 60$

$P(A) = \frac{60}{126} = \frac{10}{21}$ Ans.]

Q.19 In a hand at "whist" what is the chance that the 4 kings are held by a specified player? [Ans. $\frac{{}^4C_4 \cdot {}^{48}C_9}{{}^{52}C_{13}}$]

Q.20 I have 3 normal dice, one red, one blue and one green and I roll all three simultaneously. Let P be the probability that the sum of the numbers on the red and blue dice is equal to the number on the green die. If P is the written in lowest terms as a/b then the value of (a + b) equals

(A) 79 (B*) 77 (C) 61 (D) 57

[Sol. x denotes the number on red die

y denotes the number on blue die

then $x + y \le 6$ (as the number on green has to be less than or equal to 6)

but $x \ge 1$ and $y \ge 1$, hence $x + y \le 4$ (using beggar)

$x + y + t = 4 \Rightarrow {}^6C_2 = 15 = n(A)$

$n(S) = 216;$ $p = \dfrac{15}{216} = \dfrac{5}{72};$ $a + b = 77$ **Ans.]**

MC SIR

CLASS : XII (ABCD) **Dpp on Probability (After 2st Lecture)** ***DPP. NO.- 2***

Q.1 In throwing 3 dice, the probability that atleast 2 of the three numbers obtained are same is

(A) 1/2 (B) 1/3 (C*) 4/9 (D) none

[Hint : P(E) = 1 – P(all different) = 1 – (6/6) · (5/6) · (4/6) = 1 – (120/216) = 4/9]

Q.2 There are 4 defective items in a lot consisting of 10 items. From this lot we select 5 items at random. The probability that there will be 2 defective items among them is

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{21}$ (D*) $\frac{10}{21}$

[Hint: 10 < 4D, 6G $\xrightarrow{5}$ < 2D, 3G **[12th (26-12-2004)]**

$$p = \frac{{}^4C_2 \cdot {}^6C_3}{{}^{10}C_5} = \frac{10}{21} \Rightarrow \text{(D)} \ ]$$

Q.3 From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the ramaining cards. If

A : The event that the card drawn is an ace

H : The event that the card drawn is a heart

S : The event that the card drawn is a spade

then which of the following holds ?

(A*) 9 P(A) = 4 P(H) (B) P(S) = 4P (A ∩ H)

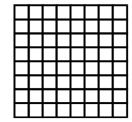
(C) 3 P(H) = 4 P(A ∪ S) (D) P(H) = 12 P(A ∩ S)

[Hint: 52 $\xrightarrow[\text{10's}]{\text{face cards}}$ 36 < 9 H, 9 S, 9 D, 9 C ;

$$P(A) = \frac{1}{9} \ ; P(H) = \frac{1}{4} \ ; P(S) = \frac{1}{4} \ ; P(A \cap H) = \frac{1}{36} \ ; \ P(A \cap S) = \frac{1}{36} \ ; P(A \cup S) = \frac{1}{3} \ ]$$

Q.4 If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is :

(A) 1/9 (B*) 1/18 (C) 2/7 (D) none



[Hint: $n(S) = {}^{64}C_2 \cdot 2$; $n(A) = \dfrac{4\cdot 2 + 6\cdot 4\cdot 3 + 36\cdot 4}{64\cdot 63}$.

<u>Alternatively</u>: $n(A) = 7\cdot 8 + 7\cdot 8 = 112$ (vertical or Horizontal)

Ask: Prob that they have a corner in common **Ans.** 7/144]

[Hint: $P(A) = \dfrac{2[2(1+2+3+4+5+6)+7]}{{}^{64}C_2} = \dfrac{7}{144}$ Ans.]

Q.5 Two red counters, three green counters and 4 blue counters are placed in a row in random order. The probability that no two blue counters are adjacent is

(A) $\dfrac{7}{99}$ (B) $\dfrac{7}{198}$ (C*) $\dfrac{5}{42}$ (D) none

[Sol. R R G G G B B B B when counters are alike **[14-8-2005, 13th]**

$n(S) = \dfrac{9!}{2!3!4!}$

$n(A) = \dfrac{5!}{3!2!}\cdot {}^6C_4$ | R | R | G | G | G |

$\therefore$ $P(A) = \left(\dfrac{5!\cdot 15}{3!2!}\right)\cdot\left(\dfrac{2!3!4!}{9!}\right) = \dfrac{6!\cdot 60}{9\cdot 8\cdot 7\cdot 6!} = \dfrac{60}{7\cdot 8\cdot 9} = \dfrac{15}{7\cdot 2\cdot 9} = \dfrac{5}{42}$

Alternatively : $n(S) = 9!$ $R_1R_2G_1G_2G_3B_1B_2B_3B_4$

$n(A) = 5!\cdot {}^6C_4\cdot 4!$ when counters are different

$p = \dfrac{5!\cdot 6\cdot 5\cdot 4\cdot 3}{9!} = \dfrac{5\cdot 4\cdot 3}{9\cdot 8\cdot 7} = \dfrac{5}{42}$]

Q.6 South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A*) $\dfrac{7}{2^{13}}$ (B) $\dfrac{1}{2^{13}}$ (C) $\dfrac{13}{2^{14}}$ (D) $\dfrac{13}{2^{13}}$

[Hint: L and W can be filled at 14 places in 2^{14} ways.

$\therefore$ $n(S) = 2^{14}$.

Now 13 L's and 1W can be arranged at 14 places in 14 ways.

Hence $n(A) = 14$

$\therefore$ $p = \dfrac{14}{2^{14}} = \dfrac{7}{2^{13}}$]

Q.7 There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is

(A*) 1/4 (B) 1/3 (C) 1/12 (D) 1/10

[Hint: $n(S) = {}^{10}C_7 = 120$

$n(A) = {}^{5}C_4 \cdot {}^{3}C_2 \cdot {}^{2}C_1$

$P(E) = \frac{5 \cdot 3 \cdot 2}{120} = \frac{1}{4}$ Ans.]

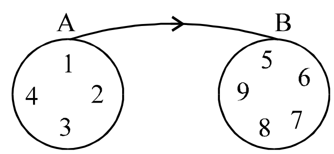
Q.8 Of all the mappings that can be defined from the set $A : \{1, 2, 3, 4\} \to B(5, 6, 7, 8, 9\}$, a mapping is randomly selected. The chance that the selected mapping is strictly monotonic, is

(A) $\frac{1}{125}$ (B*) $\frac{2}{125}$ (C) $\frac{5}{4096}$ (D) $\frac{5}{2048}$

[Sol. $n(S) = 5^4 = 625$

$n(A) = 2 \cdot {}^{5}C_4 = 10$

(either by increasing or decreasing)



$\therefore \quad P(A) = \frac{2 \cdot 5}{625} = \frac{2}{125} \Rightarrow$ (B)]

Q.9 If m/n, in lowest terms, be the probability that a randomly chosen positive divisor of 10^{99} is an integral multiple of 10^{88} then $(m + n)$ is equal to

(A*) 634 (B) 643 (C) 632 (D) 692

[Sol. $N = 10^{99} = 2^{99} \cdot 5^{99}$

$\therefore$ number of divisors of $N = (100)(100) = 10^4$

now $10^{88} = 2^{88} \cdot 5^{88}$

Hence divisors which are integral multiple of $2^{88} \cdot 5^{88}$ must be of the form of $2^a \cdot 5^b$ where $88 \le a, b \le 99$. Thus there are 12×12 ways to choose a and b and hence there are 12×12 divisors which are integral multiple if $2^{88} \cdot 5^{88}$.

Hence $p = \frac{144}{10000} = \frac{9}{625}$

$\therefore$ $m + n = 634$ **Ans.**]

Q.10 A coin is tossed and a die is thrown. Find the probability that the outcome will be a head or a number greater than 4. [**Ans.** 2/3]

[Hint: $P(H \text{ or } A) = P(H) + P(A) - P(H \cap A)$ where $A \equiv$ number > 4]

Q.11 Let A and B be events such that $P(\overline{A}) = 4/5$, $P(B) = 1/3$, $P(A/B) = 1/6$, then

(a) $P(A \cap B)$; (b) $P(A \cup B)$; (c) $P(B/A)$; (d) Are A and B independent?

[Ans. (a) 1/18, (b) 43/90, (c) 5/18, (d) NO]

[Sol. (a) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{6} \Rightarrow P(A \cap B) = \dfrac{P(B)}{6} = \dfrac{1}{18}$ Ans.]

(b) $P(A \cup B) = \dfrac{1}{5} + \dfrac{1}{3} - \dfrac{1}{18} = \dfrac{18+30-5}{90} = \dfrac{43}{90}$ Ans.

(c) $P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1}{18} \cdot \dfrac{5}{1} = \dfrac{5}{18}$ Ans.

(d) $P(A) \cdot P(B) = \dfrac{1}{5} \cdot \dfrac{1}{3} = \dfrac{1}{15} \neq P(A \cap B)$. A & B are not independent]

Q.12 If A and B are two events such that $P(A) = \dfrac{1}{4}$, $P(B) = \dfrac{1}{2}$ and $P(A \text{ and } B) = \dfrac{1}{8}$, find

(i) P (A or B), (ii) P (not A and not B) [**Ans.** (i) 5/8, (ii) 3/8]

Q.13 Given two independent events A, B such that P (A) = 0.3, P (B) = 0.6. Determine

(i) P (A and B) (ii) P (A and not B) (iii) P (not A and B)

(iv) P (neither A nor B) (v) P (A or B)

[**Ans.** (i) 0.18, (ii) 0.12, (iii) 0.42, (iv) 0.28, (v) 0.72]

Q.14 The probabilities that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive

(i) not an A grade (ii) B or C grade (iii) at most C grade

[**Ans.** (i) 0.6, (ii) 0.5, (iii) 0.25]

Q.15 In a single throw of three dice, determine the probability of getting

(i) a total of 5 (ii) a total of at most 5 (iii) a total of at least 5.

[**Ans.** (i) 1/36, (ii) 5/108, (iii) 53/54]

Q.16 A natural number x is randomly selected from the set of first 100 natural numbers. Find the probability that it satisfies the inequality. $x + \frac{100}{x} > 50$ [Ans: 55/100 = 11/20]

[Hint: Note: {1, 2, 48, 49, 50, ... ,100 } $\left[\text{wrong Ans given by students } \frac{1}{50}, \frac{27}{50}, \frac{53}{100}\right]$]

Q.17 3 students A and B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins. Assume no two reach the winning point simultaneously. [Ans. 3/5]

[Sol. $P(C) = p$; $P(A) = 2p$; $P(B) = 2p$

$\therefore$ $5p = 1 \Rightarrow p = 1/5$

$$P(B \text{ or } C) = P(B) + P(C) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$]

Q.18 A box contains 7 tickets, numbered from 1 to 7 inclusive. If 3 tickets are drawn from the box without replacement, one at a time, determine the probability that they are alternatively either odd-even-odd or even-odd-even. [Ans. 2/7]

[Hint: $p = \frac{4\cdot3\cdot3 + 3\cdot4\cdot2}{7\cdot6\cdot5} = \frac{6}{210} =$]

Q.19 5 different marbles are placed in 5 different boxes randomly. Find the probability that exactly two boxes remain empty. Given each box can hold any number of marbles. [Ans. 12/25]

[Sol. $n(S) = 5^5$; For computing favourable outcomes.

2 boxes which are to remain empty, can be selected in ${}^5C_2$ ways and 5 marbles can be placed in the remaining 3 boxes in groups of 221 or 311 in

$$3!\left[\frac{5!}{2!2!2!} + \frac{5!}{3!2!}\right] = 150 \text{ ways} \Rightarrow n(A) = {}^5C_2 \cdot 150$$

Hence $P(E) = {}^5C_2 \cdot \frac{150}{5^5} = \frac{60}{125} = \frac{12}{25}$ Ans.]

Q.1 Let A & B be two events. Suppose $P(A) = 0.4$, $P(B) = p$ & $P(A \cup B) = 0.7$. The value of p for which A & B are independent is :

(A) 1/3 (B) 1/4 (C*) 1/2 (D) 1/5

[Sol. $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$0.7 = 0.4 + p - 0.4p$

$\therefore \quad 0.6p = 0.3 \Rightarrow p = 1/2$]

Q.2 A pair of numbers is picked up randomly (without replacement) from the set {1, 2, 3, 5, 7, 11, 12, 13, 17, 19}. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(A) 0.1 (B) 0.125 (C*) 0.24 (D) 0.18

[Hint : $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{^7C_1}{^8C_2 + 1} = \dfrac{7}{29}$; A : 11 is picked , B : sum is even]

Q.3 For a biased die the probabilities for the diffferent faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(A) 1/6 (B) 1/10 (C) 5/49 (D*) 5/21

[Hint : $P(A/A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A) + P(B)} = \dfrac{0.10}{0.10 + 0.32}$]

Q.4 A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is :

(A) 3/16 (B) 6/16 (C) 10/16 (D*) 13/16

[Hint : 1 – P (Determinant has negative value)

$1 - \dfrac{3}{16} = \dfrac{13}{16} \left(\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} ; \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} ; \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \right)$]

Q.5 15 coupons are numbered 1, 2, 3,..... , 15 respectively. 7 coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is :

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D*) $\frac{9^7-8^7}{15^7}$

[Hint: n(S) = × × × × × × × = 15^7; n (A) = $9^7 - 8^7$]

Q.6 A card is drawn & replaced in an ordinary pack of 52 playing cards. Minimum number of times must a card be drawn so that there is atleast an even chance of drawing a heart, is

(A) 2 (B*) 3 (C) 4 (D) more than four

[Hint: Even chance means probability is half. Suppose *n* cards are drawn

P(E) = P(S or FS or FFS...... n terms)

$$= \frac{P(S)[1-(PF)^n]}{1-P(F)} = 1 - \left(\frac{3}{4}\right)^n \geq \frac{1}{2} \quad \Rightarrow \quad \left(\frac{3}{4}\right)^n \leq \frac{1}{2}$$

$n_{min} = 3$ Ans.]

Q.7 A license plate is 3 capital letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is

(A*) $\frac{7}{52}$ (B) $\frac{9}{65}$ (C) $\frac{8}{65}$ (D) none

[Sol. Let A : event that the three letters are palindrome **[19-2-2006, 12th & 13th]**

B : event that the three digits are palindrome

$$P(A) = \frac{26^2}{26^3} = \frac{1}{26} \quad (L_1L_2L_1)\ ; \ |||ly\ \ P(B) = \frac{10^2}{10^3} = \frac{1}{10} \quad \begin{matrix}abc\\ (\text{there are 10 digits } 0-9)\end{matrix}$$

hence, $P\,(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = \frac{1}{26} + \frac{1}{10} - \frac{1}{26\cdot 10} = \frac{10+26-1}{260} = \frac{7}{52}$ Ans.]

Q.8 Whenever horses a, b, c race together, their respective probabilities of winning the race are 0.3, 0.5 and 0.2 respectively. If they race three times the probability that "the same horse wins all the three races" and the probablity that a, b, c each wins one race, are respectively (Assume no dead heat)

(A*) $\frac{8}{50}; \frac{9}{50}$ (B) $\frac{16}{100}, \frac{3}{100}$ (C) $\frac{12}{50}; \frac{15}{50}$ (D) $\frac{10}{50}; \frac{8}{50}$

[Sol. $P(a) = 0.3$; $P(b) = 0.5$; $P(c) = 0.2$ $\Rightarrow$ a, b, c are exhaustive

P(same horse wins all the three races) = P(aaa or bbb or ccc)

$$= (0.3)^3 + (0.5)^3 + (0.2)^3 = \frac{27+125+8}{1000} = \frac{160}{1000} = \frac{4}{25}$$

P(each horse wins exactly one race)

$$= P(\text{abc or acb or bca or bac or cab or cba}) = 0.3\times 0.5 \times 0.2 \times 6 = 0.18 = \frac{9}{50}$$]

Q.9 Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is

(A) 1 (B) 2 (C*) 3 (D) 4

[Sol. Let the number of red faces on the 2nd cube = x **[08-01-2006, 12 & 13]**

number of blue faces = (6 – x)

P (R R or B B) = 1/2

$$\frac{5}{6}\cdot\frac{x}{6} + \frac{1}{6}\cdot\frac{6-x}{6} = \frac{1}{2}$$

$5x + 6 - x = 18$

$4x = 12$ $\Rightarrow$ $x = 3$ Ans.]

Q.10 A committee of three persons is to be randomly selected from a group of three men and two women and the chair person will be randomly selected from the committee. The probability that the committee will have exactly two women and one man, and that the chair person will be a woman, is/are

(A*) 1/5 (B) 8/15 (C) 2/3 (D) 3/10

[Sol. $5 \begin{cases} 3M \\ 2W \end{cases}$; $n(S) = {}^5C_3 = 10$ **[12th & 13th 07-01-2007]**

$n(A) = {}^3C_1 \cdot {}^2C_2 = 3$

$\therefore$ P(2W and 1M) = 3/10

So, P(2W and 1M & chair person is woman) $= \frac{3}{10}\cdot\frac{2}{3} = \frac{1}{5}$ Ans.]

Q.11 An urn contains 3 red balls and *n* white balls.

Mr. A draws two balls together from the urn. The probability that they have the same colour is $1/2$.

Mr. B draws one ball from the urn, notes its colour and replaces it. He then draws a second ball from the urn and finds that both balls have the same colour is, $5/8$. The possible value of n is

(A) 9 (B) 6 (C) 5 (D*) 1

[Sol. In the 1st case Urn $<$ 3R, n white **[12th, 09-11-2008]**

P(they match) $\dfrac{{}^3C_2 + {}^nC_2}{{}^{n+3}C_2} = \dfrac{1}{2}$; $\dfrac{6+n(n-1)}{(n+3)(n+2)} = \dfrac{1}{2}$ $\Rightarrow$ $2(n^2 - n + 6) = n^2 + 5n + 6$

$\Rightarrow$ $n^2 - 7n + 6 = 0$ $\Rightarrow$ $n = 1$ or 6 ....(1)

In the 2nd case, $\dfrac{3}{n+3}\cdot\dfrac{3}{n+3} + \dfrac{n}{n+3}\cdot\dfrac{n}{n+3} = \dfrac{5}{8}$

solving $n^2 - 10n + 9 = 0$

$n = 9$ or 1 ....(2)

from (1) and (2) $\Rightarrow$ $n = 1$ **Ans.**]

Q.12 The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(A) 0.3 (B*) 0.4 (C) 0.5 (D) 0.6

[Hint: $P(S \cap F) = 0.0006$, where S : moter cycle is stolen ; F : moter cycle found

$P(S) = 0.0015$

$P(F/S) = \dfrac{P(F \cap S)}{P(S)} = \dfrac{6\times10^{-4}}{15\times10^{-4}} = \dfrac{2}{5}$ $\Rightarrow$ (B)]

[REASONING TYPE]

Q.13 In one day test match between India and Australia the umpire continues tossing a fair coin until the two consecutive throws either H T or T T are obtained for the first time. If it is H T, India wins and if it is T T, Australia wins.

Statement-1: Both India and Australia have equal probability of winning the toss.

because

Statement-2: If a coin is tossed twice then the events HT or TT are equiprobable.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false. (D*) Statement-1 is false, statement-2 is true.

[Sol. If T comes to start then Australia can win only in one case that is TT **[13th, 23-11-2008]**

$\Rightarrow$ Probability Australia wins = 1/4

[Note that if the starting toss is a head then Australia can not win (think!)]

$\Rightarrow$ India wins = 3/4]

Alternatively:

India can win if H T or H H T or H H H T or..........

or T H T or T H H T or T H H H T or............]

[SUBJECTIVE]

Q.14 A certain team wins with probability 0.7, loses with probability 0.2 and ties with probability 0.1. The team plays three games. Find the probability

(i) that the team wins at least two of the games, but lose none.

(ii) that the team wins at least one game. [**Ans.** (i) 0.49 ; (ii) 0.973]

[Sol. P (W) = 0.7 ; P (L) = 0.2 ; P (T) = 0.1

E : winning at least 2 games but lose none

P (E) = P (W W T or W T W or T W W or W W W)

$= 3 \times 0.7 \times 0.7 \times 0.1 + (0.7)^3 = 0.7 \times 0.7\ [0.3 + 0.7] = 0.49$

F : wining at least 1 game

A = L or T $\Rightarrow$ P (A) = 0.3 ; P (F) = 1 – P (A A A) $= 1 - (0.3)^3 = 1 - 0.027 = 0.973$]

Q.15 The probability that a person will get an electric contract is $2/5$ and the probability that he will not get plumbing contract is $4/7$. If the probability of getting at least one contract is $2/3$, what is the probability that he will get both? [**Ans.** 17/105]

[Sol. $P(E) = \frac{2}{5}$; $P(F) = P(\text{plumbing}) = 1 - \frac{4}{7} = \frac{3}{7}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\frac{2}{3} = \frac{2}{5} + \frac{3}{7} - x \quad \Rightarrow \quad x = \frac{17}{105}$ Ans.]

Q.16 Five horses compete in a race. John picks two horses at random and bets on them. Find the probability that John picked the winner. Assume no dead heat. [**Ans.** 2/5]

[Sol. $n(S) = {}^5C_2 = 10$

$n(A) = 1 \cdot {}^4C_1 = 4$ $\Rightarrow$ $p = \frac{2}{5}$]

Q.17 There are 6 red balls and 6 green balls in a bag. Five balls are drawn out at random and placed in a red box. The remaining seven balls are put in a green box. If the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number, is $\frac{p}{q}$ where p and q are relatively prime, then find the value of $(p+q)$ [Ans. 37]

[Sol. 12 < 6G, 6R → 5 drawn **[13th for jaipur and ajmer 07-01-2007]**

Red Box		Green Box	
5R	0G	1R	6G
4R	1G	2R	5G
3R	2G	3R	4G
2R	3G	4R	3G
1R	4G	5R	2G
0R	5G	6R	1G

Let E is event as desired then

$$P(E) = \frac{{}^6C_0 \cdot {}^6C_5 + {}^6C_4 \cdot {}^6C_1}{{}^{12}C_5} = \frac{{}^6C_1 + {}^6C_4 \cdot {}^6C_1}{11 \cdot 9 \cdot 8} = \frac{6+90}{11 \cdot 9 \cdot 8} = \frac{96}{11 \cdot 9 \cdot 8} = \frac{4}{33}$$

hence $p + q = 4 + 33 = 37$ Ans.]

Q.18 The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3, and 3 to 4 respectively. What is the probability that of the three reviews a majority will be favourable?

[Ans. 209/343]

Q.19 When three cards are drawn from a standard 52-card deck, what is the probability they are all of the same rank? (e.g. all three are kings). [Ans. 1/425]

[Sol. What matter is that the last two cards are the same as the first one. the probability for the second is $\frac{3}{51}$; for the third is $\frac{2}{50}$.

$$\frac{1}{17} \cdot \frac{2}{50} = \frac{1}{425}$$ Ans.]

Q.20 A and B in order draw alternatively from a purse containing 3 rupees and 4 nP's, find their respective chances of first drawing a rupee, the coins once drawn not being replaced. [Ans. 22/35, 13/35]

MC SIR

CLASS : XII (ABCD) **Dpp on Probability (After 4th Lecture)** ***DPP. NO.- 4***

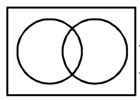
Q.1 If E & F are events with $P(E) \le P(F)$ & $P(E \cap F) > 0$, then :

(A) occurrence of E $\Rightarrow$ occurrence of F

(B) occurrence of F $\Rightarrow$ occurrence of E

(C) non $-$ occurrence of E $\Rightarrow$ non $-$ occurrence of F

(D*) none of the above implications holds.

[Hint: E: 11, 24, 33, 44, 55, 66 ]

Q.2 One bag contains 3 white & 2 black balls, and another contains 2 white & 3 black balls. A ball is drawn from the second bag & placed in the first, then a ball is drawn from the first bag & placed in the second. When the pair of the operations is repeated, the probability that the first bag will contain 5 white balls is:

(A) 1/25 (B) 1/125 (C*) 1/225 (D) 2/15

[Hint: Bag 1 < 3 W, 2 B   Bag 2 < 2 W, 3 B

$$P(E) = P\,[W\ B\ W\ B] = \frac{2}{5}\cdot\frac{2}{6}\cdot\frac{1}{5}\cdot\frac{1}{6} = \frac{1}{225}$$]

Q.3 A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is:

(A) $\frac{145}{1296}$ (B) $\frac{146}{1296}$ (C) $\frac{147}{1296}$ (D*) $\frac{148}{1296}$

[Hint: P (6) = { (51, 15, 24, 42) } or { 11 & (22 or 13 or 31) or (22 & 11) }]

Q.4 A person draws a card from a pack of 52 cards, replaces it & shuffles the pack. He continues doing this till he draws a spade. The probability that he will fail exactly the first two times is :

(A) 1/64 (B*) 9/64 (C) 36/64 (D) 60/64

[Hint: P(E) = P(FFS) = 3/4.3/4.1/4]

Q.5 Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5; P (B) = 1/6; $P(A \cap C) = 1/20$; $P(B \cup C) = 3/8$ then

(A*) events B and C are independent

(B) events B and C are mutually exclusive

(C) events B and C are neither independent nor mutually exclusive

(D) events A and C are equiprobable

[Hint: $P(A \cap C) = P(A) \cdot P(C)$

$$\frac{1}{20} = \frac{1}{5} \cdot P(C) \Rightarrow P(C) = \frac{1}{4}$$

now $P(B \cup C) = \frac{1}{6} + \frac{1}{4} - P(B \cap C)$, hence $P(B \cap C) = \frac{3}{8} - \frac{1}{3} = \frac{1}{24} = P(B) \cdot P(C) \Rightarrow$ (A)]

Q.6 An unbaised cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(A) $\frac{16}{216}$ (B*) $\frac{50}{216}$ (C) $\frac{60}{216}$ (D) none

[Hint: 1 , 2 , 2 , 3 , 3 , 3 (thrown 3 times)

$P(1) = \frac{1}{6}$; $P(2) = \frac{2}{6}$; $P(3) = \frac{3}{6}$

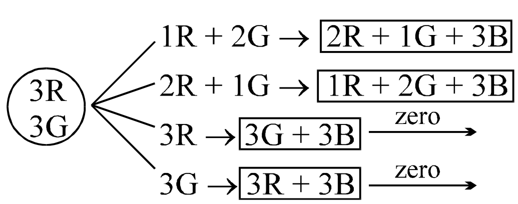
P(S) = P(4 or 6) = P(112 (3 cases) or 123 (6 cases) or 222)

$$= 3.\frac{1}{6}.\frac{1}{6}.\frac{2}{1} + 6\frac{1}{6}.\frac{2}{6}.\frac{3}{1} + \frac{2}{6}.\frac{2}{6}.\frac{2}{6} = \frac{6+36+8}{216} = \frac{50}{216} = \frac{25}{108}$$]

Q.7 A bag contains 3 R & 3 G balls and a person draws out 3 at random. He then drops 3 blue balls into the bag & again draws out 3 at random. The chance that the 3 later balls being all of different colours is

(A) 15% (B) 20% (C*) 27% (D) 40%

[Sol.



; $\frac{{}^3C_1 \cdot {}^3C_2}{{}^6C_2} \cdot \frac{{}^2C_1 \cdot {}^1C_1 \cdot {}^3C_1}{{}^6C_3} + \frac{{}^3C_2 \cdot {}^3C_1}{{}^6C_3} \cdot \frac{{}^1C_1 \cdot {}^2C_1 \cdot {}^3C_1}{{}^6C_3}$]

Q.8 A biased coin with probability P, $0 < P < 1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then the value of P is

(A) 1/4 (B) 1/6 (C*) 1/3 (D) 1/2

[Hint: $P(T\,H \text{ or } T\,T\,T\,H \text{ or } T\,T\,T\,T\,T\,H \text{ or } ....) = \frac{2}{5}$

$$\frac{p(1-p)}{1-(1-p)^2} = \frac{2}{5} \Rightarrow \frac{\not{p}(1-p)}{(2-p)\not{p}} = \frac{2}{5} \Rightarrow 5(1-p) = 2(2-p) \Rightarrow 3p = 1 \Rightarrow p = \frac{1}{3}]$$

Q.9 Two numbers a and b are selected from the set of natural number then the probability that $a^2 + b^2$ is divisible by 5 is

(A*) $\frac{9}{25}$ (B) $\frac{7}{18}$ (C) $\frac{11}{36}$ (D) $\frac{17}{81}$

[Hint: Square of a number ends in 0, 1, 4, 5, 6 and 9 favourable ordered pairs of (a^2, b^2) can be (0, 0) ; (0, 5), (5, 0), (5, 5) ; (1, 4) , (4, 1) ; (1, 9) , (9, 1) ; (4, 6) , (6, 4) ; (6, 9) , (9, 6) and $P(0) = 1/10 = P(5)$; $P(1) = P(4) = P(6) = P(9) = 2/10$]

Q.10 In an examination, one hundred candidates took paper in Physics and Chemistry. Twenty five candidates failed in Physics only. Twenty candidates failed in chemistry only. Fifteen failed in both Physics and Chemistry. A candidate is selected at random. The probability that he failed either in Physics or in Chemistry but not in both is

(A*) $\frac{9}{20}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{11}{20}$

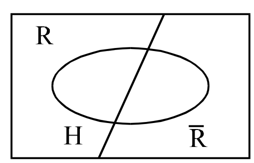
Q.11 When a missile is fired from a ship, the probability that it is intercepted is 1/3. The probability that the missile hits the target, given that it is not intercepted is 3/4. If three missiles are fired independently from the ship, the probability that all three hits the target, is

(A) 1/12 (B*) 1/8 (C) 3/8 (D) 3/4

[Sol. R: Missile is intercepted

$$P(R) = \frac{1}{3}; \quad P(\overline{R}) = \frac{2}{3}; \quad P(H/\overline{R}) = \frac{3}{4}$$

H: Missile hits the target



$$P(H) = P(H \cap R) + P(H \cap \overline{R}) = P(R) \cdot P(H/R) + P(\overline{R}) \cdot P(H/\overline{R})$$

$$= \frac{1}{3} \cdot (0) + \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$$

Hence $P(H\,H\,H) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ Ans.]

Q.12 An urn contains 10 balls coloured either black or red. When selecting two balls from the urn at random, the probability that a ball of each colour is selected is $8/15$. Assuming that the urn contains more black balls than red balls, the probability that at least one black ball is selected, when selecting two balls, is

(A) $\frac{18}{45}$ (B) $\frac{30}{45}$ (C*) $\frac{39}{45}$ (D) $\frac{41}{45}$

[Sol. $10 \begin{cases} x \text{ black} \\ 10 - x \text{ red} \end{cases}$

$$\therefore \quad \frac{{}^{x}C_1 \cdot {}^{10-x}C_1}{{}^{10}C_2} = \frac{8}{15} \Rightarrow \frac{x(10-x)}{45} = \frac{8}{15} \Rightarrow x^2 - 10x + 24 = 0 \Rightarrow x = 6 \text{ or } x = 4$$

since given that no. of black balls is more than red balls

$\therefore$ number of BB = 6

number of RB = 4

now P(E) = 1 – P(R R)

$$= 1 - \frac{{}^{4}C_2}{{}^{10}C_2} = \frac{39}{45} \text{ Ans.]}$$

Q.13 A fair die is tossed repeatidly. Mr. A wins if it is 1 or 2 on two consecutive tosses and Mr. B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely, is

(A) $\frac{1}{3}$ (B*) $\frac{5}{21}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

[Sol. Let P(S) = P(1 or 2) = 1/3 (Note: game can start with S and F)

P(F) = P(3 or 4 or 5 or 6) = 2/3

P(A wins) = P[(S S or S F S S or S F S F S S or ....) or (F S S or F S F S S or ....)]

$$= \frac{\frac{1}{9}}{1-\frac{2}{9}} + \frac{\frac{2}{27}}{1-\frac{2}{9}} = \frac{1}{9}\times\frac{9}{7} + \frac{2}{27}\times\frac{9}{7} = \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$$

$$P(\text{A winning}) = \frac{5}{21} \;;\; P(\text{B winning}) = \frac{16}{21} \text{ Ans.]}$$

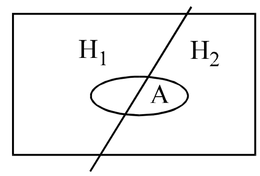
Q.14 An unbiased die with the numbers 1, 2, 3, 4, 6 and 8 on its six faces is rolled. After this roll if an odd number appeares on the top face, all odd numbers on the die are doubled. If an even number appears on the top face, all the even numbers are halved. If the given die changes in this way then the probability that the face 2 will appear on the second roll is

(A) 2/18 (B) 3/18 (C*) 2/9 (D) 5/18

[Sol. H_1: event that die shows up odd $P(H_1) = 1/3$ $S = \{1, 2, 3, 4, 6, 8\}$

H_2: event that die shows up even $P(H_2) = 2/3$

A = 2^{nd} roll shows up 2



if H_1 occurs then the faces becomes 2, 2, 6, 4, 6, 8

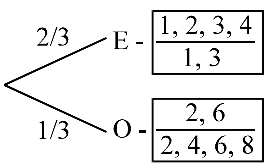
if H_2 occurs then the faces becomes 1, 1, 3, 2, 3, 4

Now, $P(A) = P(A \cap H_1) + P(A \cap H_2)$

$= P(H_1) \cdot P(A / H_1) + P(H_2) \cdot P(A/H_2)$

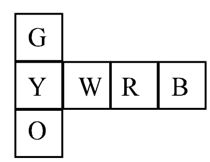
$= \frac{1}{3} \cdot \frac{2}{6} + \frac{2}{3} \cdot \frac{1}{6} = \frac{4}{18} = \frac{2}{9}$ **Ans.**

Alternatively: make a tree diagram



; $\frac{2}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}+\frac{1}{9} = \frac{2}{9}$ **Ans.**]

Q.15 A butterfly randomly lands on one of the six squares of the T-shaped figure shown and then randomly moves to an adjacent square. The probability that the butterfly ends up on the R square is



(A*) 1/4 (B) 1/3 (C) 2/3 (D) 1/6

[Sol. Pr (Ending on R in the second step)

= Pr (landing on B then move to R) + Pr (landing on W then move to R)

$= \frac{1}{6} . 1 + \frac{1}{6} . \frac{1}{2} = \frac{1}{4}$ **Ans.**]

Q.16 A fair coin is tossed a large number of times. Assuming the tosses are independent which one of the following statement, is True?

(A) Once the number of flips is large enough, the number of heads will always be exactly half of the total number of tosses. For example, after 10,000 tosses one should have exactly 5,000 heads.

(B*) The proportion of heads will be about 1/2 and this proportion will tend to get closer to 1/2 as the number of tosses inreases

(C) As the number of tosses increases, any long run of heads will be balanced by a corresponding run of tails so that the overall proportion of heads is exactly 1/2

(D) All of the above

[MULTIPLE OBJECTIVE TYPE]

Q.17 Which of the following statement(s) is/are correct?

(A) 3 coins are tossed once. Two of them atleast must land the same way. No mater whether they land heads or tails, the third coin is equally likely to land either the same way or oppositely. So, the chance that all the three coins land the same way is 1/2.

(B*) Let $0 < P(B) < 1$ and $P(A/B) = P(A/B^c)$ then A and B are independent.

(C*) Suppose an urn contains 'w' white and 'b' black balls and a ball is drawn from it and is replaced along with 'd' additional balls of the same colour. Now a second ball is drawn from it. The probability that the second drawn ball is white is independent of the value of 'd'.

(D*) A, B, C simultaneously satisfy $P(ABC) = P(A)\cdot P(B)\cdot P(C)$ and $P(AB\overline{C}) = P(A)\cdot P(B)\cdot P(\overline{C})$ and $P(A\overline{B}C) = P(A)\cdot P(\overline{B})\cdot P(C)$ and $P(\overline{A}BC) = P(\overline{A})\cdot P(B)\cdot P(C)$ then A, B, C are independent.

[Sol.

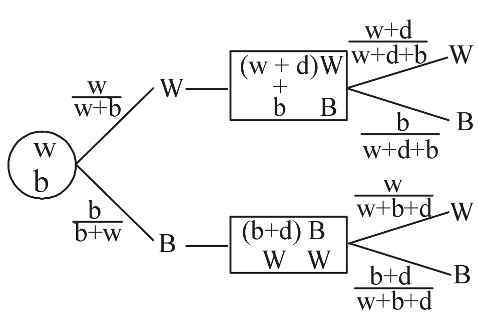
(A) False; $P(\text{TTT or HHH}) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

(B) $\frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B^c)}{1-P(B)} = \frac{P(A)-P(A\cap B)}{1-P(B)}$

$P(A\cap B)[1 - P(B)] = P(B)\cdot P(A) - P(B)\cdot P(A\cap B)$

$P(A\cap B) = P(A)\cdot P(B)$ $\Rightarrow$ **True**

(C)

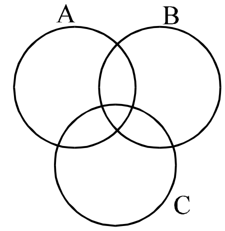


$$P(W) = \frac{w}{w+b}\cdot\frac{w+d}{w+d+b} + \frac{b}{b+w}\cdot\frac{w}{w+b+d} = \frac{w}{(b+w)(w+d+b)}(w+d+b) = \frac{w}{w+b}$$

Hence 2^{nd} ball drawn is white is independent of 'd'.

(D) To prove that A, B, C are pairwise independent also

now $P(A \cap B) = P(A \cap B \cap \overline{C} \cup A \cap B \cap C)$ (from the venn diagram)



$P(A \cap B) = P(A \cap B \cap \overline{C}) + P(A \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(\overline{C}) + P(A) \cdot P(B) \cdot P(C)$ (given)

$= P(A) \cdot P(B)\,[P(C) + P(\overline{C})]$

$= P(A) \cdot P(B)$

|||ly for other two $\Rightarrow$ **(D)** is correct]

Q.18 In each of a set of games it is 2 to 1 in favour of the winner of the previous game. What is the chance that the player who wins the first game shall win three at least, of the next four? **[Ans. 4/9]**

[Hint: $P(W/W) = \frac{2}{3}$; $P(L/W) = \frac{1}{3}$; $P(W/L) = \frac{1}{3}$; $P(L/L) = \frac{2}{3}$]

Q.19 A normal coin is continued tossing unless a head is obtained for the first time. Find the probability that

(a) number of tosses needed are at most 3.

(b) number of tosses are even. **[Ans. (a) 7/8, (b) 1/3]**

[Sol. (a) P (H or T H or T T H);

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} \rightarrow$ probability that H appears for the 1^{st} time on atmost 3 tosses

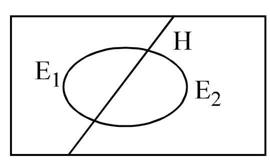
(b) T H or T T T H or .... ; $P(E) = \frac{1/4}{1-1/4} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$]

Q.20 Before a race the chance of three runners, A, B, C were estimated to be proportional to 5, 3, 2, but during the race A meets with an accident which reduces his chance to 1/3. What are the respective chance of B and C now? [Ans. B = 2/5 ; C = 4/15]

Q.21 A is one of the 6 horses entered for a race, and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win; if C rides A, his chance is trebled, what are the odds against his winning? [Ans. 13 to 5]

[Hint: 'A' is horse; B and C joceky

H: Horse 'A' wins the race



E_1: 'B' rides 'A' ; $P(E_1) = 2/3$

E_2: 'C' rides 'A' ; $P(E_2) = 1/3$

$P(H / E_1) = 1/6$; $P(H / E_2) = 3/6$

$P(H) = P(H \cap E_1) + P(H \cap E_2) = \frac{2}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{3}{6}$]

Q.1 Indicate the correct order sequence in respect of the following :

I. If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

II. A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

III. Given the events A and B in a sample space. If P(A) = 1, then A and B are independent.

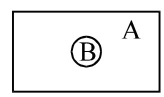
IV. When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

(A) FTFT (B*) FTTT (C) TFTF (D) TFFF

[Hint: **I.** $P(X = 1) = {}^{100}C_1\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{100}$ **[18-12-2005, 12 & 13]**

II. Every key that fits have the same probability = 1/10

III. Consider $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



but $P(A \cup B) = P(A) = 1$

$1 = 1 + P(B) - P(A \cap B)$

$P(A \cap B) = P(B) = P(B) \cdot P(A)$ $\quad (P(A) = 1)$

IV. Each product 1 2 3 4 5 ; 1 2 3 4 6 ; 1 2 3 5 6 ; 1 2 4 5 6 ; 1 3 4 5 6 ; 2 3 4 5 6 is divisible by six.]

Q.2 If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1, 2, 3, 4, 5}, the probability that (ab + c) is even, is

(A) $\frac{35}{125}$ (B*) $\frac{59}{125}$ (C) $\frac{64}{125}$ (D) $\frac{75}{125}$

[Sol. P (number chosen is odd) = 3/5

P (number chosen is even) = 2/5 ab + c is even < a,b,c, are all odd / c is even and atleast a or b is even

E: (ab + c) is even ;

note that event E can occus in two cases

E_1 : all the three number a, b and c are odd; $P(E_1) = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$

E_2 : c is even and atleast one of a or b is even

$$P(E_2) = \frac{2}{5} \cdot \left(1 - \frac{9}{25}\right) = \frac{2}{5} \cdot \frac{16}{25} = \frac{32}{125}$$

$P(E) = P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \frac{59}{125}$ Ans.]

Q.3 A examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to

(A) $(0.8)^8$ (B) $3\,(0.8)^8$ (C) $1 - (0.8)^8$ (D*) $1 - 3\,(0.8)^8$

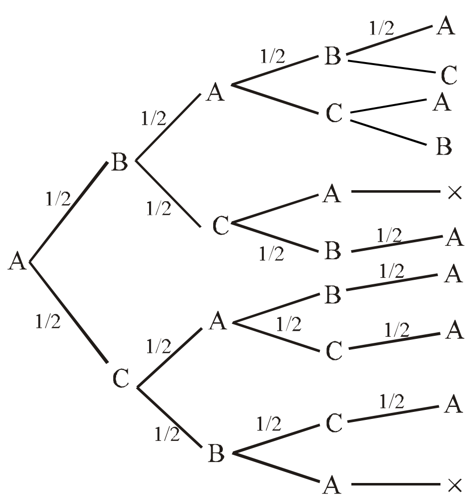
[Hint: $p = \frac{1}{5} = 0.2$; $q = 0.8$; $P(E) = 1 - P(0 \text{ or } 1)$]

Q.4 An ant is situated at the vertex A of the triangle ABC. Every movement of the ant consists of moving to one of other two adjacent vertices from the vertex where it is situated. The probability of going to any of the other two adjacent vertices of the triangle is equal. The probability that at the end of the fourth movement the ant will be back to the vertex A, is :

(A) $\frac{4}{16}$ (B*) $\frac{6}{16}$ (C) $\frac{7}{16}$ (D) $\frac{8}{16}$

[13th, 25-1-2009] [Dpp, prob] DONE

[Sol.



$\longrightarrow \frac{6}{16} \Rightarrow$ **[B]**]

Q.5 A key to room number C_3 is dropped into a jar with five other keys, and the jar is throughly mixed. If keys are randomly drawn from the jar without replacement until the key to room C_3 is chosen, then what are the odds in favour that the key to room C_3 will be obtained on the 2^{nd} try?

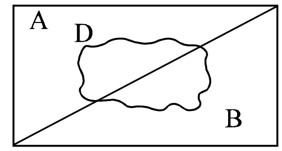
(A) 1:4 (B*) 1:5 (C) 1:6 (D) 5:6

[Sol. We want to fail the first try, so we have $\frac{5}{6} \cdot \frac{1}{5} = \frac{1}{6}$ for the probability. The odds are therefore 1 : 5.]

Q.6 Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(A) 1/3 (B) 2/5 (C*) 8/25 (D) none

[Hint : A = event that the item came from lot A ; $P(A) = \frac{3}{3+2} = \frac{3}{5}$



B = item came from B ; P (B) = 2/5

D = item from mixed lot ' C ' is defective

$P(D) = P(D \cap A) + P(D \cap B)$

$= P(A).\ P(D/A) + P(B).\ P(D/A) = \frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5}=\frac{8}{25}$ Ans.]

Q.7 Mr. A and Mr. B each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is

(A) 1/6 (B) 1/5 (C*) 1/3 (D) 1/2

[Sol. $D[(R \cap R) + (B \cap B) + (G \cap G) + (O \cap O) + (V \cap V)]$

$P(R) \cdot P(R/R) + P(B) \cdot P(B/B) + \ldots\ldots\ldots$

$\frac{1}{5}\left[\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}\right] = \frac{1}{5}\cdot\frac{5}{3} = \frac{1}{3}$ Ans.

<u>Alternatively</u>: any ball from A can go to B. For the contents of the two bag to be the same the ball of the same colour must return. Hence p = 2/6 = 1/3 Ans.]

Q.8 On a Saturday night 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is

(A) 3/7 (B) 4/7 (C*) 5/7 (D) 6/7

[Hint: A : car met with an accident

B_1: driver was alcoholic, $P(B_1) = 1/5$

B_2: driver was sober, $P(B_2) = 4/5$

$P(A/B_1) = 0.001$; $P(A/B_2) = 0.0001$

$P(B_1/A) = \frac{(.2)(.001)}{(.2)(.001)+(.8)(.0001)} = 5/7$ **Ans.]**

Q.9 A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occassions. The chance that the die chosen was a rigged die, is

(A) $\frac{216}{217}$ (B) $\frac{215}{219}$ (C*) $\frac{216}{219}$ (D) none

[Sol. 4 < 3 *normal die* / 1 *rigged die with all faces marked 5* **[27-11-2005, 12th]**

A : die shows up the face 5

B_1 : it is a rigged die ; $P(B_1) = 1/4$

B_2 : it is a normal die ; $P(B_2) = 3/4$

$P(A/B_1) = 1$; $P(A/B_2) = \frac{1}{216}$; $P(B_1/A) = \frac{\frac{1}{4}\cdot 1}{\frac{1}{4}\cdot 1 + \frac{3}{4}\cdot\frac{1}{216}} = \frac{216}{219}$ Ans.]

Q.10 A real estate man has eight master keys to open several new houses. Only one master key will open a given house. If 40% of these homes are usually left unlocked, the probability that the real estate man can get into a specific home if he selects three master keys at random, is

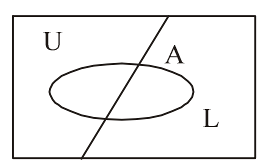
(A) 1/2 (B*) 5/8 (C) 2/3 (D) 3/4

[Sol. U : Home is unlocked: $P(U) = 2/5$ **[13th, 20-01-2008]**

L : Home is locked; $P(L) = 3/5$

A : room is opened by any of the 3 keys

$P(A) = P(A \cap U) + P(A \cap L) = P(U) \cdot P(A/U) + P(L) \cdot P(A/L)$



$P(A) = \frac{2}{5} \cdot 1 + \frac{3}{5} \cdot P(A/L)$

$P(A/L) = \frac{{}^7C_2}{{}^8C_3} = \frac{21}{56} = \frac{3}{8}$ $[k_1\, k_2\, k_3\, k_4\, k_5\, k_6\, k_7\, k_8]$

$\therefore \quad P(A) = \frac{2}{5} + \frac{3}{5}\cdot\frac{3}{8} = \frac{2}{5} + \frac{9}{40} = \frac{16+9}{40} = \frac{5}{8}$ Ans.]

Q.11 A purse contains 100 coins of unknown value, a coin drawn at random is found to be a rupee, The chance that it is the only rupee in the purse, is (Assume all numbers of rupee coins in the purse to be equally likely.)

(A*) $\frac{1}{5050}$ (B) $\frac{2}{5151}$ (C) $\frac{1}{4950}$ (D) $\frac{2}{4950}$

[Sol. A: coin drawn found to be rupee

B_0: 0R + n other

B_1: 1R + (n – 1) other

B_2: 2R + (n – 2) other

$\vdots$

B_n : nR + 0 other

$$P(B_1) = \frac{1}{n+1}$$

$$P(B_1/A) = \frac{P(A\cap B_1)}{P(A)} = \frac{P(B_1)\cdot P(A/B_1)}{P(A)} = \frac{\frac{1}{n}}{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+.....+\frac{n}{n}} = \frac{1}{1+2+3+.....+n}$$

$$= \frac{2}{n(n+1)} = \frac{2}{100\cdot 101} = \frac{1}{5050}$$ **Ans.**]

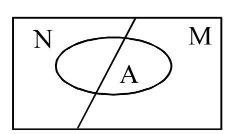
Q.12 A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(A*) $\frac{1}{7}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{24}$

[Sol. N = Normal die ; P(N) = 1/4 **[08-01-2006, 12 & 13]**

M = magnetic die ; P(M) = 3/4

A = die shows up 3



$P(A) = P(A \cap N) + P(A \cap M)$

$= P(N)\, P(A/N) + P(M) \cdot P(A/M)$

$$= \frac{1}{4}\cdot\frac{1}{6}+\frac{3}{4}\cdot\frac{2}{6} = \frac{7}{24}$$

$$P(N/A) = \frac{P(N\cap A)}{P(A)} = \frac{(1/4)\cdot(1/6)}{7/24} = \frac{1}{7}$$ Ans.]

Q.13 An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(A) 1/7 (B*) 2/7 (C) 3/7 (D) 4/7

[Hint: A : the instrument has failed

B_1 : first unit fails and second is healthy

B_2 : first unit healthy and second unit fails

B_3 : both fails

B_4 : both healthy

$P(B_1) = 0.1 \times 0.8 = 0.08$

$P(B_2) = 0.2 \times 0.9 = 0.18$

$P(B_3) = 0.1 \times 0.2 = 0.02$

$P(B_4) = 0.9 \times 0.8 = 0.72$

$P(A/B_1) = P(A/B_2) = P(A/B_3) = 1$

$P(A/B_4) = 0$

Now compute $P(B_1/A)$]

Q.14 A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "difference between the first drawn ticket number and the second is not less than 4" is

(A*) $\frac{7}{30}$ (B) $\frac{14}{30}$ (C) $\frac{11}{30}$ (D) $\frac{10}{30}$

[Sol. 1 2 3 4 <u>5 6 7 8 9 10</u> **[12th, 09-11-2008]**

1^{st} drawn is 5 then 2^{nd} drawn can be 1 only. If 1^{st} is 6 then 2^{nd} is 1 or 2

$$\therefore \quad P(E) = \frac{1}{10}\left[\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{4}{9}+\frac{5}{9}+\frac{6}{9}\right] = \frac{1}{90}\left[\frac{6\cdot 7}{2}\right] = \frac{7}{30} \text{ **Ans.** }]$$

Paragraph for question nos. 15 to 17

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

Q.15 The chance she will be successful, is

(A) 0.28 (B) 0.38 (C*) 0.48 (D) 0.58

Q.16 Given that she is successful, the chance she studied for 4 hours, is

(A) $\frac{6}{12}$ (B*) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

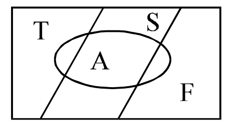
Q.17 Given that she does not achieve success, the chance she studied for 4 hour, is

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D*) $\frac{21}{26}$

[Sol. A : She get a success **[18-12-2005, 12 & 13]**

T : She studies 10 hrs : P(T) = 0.1

S : She studies 7 hrs : P(S) = 0.2



F : She studies 4 hrs : P(F) = 0.7

P(A/T) = 0.8 ; P(A/S) = 0.6 ; P(A/F) = 0.4

$P(A) = P(A \cap T) + P(A \cap S) + P(A \cap F)$

$= P(T) \cdot P(A/T) + P(S) \cdot P(A/S) + P(F) \cdot P(A/F)$

$= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)$

$= 0.08 + 0.12 + 0.28 = 0.48$ **Ans.(15)**

$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{(0.7)(0.4)}{0.48} = \frac{0.28}{0.48} = \frac{7}{12}$ **Ans.(16)**

$P(F/\overline{A}) = \frac{P(F \cap \overline{A})}{P(\overline{A})} = \frac{P(F) - P(F \cap A)}{0.52} = \frac{(0.7) - 0.28}{0.52} = \frac{0.42}{0.52} = \frac{21}{26}$ **Ans.(17)**]

[SUBJECTIVE]

Q.18 A and B each throw simultaneously a pair of dice. Find the probability that they obtain the same score.

Hint: [P [(2&2) or (3&3) or (4&4) ...] [Ans: 73/648]

Q.19 If *mn* coins have been distributed into *m* purses, n into each find

(1) the chance that two specified coins will be found in the same purse, and

(2) what the chance becomes when r purses have been examined and found not to contain either of the specified coins. [Ans. (1) $\frac{n-1}{mn-1}$, (2) $\frac{n-1}{mn-rn-1}$]

Q.20 A, B are two inaccurate arithmeticians whose chance of solving a given question correctly are (1/8) and

(1/12) respectively. They solve a problem and obtained the same result. If it is 1000 to 1 against their making the same mistake, find the chance that the result is correct. [Ans. 13/14]

[Hint: R : they obtained the same result

$B_1 : A \cap \overline{B}$; $P(B_1) = \frac{1}{8} \cdot \frac{11}{12}$ Now $P(R/B_1) = 0$

$B_2 : \overline{A} \cap B$; $P(B_1) = \frac{7}{8} \cdot \frac{1}{12}$ $P(R/B_2) = 0$

$B_3 : A \cap B$; $P(B_3) = \frac{1}{8} \cdot \frac{1}{12}$ $P(R/B_3) = 1$

$B_4 : \overline{A} \cap \overline{B}$; $P(B_4) = \frac{7}{8} \cdot \frac{11}{12}$ $P(R/B_4) = \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}$

Now $P(B_3/A) = \dfrac{\frac{1}{8} \cdot \frac{1}{12}}{\frac{1}{8} \cdot \frac{1}{12} + \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}} = \frac{13}{14}$

Alternatively: H_1 : both solve correctly

H_2 : both solve in correctly and take a common mistake

$P(H_1) = \frac{1}{8} \cdot \frac{1}{12}$; $P(H_2) = \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}$

$P(H_1/H_1 \cup H_2) = \dfrac{\frac{1}{8} \cdot \frac{1}{12}}{\frac{1}{8} \cdot \frac{1}{12} + \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}}$]

Q.1 A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is
(A*) 2/3 (B) 1/4 (C) 5/12 (D) 5/8

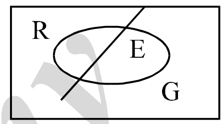
[Hint: 9 < 6R, 3G **[27-11-2005, 12th]**

E : Event that the 2nd drawn marble is red; R : 1st drawn is red; G = 1st drawn is green

$P(E) = P(E \cap R) + P(E \cap G)$

$= P(R) \cdot P(E/R) + P(G) \cdot P(E/G)$

$= \frac{6}{9} \cdot \frac{5}{8} + \frac{3}{9} \cdot \frac{6}{8} = \frac{48}{72} = \frac{2}{3}$]



Q.2 The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is :
(A) $1 - p^n$ (B*) $1 - (1 - p)^n$ (C) p^n (D) $p(1 - p)^{n-1}$

[Hint: P (A) = p

p (object is not detected in one cycle) = $1 - p$

p (object is not detected in *n* cycle) = $(1 - p)^n$

p (object will be detected) = $1 - (1 - p)^n$]

Q.3 In a certain factory, machines A, B and C produce bolts. Of their production, machines A, B, and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts, machine B produces 25% and machine C produces 40%. A bolts is chosen at random from the factory's production and is found to be defective. The probability it was produced on machine C, is

(A) $\frac{6}{11}$ (B) $\frac{23}{45}$ (C*) $\frac{24}{43}$ (D) $\frac{3}{11}$

[Sol. $P(C/D) = \frac{P(C \cap D)}{P(D)}$ **[12th, 21-10-2007]**

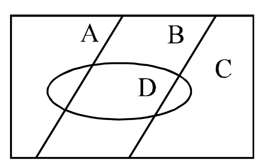
$P(C/D) = \frac{P(C) \cdot P(D/C)}{P(D)}$ ....(1)

$P(D) = P(A \cap D) + P(B/D) + P(C / D)$

$= P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)$

$= (0.35)(0.02) + (0.25)(0.01) + (0.40)(0.03)$

$= 0.0070 + 0.0025 + 0.0120 = 0.0215$



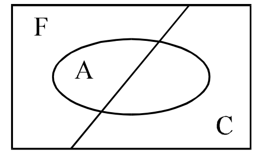
$P(C/D) = \frac{0.0120}{0.0215} = \frac{120}{215} = \frac{24}{43}$ Ans.]

Q.4 Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(A) 0.14 (B) 0.24 (C*) 0.34 (D) 0.44

[Sol. P(F/F) = 0.9 ; P(C/F) = 0.1 ; P(C/C) = 0.8 ; P(F/C) = 0.2

$P(F) = \frac{3}{10}$; $P(C) = \frac{7}{10}$



A : Wine tasted was French

B_1 : It is a Californian wine ; $P(B_1) = \frac{7}{10}$

B_2 : It is a French wine ; $P(B_2) = \frac{3}{10}$

$P(A/B_1) = 0.2$; $P(A/B_2) = 0.9$

$$P(B_1/A) = \frac{0.7 \times 0.2}{0.7 \times 0.2 + 0.3 \times 0.9} = \frac{0.14}{0.14 + 0.27} = \frac{14}{41} \text{ Ans.]}$$

Q.5 Three numbers are chosen at random without replacement from {1, 2, 3,...... , 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

(A) 1/2 (B) 1/3 (C) 1/4 (D*) 11/40

[Hint: N = {1, 2,.......10} $\rightarrow$ 3 are drawn

A = minimum of the chosen number is 3

B = maximum number of the chosen number is 7.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = \frac{{}^7C_2 + {}^6C_2 - {}^3C_1}{{}^{10}C_3} \quad]$$

Q.6 Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

(ii) bus A is late given that bus B is late, are respectively

(A) 2/25 and 12/28 (B) 18/25 and 22/28 (C*) 7/10 and 18/28 (D) 12/25 and 2/28

[Hint : (i) $P(A) = \frac{1}{5}$; $P(B) = \frac{7}{25}$; $P(B/A) = \frac{9}{10}$

$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - P(A) \cdot P(B/A)\right]$$

$$= 1 - \left[\frac{1}{5} + \frac{7}{25} - \frac{1}{5} \cdot \frac{9}{10}\right] = \frac{7}{10} \text{ Ans.]}$$

(ii) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B/A)}{P(B)}$$

$$= \frac{\frac{1}{5} \cdot \frac{9}{10}}{\frac{7}{25}} = \frac{9}{50} \times \frac{25}{7} = \frac{9}{14} = \frac{18}{28} \text{ Ans. } \quad]$$

Q.7 If at least one child in a family with 3 children is a boy then the probability that exactly 2 of the children are boys, is

(A*) 3/7 (B) 4/7 (C) 1/3 (D) 3/8

[Hint: n (S) = B G G (3) ; B B G (3) ; B B B (1) ; hence n(S) = 7

n (A) = B B G (3) $\Rightarrow$ $p = \frac{3}{7}$]

Q.8 From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

(A) 4/5 (B*) 11/15 (C) 11/30 (D) 2/5

[Sol. Total number of possible cases = 3 (either 2 or 4 or 6 are drawn)

Hence required probability = $\frac{1}{3}\left(\frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} + \frac{{}^3C_2 \times {}^3C_2}{{}^6C_4} + \frac{{}^3C_3 \times {}^3C_3}{{}^6C_6}\right) = \frac{11}{15} \Rightarrow$ (B)]

Q.9 There are three main political parties namely 1, 2, 3. If in the adjoining table p_{ij} , (i, j=1, 2, 3) denote the probability that party j wins the general elections contested when party i is in the power. What is the probability that the party 2 will be in power after the next two elections, given that the party 1 is in the power?

P_{11}=0.7	P_{12}=0.2	P_{13}=0.1
P_{21}=0.5	P_{22}=0.3	P_{23}=0.2
P_{31}=0.3	P_{32}=0.4	P_{33}=0.3

(A) 0.27 (B*) 0.24 (C) 0.14 (D) 0.06

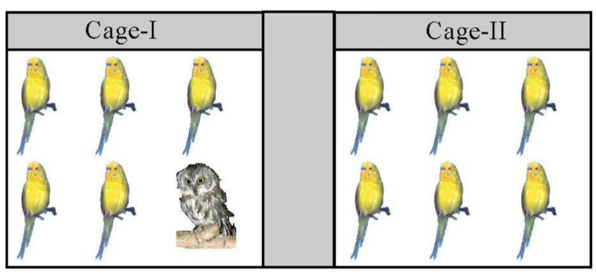
[Hint: $P(E) = P_{11} \cdot P_{12} + P_{12} \cdot P_{22} + P_{13} \cdot P_{32}$ **[12th, 06-01-2008]**

$P_{11} \cdot P_{12}$ = Party-1 in power and Party-1 wins in the 1st and party-1 in power and party-2 wins

or $P_{12} \cdot P_{22}$ = Party-1 in power and Party-2 wins in the 1st and party-2 in power and party-2 wins

or $P_{13} \cdot P_{32}$ = Party-1 in power and party-3 wins]

Q.10 Shalu bought two cages of birds : Cage-I contains 5 parrots and 1 owl, and Cage-II contains 6 parrots, as shown



Birds like to fly

One day Shalu forgot to lock both cages and two birds flew from Cage-I to Cage-II. Then two birds flew back from Cage-II to Cage-I. Assume that all birds have equal chance of flying, the probability that the Owl is still in Cage-I, is

(A) 1/6 (B) 1/3 (C) 2/3 (D*) 3/4

[19-2-2006, 12th & 13th]

[REASONING TYPE]

Q.11 From a well shuffled pack of 52 playing cards a card is drawn at random. Two events A and B are defined as

A: Red card is drawn.

B: Card drawn is either a Diamond or Heart

Statement-1: P(A + B) = P(AB)

because

Statement-2: $A \subseteq B$ and $B \subseteq A$

(A*) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

[Hint: A and B are equivalent statements.] **[12th, 21-10-2007]**

Paragraph for question nos. 14 to 16

A box contains b red balls, '2b' white balls and '3b' blue balls where *b* is a positive integer. 3 balls are selected at random from the box.

Q.12 If balls are drawn without replacement and 'A' denotes the event that "No two of the selected balls have the same colour" then

(A) there is no value of b for which P(A) = 0.3

(B*) There is exactly one value of b for which P(A) = 0.3 and this value is less than 10.

(C) There is exactly one value of b for which P(A) = 0.3 and this value is greater than 10.

(D) There is more than one value of b for which P(A) = 0.3

Q.13 If balls are drawn without replacement and 'B' denotes the event that "No two of the 3 drawn balls are blue" then

(A) $P(B) = \frac{1}{3}$ if b = 1 (B) $P(B) = \frac{2}{3}$ if b = 2

(C) $P(B) = \frac{1}{4}$ if b = 4 (D*) $P(B) = \frac{1}{2}$ for all value of b.

Q.14 If P(A) = 0.3, then the value of P(A/B) equals

(A*) 3/5 (B) 3/10 (C) 1/2 (D) 2/3

[Sol. Bag — b red balls, 2b white balls, 3b blue balls **[13th, 20-01-2008]**

(12) $P(A) = P\text{ (all different colour)} = \frac{{}^{b}C_1 \cdot {}^{2b}C_1 \cdot {}^{3b}C_1}{{}^{6b}C_3} = \frac{6b^3 \cdot 6}{6b(6b-1)(6b-2)} = P(A)$

$$P(A) = \frac{6b^2}{(6b-1)(6b-2)} = \frac{3b^2}{(6b-1)(3b-1)} = \frac{3}{10}$$

$10b^2 = 18b^2 - 9b + 1$

$8b^2 - 9b + 1 = 0 \quad \Rightarrow \quad (b-1)(8b-1) = 0$

b = 1 Ans.

(13) P(B) = P(no two of them are blue)

= 1 – P[(B B B) or B B and one R or W) i.e. BBB or BBR or BBW]

$$= 1 - \left[\frac{{}^{3b}C_3}{{}^{6b}C_3} + \frac{{}^{3b}C_2 \cdot {}^{3b}C_1}{{}^{6b}C_3}\right] = 1 - \left[\frac{3b(3b-1)(3b-2)}{6b(6b-1)(6b-2)} + \frac{3b(3b-1)3b \cdot 6}{2 \cdot 6b(6b-1)(6b-2)}\right]$$

$$= 1 - \frac{1}{4} \cdot \frac{3b-2}{4(6b-1)}[3b - 2 + 9b]$$

$$= 1 - \frac{(6b-1)}{2(6b-1)} = \frac{1}{2}$$

hence P(B) is independent b

(14) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$ (think !)

$$= \frac{3}{10} \cdot \frac{2}{1} = \frac{3}{5} \text{ Ans.]}$$

Pagragraph for question nos. 15 to 17

Urn-I contains 5 Red balls and 1 Blue ball,
Urn-II contains 2 Red balls and 4 Blue balls.
A fair die is tossed. If it results in an even number, balls are repeatedly withdrawn one at a time with replacement from urn-I. If it is an odd number, balls are repeatedly withdrawn one at a time with replacement from urn-II. Given that the first two draws both have resulted in a blue ball.

Q.15 Conditional probability that the first two draws have resulted in blue balls given urn-II is used is
(A) 1/2 (B*) 4/9 (C) 1/3 (D) None

Q.16 If the probability that the urn-I is being used is *p*, and *q* is the corresponding figure for urn-II then
(A*) q = 16p (B) q = 4p (C) q = 2p (D) q = 3p

Q.17 The probability of getting a red ball in the third draw, is
(A) 1/3 (B) 1/2 (C*) 37/102 (D) 41/102

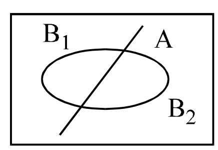
[Sol. Urn-I < 5R, 1B **[12th, 04-01-2009]**

Urn-II < 2R, 4B

A: first two draws resulted in a blue ball.

B_1 : urn-I is used $P(B_1) = \frac{1}{2}$

B_2 : urn-II is used $P(B_2) = \frac{1}{2}$



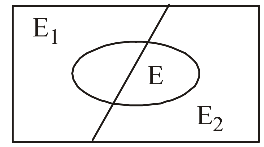
$$P(A/B_1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(A/B_2) = \frac{4}{6} \cdot \frac{4}{6} = \frac{16}{36} = \frac{4}{9} \textbf{ Ans.(i)}$$

$$\underbrace{P(B_1/A)}_{E_1} = \frac{\frac{1}{2}\cdot\frac{1}{36}}{\frac{1}{2}\cdot\frac{1}{36}+\frac{1}{2}\cdot\frac{16}{36}} = \frac{1}{17}$$

$$\underbrace{P(B_2/A)}_{E_2} = \frac{\frac{1}{2}\cdot\frac{16}{36}}{\frac{1}{2}\cdot\frac{16}{36}+\frac{1}{2}\cdot\frac{1}{36}} = \frac{16}{17}$$

$\Rightarrow$ **Ans.(ii)**



E : third ball drawn is red

$P(E) = P(E \cap E_1) + P(E \cap E_2)$

$$= \frac{1}{17}\cdot\frac{5}{6}+\frac{16}{17}\cdot\frac{2}{6} = \frac{5}{102}+\frac{32}{102} = \frac{37}{102}$$ **Ans.(iii)]**

[MULTIPLE OBJECTIVE TYPE]

Q.18 Two whole numbers are randomly selected and multiplied. Consider two events E_1 and E_2 defined as

E_1 : Their product is divisible by 5

E_2: Unit's place in their product is 5.

Which of the following statement(s) is/are correct?

(A) E_1 is twice as likely to occur as E_2.
(B) E_1 and E_2 are disjoint
(C*) $P(E_2/E_1) = 1/4$
(D*) $P(E_1/E_2) = 1$

[Sol. $P(E_1) = 1 - P$(unit's place in both is 1, 2, 3, 4, 6, 7, 8, 9) **[12th, 09-11-2008]**

$$P(E_1 = 0 \text{ or } 5) = 1-\left(\frac{4}{5}\right)^2 = \frac{9}{25}$$

$P(E_2 : 5) = P(1\ 3\ 5\ 7\ 9) - P(1\ 3\ 7\ 9)$ for 2 numbers

$$= \frac{1}{4}-\frac{4}{25} = \frac{25-16}{100} = \frac{9}{100}$$

$$\frac{P(E_2)}{P(E_1)} = \frac{9}{100}\cdot\frac{9}{100}\cdot\frac{25}{9} = \frac{1}{4}$$

$P(E_1) = 4\,P(E_2)$ $\Rightarrow$ A is not correct

$$P(E_2 / E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{9}{100}\cdot\frac{25}{9} = \frac{1}{4} \Rightarrow \textbf{(C)}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_1)}{P(E_2)} = 1 \Rightarrow \textbf{(D)}$$]

[MATCH THE COLUMN]

Q.19

Column-I	Column-II
(A)$_{806/prob}$ The probability of a bomb hitting a bridge is 1/2. Two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9, is	(P) 4
(B)$_{93/prob}$ A bag contains 2 red, 3 white and 5 black balls, a ball is drawn its colour is noted and replaced. Minimum number of times, a ball must be drawn so that	(Q) 5

the probability of getting a red ball for the first time is at least even, is

(C)$_{72/5}$ A hunter knows that a deer is hidden in one of the two near by bushes, the probability of its being hidden in bush-I being 4/5. The hunter having a rifle containing 10 bullets decides to fire them all at bush-I or II. It is known that each shot may hit one of the two bushes, independently of the other with probability 1/2. Number of bullets must he fire on bush-I to hit the animal with maximum probability is (Assume that the bullet hitting the bush also hits the animal). (R) 6 (S) 7

[Ans. (A) S; (B) P; (C) R]

[Sol. **[12th, 09-11-2008]**

(A) P(S) = 1/2 ; P(F) = 1/2

Let 'n' be the least number of bombs to be dropped

E : bridge is destroyed $\Rightarrow$ P(E) = 1 – P(0 or 1 successes)

$$= 1 - \left(\left(\frac{1}{2}\right)^n + {}^nC_1 \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{n-1}\right) = 1 - \left(\frac{1}{2^n} + \frac{n}{2^n}\right) \geq 0.9$$

or $\frac{1}{10} \geq \frac{n+1}{2^n}$ or $\frac{2^n}{10(n+1)} \geq 1$

The value of n consistent with n = 7 or draw graph between $y = 2^x$ and y = 10 (x + 1).

(B) Bag $\begin{cases} 2\text{ R} \\ 3\text{ B} \\ 5\text{ B} \end{cases}$; $P(S) = \frac{1}{5}$; $P(F) = \frac{4}{5}$; E : getting a red ball

$P(E) = P(\text{S or F S or F F S or ....}] \geq \frac{1}{2}$; hence $\frac{P(S)[1-(P(F))^n]}{1-P(F)} \geq \frac{1}{2}$

$P(F)^n \leq \frac{1}{2}$; $\left(\frac{4}{5}\right)^n \leq \frac{1}{2}$

The value of n consistent within is 4 $\Rightarrow$ (P)

(C) B_1 : animal hides in Bush I

B_2 : animal hides in Bush II

$P(B_1) = \frac{4}{5}$; $P(B_2) = \frac{1}{5}$; $P(H) = \frac{1}{2}$

Let x bullets $\rightarrow$ Bush I;

Let 10 – x bullets $\rightarrow$ Bush II;

$$\text{To maximum probability} = \frac{4}{5}\underbrace{\left[1-\left(\frac{1}{2}\right)^x\right]}_{P(\text{at least one hit})} + \frac{1}{5}\underbrace{\left[1-\left(\frac{1}{2}\right)^{10-x}\right]}_{P(\text{at least one hit})}$$

$$\frac{dP}{dx} = -\frac{4}{5}\left(\frac{1}{2}\right)^x \ln\frac{1}{2} + \frac{1}{5}\left(\frac{1}{2}\right)^{10-x} \ln\left(\frac{1}{2}\right) = 0 \quad \Rightarrow \quad x = 6 \;]$$

[SUBJECTIVE]

Q.20$_{48/5}$ A lot contains 50 defective & 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :

A = { the first bulb is defective} ; B = { the second bulb is non defective}
C = { the two bulbs are both defective or both non defective}
Determine whether (i) A,B,C are pair wise independent (ii) A,B,C are independent
[Ans: **(i)** A,B,C are pairwise independent **(ii)** A,B,C are not independent.]

[Sol. Let < 50 defective, 50 good

A : first bulb is defective; P (A) = 1/2
B : second bulb is good; P (B) = 1/2
C : two bulbs are either both good or both defective; P (C) = 1/4

$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $P(B \cap C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$; $P(C \cap A) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Since

$$\left.\begin{array}{l} P(A \cap B) = P(A) \cdot P(B) \\ P(B \cap C) = P(B) \cdot P(C) \\ P(C \cap A) = P(C) \cdot P(A) \end{array}\right\}$$ Hence the events are pairwise independent

$P(A \cap B \cap C) = 0 \rightarrow$ Hence, A, B, C are not independent]

Q.1 Suppose families always have one, two or three children, with probabilities $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. Assume everyone eventually gets married and has children, the probability of a couple having exactly four grandchildren is

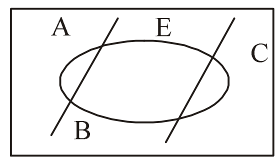
(A*) $\frac{27}{128}$ (B) $\frac{37}{128}$ (C) $\frac{25}{128}$ (D) $\frac{20}{128}$

[Sol. A : exactly one child **[13th, 08-03-2009, P-1]**
B : exactly two children
C : exactly 3 children

$P(A) = \frac{1}{4}$; $P(B) = \frac{1}{2}$; $P(C) = \frac{1}{4}$

E : couple has exactly 4 grandchildren
P(E) = P(A) · P(E/A) + P(B) · P(E/B) + P(C) · P(E/C)

$$= \frac{1}{4}\cdot 0 + \frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^2}_{2/2} + \underbrace{\frac{1}{4}\cdot\frac{1}{4}\cdot 2}_{(1,3)}\right] + \frac{1}{4}\left[3\underbrace{\left(\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{2}\right)}_{1\quad 1\quad 2}\right]$$



$= \frac{1}{8} + \frac{1}{16} + \frac{3}{128} = \frac{27}{128}$ **Ans.**

|||ly 2 / 2 denotes each child having two children

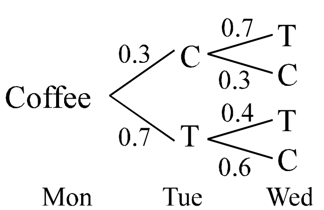
$2\cdot\frac{1}{4}\cdot\frac{1}{4}$ denotes each child having 1 and 3 or 3 and 1 children

$= \frac{16}{128} + \frac{8}{128} + \frac{3}{128} = \frac{27}{128}$ **Ans.]**

Q.2 Miss C has either Tea or Coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has coffee one morning, the probability she has coffee next morning is 0.3. Suppose she has coffee on a Monday morning. The probability that she has tea on the following Wednesday morning is

(A) 0.46 (B*) 0.49 (C) 0.51 (D) 0.61

[Sol. **[12th, 02-12-2007]**



P(E) = C C T or C T T
= (0.3) (0.7) + (0.7)(0.4)
= 0.21 + 28 = 0.49 Ans.]

Q.3 In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then :

(A) $p = q = 1$ (B) $p = q = 1/2$ (C) $p = 1, q = 0$ (D*) $p = 1, q = 1/2$

[Hint: $p(S) = P\big(A \text{ and } (B \text{ or } C)\big) = p \cdot \frac{1}{2}\left(q + \frac{1}{2}\right)$

$\frac{1}{2} = \frac{p}{2}\left(q + \frac{1}{2}\right)$; $1 = p\left(q + \frac{1}{2}\right) \Rightarrow$ (D)]

Q.4 A box contains 100 tickets numbered 1, 2, 3,.... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5, with probability

(A*) $\frac{1}{9}$ (B) $\frac{2}{11}$ (C) $\frac{3}{19}$ (D) none

[Hint: $N = \{1, 2, ...5, 6, 7, 8, 9, 10, ...100\}$
two tickets are drawn
A : maximum number on the two chosen ticket is $\le 10 \Rightarrow$ $n(S) = 10$
B : minimum number on the two chosen ticket is 5

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^5C_1}{{}^{10}C_2} = \frac{5}{45} = \frac{1}{9}$ [one of the ticket is 5 and one is frm 6, 7, 8, 9, 10]]

Q.5 Sixteen players $s_1, s_2, ..., s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(A) $\frac{4}{15}$ (B) $\frac{7}{15}$ (C*) $\frac{8}{15}$ (D) $\frac{9}{15}$

[Hint: 7 players (leaving S_1 & S_2) out of 14 can be selected in ${}^{14}C_7$ and the 8[th] player can be chosen in two ways i.e. either s_1 or s_2. Hence the total ways $= {}^{14}C_7 . 2$

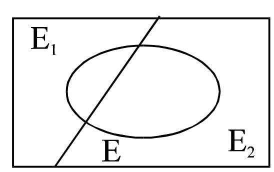
Therefore $p = \frac{2 \cdot {}^{14}C_7}{{}^{16}C_8} = \frac{8}{15}$]

[Alternatively: Let E_1 : S_1 and S_2 are in the same group
E_2 : S_1 and S_2 are in the different group
E : exactly one of the two players S_1 & S_2 is among the eight winners.

$E = (E \cap E_1) + (E \cap E_2)$
$P(E) = P(E \cap E_1) + p(E \cap E_2)$
$P(E) = P(E_1).P(E/E_1) + p(E_2).P(E/E_2)$ ....(1)

Now $P(E_1) = \frac{\frac{(14)!}{(2)^7 . 7!}}{\frac{16!}{2^8 . 8!}} = \frac{1}{15}$

$P(E_2) = 1 - \frac{1}{15} = \frac{14}{15}$



$P(E) = \frac{1}{15} . 1 + \frac{14}{15} \cdot P$ (exactly one of either S_1 & S_2 wins)

$$= \frac{1}{15} + \frac{14}{15} \cdot \left(\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{15} + \frac{1}{14} \cdot \frac{1}{2} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15} \text{ Ans]}$$

Q.6 The number 'a' is randomly selected from the set {0, 1, 2, 3, ... 98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(A) $\frac{1}{16}$ (B) $\frac{2}{16}$ (C) $\frac{4}{16}$ (D*) $\frac{3}{16}$

[Hint: **[27-11-2005, 12th]**

3^a ends in → 7^b ends in ↓	1	3	7	9
1			8	
3				
7	8			
9				8

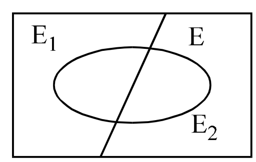
Out of 16 case 3 are favorable ⇒ $p = \frac{3}{16}$]

Q.7 On a normal standard die one of the 21 dots from any one of the six faces is removed at random with each dot equally likely to be chosen. The die is then rolled. The probability that the top face has an odd number of dots is

(A) $\frac{5}{11}$ (B) $\frac{5}{12}$ (C*) $\frac{11}{21}$ (D) $\frac{6}{11}$

[Sol. E_1 : event that the dot is removed from an odd face
E_2 : dot is removed from the even face
E : die thrown has an odd number of dots on its top face

$$P(E) = P(E \cap E_1) + P(E \cap E_2)$$
$$= P(E_1) \cdot P(E / E_1) + P(E_2) \cdot P(E / E_2)$$
$$= \left(\frac{1+3+5}{21}\right) \cdot \frac{2}{6} + \left(\frac{2+4+6}{21}\right) \cdot \frac{4}{6}$$



$$= \frac{9}{21} \cdot \frac{1}{3} + \frac{12}{21} \cdot \frac{2}{3} = \frac{3}{21} + \frac{8}{21} = \frac{11}{21} \text{ Ans.]}$$

Q.8 Two boys A and B find the jumble of n ropes lying on the floor. Each takes hold of one loose end randomly. If the probability that they are both holding the same rope is $\frac{1}{101}$ then the number of ropes is equal to

(A) 101 (B) 100 (C*) 51 (D) 50

[Sol. The n strings have a total of 2n ends. One boy picks up one end, this leaves (2n – 1) ends for the second boy to choose, of which only one is correct.

$$\therefore \quad p = \frac{1}{2n-1} \Rightarrow \frac{1}{2n-1} = \frac{1}{101} \Rightarrow 2n - 1 = 101 \Rightarrow n = 51 \text{]}$$ **[08-01-2006, 12 & 13]**

[REASONING TYPE]

Q.9 A fair coin is tossed 3 times consider the events

A : first toss is head
B : second toss is head
C : exactly two consecutive heads or exactly two consecutive tails.

Statement-1: A, B, C are independent events.

because

Statement-2: A, B, C are pairwise independent.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B*) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. **[12th, 21-10-2007]**

Q.10 Let a sample space S contains *n* elements. Two events A and B are defined on S, and $B \neq \phi$.

Statement-1: The conditional probability of the event A given B, is the ratio of the number of elements in AB divided by the number of elements in B.

because

Statement-2: The conditional probability model given B, is equally likely model on B.

(A*) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

[Sol. $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{n(AB)/N}{n(B)/N} = \dfrac{n(AB)}{n(B)}$ **[12th, 09-11-2008]**

thus for P(A/B) the sample space is the set B. That is, the conditional probability model, given B, assign $\dfrac{1}{n(B)}$ to element of B and zero to each elements of B^c]

Q.11 A bag contains 6 balls of 3 different colours namely White, Green and Red, atleast one ball of each different colour. Assume all possible probability distributions are equally likely.

(a) The probability that the bag contains 2 balls of each colour, is

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C*) $\frac{1}{10}$ (D) $\frac{1}{4}$

(b) Three balls are picked up at random from the bag and found to be one of each different colour. The probability that the bag contained 4 Red balls is

(A*) $\frac{1}{14}$ (B) $\frac{2}{14}$ (C) $\frac{3}{14}$ (D) $\frac{4}{14}$

(c) Three balls are picked at random from the bag and found to be one of each different colour. The probability that the bag contained equal number of White and Green balls, is

(A) $\frac{4}{14}$ (B*) $\frac{3}{14}$ (C) $\frac{2}{14}$ (D) $\frac{5}{14}$

[Sol. **[13th, 01-02-2009, P-1]**

(a) A: 3 balls drawn found to be one each of different colours.

B_1 : 1(W) + 1(G) + 4(R) are drawn; $P(B_1) = \dfrac{1}{10}$

B_2 : 1(W) + 4(G) + 1(R) are drawn; $P(B_2) = \dfrac{1}{10}$

B_3 : 4(W) + 1(G) + 1(R) are drawn; $P(B_3) = \dfrac{1}{10}$

B_4: They are drawn in groups of 1, 2, 3 (WGR) – (6 cases); $P(B_4) = \frac{6}{10}$

B_5: 2(W) + 2(G) + 2(R); $P(B_5) = \frac{1}{10}$ **Ans.**

$$P(A/B_1) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20} \qquad \text{W G R R R R}$$

$$P(A/B_2) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20} \qquad \text{W G G G G R}$$

$$P(A/B_3) = \frac{{}^4C_1}{{}^6C_3} = \frac{4}{20} \qquad \text{W W W W G R}$$

$$P(A/B_4) = 6 \cdot \frac{{}^1C_1 \cdot {}^2C_1 \cdot {}^3C_1}{{}^6C_3} = \frac{36}{20} \qquad \text{W G G R R R,}$$

$$P(A/B_5) = \frac{{}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1}{{}^6C_3} = \frac{8}{20} \qquad \text{W W G G R R}$$

$$\sum_{r=1}^{5} P(B_i) \cdot P(A/B_i) = \frac{1}{10} \cdot \frac{4}{20} + \frac{1}{10} \cdot \frac{4}{20} + \frac{1}{10} \cdot \frac{4}{20} + \frac{1}{10} \cdot \frac{36}{20} + \frac{1}{10} \cdot \frac{8}{20} = \frac{56}{200}$$

(b) $P(B_1/A) = \dfrac{\frac{1}{10} \cdot \frac{4}{20}}{\frac{56}{200}} = \frac{4}{56} = \frac{1}{14}$ **Ans.**

(c) $P(B_5/A) = \dfrac{\frac{1}{10} \cdot \frac{8}{20}}{\frac{56}{200}} = \frac{8}{56} = \frac{2}{14}$

Hence P(bag had equals number of W and G balls / A)

$= P(B_1 / A) + P(B_5 / A) = \frac{1}{14} + \frac{2}{14} = \frac{3}{14}$ **Ans.**]

Q.12 Two fair dice are rolled. Let $P(A_i) > 0$ denotes the event that the sum of the number appearing on the faces of the dice is divisible by i.

(a) Which one of the following events is most probable?
(A*) A_3 (B) A_4 (C) A_5 (D) A_6

(b) For which one of the following pairs (i, j) are the events A_i and A_j are independent?
(A) (3, 4) (B) (4, 6) (C*) (2, 3) (D) (4, 2)

(c) Number of all possible ordered pairs (i, j) for which the events A_i and A_j are independent.
(A) 6 (B) 12 (C) 13 (D*) 25

[Sol.**(a)** $P(A_2) = \frac{18}{36}$; $P(A_3) = \frac{1}{3} = \frac{12}{36}$; $P(A_4) = \frac{1}{4} = \frac{9}{36}$; $P(A_5) = \frac{7}{36} = \frac{7}{36}$; $P(A_6) = \frac{6}{36} = \frac{6}{36}$

$\Rightarrow$ A_3 is most probable

(b) $P(A_2) = \frac{1}{2}$; $P(A_3) = \frac{1}{3}$; $P(A_6) = \frac{1}{6}$ **[12th, 09-11-2008]**

$\therefore$ $P(A_2 \cap A_3) = P(A_2) \cdot P(A_3)$

$\Rightarrow$ $P(A_6) = P(A_2) \cdot P(A_3)$

$\frac{6}{36} = \frac{1}{2} \times \frac{1}{3}$ $\Rightarrow$ A_2 and A_3 are independent

(c) Note A_1 is independent with all events A_1, A_2, A_3, A_4............., A_{12}
now total ordered pairs

$$\underbrace{(1,1),(1,2),\ (1,3),\ ....,(1,11)}_{21} + \underbrace{(1,12)}_{2} = 23 \text{ pairs}$$

Also A_2, A_3 and A_3, A_2 are independent $\Rightarrow$ 25 ordered pairs.]

Q.13 A multiple choice test question has five alternative answers, of which only one is correct. If a student has done his home work, then he is sure to identify the correct answer; otherwise, he chooses an answer at random.
Let E : denotes the event that a student does his home work with P(E) = p and
F : denotes the event that he answer the question correctly.

(a) If p = 0.75 the value of P(E/F) equals

(A) $\frac{8}{16}$ (B) $\frac{10}{16}$ (C) $\frac{12}{16}$ (D*) $\frac{15}{16}$

(b) The relation $P(E/F) \geq P(E)$ holds good for

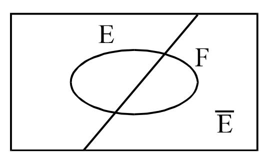
(A*) all values of p in [0, 1] (B) all values of p in (0, 1) only
(C) all values of p in [0.5, 1] only (D) no value of p.

(c) Suppose that each question has *n* alternative answers of which only one is correct, and p is fixed but not equal to 0 or 1 then P(E/F)

(A) decreases as *n* increases for all $p \in (0, 1)$
(B*) increases as *n* increases for all $p \in (0, 1)$
(C) remains constant for all $p \in (0, 1)$
(D) decreases if $p \in (0, 0.5)$ and increases if $p \in (0.5, 1)$ as *n* increases

[Sol. P(E) = p **[12th, 07-12-2008]**

$P(F) = P(E \cap F) + P(\overline{E} \cap F)$



$P(F) = P(E)\, P(F/E) + P(\overline{E})\, P(F/\overline{E})$

$= p \cdot 1 + (1 - p) \cdot \frac{1}{5} = \frac{4p}{5} + \frac{1}{5}$

(a) if p = 0.75

$P(F) = \frac{1}{5}(4p + 1) = \frac{1}{5}(4) = 0.8$

$\therefore$ $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0.75}{0.80} = \frac{15}{16}$ **Ans.**

(b) now $P(E/F) = \frac{5p}{(4p+1)} \geq p$

equality holds for p = 0 or p = 1
for all others value of $p \in (0, 1)$, LHS > RHS, hence **(A)**

(c) If each questions has n alternatives then

$$P(F) = p + (1-p)\frac{1}{n} = P\left(1-\frac{1}{n}\right)+\frac{1}{n} = \frac{(n-1)p+1}{n}$$

$\therefore$ $P(E/F) = \dfrac{np}{(n-1)p+1}$ which increases as *n* increases for a fixed p $\Rightarrow$ **(B)**]

[MULTIPLE OBJECTIVE TYPE]

Q.14 A boy has a collection of blue and green marbles. The number of blue marbles belong to the sets $\{2, 3, 4, ... 13\}$. If two marbles are chosen simultaneously and at random from his collection, then the probability that they have different colour is $1/2$. Possible number of blue marbles is :

(A) 2 (B*) 3 (C*) 6 (D*) 10

[Sol. Let number of blue marbles is b and number of green marbles is g

Hence $\dfrac{bg}{{}^{b+g}C_2} = \dfrac{1}{2}$ **[13th, 08-03-2009, P-1] [Dpp, prob] done**

$(b + g)(g + b - 1) = 4bg$
$(b + g)^2 - (b + g) = 4bg$
$b^2 + g^2 + 2bg - b - g = 4bg$
$g^2 - 2bg - g + b^2 - b = 0$
$g^2 - (2b + 1)g + b^2 - b = 0$
$D = (2b + 1)^2 - 4(b^2 - b)$
$= 8b + 1$ must a perfect square. Hence possible values of b are 3,6,10 $\Rightarrow$ **[B,C,D]**]

Q.15 If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementry to B, then

(A*) $P\left(A/B^C\right) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(B*) $P(A \cap B) \geq P(A) + P(B) - 1$

(C*) $P(A) > < P(A/B)$ according as $P\left(A/B^C\right) > < P(A)$

(D*) $P\left(A/B^C\right) + P\left(A^C/B^C\right) = 1$

[Sol. (B) $1 \geq P(A) + P(B) - P(A\cap B)$ or $P(A \cup B) \leq 1 \Rightarrow$ (B)

(C) Let $P(A) > P(A/B)$

or $P(A) > \dfrac{P(A \cap B)}{P(B)}$

$P(A).\ P(B) > P(A\cap B)$ ....(1)

TPT $P(A/B^C) > P(A)$

$\dfrac{P(A \cap B^c)}{P(B^c)} > P(A)$

$P(A) - P(A\cap B) > P(A)\,[1-P(B)]$
$- P(A\cap B) > - P(A).\ P(B)$

or $P(A).\ P(B) > P(A\cap B)$ ....(2)

from (1) and (2) $P(A) > P(A/B) \Rightarrow P(A/B^c) > P(A)$]

Q.16$_{24prob}$ For $P(A) = \dfrac{3}{8}$; $P(B) = \dfrac{1}{2}$; $P(A \cup B) = \dfrac{5}{8}$ which of the following do/does hold good?

(A*) $P\left(A^c/B\right) = 2P\left(A/B^c\right)$ (B*) $P(B) = P\left(A/B\right)$

(C) $15P\left(A^c/B^c\right) = 8P\left(B/A^c\right)$ (D*) $P\left(A/B^c\right) = P\left(A\cap B\right)$

[Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **[12th, 09-11-2008]**

$$\frac{5}{8} = \frac{3}{8} + \frac{4}{8} - P(A \cap B) \quad \Rightarrow \quad P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

Now $P\left(A^c/B\right) = \dfrac{P\left(A^c \cap B\right)}{P(B)} = \dfrac{P(B) - P\left(A \cap B\right)}{P(B)}$

$$= 1 - \frac{2}{8} \cdot \frac{8}{4} = \frac{1}{2}$$

$$2P\left(A/B^c\right) = \frac{2P\left(A \cap B^c\right)}{P(B^c)} = \frac{2\left(P(A) - P\left(A \cap B\right)\right)}{1 - P(B)} = 4\left(\frac{3}{8} - \frac{2}{8}\right) = \frac{1}{2} \quad \Rightarrow$$ **(A)** is correct

$$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} = P(B) \quad \Rightarrow$$ **(B)** is correct

again $P\left(A^c/B^c\right) = \dfrac{P\left(A^c \cap B^c\right)}{P(B^c)} = \dfrac{1 - P\left(A \cup B\right)}{1 - P(B)} = 2\left(1 - \dfrac{5}{8}\right) = \dfrac{3}{4}$

$$P\left(B/A^c\right) = \frac{P(B \cap A^c)}{1 - P(A)} = \frac{P(B) - P(A \cap B)}{5/8} = \frac{\frac{1}{2} - \frac{1}{4}}{\frac{5}{8}} = \frac{1}{4} \cdot \frac{8}{5} = \frac{2}{5}$$

Hence $8P\left(A^c/B^c\right) = 15P\left(B/A^c\right) \quad \Rightarrow$ **(C)** is **not** correct

again $2P\left(A/B^c\right) = \dfrac{1}{2}$ from (1) $\Rightarrow$ $P\left(A/B^c\right) = \dfrac{1}{4} = P(A \cap B)$

hence **(D)** is correct]

Q.17 If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$
(A*) then E_1 and E_2 are independent
(B) E_1 and E_2 are exhaustive
(C*) E_2 is twice as likely to occur as E_1
(D*) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

[Hint: $P\left(E_2/E_1\right) = \dfrac{P\left(E_1 \cap E_2\right)}{P\left(E_1\right)}$

$$\frac{1}{2} = \frac{P\left(E_1 \cap E_2\right)}{1/4} \quad \Rightarrow \quad P\left(E_1 \cap E_2\right) = \frac{1}{8} = P(E_2).\ P\left(E_1/E_2\right)$$

$\frac{1}{8} = P(E_2) \cdot \frac{1}{4} \quad \Rightarrow \quad P(E_2) = \frac{1}{2}$

Since $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1).\ P(E_2) \quad \Rightarrow$ events are independent

Also $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8} \quad \Rightarrow \quad E_1$ & E_2 are non exhaustive]

Q.18 Two events A and B are such that the probability that at least one of them occurs is 5/6 and both of them occurring simultaneously is 1/3. If the probability of not occurrence of B is 1/2 then
(A) A and B are equally likely
(B*) A and B are independent
(C*) P(A/B) = 2/3
(D*) 3 P(A) = 4 P(B)

[Sol. $P(A \cup B) = \frac{5}{6}$; $\quad P(A \cap B) = \frac{1}{3}$; $\quad P(B) = \frac{1}{2}$ **[12th, 02-12-2007]**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3} \quad \Rightarrow \quad P(A) = \frac{2}{3}$

$P(A \cap B) = \frac{1}{3} = P(A) \cdot P(B) \quad \Rightarrow$ **(B)**

hence $P(A / B) = P(A) = \frac{2}{3} \quad \Rightarrow$ **(C)**

Also $\frac{P(B)}{P(A)} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4} \quad \Rightarrow \quad 3P(A) = 4P(B) \quad \Rightarrow$ **(D)**]

Q.19 The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are
(A*) mutually exclusive
(B) independent
(C) such that one of them must occur
(D*) such that one is twice as likely as the other

[Hint: $P(A \cap B)$, $P(A)$, $P(B)$, $P(A \cup B)$ are in A.P. with $d = P(A)$

$\therefore \quad P(A) - P(A \cap B) = P(A) \quad \Rightarrow \quad P(A \cap B) = 0 \quad \Rightarrow$ A & B are ME

also $\quad P(B) - P(A) = P(A) \quad \Rightarrow \quad 2P(A) = P(B)$

$\therefore \quad$ if $P(A) = p$; $P(B) = 2p \quad \Rightarrow$ (D) compatible means whcih can happen simultaneously]

Q.20 A box contains 11 tickets numbered from 1 to 11. Six tickets are drawn simultaneously at random.
Let E_1 denotes the event that the sum of the numbers on the tickets drawn is even
and E_2 denotes the event that the sum of the numbers on the tickets drawn is odd
Which of the following hold good?
(A) E_1 and E_2 are equally likely
(B*) E_1 and E_2 are exhaustive
(C*) $P(E_2) > P(E_1)$
(D*) $P(E_1/E_2) = P(E_2 / E_1)$

[Hint: $P(E_2) = \frac{118}{231}$ and $P(E_1) = \frac{113}{231}$; $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$E_2 \quad \Rightarrow$ 1 odd + 5 even or 3 odd + 3 even **[12th, 21-10-2007]**
or 5 odd + one even

as $P(E_1 \cap E_2) = 0 \quad \Rightarrow \quad P(E_1 / E_2) = P(E_2/E_1) = 0$]

Q.21 If $\overline{E}$ & $\overline{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then :
(A*) $P(E|F) + P(\overline{E}|F) = 1$
(B) $P(E|F) + P(E|\overline{F}) = 1$

(C) $P(\overline{E}\,|\,F) + P(E\,|\,\overline{F}) = 1$ (D*) $P(E\,|\,\overline{F}) + P(\overline{E}\,|\,\overline{F}) = 1$ **[JEE '98, 2]**

Q.22 Probability of ***n*** heads in ***2n*** tosses of a fair coin can be given by

(A*) $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)$ (B) $\prod_{r=1}^{n}\left(\frac{n+r}{2r}\right)$ (C*) $\sum_{r=0}^{n}\left(\frac{{}^{n}C_r}{2^n}\right)^2$ (D*) $\frac{\sum_{r=0}^{n}\left({}^{n}C_r\right)^2}{\left(\sum_{r=0}^{n}{}^{n}C_r\right)^2}$

[Sol. $P(E) = {}^{2n}C_n \cdot \frac{1}{2^{2n}} = \frac{(2n)!}{n!\cdot n!\cdot 2^n \cdot 2^n}$

verify all the alternatives, noting the fact that $C_0 + C_1 + C_2 + \ldots\ldots + C_n = 2^n$ and $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n$] **[12th, 18-10-2008]**

Q.23 Which of the following statements is/are True?

(A*) A fair coin is tossed *n* times where *n* is a positive integer. The probability that n^{th} toss results in head is 1/2.

(B) The conditional probability that the n^{th} toss results in head given that first $(n-1)$ tosses results in head is $1/2^n$

(C) Let E and F be the events such that F is neither impossible nor sure.
If $P(E/F) > P(E)$ then $P(E/F^c) > P(E)$

(D*) If A, B and C are independent then the events $(A \cup B)$ and C are independent.

[Sol. (D) to prove that $P\big(C \cap (A \cup B)\big) = P(C) \cdot P(A \cup B)$

$P\big((C \cap A) + (C \cap B)\big)$

$= P(C \cap A) + P(C \cap B) - P(A \cap B \cap C)$

$= P(C) \cdot P(A) + P(C) \cdot P(B) - P(A) \cdot P(B)\,P(C)$

$= P(C) \cdot [P(A) + P(B) - P(A \cap B)]$

$= P(C) \cdot P(A \cup B) \Rightarrow$ C and $A \cup B$ are independent]

[MATCH THE COLUMN]

Q.25

	Column-I	Column-II
(A)	Two different numbers are taken from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The probability that their sum and positive difference, are both multiple of 4, is $x/55$ then *x* equals	(P) 4 (Q) 6
(B)	There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same colour is 1/5 then the number of green socks are	(R) 8
(C)	A drawer contains a mixture of red socks and blue socks, at most 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly $1/2$ that both are red or both are blue. The largest possible number of red socks in the drawer that is consistent with this data, is	(S) 10

[Ans. (A) Q; (B) P; (C) S]

[Sol.**(A)** Let the two numbers are 'a' and 'b' **[12th, 09-11-2008]**

$\left.\begin{array}{l} a+b=4p \\ a-b=4q \end{array}\right]$ $p, q \in I$

$2a = 4(p+q) \Rightarrow a = 2I_1$

$2b = 4(p - q) \Rightarrow \quad b = 2I_2$

Hence both *a* and *b* must be even. Also note that if $(a - b)$ is a multiple of 4 then $(a + b)$ will automatically be a multiple of 4.

Hence $n(S) = {}^{11}C_2$

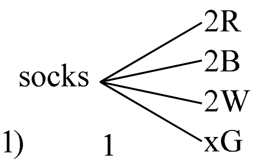
$n(A) = (0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10) = 6$

$\therefore \quad P(A) = \dfrac{6}{{}^{11}C_2}$

(B) Let the number of green socks are $x > 0$

E : two socks drawn are of the same colour

P(E) = P(R R or B B or W W or G G)



$$= \frac{3}{{}^{6+x}C_2} + \frac{{}^{x}C_2}{{}^{6+x}C_2} = \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)} = \frac{1}{5}$$

$5(x^2 - x + 6) = x^2 + 11x + 30$

$4x^2 - 16x = 0 \Rightarrow \quad x = 4$ **Ans.**

(C) Let there be x red socks and y blue socks. Then $\dfrac{{}^{x}C_2 + {}^{y}C_2}{{}^{x+y}C_2} = \dfrac{1}{2}$

let $\quad x > y$

or $\quad \dfrac{x(x-1) + y(y-1)}{(x+y)(x+y-1)} = \dfrac{1}{2}$

Multiplying both sides by $2(x + y)(x + y - 1)$ and expanding,

we find that $\quad 2x^2 - 2x + 2y^2 - 2y = x^2 + 2xy + y^2 - x - y.$

Rearranging, we have $\quad x^2 - 2xy + y^2 = x + y \Rightarrow \quad (x - y)^2 = x + y \Rightarrow |x - y| = x + y$

Since $x + y \le 17$, $\ x - y \le \sqrt{17}$. as $x - y$ must be an integer $\Rightarrow x - y = 4$

$\therefore \quad x + y = 16$

Adding both together and dividing by two yields $x \le 10$ **Ans.**]

Q.1 **Column-I** **Column-II**

(A) In a knockout tournament 2^n equally skilled players; $S_1, S_2, \ldots\ldots\ldots\ldots S_{2^n}$ are participating. In each round players are divided in pair at random and winner from each pair moves in the next round. If S_2 reaches the semifinal then the probability that S_1 wins the tournament is $\dfrac{1}{20}$. The value of 'n' equals

(B) In a multiple choice question there are four alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks all the correct answers. The candidate ticks the answers at random. If the probability of the candidate getting marks on the question is to be greater than or equal to 1/3 the least number of chances he should be allowed is

(C) All the face cards from a pack of 52 playing cards are removed. From the remaining pack half of the cards are randomly removed without looking at them and then randomly drawn two cards simultaneously from the remaining. If the probability that, two cards drawn are both aces, is $\dfrac{p({}^{38}C_{20})}{{}^{40}C_{20} \cdot {}^{20}C_2}$, then the value of p is

(D) An unbiased normal coin is tossed 'n' times. Let

(P) 3

(Q) 4

(R) 5

(S) 6

E_1 : event that both **Heads and Tails** are present in 'n' tosses.
E_2 : event that the coin shows up **Heads** atmost once.
The value of 'n' for which E_1 and E_2 are independent, is
[Ans. (A) Q; (B) R; (C) S; (D) P] **[13th, 25-01-2009]**

[Sol.
(A) number of ways in which S_2 and 3 other players out of $2^n - 1 = \lambda$ say can be taken for semifinals = ${}^{\lambda}C_3$
number of ways in which S_1, S_2 and 2 others out of $(\lambda - 1)$ can be taken for semifinals = ${}^{\lambda-1}C_2$

$\therefore$ $$P(S_1S_2 \text{ and two others reach semifinals}) = \frac{{}^{\lambda-1}C_2}{{}^{\lambda}C_3} = \frac{(\lambda-1)!}{2!(\lambda-3)!}\,\frac{3!(\lambda-3)!}{\lambda!} = \frac{3}{\lambda} = \frac{3}{(2^n-1)}$$

now $S_1 \cdot S_2$ and two others reach the semifinals with probability $\frac{3}{2^n-1}$

Probability (S_1 wints the tournament) $= \frac{3}{(2^n-1)}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4}\cdot\frac{1}{(2^n-1)} = \frac{1}{20}$ $\Rightarrow$ n = 4 **Ans.**

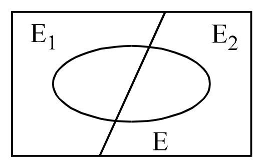
(B) P(E) = P(S or FS or FFS or ....)
= P(S) + P(FS) + P(FFS) + P(FFFS) + ...

$$= \frac{1}{15} + \frac{14}{15}\cdot\frac{1}{14} + \frac{14}{15}\cdot\frac{13}{14}\cdot\frac{1}{13} + \text{............}$$ **[13th, 25-1-2009]** **[Dpp, prob] to be put**

$\Rightarrow$ $\frac{n}{15} \ge \frac{1}{3}$ $\Rightarrow$ $n \ge \frac{15}{3}$ $\Rightarrow$ $n \ge 5$ **Ans.** **Note: in place C take Q with ans 6**

(C) $52 \xrightarrow{\text{face card removed}} 40 \xrightarrow{\text{20 drawn randomly}}$ **[13th, quiz]**

Let E_0: 20 cards randomly removed has no aces.
E_1: 20 cards randomly removed has exactly one ace.
E_2: 20 cards randomly removed has exactly 2 aces.
E : event that 2 drawn from the remaining 20 cards has both the aces.
$P(E) = P(E \cap E_0) + P(E \cap E_1) + P(E \cap E_2)$
$= P(E_0)\cdot P(E/E_0) + P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2)$



= 40 ⟨ 4 aces, 36 other

$$= \frac{{}^4C_0\cdot{}^{36}C_{20}}{{}^{40}C_{20}}\cdot\frac{{}^4C_2}{{}^{20}C_2} + \frac{{}^4C_1\cdot{}^{36}C_{19}}{{}^{40}C_{20}}\cdot\frac{{}^3C_2}{{}^{20}C_2} + \frac{{}^4C_2\cdot{}^{36}C_{18}}{{}^{40}C_{20}}\cdot\frac{{}^2C_2}{{}^{20}C_2}$$

$$= \frac{{}^{36}C_{20}\cdot{}^4C_2 + {}^4C_1\cdot{}^{36}C_{19}\cdot{}^3C_2 + {}^4C_2\cdot{}^{36}C_{18}\cdot{}^2C_2}{{}^{40}C_{20}\cdot{}^{20}C_2}$$

$$= \frac{6\cdot{}^{36}C_{20} + 12\cdot{}^{36}C_{19} + 6\cdot{}^{36}C_{18}}{{}^{40}C_{20}\cdot{}^{20}C_2} = \frac{6[{}^{36}C_{20} + {}^{36}C_{19} + {}^{36}C_{19} + {}^{36}C_{18}]}{{}^{40}C_{20}\cdot{}^{20}C_2}$$

$$= \frac{6({}^{37}C_{20} + {}^{37}C_{19})}{{}^{40}C_{20}\cdot{}^{20}C_2} = \frac{6({}^{38}C_{20})}{{}^{40}C_{20}\cdot{}^{20}C_2}$$ $\Rightarrow$ p = 6 Ans.]

(D) $P(E_1) = 1 - [P(\text{all heads}) + P(\text{all tails})]$

$$= 1 - \left[\frac{1}{2^n} + \frac{1}{2^n}\right] = 1 - \frac{1}{2^{n-1}}$$

$P(E_2) = P(\text{no head}) + P(\text{exactly one head})$

$$= \frac{1}{2^n} + {}^nC_1 \cdot \frac{1}{2^n} = \frac{n+1}{2^n}$$

$P(E_1 \cap E_2) = (\text{exactly one head } \& \ (n-1) \text{ tail})$

$$= {}^nC_1 \cdot \frac{1}{2} \cdot \frac{1}{2^{n-1}} = \frac{n}{2^n}$$

If E_1 & E_2 are independent, then $\frac{n}{2^n} = \left(1 - \frac{1}{2^{n-1}}\right)\left(\frac{n+1}{2^n}\right)$

$$\frac{n}{2^n} = \frac{n}{2^n} + \frac{1}{2^n} - \frac{n}{2^{2n-1}} - \frac{1}{2^{2n-1}}$$

$$n + 1 = \frac{2^{2n-1}}{2^n} = 2^{n-1} \qquad \Rightarrow \quad n = 3 \textbf{ Ans.}]$$

ANSWER KEY

DPP-1

Q.1 A Q.2 C Q.3 A Q.4 A Q.5 C
Q.6 (i) 7/13, (ii) 1/2, (iii) 2/13, (iv) 2/13, (v) 1/2, (vi) 9/13 Q.7 1/56 Q.8 1/2 ; 1/2
Q.9 5 : 1 Q.10 952 to 715 Q.11 3/140 Q.12 $n(S) = 8^5$; $n(A) = {}^8C_5 \cdot 5!$ Q.13 4/21
Q.14 **(a)** 2/3, **(b)** 1/2 Q.15 B Q.16 3/4, 1/4; 15/16
Q.17 1/7 Q.18 B Q.19 $\frac{{}^4C_4 \cdot {}^{48}C_9}{{}^{52}C_{13}}$ Q.20 B

DPP-2

Q.1 C Q.2 D Q.3 A Q.4 B Q.5 C
Q.6 A Q.7 A Q.8 B Q.9 A Q.10 2/3
Q.11 (a) 1/18, (b) 43/90, (c) 5/18, (d) NO Q.12 (i) 5/8, (ii) 3/8
Q.13 (i) 0.18, (ii) 0.12, (iii) 0.42, (iv) 0.28, (v) 0.72
Q.14 (i) 0.6, (ii) 0.5, (iii) 0.25 Q.15 (i) 1/36, (ii) 5/108, (iii) 53/54
Q.16 11/20 Q.17 3/5 Q.18 2/7 Q.19 12/25

DPP-3

Q.1 C Q.2 C Q.3 D Q.4 D Q.5 D Q.6 B Q.7 A
Q.8 A Q.9 C Q.10 A Q.11 D Q.12 B Q.13 D
Q.14 (i) 0.49 ; (ii) 0.973 Q.15 17/105 Q.16 2/5 Q.17 37 Q.18 209/343
Q.19 1/425 Q.20 22/35, 13/35

DPP-4

Q.1 D Q.2 C Q.3 D Q.4 B Q.5 A Q.6 B Q.7 C
Q.8 C Q.9 A Q.10 A Q.11 B Q.12 C Q.13 B Q.14 C
Q.15 A Q.16 B Q.17 B, C, D Q.18 4/9 Q.19 (a) 7/8, (b) 1/3
Q.20 B = 2/5 ; C = 4/15 Q.21 13 to 5

DPP-5

Q.1 B Q.2 B Q.3 D Q.4 B Q.5 B Q.6 C Q.7 C
Q.8 C Q.9 C Q.10 B Q.11 A Q.12 A Q.13 B Q.14 A
Q.15 C Q.16 B Q.17 D Q.18 73/648 Q.19 (1) $\frac{n-1}{mn-1}$, (2) $\frac{n-1}{mn-rn-1}$
Q.20 13/14

DPP-6

Q.1 A Q.2 B Q.3 C Q.4 C Q.5 D Q.6 C
Q.7 A Q.8 B Q.9 B Q.10 D Q.11 A Q.12 B
Q.13 D Q.14 A Q.15 B Q.16 A Q.17 C Q.18 C, D
Q.19 (A) S; (B) P; (C) R Q.20 **(i)** A,B,C are pairwise independent **(ii)** A,B,C are not independent

DPP-7

Q.1 A Q.2 B Q.3 D Q.4 A Q.5 C Q.6 D
Q.7 C Q.8 C Q.9 B Q.10 A
Q.11 **(a)** C, **(b)** A, **(c)** B Q.12 **(a)** A **(b)** C **(c)** D
Q.13 **(a)** D **(b)** A **(c)** B Q.14 B,C,D Q.15 A,B,C,D
Q.16 A,B,D Q.17 A,C,D Q.18 B,C,D Q.19 A,D Q.20 B,C,D Q.21 A,D
Q.22 A, C, D Q.23 A,D Q.24 (A) Q; (B) P; (C) S
Q.25 (A) Q; (B) R; (C) S; (D) P