ETOOS
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PROBABILITY

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KEY CONCEPTS

THINGS TO REMEMBER :

RESULT – 1

(i) ***SAMPLE–SPACE* :** The set of all possible outcomes of an experiment is called the **SAMPLE–SPACE(S)**.

(ii) ***EVENT* :** A sub set of sample–space is called an **EVENT**.

(iii) ***COMPLEMENT OF AN EVENT A* :** The set of all out comes which are in S but not in A is called the **COMPLEMENT OF THE EVENT A DENOTED BY** $\overline{A}$ **OR** A^c.

(iv) ***COMPOUND EVENT* :** If A & B are two given events then $A \cap B$ is called **COMPOUND EVENT** and is denoted by $A \cap B$ or AB or A & B .

(v) ***MUTUALLY EXCLUSIVE EVENTS* :** Two events are said to be **MUTUALLY EXCLUSIVE** (or disjoint or incompatible) if the occurence of one precludes (rules out) the simultaneous occurence of the other . If A & B are two mutually exclusive events then P (A & B) = 0.

(vi) ***EQUALLY LIKELY EVENTS* :** Events are said to be **EQUALLY LIKELY** when each event is as likely to occur as any other event.

(vii) ***EXHAUSTIVE EVENTS* :** Events A,B,C .... L are said to be **EXHAUSTIVE EVENTS** if no event outside this set can result as an outcome of an experiment . For example, if A & B are two events defined on a sample space S, then A & B are exhaustive $\Rightarrow A \cup B = S \Rightarrow P(A \cup B) = 1$.

(viii) ***CLASSICAL DEF. OF PROBABILITY* :** If n represents the total number of equally likely , mutually exclusive and exhaustive outcomes of an experiment and m of them are favourable to the happening of the event A, then the probability of happening of the event A is given by P(A) = m/n .

Note : **(1)** $0 \le P(A) \le 1$

(2) $P(A) + P(\overline{A}) = 1$, Where $\overline{A}$ = Not A .

(3) If x cases are favourable to A & y cases are favourable to $\overline{A}$ then $P(A) = \frac{x}{(x+y)}$ and $P(\overline{A}) = \frac{y}{(x+y)}$ We say that **ODDS IN FAVOUR OF A** are x: y & odds against A are y : x

Comparative study of Equally likely , Mutually Exclusive and Exhaustive events.

Experiment	Events	E/L	M/E	Exhaustive
1. Throwing of a die	A : throwing an odd face {1, 3, 5} B : throwing a composite face {4,. 6}	No	Yes	No
2. A ball is drawn from an urn containing 2W, 3R and 4G balls	E_1 : getting a W ball E_2 : getting a R ball E_3 : getting a G ball	No	Yes	Yes
3. Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more {46, 64, 55, 56, 65, 66}	Yes	No	No
4. From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	Yes	Yes	Yes
5. From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	No	No	No

RESULT – 2

AUB = A+ B = A or B denotes occurence of at least A or B. For 2 events A & B : **(See fig.1)**

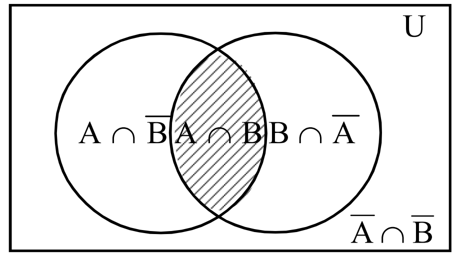

Fig . 1

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$P(A.\overline{B}) + P(\overline{A}.B) + P(A.B) = 1 - P(\overline{A}.\overline{B})$

(ii) Opposite of' "atleast A or B" is NIETHER **A** NOR **B**

i.e. $\overline{A+B}$ = 1-(A or B) = $\overline{A} \cap \overline{B}$

Note that $P(A+B) + P(\overline{A} \cap \overline{B}) = 1$.

(iii) If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

(iv) For any two events A & B, P(exactly one of A , B occurs)

$$= P(A \cap \overline{B}) + P(B \cap \overline{A}) = P(A) + P(B) - 2P(A \cap B)$$
$$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$$

(v) If A & B are any two events $P(A \cap B) = P(A).P(B/A) = P(B).P(A/B)$, Where P(B/A) means conditional probability of B given A & P(A/B) means conditional probability of A given B. (This can be easily seen from the figure)

(vi) **DE MORGAN'S LAW** : – If A & B are two subsets of a universal set U , then

(a) $(A \cup B)^c = A^c \cap B^c$ & **(b)** $(A \cap B)^c = A^c \cup B^c$

(vii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

RESULT – 3

For any three events A,B and C we have **(See Fig. 2)**

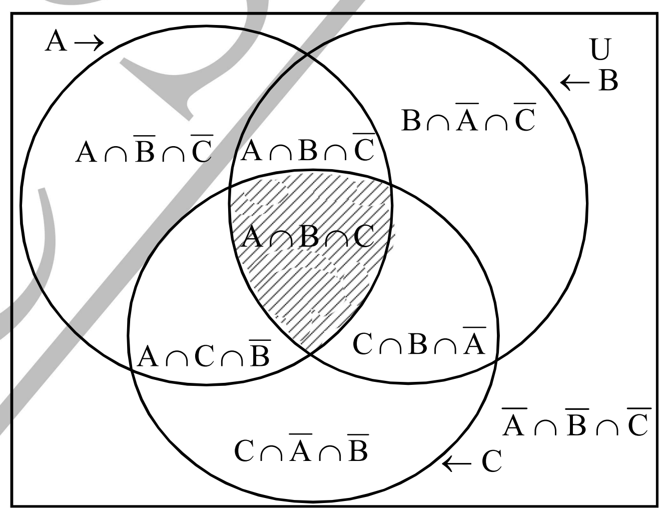

Fig. 2

(i) P(A or B or C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A,B,C occurs) =
$P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

NOTE :

If three events A, B and C are pair wise mutually exclusive then they must be mutually exclusive. i.e $P(A \cap B) = P(B \cap C) = P(C \cap A) = 0 \Rightarrow P(A \cap B \cap C) = 0$. However the converse of this is not true.

RESULT – 4

INDEPENDENT EVENTS **:** Two events A & B are said to be independent if occurence or non occurence of one does not effect the probability of the occurence or non occurence of other.

(i) If the occurence of one event affects the probability of the occurence of the other event then the events are said to be **DEPENDENT** or **CONTINGENT** . For two independent events A and B : $P(A\cap B) = P(A).\ P(B)$. Often this is taken as the definition of independent events.

(ii) Three events A , B & C are independent if & only if all the following conditions hold ;

$P(A\cap B) = P(A)\ .\ P(B)$; $P(B\cap C) = P(B)\ .\ P(C)$

$P(C\cap A) = P(C)\ .\ P(A)$ & $P(A\cap B\cap C) = P(A)\ .\ P(B)\ .\ P(C)$

i.e. they must be pairwise as well as mutually independent .

Similarly for n events $A_1, A_2, A_3, \ldots\ldots A_n$ to be independent , the number of these conditions is equal to ${}^nc_2 + {}^nc_3 + \ldots\ldots + {}^nc_n = 2^n - n - 1$.

(iii) The probability of getting exactly r success in n independent trials is given by

$P(r) = {}^nC_r\, p^r\, q^{n-r}$ where : p = probability of success in a single trial .

q = probability of failure in a single trial. note : p + q = 1 .

Note : Independent events are not in general mutually exclusive & vice versa.

Mutually exclusiveness can be used when the events are taken from the same experiment & independence can be used when the events are taken from different experiments .

RESULT – 5 : BAYE'S THEOREM OR TOTAL PROBABILITY THEOREM :

If an event A can occur only with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots. B_n$ & the probabilities $P(A/B_1)$, $P(A/B_2)$ .... $P(A/B_n)$ are known then,

$$P(B_1/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

PROOF :

The events A occurs with one of the n mutually exclusive & exhaustive events $B_1, B_2, B_3, \ldots\ldots\ldots B_n$

$A = AB_1 + AB_2 + AB_3 + \ldots\ldots. + AB_n$

$$P(A) = P(AB_1) + P(AB_2) + \ldots\ldots. + P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

NOTE : A $\equiv$ event what we have ; B_1 $\equiv$ event what we want ;

$B_2, B_3, \ldots.B_n$ are alternative event .

Now,

$P(AB_i) = P(A)\ .\ P(B_i/A) = P(B_i)\ .\ P(A/B_i)$

$$P(B_i/A) = \frac{P(B_i)\ .\ P(A/B_i)}{P(A)} = \frac{P(B_i)\ .\ P(A/B_i)}{\sum_{i=1}^{n} P(AB_i)}$$

$$\boxed{P(B_i/A) = \frac{P(B_i)\ .\ P(A/B_i)}{\sum P(B_i)\ .\ P(A/B_i)}}$$

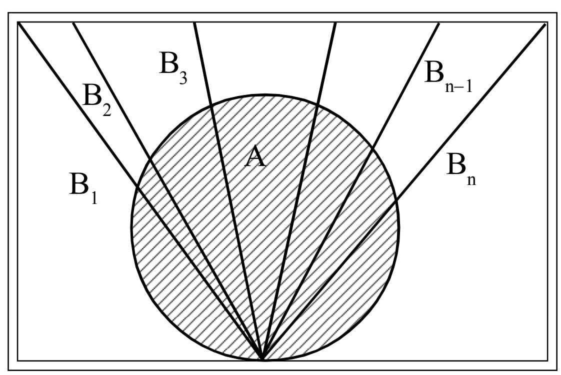

Fig . 3

RESULT – 6

If p_1 and p_2 are the probabilities of speaking the truth of two indenpendent witnesses A and B then

P (their combined statement is true) = $\dfrac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)}$.

In this case it has been assumed that we have no knowledge of the event except the statement made by A and B.

However if p is the probability of the happening of the event before their statement then

P (their combined statement is true) = $\dfrac{p p_1 p_2}{p p_1 p_2 + (1-p)(1-p_1)(1-p_2)}$.

Here it has been assumed that the statement given by all the independent witnesses can be given in two ways only, so that if all the witnesses tell falsehoods they agree in telling the same falsehood.

If this is not the case and c is the chance of their coincidence testimony then the

Pr. that the statement is true = $P p_1 p_2$

Pr. that the statement is false = $(1-p).c\,(1-p_1)(1-p_2)$

However chance of coincidence testimony is taken only if the joint statement is not contradicted by any witness.

RESULT – 7

(i) A **Probability Distribution** spells out how a total probability of 1 is distributed over several values of a random variable .

(ii) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\sum p_i x_i}{\sum p_i} = \sum p_i x_i \quad (\text{Since } \Sigma p_i = 1)$$

(iii) Variance of a random variable is given by, $\sigma^2 = \sum (x_i - \mu)^2 . p_i$

$$\boxed{\sigma^2 = \sum p_i x^2_i - \mu^2} \qquad (\text{Note that SD} = +\sqrt{\sigma^2})$$

(iv) The probability distribution for a binomial variate 'X' is given by ;

$P(X = r) = {}^nC_r p^r q^{n-r}$ where all symbols have the same meaning as given in result 4.

The recurrence formula $\dfrac{P(r+1)}{P(r)} = \dfrac{n-r}{r+1} . \dfrac{p}{q}$, is very helpful for quickly computing

P(1) , P(2). P(3) etc. if P(0) is known .

(v) Mean of BPD = np ; variance of BPD = npq .

(vi) If p represents a persons chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

$$\boxed{\text{expectations} = pM}$$

RESULT – 8 : Geometrical Applications :

The following statements are axiomatic :

(i) If a point is taken at random on a given staright line AB, the chance that it falls on a particular segment PQ of the line is PQ/AB .

(ii) If a point is taken at random on the area S which includes an area σ, the chance that the point falls on σ is σ/S .

EXERCISE-I

Q.1 In a box, there are 8 alphabets cards with the letters: S, S, A, A, A, H, H, H. Find the probability that the word 'ASH' will form if:

(i) the three cards are drawn one by one & placed on the table in the same order that they are drawn.

(ii) the three cards are drawn simultaneously.

Q.2 Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02, ..., 99 with replacement. An event E occurs if & only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

Q.3 To pass a test a child has to perform successfully in two consecutive tasks, one easy and one hard task. The easy task he can perform successfully with probability 'e' and the hard task he can perfrom successfully with probability 'h', where $h < e$. He is allowed 3 attempts, either in the order (Easy, Hard, Easy) (option A) or in the order (Hard, Easy, Hard) (option B) whatever may be the order, he must be successful twice in a row. Assuming that his attempts are independent, in what order he choses to take the tasks, in order to maximise his probability of passing the test.

Q.4 There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects. An unbiased die is cast. If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd} group. Find the probability that an engg. subject is selected.

Q.5 A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

Q.6 In a building programme the event that all the materials will be delivered at the correct time is M, and the event that the building programme will be completed on time is F. Given that $P(M) = 0.8$ and $P(M \cap F) = 0.65$, find $P(F/M)$. If $P(F) = 0.7$, find the probability that the building programme will be completed on time if all the materials are not delivered at the correct time.

Q.7 In a given race, the odds in favour of four horses A, B, C & D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

Q.8 A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

Q.9 The probability that an archer hits the target when it is windy is 0.4; when it is not windy, her probability of hitting the target is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that

(a) She hit the target on first shot

(b) Hits the target exactly once in two shots

Q.10 There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ ($i = 1, 2, 3, 4$). If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where p and $q \in N$ are in their lowest form. Find $(p + q)$.

Q.11 A room has three electric lamps . From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.

Q.12 A bomber wants to destroy a bridge. Two bombs are sufficient to destroy it.
If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.

Q.13 The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd} . Find the chances of each. (assume that both events are neither sure nor impossible).

Q.14 A box contains 5 radio tubes of which 2 are defective . The tubes are tested one after the other until the 2 defective tubes are discovered . Find the probability that the process stopped on the
(i) Second test; **(ii)** Third test. If the process stopped on the third test , find the probability that the first tube is non defective.

Q.15 Anand plays with Karpov 3 games of chess. The probability that he wins a game is 0.5, looses with probability 0.3 and ties with probability 0.2. If he plays 3 games then find the probability that he wins atleast two games.

Q.16 An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane .

Q.17 In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective , the whole batch is rejected Find the probability that the batch will be rejected.

Q.18 A game is played with a special fair cubic die which has one red side, two blue sides, and three green sides. The <u>result</u> is the colour of the top side after the die has been rolled. If the die is rolled repeatedly, the probability that the <u>second blue result</u> occurs on or before the tenth roll, can be expressed in the form $\dfrac{3^p - 2^q}{3^r}$ where p, q, r are positive integers, find the value of $p^2 + q^2 + r^2$.

Q.19 One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world and 30 read Business Today. Five students read all the three magazines. A student was selected randomly. Find the probability that he reads exactly two magazines.

Q.20 An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

Q.21 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, Which one is more likely to solve the puzzle correctly.

Q.22 A uniform unbised die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(i) exactly 6 on each of 3 successive throws.

(ii) more than 4 on at least one of the three successive throws.

Q.23 Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that one of them is a red card & the other is a queen.

Q.24 A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

Q.25 Consider the following events for a family with children

A = {of both the genders} ; B = {at most one boy}

In which of the following (are/is) the events A and B are independent.

(a) if a family has 3 children (b) if a family has 2 children

Assume that the birth of a boy or a girl is equally likely mutually exclusive and exhaustive.

Q.26 A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}) \qquad n \geq 3$$

Also compute P_1 and P_2 and hence deduce the pr that he scores exactly 4.

Q.27 Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being p_o. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

Q.28 A jar contains $2n$ throughly mixed balls, n white and n black balls. n persons each of whom draw 2 balls simultaneously from the bag without replacement.

(a) If the probability that each of the n person draw both balls of different colours is $8/35$, then find the value of n.

(b) If n = 4 then find the probability that each of the 4 persons draw balls of the same colour.

(c) If n = 7 then the probability that each of the 7 persons draw balls of same colour, lies in the interval.

*Q.29 Two bad eggs are accidently mixed with ten good ones. Three eggs are drawn at random without replacement, from this lot. Compute mean & S.D. for the number of bad eggs drawn.

Q.30 16 players take part in a tennis tournament. The order of the matches is chosen at random. There is always a player better than another one, the better wins. Find

(a) The probability that all the 4 best players reach the semifinals.

(b) The probability that the sixth best reaches the semifinals.

NOTE: * C.B.S.E.

EXERCISE-II

Q.1 The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)

(a) Find the probability that they all : **(i)** hit the target ; **(ii)** miss the target

(b) Find the probability that the target is hit : **(i)** at least once, **(ii)** exactly once.

(c) If only one hits the target, what is the probability that it was the first man?

Q.2 Let A & B be two events defined on a sample space . Given P(A) = 0.4 ; P(B) = 0.80 and $P\left(\overline{A}/\overline{B}\right) = 0.10$. Then find ; (i) $P\left(\overline{A}\cup B\right)$ & $P\left[\left(\overline{A}\cap B\right)\cup\left(A\cap\overline{B}\right)\right]$.

Q.3 Three shots are fired independently at a target in succession. The probabilities that the target is hit in the first shot is 1/2 , in the second 2/3 and in the third shot is 3/4. In case of exactly one hit , the probability of destroying the target is 1/3 and in the case of exactly two hits, 7/11 and in the case of three hits is 1.0. Find the probability of destroying the target in three shots.

Q.4 In a game of chance each player throws two unbiased dice and scores the difference between the larger and smaller number which arise . Two players compete and one or the other wins if and only if he scores atleast 4 more than his opponent . Find the probability that neither player wins .

Q.5 A certain drug , manufactured by a Company is tested chemically for its toxic nature. Let the event **"THE DRUG IS TOXIC"** be denoted by H & the event **"THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC"** be denoted by S. Let P(H) = a, $P\left(S/H\right) = P\left(\overline{S}/\overline{H}\right) = 1 - a$. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic, is free from 'a'.

Q.6 A plane is landing. If the weather is favourable, the pilot landing the plane can see the runway. In this case the probability of a safe landing is p_1. If there is a low cloud ceiling, the pilot has to make a blind landing by instruments. The reliability (the probability of failure free functioning) of the instruments needed for a blind landing is P. If the blind landing instruments function normally, the plane makes a safe landing with the same probability p_1 as in the case of a visual landing. If the blind landing instruments fail, then the pilot may make a safe landing with probability $p_2 < p_1$. Compute the probability of a safe landing if it is known that in K percent of the cases there is a low cloud ceiling. Also find the probability that the pilot used the blind landing instrument, if the plane landed safely.

Q.7 A train consists of n carriages , each of which may have a defect with probability p. All the carriages are inspected , independently of one another , by two inspectors ; the first detects defects (if any) with probability p_1 , & the second with probability p_2 . If none of the carriages is found to have a defect, the train departs . Find the probability of the event ; **"THE TRAIN DEPARTS WITH ATLEAST ONE DEFECTIVE CARRIAGE ".**

Q.8 A is a set containing n distinct elements. A non-zero subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A non-zero subset Q of A is again chosen at random. Find the probability that P & Q have no common elements.

Q.9 During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the photograph says he is guilty is a/b where *a* and *b* are relatively prime, find the value of (a + b).

Q.10 n people are asked a question successively in a random order & exactly 2 of the n people know the answer :

(a) If $n > 5$, find the probability that the first four of those asked do not know the answer.

(b) Show that the probability that the r^{th} person asked is the first person to know the answer is : $\left[\frac{2(n-r)}{n(n-1)}\right]$, if $1 < r < n$.

Q.11 A box contains three coins two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it is two headed coin.

(iii) Find the probability that tail appears twice.

Q.12 The ratio of the number of trucks along a highway, on which a petrol pump is located, to the number of cars running along the same highway is 3 : 2. It is known that an average of one truck in thirty trucks and two cars in fifty cars stop at the petrol pump to be filled up with the fuel. If a vehicle stops at the petrol pump to be filled up with the fuel, find the probability that it is a car.

Q.13 A batch of fifty radio sets was purchased from three different companies A, B and C. Eighteen of them were manufactured by A, twenty of them by B and the rest were manufactured by C.
The companies A and C produce excellent quality radio sets with probability equal to 0.9 ; B produces the same with the probability equal to 0.6.
What is the probability of the event that the excellent quality radio set chosen at random is manufactured by the company B?

Q.14 Integers a, b, c and d not necessarily distinct, are chosen independently and at random from the set $S = \{0, 1, 2, 3, ... 2006, 2007\}$. If the probability that $|ad - bc|$ is even, is $\frac{p}{q}$ where p and q are relatively prime the find the value of $(p + q)$.

Q.15 Eight players $P_1, P_2, P_3, ...P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final.

Q.16 A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighbourhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M.
A well known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flu also develop a rash, with conditional probability 0.08.
Upon examination the child, the doctor finds a rash. What is the probability that the child has the measles?
If the probability can be expressed in the form of p/q where $p, q \in N$ and are in their lowest form, find $(p + q)$

Q.17 Two cards are randomly drawn from a well shuffled pack of 52 playing cards, without replacement. Let x be the first number and y be the second number.
Suppose that Ace is denoted by the number 1; Jack is denoted by the number 11 ; Queen is denoted by the number 12 ; King is denoted by the number 13.
Find the probability that x and y satisfy $\log_3(x + y) - \log_3 x - \log_3 y + 1 = 0$.

Q.18(a) Two numbers x & y are chosen at random from the set $\{1,2,3,4,....3n\}$. Find the probability that $x^2 - y^2$ is divisible by 3.

(b) If two whole numbers x and y are randomly selected from the set of natural numbers, then find the probability that $x^3 + y^3$ is divisible by 8.

Q.19 A hunter's chance of shooting an animal at a distance r is $\frac{a^2}{r^2}$ $(r > a)$. He fires when $r = 2a$ & if he misses he reloads & fires when $r = 3a, 4a, ...$ If he misses at a distance 'na', the animal escapes. Find the odds against the hunter.

Q.20 A hotel packed breakfast for each of the three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls and once warpped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. If the probability that each guset got one roll of each type is m/n where *m* and *n* are relatively prime integers, find the value of $(m + n)$.

Q.21 A coin is tossed $(m + n)$ times $(m>n)$. Show that the probability of at least m consecutive heads is $\frac{n+2}{2^{m+1}}$

Q.22 There are two lots of identical articles with different amount of standard and defective articles. There are N articles in the first lot, n of which are defective and M articles in the second lot, m of which are defective. K articles are selected from the first lot and L articles from the second and a new lot results. Find the probability that an article selected at random from the new lot is defective.

Q.23 *m* red socks and *n* blue socks $(m > n)$ in a cupboard are well mixed up, where $m + n \le 101$. If two socks are taken out at random, the chance that they have the same colour is 1/2. Find the largest value of *m*.

Q.24 With respect to a particular question on a multiple choice test (having 4 alternatives with only 1 correct) a student knows the answer and therefore can eliminate 3 of the 4 choices from consideration with probability 2/3, can eliminate 2 of the 4 choices from consideration with probability 1/6, can eliminate 1 choice from consideration with probability 1/9, and can eliminate none with probability 1/18. If the student knows the answer, he answers correctly, otherwise he guesses from among the choices not eliminated.
If the answer given by the student was found correct, then the probability that he knew the answer is $\frac{a}{b}$ where a and b are relatively prime. Find the value of $(a + b)$.

Q.25 A match between two players A and B is won by the player who first wins two games. A's chance of winning, drawing or losing any particular games are 1/2, 1/6 or 1/3 respectively. If the probability of A's winning the match can be expressed in the form p/q, find $(p + q)$.

EXERCISE-III

Q.1 A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that,

$$p_1 = 1\ ,\ p_2 = 1 - p^2\ \&\ p_n = (1-p)\,p_{n-1} + p\,(1-p)\,p_{n-2}\ ,\ \text{for all } n \geq 3.$$

[JEE ' 2000 (Mains), 5]

Q.2 A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately. [REE ' 2000 (Mains), 3]

Q.3 Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. [REE ' 2001 (Mains), 3]

Q.4(a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. [JEE ' 2001 (Mains), 5 + 5]

Q.5 A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? [JEE ' 2002 (mains)]

Q.6(a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3 , 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. Find the probability that B hits his target whereas C does not. [JEE' 2003, Mains-2 + 2 out of 60]

Q.7(a) Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(A) $\frac{4}{25}$ (B) $\frac{4}{35}$ (C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

(b) If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \leq P(C)$, where C is an event defined that exactly one of A or B occurs.

(c) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^{n}C_{r}$). [JEE 2004, 3 + 2 + 4]

Q.8(a) A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even number of trials is
(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6
[JEE 2005 (Scr)]

(b) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.
[JEE 2005 (Mains), 2]

Comprehension (3 questions)

There are n urns each containing $n + 1$ balls such that the i^{th} urn contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and w denotes the event of getting a white ball.

Q.9(a) If $P(u_i) \propto i$ where $i = 1, 2, 3,....., n$ then $\lim_{n\to\infty} P(w)$ is equal to
(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

(b) If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to
(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn ($P(u_i) = \frac{1}{n}$), then the value of $P(w/E)$, is
(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (B) $\frac{1}{n+1}$
[JEE 2006, 5 marks each]

Q.10(a) One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is
(A) 1/2 (B) 1/3 (C) 2/5 (D) 1/5

(b) Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c \mid G)$ equals
(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$
(C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

(c) Let $H_1, H_2, ... , H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0, i = 1, 2, ..., n$. Let E be any other event with $0 < P(E) < 1$.
Statement-1: $P(H_i / E) > P(E / H_i) \cdot P(H_i)$ for $i = 1, 2, ..., n$.
because
Statement-2: $\sum_{i=1}^{n} P(H_i) = 1$
(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.
[JEE 2007, 3+3+3]

Q.11(a) An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8 (B) 3, 6, or 9 (C) 4 or 8 (D) 5 or 10

(b) Consider the system of equations

$ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

STATEMENT-1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT-2 : The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True [JEE 2008, 3+3]

Comprehension (3 questions)

Q.12 A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

(a) The probability that $X = 3$ equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

(b) The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

(c) The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

[JEE 2009, 4+4+4]

ANSWER KEY

EXERCISE-I

Q.1 **(i)** 3/56 **(ii)** 9/28 **Q.2** $97/(25)^4$ **Q.3** Option B

Q.4 13/24 **Q.5** 5/9 **Q.6** $P(F/M) = \frac{13}{16}$; $P(F/\overline{M}) = \frac{1}{4}$

Q.7 319/420 **Q.8** 120 **Q.9** **(a)** 0.61; **(b)** 0.4758

Q.10 2065 **Q.11** $\frac{29}{30}$ **Q.12** $\frac{328}{625}$ **Q.13** $\frac{1}{9}, \frac{1}{3}$

Q 14. **(i)** 1/10, **(ii)** 3/10, **(iii)** 2/3 **Q.15** 1/2 **Q.16** 0.6976

Q.17 19/42 **Q.18** 283 **Q.19** 1/2

Q.20 407/576 **Q.21** Both are equally likely **Q.22** (i) $\frac{125}{16^3}$; (ii) $\frac{63}{64}$

Q.23 101/1326 **Q.24** $\frac{{}^{24}C_2}{{}^{64}C_2}$ or $\frac{23}{168}$

Q.25 Independent in (a) and not independent in (b) **Q.26** $P_1 = 1/2$, $P_2 = 3/4$

Q.27 $(1-p)^{n-1} \cdot [\,p_o(1-p) + np(1-p_0)]$ **Q.28** (a) 4; (b) 3/35 ; (c) [0, 0.1]

Q.29 mean = 0.5 **Q.30** (a) $\frac{64}{455}$; (b) $\frac{24}{91}$

EXERCISE-II

Q.1 (a) 6%, 21% ; (b) 79%, 44%, (c) 9/44 ≈ 20.45% **Q.2** **(i)** 0.82, **(ii)** 0.76 **Q.3** $\frac{5}{8}$

Q.4 74/81 **Q.5** $P(\overline{H}/S) = 1/2$

Q.6 $P(E) = (1-\frac{K}{100})p_1 + \frac{K}{100}[P\,p_1 + (1-P)p_2]$; $P(H_2/A) = \dfrac{\frac{K}{100}[P\,p_1+(1-P)p_2]}{\left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}[P\,p_1+(1-P)p_2]}$

Q.7 $1 - [\,1-p(1-p_1)(1-p_2)]^n$ **Q.8** $(3^n-2^{n+1}+1)/(4^n-2^{n+1}+1)$ **Q.9** 179

Q.10 **(a)** $\frac{(n-4)(n-5)}{n((n-1)}$ **Q.11** 1/2, 1/2, 1/12 **Q.12** $\frac{4}{9}$

Q.13 $\frac{4}{13}$ **Q.14** 13 **Q.15** 4/35

Q.16 262 **Q.17** $\frac{11}{663}$ **Q.18 (a)** $\frac{(5n-3)}{(9n-3)}$ **(b)** $\frac{5}{16}$

Q.19 n+1 : n−1 **Q.20** 79 **Q.22** $\frac{KnM+LmN}{MN(K+L)}$

Q.23 55 **Q.24** 317 **Q.25** 206

EXERCISE-III

Q.2 $\frac{1}{2}$ & $\frac{1}{3}$ or $\frac{1}{3}$ & $\frac{1}{2}$

Q.3 $\frac{1}{26}$ **Q.4** **(a)** $\frac{m}{m+n}$; **(b)** $\frac{{}^{6}C_{3}\left(3^{n}-3.2^{n}+3\right)}{6^{n}}$ **Q.5** $\frac{9m}{m+8N}$

Q.6 (a) $p^2(2-p)$; (b) 1/2 **Q.7** **(a)** D , **(c)** $\frac{{}^{12}C_{2}\,{}^{6}C_{4}\,{}^{10}C_{1}\,{}^{2}C_{1}+{}^{12}C_{1}\,{}^{6}C_{5}\,{}^{11}C_{1}\,{}^{1}C_{1}}{{}^{12}C_{2}\left({}^{12}C_{2}\,{}^{6}C_{4}+{}^{12}C_{1}\,{}^{6}C_{5}+{}^{12}C_{0}\,{}^{6}C_{6}\right)}$

Q.8 **(a)** A, **(b)** $\frac{1}{7}$ **Q.9** (a) B, (b) A, (c) B **Q.10** **(a)** C; **(b)** C; **(c)** D

Q.11 **(a)** D, **(b)** B **Q.12** (a) A, (b) B, (c) D