PROGRESSION

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KEY CONCEPTS (SEQUENCE & PROGRESSION)

DEFINITION :

A sequence is a set of terms in a definite order with a rule for obtaining the terms. e.g. 1 , 1/2 , 1/3 , ... , 1/n , ... is a sequence.

AN ARITHMETIC PROGRESSION (AP) :

AP is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then AP can be written as a, a + d, a + 2 d, ... a + (n – 1)d, ...

n^{th} term of this AP $t_n = a + (n-1)d$, where $d = a_n - a_{n-1}$.

The sum of the first n terms of the AP is given by ; $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$.

where l is the last term.

NOTES :

(i) If each term of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an AP.

(ii) Three numbers in AP can be taken as a – d , a , a + d ; four numbers in AP can be taken as a – 3d, a – d, a + d, a + 3d ; five numbers in AP are a – 2d , a – d , a, a + d, a + 2d & six terms in AP are a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d etc.

(iii) The common difference can be zero, positive or negative.

(iv) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms.

(v) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.

(vi) $t_r = S_r - S_{r-1}$

(vii) If a , b , c are in AP $\Rightarrow 2b = a + c$.

GEOMETRIC PROGRESSION (GP) :

GP is a sequence of numbers whose first term is non zero & each of the succeeding terms is equal to the proceeding terms multiplied by a constant . Thus in a GP the ratio of successive terms is constant. This constant factor is called the **COMMON RATIO** of the series & is obtained by dividing any term by that which immediately proceeds it. Therefore a, ar, ar^2, ar^3, ar^4, ... is a GP with a as the first term & r as common ratio.

(i) n^{th} term = $a\,r^{n-1}$

(ii) Sum of the I^{st} n terms i.e. $S_n = \frac{a\left(r^n - 1\right)}{r-1}$, if $r \neq 1$.

(iii) Sum of an infinite GP when $|r| < 1$ when $n \to \infty$ $r^n \to 0$ if $|r| < 1$ therefore,

$$S_\infty = \frac{a}{1-r}\,(|r| < 1)\ .$$

(iv) If each term of a GP be multiplied or divided by the same non-zero quantity, the resulting sequence is also a GP.

(v) Any 3 consecutive terms of a GP can be taken as a/r, a, ar ; any 4 consecutive terms of a GP can be taken as a/r^3, a/r, ar, ar^3 & so on.

(vi) If a, b, c are in GP $\Rightarrow b^2 = ac$.

HARMONIC PROGRESSION (HP) :

A sequence is said to HP if the reciprocals of its terms are in AP.

If the sequence $a_1, a_2, a_3, \ldots, a_n$ is an HP then $1/a_1, 1/a_2, \ldots, 1/a_n$ is an AP & converse. Here we do not have the formula for the sum of the n terms of an HP. For HP whose first term is a & second term is b, the n^{th} term is $t_n = \dfrac{ab}{b + (n-1)(a-b)}$.

If a, b, c are in HP $\Rightarrow b = \dfrac{2ac}{a+c}$ or $\dfrac{a}{c} = \dfrac{a-b}{b-c}$.

MEANS

ARITHMETIC MEAN :

If three terms are in AP then the middle term is called the AM between the other two, so if a, b, c are in AP, b is AM of a & c .

AM for any n positive number $a_1, a_2, \ldots, a_n$ is ; $A = \dfrac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$.

n - ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a, b are any two given numbers & $a, A_1, A_2, \ldots, A_n, b$ are in AP then $A_1, A_2, \ldots A_n$ are the n AM's between a & b .

$A_1 = a + \dfrac{b-a}{n+1}$, $A_2 = a + \dfrac{2(b-a)}{n+1}$, ... , $A_n = a + \dfrac{n(b-a)}{n+1}$

$= a + d$, $= a + 2d$, ... , $A_n = a + nd$, where $d = \dfrac{b-a}{n+1}$

NOTE : Sum of n AM's inserted between a & b is equal to n times the single AM between a & b

i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single AM between a & b.

GEOMETRIC MEANS :

If a, b, c are in GP, b is the GM between a & c.

$b^2 = ac$, therefore $b = \sqrt{ac}$; $a > 0$, $c > 0$.

n-GEOMETRIC MEANS BETWEEN a, b :

If a, b are two given numbers & $a, G_1, G_2, \ldots, G_n, b$ are in GP. Then $G_1, G_2, G_3, \ldots, G_n$ are n GMs between a & b .

$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$, ... , $G_n = a(b/a)^{n/n+1}$

$= ar$, $= ar^2$, ... $= ar^n$, where $r = (b/a)^{1/n+1}$

NOTE : The product of n GMs between a & b is equal to the n^{th} power of the single GM between a & b

i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single GM between a & b.

HARMONIC MEAN :

If a, b, c are in HP, b is the HM between a & c, then $b = 2ac/[a+c]$.

THEOREM :

If A, G, H are respectively AM, GM, HM between a & b both being unequal & positive then,

(i) $G^2 = AH$

(ii) $A > G > H$ $(G > 0)$. Note that A, G, H constitute a GP.

ARITHMETICO-GEOMETRIC SERIES :

A series each term of which is formed by multiplying the corresponding term of an AP & GP is called the **Arithmetico-Geometric Series**. e.g. $1 + 3x + 5x^2 + 7x^3 + ...$
Here 1, 3, 5, ... are in AP & $1, x, x^2, x^3$ ..... are in GP.

Standart appearance of an Arithmetico-Geometric Series is
Let $S_n = a + (a + d) r + (a + 2 d) r^2 + ... + [a + (n - 1)d]\ r^{n-1}$

SIGMA NOTATIONS

THEOREMS :

(i) $\sum\limits_{r=1}^{n} (a_r \pm b_r) = \sum\limits_{r=1}^{n} a_r \pm \sum\limits_{r=1}^{n} b_r.$

(ii) $\sum\limits_{r=1}^{n} k\, a_r = k \sum\limits_{r=1}^{n} a_r.$

(iii) $\sum\limits_{r=1}^{n} k = nk$; where k is a constant.

RESULTS

(i) $\sum\limits_{r=1}^{n} r = \frac{n\,(n+1)}{2}$ (sum of the first n natural nos.)

(ii) $\sum\limits_{r=1}^{n} r^2 = \frac{n\,(n+1)\,(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(iii) $\sum\limits_{r=1}^{n} r^3 = \frac{n^2\,(n+1)^2}{4} \left[\sum\limits_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

METHOD OF DIFFERENCE :

If $T_1, T_2, T_3, ... , T_n$ are the terms of a sequence then some times the terms $T_2 - T_1, T_3 - T_2$, ... constitute an AP/GP. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Remember that to find the sum of n terms of a series each term of which is composed of r factors in AP, the first factors of several terms being in the same AP, we "write down the nth term, affix the next factor at the end, divide by the number of factors thus increased and by the common difference and add a constant. Determine the value of the constant by applying the initial conditions".

EXERCISE–I

Q.1 The sum of *n* terms of two arithmetic series are in the ratio of $(7n+1):(4n+27)$. Find the ratio of their n^{th} term.

Q.2 In an AP of which 'a' is the Ist term, if the sum of the Ist p terms is equal to zero, show that the sum of the next q terms is $-\left(\frac{aq(p+q)}{p-1}\right)$.

Q.3(a) The interior angles of a polygon are in AP. The smallest angle is 120° & the common difference is 5°. Find the number of sides of the polygon.

(b) The interior angles of a convex polygon form an arithmetic progression with a common difference of 4°. Determine the number of sides of the polygon if its largest interior angle is 172°.

Q.4 Show that $ln\,(4 \times 12 \times 36 \times 108 \times$ .... up to *n* terms$) = 2n\, ln\, 2 + \frac{n(n-1)}{2} ln\, 3$

Q.5 There are n AM's between 1 & 31 such that 7th mean : $(n - 1)^{th}$ mean = 5 : 9, then find the value of n.

Q.6 Prove that the average of the numbers $n \sin n°$, $n = 2, 4, 6, \ldots\ldots, 180$, is $\cot 1°$.

Q.7 Find the value of the sum $\sum_{k=0}^{359} k \cdot \cos k°$.

Q.8 The first term of an arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression coincide with the first and the ninth term of the arithmetic progression. Find the seventh term of the geometric progression.

Q.9 In a set of four numbers, the first three are in GP & the last three are in AP, with common difference 6. If the first number is the same as the fourth, find the four numbers.

Q.10 The 1^{st}, 2^{nd} and 3^{rd} terms of an arithmetic series are a, b and a^2 where 'a' is negative. The 1^{st}, 2^{nd} and 3^{rd} terms of a geometric series are a, a^2 and b find the

(a) value of a and b

(b) sum of infinite geometric series if it exists. If no then find the sum to n terms of the G.P.

(c) sum of the 40 term of the arithmetic series.

Q.11 Let 'X' denotes the value of the product

$$(1 + a + a^2 + a^3 + \ldots\ldots \infty)(1 + b + b^2 + b^3 + \ldots\ldots \infty)$$

where 'a' and 'b' are the roots of the quadratic equation $11x^2 - 4x - 2 = 0$

and 'Y' denotes the numerical value of the infinite series

$$(\log_b 2)^0\left(\log_b 5^{4^0}\right) + (\log_b 2)^1\left(\log_b 5^{4^1}\right) + (\log_b 2)^2\left(\log_b 5^{4^2}\right) + (\log_b 2)^3\left(\log_b 5^{4^3}\right) + \ldots\ldots\infty$$

where $b = 2000$. Find (XY).

Q.12 Find three numbers a , b , c between 2 & 18 such that;

(i) their sum is 25

(ii) the numbers 2, a, b are consecutive terms of an AP &

(iii) the numbers b , c , 18 are consecutive terms of a GP .

Q.13 If one AM 'a' and two GM's p and q be inserted between any two given numbers then show that $p^3 + q^3 = 2apq$.

Q.14 If $S_1, S_2, S_3, \ldots S_n, \ldots$ are the sums of infinite geometric series whose first terms are 1, 2, 3, ... n, ... and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n+1}, \ldots$ respectively, then find the value of $\sum_{r=1}^{2n-1} S_r^2$.

Q.15 Find the sum of the first n terms of the sequence : $1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + \ldots\ldots\ldots\ldots$

Q.16 Find the nth term and the sum to n terms of the sequence:

(i) $1 + 5 + 13 + 29 + 61 + \ldots\ldots$ (ii) $6 + 13 + 22 + 33 + \ldots\ldots$

Q.17 Sum the following series to n terms and to infinity :

(i) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \ldots\ldots\ldots$ (ii) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$

(iii) $\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$ (iv) $\frac{1}{4} + \frac{1.3}{4.6} + \frac{1.3.5}{4.6.8} + \ldots\ldots\ldots\ldots$

Q.18 Find the sum of the n terms of the sequence $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots\ldots\ldots\ldots\ldots$

Q.19 Let 'σ' denotes the sum of the infinite series $\sum_{n=1}^{\infty}\left(\frac{n^2+2n+3}{2^n}\right)$.

Compute the value of $(1^3+2^3+3^3+\ldots\ldots+\sigma^3)$.

Q.20 If the sum $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots\ldots+\sqrt{1+\frac{1}{(1999)^2}+\frac{1}{(2000)^2}}$ equal to $n-\frac{1}{n}$ where $n \in N$. Find n.

Q.21 If the 10^{th} term of an HP is 21 and 21^{st} term of the same HP is 10, then find the 210^{th} term.

Q.22 The p^{th} term T_p of H.P. is $q(p+q)$ and q^{th} term T_q is $p(p+q)$ when $p > 2$, $q > 2$. Prove that
(a) $T_{p+q} = pq$; (b) $T_{pq} = p+q$; (c) $T_{p+q} > T_{pq}$

Q.23 The harmonic mean of two numbers is 4. The airthmetic mean A & the geometric mean G satisfy the relation $2A+G^2=27$. Find the two numbers.

Q.24 The AM of two numbers exceeds their GM by 15 & HM by 27. Find the numbers.

Q.25 In the quadratic equation $A\left(\sqrt{3}-\sqrt{2}\right)x^2+\frac{B}{\left(\sqrt{3}+\sqrt{2}\right)}x+C=0$ with α, β as its roots.

If $A=\left(49+20\sqrt{6}\right)^{1/4}$; B = sum of the infinite G.P. as $8\sqrt{3}+\frac{8\sqrt{6}}{\sqrt{3}}+\frac{16}{\sqrt{3}}+\ldots\ldots\infty$

and $|\alpha-\beta| = \left(6\sqrt{6}\right)^k$ where $k=\log_6 10-2\log_6\sqrt{5}+\log_6\sqrt{(\log_6 18+\log_6 72)}$,
then find the value of C.

EXERCISE–II

Q.1 If $\sin x$, $\sin^2 2x$ and $\cos x \cdot \sin 4x$ form an increasing geometric sequence, find the numerial value of $\cos 2x$. Also find the common ratio of geometric sequence.

Q.2 If the first 3 consecutive terms of a geometrical progression are the real roots of the equation $2x^3-19x^2+57x-54=0$ find the sum to infinite number of terms of G.P.

Q.3 Find the sum of the infinite series $\frac{1.3}{2}+\frac{3.5}{2^2}+\frac{5.7}{2^3}+\frac{7.9}{2^4}+\ldots\ldots\ldots\infty$.

Q.4 Two distinct, real, infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is $1/8$. If the second term of both the series can be written in the form $\frac{\sqrt{m}-n}{p}$, where *m*, *n* and *p* are positive integers and *m* is not divisible by the square of any prime, find the value of $100m+10n+p$.

Q.5 One of the roots of the equation $2000x^6+100x^5+10x^3+x-2=0$ is of the form $\frac{m+\sqrt{n}}{r}$, where *m* is non zero integer and *n* and *r* are relatively prime natural numbers. Find the value of $m+n+r$.

Q.6 Find the condition that the roots of the equation $x^3-px^2+qx-r=0$ are in A.P. and hence solve the equation $x^3-12x^2+39x-28=0$.

Q.7 If a, b, c, d, e be 5 numbers such that a, b, c are in AP ; b, c, d are in GP & c, d, e are in HP then:

(i) Prove that a, c, e are in GP .

(ii) Prove that $e = (2b - a)^2/a$.

(iii) If $a = 2$ & $e = 18$, find all possible values of b , c , d .

Q.8 Let $f(x)$ denote the sum of the infinite trigonometric series, $f(x) = \sum_{n=1}^{\infty} \sin\frac{2x}{3^n} \sin\frac{x}{3^n}$.

Find $f(x)$ (independent of n). If the sum of the solutions of the equation $f(x) = 0$ lying in the interval (0, 629) is $2k\pi$, find k.

Q.9 A computer solved several problems in succession. The time it took the computer to solve each successive problem was the same number of times smaller than the time it took to solve the preceding problem. How many problems were suggested to the computer if it spent 63.5 min to solve all the problems except for the first, 127 min to solve all the problems except for the last one, and 31.5 min to solve all the problems except for the first two?

Q.10 If n is a root of the equation $x^2(1 - ac) - x(a^2 + c^2) - (1 + ac) = 0$ & if n HM's are inserted between a and c, show that the difference between the first & the last mean is equal to $ac(a - c)$.

Q.11 Given that the cubic $ax^3 - ax^2 + 9bx - b = 0$ $(a \neq 0)$ has all three positive roots. Find the harmonic mean of the roots independent of a and b, hence deduce that the root are all equal. Find also the minimum value of (a + b) if a and b $\in$ N.

Q.12 If $\tan\left(\frac{\pi}{12} - x\right), \tan\frac{\pi}{12}, \tan\left(\frac{\pi}{12} + x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. Find the value of k.

Q.13 In a right angled triangle, S_a and S_b denote the medians that belong to the legs of the triangle, the median belonging to the hypotenuse is S_c. Find the maximum value of the expression $\frac{S_a + S_b}{S_c}$.
(You may use the fact that R.M.S. $\geq$ A.M).

Q.14 The sequence $a_1, a_2, a_3, \ldots\ldots a_{98}$ satisfies the relation $a_{n+1} = a_n + 1$ for n = 1, 2, 3, ...97 and has the sum equal to 4949. Evaluate $\sum_{k=1}^{49} a_{2k}$.

Q.15 (a) The value of $x + y + z$ is 15 if a , x , y , z , b are in AP while the value of ; $(1/x)+(1/y)+(1/z)$ is 5/3 if a , x , y , z , b are in HP. Find a & b.

(b) The values of xyz is 15/2 or 18/5 according as the series a , x , y , z , b is an AP or HP . Find the values of a & b assuming them to be positive integer.

Q.16 Find the conditions on α and β x_1, x_2, x_3 satisfying the cubic $x^3 - x^2 + \alpha x + \beta = 0$ are in A.P.

Q.17 If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c and all the roots.

Q.18 If a , b , c be in GP & $\log_c a, \log_b c, \log_a b$ be in AP , then show that the common difference of the AP must be 3/2.

Q.19 In a GP the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the GP.

Q.20 Given a three digit number whose digits are three successive terms of a G.P. If we subtract 792 from it, we get a number written by the same digits in the reverse order. Now if we subtract four from the hundred's digit of the initial number and leave the other digits unchanged, we get a number whose digits are successive terms of an A.P. Find the number.

EXERCISE–III

Q.1(a) Consider an infinite geometric series with first term 'a' and common ratio r . If the sum is 4 and the second term is 3/4, then :

(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$ (C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

(b) If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation :

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

[JEE 2000, Screening, 1 + 1 out of 35]

(c) The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it . Prove that the resulting sum is the square of an integer.

[JEE 2000, Mains, 4 out of 100]

Q.2 Given that α, γ are roots of the equation, $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation, $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P.

[REE 2000, 5 out of 100]

Q.3 The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P.? [REE 2001, 3 out of 100]

Q.4 Solve the following equations for x and y

$\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots\ldots\ldots\ldots\ldots = y$

$\frac{5+9+13+\ldots\ldots\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots\ldots\ldots\ldots+(2y-1)} = 4\log_4 x$ [REE 2001, 5 out of 100]

Q.5(a) Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

(b) If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..., then n equals

(A) 10 (B) 12 (C) 11 (D) 13

(c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

(A) NOT in A.P./G.P./H.P. (B) in A.P.

(C) in G.P. (D) H.P. [JEE 2001, Scr, 1 + 1 + 1 out of 35]

(d) Let a_1, a_2 .... be positive real numbers in G.P. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots\ldots G_n$ in terms of $A_1, A_2 \ldots\ldots\ldots\ldots A_n, H_1, H_2, \ldots\ldots\ldots H_n$.

[JEE 2001 (Mains); 5]

Q.6(a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

[JEE 2002 (Screening), 3]

(b) Let a, b be positive real numbers. If a , A_1 , A_2 , b are in A.P. ; a , G_1 , G_2 , b are in G.P. and a , H_1 , H_2 , b are in H.P. , show that

$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$ [JEE 2002 , Mains , 5 out of 60]

Q.7 If a, b, c are in A.P., a^2 , b^2 , c^2 are in H.P. , then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P.

[JEE-03, Mains-4 out of 60]

Q.8 The first term of an infinite geometric progression is x and its sum is 5. Then

(A) $0 \le x \le 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

[JEE 2004 (Screening)]

Q.9 If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$.

[JEE 2004, 4 out of 60]

Q.10(a) In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. where α, β are the roots of $ax^2 + bx + c = 0$, then

(A) $\Delta \ne 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

[JEE 2005 (Screening)]

(b) If total number of runs scored in *n* matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \le k \le n$. Find n. [JEE 2005 (Mains), 2]

Q.11 If $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$, then find the minimum natural number n_0 such that $B_n > A_n$. $\forall\, n > n_0$. [JEE 2006, 6]

Comprehension (3 questions)

Q.12 Let V_r denote the sum of the first 'r' terms of an arithmetic progression (A.P.) whose first term is 'r' and the common difference is $(2r - 1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ...

(a) The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

(b) T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

(c) Which one of the following is a correct statement?

(A) Q_1, Q_2, Q_3, ... are in A.P. with common difference 5.

(B) Q_1, Q_2, Q_3, ... are in A.P. with common difference 6.

(C) Q_1, Q_2, Q_3, ... are in A.P. with common difference 11.

(D) $Q_1 = Q_2 = Q_3 = ...$ [JEE 2007, 4+4+4]

Comprehension (3 questions)

Q.13 Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

(a) Which one of the following statements is correct?
(A) $G_1 > G_2 > G_3 > ......$
(B) $G_1 < G_2 < G_3 < ......$
(C) $G_1 = G_2 = G_3 = ...$
(D) $G_1 < G_3 < G_5 < ......$ and $G_2 > G_4 > G_6 > ......$

(b) Which one of the following statements is correct?
(A) $A_1 > A_2 > A_3 > ......$
(B) $A_1 < A_2 < A_3 < ......$
(C) $A_1 > A_3 > A_5 > ......$ and $A_2 < A_4 < A_6 < ......$
(D) $A_1 < A_3 < A_5 < ......$ and $A_2 > A_4 > A_6 > ......$

(c) Which one of the following statements is correct?
(A) $H_1 > H_2 > H_3 > ......$
(B) $H_1 < H_2 < H_3 < ......$
(C) $H_1 > H_3 > H_5 > ......$ and $H_2 < H_4 < H_6 < ......$
(D) $H_1 < H_3 < H_5 < ......$ and $H_2 > H_4 > H_6 > ......$ [JEE 2007, 4+4+4]

Q.14**(a)** A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$ [JEE 2008, 4]

ASSERTION & REASON:

(b) Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.
STATEMENT-1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.
and
STATEMENT-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.
(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True [JEE 2008, 3 (–1)]

Q.15 If the sum of first *n* terms of an A.P. is cn^2, then the sum of squares of these *n* terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

[JEE 2009, 3 (–1)]

ANSWER KEY

SEQUENCE & PROGRESSION

EXERCISE–I

Q.1 $(14n-6)/(8n+23)$ **Q.3** (a) 9 ; (b) 12 **Q.5** $n = 14$

Q.7 -180 **Q.8** 27 **Q.9** $(8, -4, 2, 8)$

Q.10 (a) $a = -\frac{1}{2}$, $b = -\frac{1}{8}$; (b) $-\frac{1}{3}$; (c) $\frac{545}{2}$ **Q.11** $\frac{11}{15}$

Q.12 $a = 5, b = 8, c = 12$ **Q.14** $\frac{n(2n+1)(4n+1)}{3}-1$ **Q.15** n^2

Q.16 (i) $2^{n+1}-3$; $2^{n+2}-4-3n$ (ii) n^2+4n+1; $(1/6)\,n\,(n+1)\,(2n+13)+n$

Q.17 **(i)** $s_n = (1/24) - [1/\{6(3n+1)(3n+4)\}]$; $s_\infty = 1/24$ **(ii)** $(1/5)\,n\,(n+1)\,(n+2)\,(n+3)\,(n+4)$

(iii) $n/(2n+1)$ **(iv)** $S_n = 2\left[\frac{1}{2} - \frac{1.3.5.....(2n-1)(2n+1)}{2.4.6......(2n)(2n+2)}\right]$; $S_\infty = 1$

Q.18 $\frac{n(n+1)}{2(n^2+n+1)}$ **Q.19** 8281 **Q.20** $n = 2000$

Q.21 1 **Q.23** 6, 3 **Q.24** 120, 30 **Q.25** 128

EXERCISE–II

Q.1 $\frac{\sqrt{5}-1}{2}$; $\sqrt{2}$ **Q.2** $\frac{27}{2}$ **Q.3** 23 **Q.4** 518

Q.5 200 **Q.6** $2p^3 - 9pq + 27r = 0$; roots are 1, 4, 7

Q.7 (iii) $b = 4, c = 6, d = 9$ **or** $b = -2, c = -6, d = -18$

Q.8 $f(x) = \frac{1}{2}[1-\cos x]$; $S = 5050$ **Q.9** 8 problems, 127.5 minutes

Q.11 28 **Q.12** 4950 **Q.13** $\sqrt{10}$ **Q.14** 2499

Q.15 (a) $a = 1, b = 9$ OR $b = 1, a = 9$; (b) $a = 1$; $b = 3$ or vice versa **Q.16** $\alpha \le \frac{1}{3}$; $\beta \ge -\frac{1}{27}$

Q.17 $C = 9$; $(3, -3/2, -3/5)$

Q.19 $n = 38$ **Q.20** 931

EXERCISE–III

Q.1 (a) D (b) A **Q.2** $A = 3$; $B = 8$ **Q.3** A.P.

Q.4 $x = 2\sqrt{2}$ and $y = 3$

Q.5 (a) A, (b) C, (c) D , (d) $\left[\left(A_1, A_2, ... A_n\right)\left(H_1, H_2, ... H_n\right)\right]^{\frac{1}{2n}}$

Q.6 (a) D **Q.8** B **Q.10** (a) C, (b) $n = 7$

Q.11 $n_0 = 5$ **Q.12** (a) B; (b) D; (c) B **Q.13** (a) C; (b) A; (c) B

Q.14 (a) B, D; (b) C **Q.15** C