QUADRATIC EQUATION

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KEY CONCEPTS (QUADRATIC EQUATIONS)

The general form of a quadratic equation in x is, $ax^2 + bx + c = 0$, where $a, b, c \in R$ & $a \neq 0$.

RESULTS:

1. The solution of the quadratic equation, $ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $b^2 - 4ac = D$ is called the discriminant of the quadratic equation.

2. If α & β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then;

(i) $\alpha + \beta = - b/a$ (ii) $\alpha\beta = c/a$ (iii) $\alpha - \beta = \sqrt{D}/a$.

3. NATURE OF ROOTS:

(A) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in R$ & $a \neq 0$ then ;

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary .

(iv) If $p + iq$ is one root of a quadratic equation, then the other must be the conjugate $p - iq$ & vice versa. ($p, q \in R$ & $i = \sqrt{-1}$).

(B) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in Q$ & $a \neq 0$ then;

(i) If $D > 0$ & is a perfect square, then roots are rational & unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

4. A quadratic equation whose roots are α & β is $(x - \alpha)(x - \beta) = 0$ i.e.
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 -$ (sum of roots) $x +$ product of roots $= 0$.

5. Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

6. Consider the quadratic expression , $y = ax^2 + bx + c$, $a \neq 0$ & $a, b, c \in R$ then ;

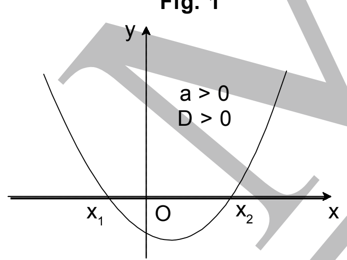
(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $\forall\ x \in R$, $y > 0$ only if $a > 0$ & $b^2 - 4ac < 0$ (figure 3) .

(iii) $\forall\ x \in R$, $y < 0$ only if $a < 0$ & $b^2 - 4ac < 0$ (figure 6) .

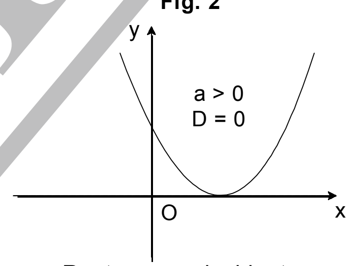
Carefully go through the 6 different shapes of the parabola given below.

Fig. 1



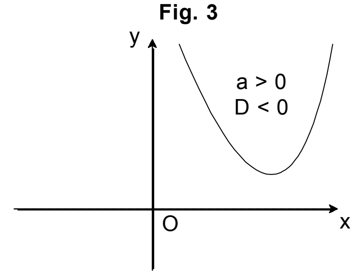
Roots are real & distinct

Fig. 2



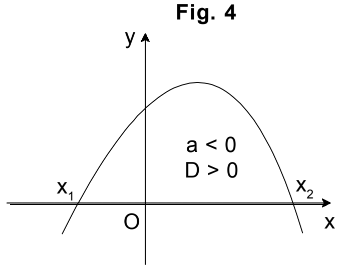
Roots are coincident

Fig. 3



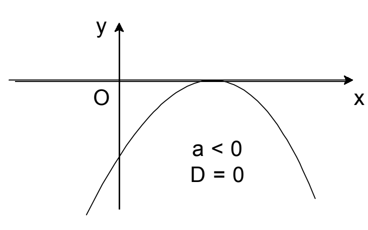
Roots are complex conjugate

Fig. 4



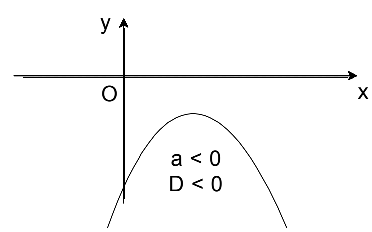
Roots are real & distinct

Fig. 5



Roots are coincident

Fig. 6



Roots are complex conjugate

7. SOLUTION OF QUADRATIC INEQUALITIES:

$ax^2 + bx + c > 0$ $(a \neq 0)$.

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $x_1 < x_2$.
Then $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_2, \infty)$
$a < 0 \Rightarrow x \in (x_1, x_2)$

(ii) If $D = 0$, then roots are equal, i.e. $x_1 = x_2$.
In that case $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_1, \infty)$
$a < 0 \Rightarrow x \in \phi$

(iii) Inequalities of the form $\dfrac{P(x)}{Q(x)} \lesseqgtr 0$ can be quickly solved using the method of intervals.

8. MAXIMUM & MINIMUM VALUE of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as ;

$a < 0$ or $a > 0$. $y \in \left[\dfrac{4ac - b^2}{4a}, \infty\right)$ if $a > 0$ & $y \in \left(-\infty, \dfrac{4ac - b^2}{4a}\right]$ if $a < 0$.

9. COMMON ROOTS OF 2 QUADRATIC EQUATIONS [ONLY ONE COMMON ROOT] :

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$. Therefore

$a\alpha^2 + b\alpha + c = 0$; $a'\alpha^2 + b'\alpha + c' = 0$. By Cramer's Rule $\dfrac{\alpha^2}{bc' - b'c} = \dfrac{\alpha}{a'c - ac'} = \dfrac{1}{ab' - a'b}$

Therefore, $\alpha = \dfrac{ca' - c'a}{ab' - a'b} = \dfrac{bc' - b'c}{a'c - ac'}$.

So the condition for a common root is $(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$.

10. The condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factors is that ;

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ **OR** $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

11. THEORY OF EQUATIONS :

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ... a_n$ are all real & $a_0 \neq 0$ then,

$\sum \alpha_1 = -\dfrac{a_1}{a_0}$, $\sum \alpha_1 \alpha_2 = +\dfrac{a_2}{a_0}$, $\sum \alpha_1 \alpha_2 \alpha_3 = -\dfrac{a_3}{a_0}$, ..., $\alpha_1 \alpha_2 \alpha_3 ....\alpha_n = (-1)^n \dfrac{a_n}{a_0}$

Note : (i) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$ and conversely .

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. **imaginary roots occur in conjugate pairs**.

(iv) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(v) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b' .

(vi) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

12. LOCATION OF ROOTS:

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'd' are $b^2 - 4ac \ge 0$; $f(d) > 0$ & $(-b/2a) > d$.

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'd' (in other words the number 'd' lies between the roots of $f(x) = 0$) is $f(d) < 0$.

(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (d, e) i.e. $d < x < e$ are $b^2 - 4ac > 0$ & $f(d) . f(e) < 0$.

(iv) Conditions that both roots of $f(x) = 0$ to be confined between the numbers p & q are $(p < q)$. $b^2 - 4ac \ge 0$; $f(p) > 0$; $f(q) > 0$ & $p < (-b/2a) < q$.

13. LOGARITHMIC INEQUALITIES

(i) For $a > 1$ the inequality $0 < x < y$ & $\log_a x < \log_a y$ are equivalent.

(ii) For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent.

(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$

(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$

(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$

(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

EXERCISE–I

Q.1 A quadratic polynomial $f(x) = x^2 + ax + b$ is formed with one of its zeros being $\frac{4+3\sqrt{3}}{2+\sqrt{3}}$ where *a* and *b* are integers. Also $g(x) = x^4 + 2x^3 - 10x^2 + 4x - 10$ is a biquadratic polynomial such that $g\left(\frac{4+3\sqrt{3}}{2+\sqrt{3}}\right) = c\sqrt{3} + d$ where *c* and *d* are also integers. Find the values of a, b, c and d.

Q.2 If $(x^2 - x\cos(A+B) + 1)$ is a factor of the expression,
$2x^4 + 4x^3 \sin A \sin B - x^2(\cos 2A + \cos 2B) + 4x \cos A \cos B - 2$. Then find the other factor.

Q.3 α, β are the roots of the equation $K(x^2 - x) + x + 5 = 0$. If K_1 & K_2 are the two values of K for which the roots α, β are connected by the relation $(\alpha/\beta) + (\beta/\alpha) = 4/5$. Find the value of $(K_1/K_2) + (K_2/K_1)$.

Q.4 If the quadratic equations, $x^2 + bx + c = 0$ and $bx^2 + cx + 1 = 0$ have a common root then prove that either $b + c + 1 = 0$ or $b^2 + c^2 + 1 = bc + b + c$.

Q.5 Let a, b be arbitrary real numbers. Find the smallest natural number 'b' for which the equation $x^2 + 2(a+b)x + (a - b + 8) = 0$ has unequal real roots for all $a \in R$.

Q.6 Find the range of values of a, such that $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

Q.7 Find the product of uncommon real roots of the two polynomials $P(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$ and $Q(x) = x^3 + 4x^2 - x - 10$.

Q.8 Let the quadratic equation $x^2 + 3x - k = 0$ has roots a, b and $x^2 + 3x - 10 = 0$ has roots c, d such that modulus of difference of the roots of the first equation is equal to twice the modulus of the difference of the roots of the second equation. If the value of 'k' can be expressed as rational number in the lowest form as m/n then find the value of $(m+n)$.

Q.9 When $y^2 + my + 2$ is divided by $(y - 1)$ then the quotient is $f(y)$ and the remainder is R_1. When $y^2 + my + 2$ is divided by $(y + 1)$ then quotient is $g(y)$ and the remainder is R_2. If $R_1 = R_2$ then find the value of m.

Q.10 Find the value of m for which the quadratic equations $x^2 - 11x + m = 0$ and $x^2 - 14x + 2m = 0$ may have common root.

Q.11 If the quadratic equations $x^2 + bx + ca = 0$ & $x^2 + cx + ab = 0$ have a common root, prove that the equation containing their other root is $x^2 + ax + bc = 0$.

Q.12 If by eleminating x between the equation $x^2 + ax + b = 0$ & $xy + l(x + y) + m = 0$, a quadratic in y is formed whose roots are the same as those of the original quadratic in x. Then prove either $a = 2l$ & $b = m$ or $b + m = al$.

Q.13(a) If α, β are the roots of the quadratic equation $ax^2+bx+c = 0$ then which of the following expressions in α, β will denote the symmetric functions of roots. Give proper reasoning.

(i) $f(\alpha, \beta) = \alpha^2 - \beta$ (ii) $f(\alpha, \beta) = \alpha^2\beta + \alpha\beta^2$

(iii) $f(\alpha, \beta) = ln \dfrac{\alpha}{\beta}$ (iv) $f(\alpha, \beta) = \cos(\alpha - \beta)$

(b) If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ & $\alpha^3\beta^2 + \alpha^2\beta^3$.

Q.14 Let $P(x) = 4x^2 + 6x + 4$ and $Q(y) = 4y^2 - 12y + 25$. Find the unique pair of real numbers (x, y) that satisfy $P(x) \cdot Q(y) = 28$.

Q.15 Find a quadratic equation whose sum and product of the roots are the values of the expressions $(\text{cosec } 10° - \sqrt{3}\sec 10°)$ and $(0.5 \text{ cosec} 10° - 2\sin 70°)$ respectively. Also express the roots of this quadratic in terms of tangent of an angle lying in $(0, \pi/2)$.

Q.16 Find the product of the real roots of the equation,

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$$

Q.17 We call 'p' a good number if the inequality $\dfrac{2x^2 + 2x + 3}{x^2 + x + 1} \le p$ is satisfied for any real x. Find the smallest integral good number.

Q.18 Find the values of 'a' for which $-3 < [(x^2 + ax - 2)/(x^2 + x + 1)] < 2$ is valid for all real x.

Q.19 If the range of the function $f(x) = \dfrac{x^2 + ax + b}{x^2 + 2x + 3}$ is $[-5, 4]$, $a, b \in N$, then find the value of $(a^2 + b^2)$.

Q.20 Suppose $a, b, c \in I$ such that greatest common divisor of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x + 1)$ and the least common multiple of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x^3 - 4x^2 + x + 6)$. Find the value of $(a + b + c)$.

Q.21 Let a, b, c and $m \in R^+$. Find the range of m (independent of a, b and c) for which atleast one of the following equations.

$$\left.\begin{aligned} ax^2+bx+cm=0 \\ bx^2+cx+am=0 \\ \text{and}\quad cx^2+ax+bm=0 \end{aligned}\right\} \text{have real roots.}$$

Q.22 If a & b are positive numbers, prove that the equation $\frac{1}{x}+\frac{1}{x-a}+\frac{1}{x+b}=0$ has two real roots, one between a/3 & 2a/3 and the other between $-2b/3$ & $-b/3$.

Q.23 If the roots of $x^2-ax+b=0$ are real & differ by a quantity which is less than c (c > 0), prove that b lies between $(1/4)(a^2-c^2)$ & $(1/4)a^2$.

Q.24 Find all real numbers x such that, $\left(x-\frac{1}{x}\right)^{\frac{1}{2}}+\left(1-\frac{1}{x}\right)^{\frac{1}{2}}=x.$

Q.25 Find the minimum value of $\dfrac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+x^3+\frac{1}{x^3}}$ for $x > 0$.

EXERCISE–II

Q.1 Let $x^2+y^2+xy+1 \ge a(x+y)\ \forall\ x, y \in R$. Find the possible integer(s) in the range of a.

Q.2 Solve the following where $x \in R$.

(a) $(x-1)\left|x^2-4x+3\right|+2x^2+3x-5=0$

(b) $3\left|x^2-4x+2\right|=5x-4$

(c) $\left|x^3+1\right|+x^2-x-2=0$

(d) $2^{|x+2|}-\left|2^{x+1}-1\right|=2^{x+1}+1$

(e) For $a \le 0$, determine all real roots of the equation $x^2-2a\left|x-a\right|-3a^2=0$.

Q.3 Let a, b, c, d be distinct real numbers and a and b are the roots of quadratic equation $x^2-2cx-5d=0$. If c and d are the roots of the quadratic equation $x^2-2ax-5b=0$ then find the numerical value of $a+b+c+d$.

Q.4 Let α, β and γ are the roots of the cubic $x^3-3x^2+1=0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}$, $\frac{\beta}{\beta-2}$ and $\frac{\gamma}{\gamma-2}$. Hence or otherwise find the value of $(\alpha-2)(\beta-2)(\gamma-2)$.

Q.5 Suppose a cubic polynomial $f(x) = x^3+px^2+qx+72$ is divisible by both x^2+ax+b and x^2+bx+a (where a, b, p, q are constants and $a \ne b$). Find the sum of the squares of the roots of the cubic polynomial.

Q.6 Let x be a positive real. Find the maximum possible value of the expression $y=\dfrac{x^2+2-\sqrt{x^4+4}}{x}$.

Q.7 Given $x, y \in R$, $x^2+y^2>0$. If the maximum and minimum value of the expression $E=\dfrac{x^2+y^2}{x^2+xy+4y^2}$ are M and m, and A denotes the average value of M and m, compute (2007)A.

Q.8 Find all numbers p for each of which the least value of the quadratic trinomial $4x^2 - 4px + p^2 - 2p + 2$ on the interval $0 \le x \le 2$ is equal to 3.

Q.9 Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (-32). Find the value of k.

Q.10 Find the complete set of real values of 'a' for which both roots of the quadratic equation $(a^2 - 6a + 5)x^2 - \sqrt{a^2 + 2a}\ x + (6a - a^2 - 8) = 0$ lie on either side of the origin.

Q.11 If α, β are the roots of the equation, $x^2 - 2x - a^2 + 1 = 0$ and γ, δ are the roots of the equation, $x^2 - 2(a + 1)x + a(a - 1) = 0$ such that $\alpha, \beta \in (\gamma, \delta)$ then find the values of 'a'.

Q.12 Find all the values of the parameter 'a' for which both roots of the quadratic equation $x^2 - ax + 2 = 0$ belong to the interval (0, 3).

Q.13 At what values of 'a' do all the zeroes of the function,

$$f(x) = (a - 2)x^2 + 2ax + a + 3 \text{ lie on the interval } (-2, 1)?$$

Q.14 Find the values of K so that the quadratic equation $x^2 + 2(K - 1)x + K + 5 = 0$ has atleast one positive root.

Q.15 Let $P(x) = x^2 + bx + c$, where b and c are integer. If P (x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

Q.16 Find the values of 'b' for which the equation $2\log_{\frac{1}{25}}(bx + 28) = -\log_5(12 - 4x - x^2)$ has only one solution.

Q.17 Find all the values of the parameters c for which the inequality has at least one solution.

$$1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) \ge \log_2\left(cx^2 + c\right).$$

Q.18 Find all the values of the parameter 'a' for which the inequality

$$a.9^x + 4(a-1)3^x + a - 1 > 0 \text{ is satisfied for all real values of } x.$$

MATCH THE COLUMN (one to many & Many to one):

Q.19 Match the conditions in Column-I with the intervals in Column-II.
Let $f(x) = x^2 - 2px + p^2 - 1$, then

	Column-I		Column-II.
(A)	both the roots of $f(x) = 0$ are less than 4, if $p \in$	(P)	$(-1, \infty)$
(B)	both the roots of $f(x) = 0$ are greater than -2 if $p \in$	(Q)	$(-\infty, 3)$
(C)	exactly one root of $f(x) = 0$ lie in $(-2, 4)$, if $p \in$	(R)	$(0, 2)$
(D)	1 lies between the roots of $f(x) = 0$, if $p \in$	(S)	$(-3, -1) \cup (3, 5)$

Q.20 It is given that α, β $(\beta \ge \alpha)$ are the roots of the equation $f(x) = ax^2 + bx + c$. Also $af(t) > 0$.
Match the condition given in column-I with their corresponding conclusions given in column-II.

	Column-I		Column-II
(A)	$a > 0$ and $b^2 > 4ac$	(P)	$t \ne \alpha$
(B)	$a > 0$ and $b^2 = 4ac$	(Q)	no solution
(C)	$a < 0$ and $b^2 > 4ac$	(R)	$\alpha < t < \beta$
(D)	$a < 0$ and $b^2 = 4ac$	(S)	$t < \alpha$ or $t > \beta$

EXERCISE–III

Solve the inequality. Where ever base is not given take it as 10.

Q.1 $(\log_2 x)^4 - \left(\log_{\frac{1}{2}} \frac{x^5}{4}\right)^2 - 20\log_2 x + 148 < 0$.

Q.2 $(\log 100x)^2 + (\log 10x)^2 + \log x \le 14$

Q.3 $\log_{1/2}(x+1) > \log_2(2-x)$.

Q.4 $\log_x 2 \,.\, \log_{2x} 2 \,.\, \log_2 4x > 1$.

Q.5 $\log_{1/5}(2x^2 + 5x + 1) < 0$.

Q.6 $\log_{1/2} x + \log_3 x > 1$.

Q.7 $\log_{x^2}(2+x) < 1$

Q.8 $(\log_{|x+6|} 2) \,.\, \log_2(x^2 - x - 2) \ge 1$

Q.9 $\log_x \frac{4x+5}{6-5x} < -1$

Q.10 $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x-5|} \ge 0$

Q.11 Find the maximum possible value of $8 \cdot 27^{\log_6 x} + 27 \cdot 8^{\log_6 x} - x^3$, where $x > 0$

Q.12 Find the set of values of k for which the equation, $3\tan 3x = (3\log^2 K - 4\log K + 2)\tan x$ $(x \ne n\pi, n \in I)$ has a solution. The base of the logarithm is 8.

Q.13 Find out the values of 'a' for which any solution of the inequality, $\frac{\log_3(x^2 - 3x + 7)}{\log_3(3x+2)} < 1$ is also a solution of the inequality, $x^2 + (5 - 2a)x \le 10a$.

Q.14 Solve the inequality $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$.

Q.15 Find the set of values of 'y' for which the inequality, $2\log_{0.5} y^2 - 3 + 2x\log_{0.5} y^2 - x^2 > 0$ is valid for atleast one real value of 'x'.

EXERCISE–IV

Q.1 If α, β are the roots of the equation, $(x - a)(x - b) + c = 0$, find the roots of the equation, $(x - \alpha)(x - \beta) = c$. [REE 2000 (Mains), 3]

Q.2(a) For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:
(A) 1/3 (B) 1 (C) 3 (D) 2/3

(b) If α & β $(\alpha < \beta)$, are the roots of the equation, $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$
(B) $\alpha < 0 < \beta < |\alpha|$
(C) $\alpha < \beta < 0$
(D) $\alpha < 0 < |\alpha| < \beta$

(c) If $b > a$, then the equation, $(x - a)(x - b) - 1 = 0$, has :
(A) both roots in $[a, b]$
(B) both roots in $(-\infty, a)$
(C) both roots in $[b, \infty)$
(D) one root in $(-\infty, a)$ & other in $(b, +\infty)$
[JEE 2000 Screening, 1 + 1 + 1 out of 35]

(d) If α, β are the roots of $ax^2 + bx + c = 0$, $(a \ne 0)$ and $\alpha + \delta$, $\beta + \delta$, are the roots of, $Ax^2 + Bx + C = 0$, $(A \ne 0)$ for some constant δ, then prove that,

$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}.$$

[JEE 2000, Mains, 4 out of 100]

Q.3 Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.

[JEE 2001, Mains, 5 out of 100]

Q.4 The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is

(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(C) $(-\infty, -1) \cup (1, \infty)$ (D) $(\sqrt{2}, \infty)$ [JEE 2002 (screening), 3]

Q.5 If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. [JEE 2003, Mains-4 out of 60]

Q.6(a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(A) $p^3 + q^2 - q(3p + 1) = 0$ (B) $p^3 + q^2 + q(1 + 3p) = 0$

(C) $p^3 + q^2 + q(3p - 1) = 0$ (D) $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then

(A) $-5 < a < 2$ (B) $a < -5$ (C) $a > 5$ (D) $2 < a < 5$

[JEE 2004 (Screening)]

Q.7 Find the range of values of *t* for which $2 \sin t = \dfrac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

[JEE 2005(Mains), 2]

Q.8(a) Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then

(A) $\lambda < \dfrac{4}{3}$ (B) $\lambda > \dfrac{5}{3}$ (C) $\lambda \in \left(\dfrac{1}{3}, \dfrac{5}{3}\right)$ (D) $\lambda \in \left(\dfrac{4}{3}, \dfrac{5}{3}\right)$

[JEE 2006, 3]

(b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then find the value of $a + b + c + d$. (a, b, c and d are distinct numbers) [JEE 2006, 6]

Q.9(a) Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of 'r' is

(A) $\dfrac{2}{9}(p-q)(2q-p)$ (B) $\dfrac{2}{9}(q-p)(2p-q)$ (C) $\dfrac{2}{9}(q-2p)(2q-p)$ (D) $\dfrac{2}{9}(2p-q)(2q-p)$

MATCH THE COLUMN:

(b) Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions / statements in **Column I** with expressions / statements in **Column II**.

	Column I		**Column II**
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(P)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, the $f(x)$ satisfies	(Q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(R)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(S)	$f(x) < 1$

[JEE 2007, 3+6]

ASSERTION & REASON:

Q.10 Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, 1/\beta$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

STATEMENT-1 : $(p^2 - q)(b^2 - ac) \geq 0$

and

STATEMENT-2 : $b \neq pa$ or $c \neq qa$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True [JEE 2008, 3 (–1)]

Q.11 The smallest value of k, for which both the roots of the equation, $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is [JEE 2009, 4 (–1)]

ANSWER KEY

EXERCISE–I

Q.1 $a = 2, b = -11, c = 4, d = -1$ **Q.2** $2x^2 + 2x\cos(A - B) - 2$ **Q.3** 254

Q.5 5 **Q.6** $a \in \left(-\infty, -\frac{1}{2}\right)$ **Q.7** 6 **Q.8** 191 **Q.9** 0

Q.10 0 or 24 **Q.13** **(a)** (ii) and (iv) ; **(b)** $x^2 - p(p^4 - 5p^2q + 5q^2)x + p^2q^2(p^2 - 4q)(p^2 - q) = 0$

Q.14 $\left(-\frac{3}{4}, \frac{3}{2}\right)$ **Q.15** $x^2 - 4x + 1 = 0; \alpha = \tan\left(\frac{\pi}{12}\right); \beta = \tan\left(\frac{5\pi}{12}\right)$ **Q.16** 20 **Q.17** 4

Q.18 $-2 < a < 1$ **Q.19** 277 **Q.20** -6 **Q.21** $m \in \left(0, \frac{1}{4}\right]$ **Q.24** $x = \frac{\sqrt{5}+1}{2}$

Q.25 $y_{min} = 6$

EXERCISE–II

Q.1 $-1, 0, 1$ **Q.2 (a)** $x = 1$; **(b)** $x=2$ or 5; **(c)** $x = -1$ or 1; **(d)** $x \geq -1$ or $x = -3$; **(e)** $x = (1-\sqrt{2})a$ or $(\sqrt{6}-1)a$

Q.3 30 **Q.4** $3y^3 - 9y^2 - 3y + 1 = 0$; $(\alpha - 2)(\beta - 2)(\gamma - 2) = 3$ **Q.5** 146

Q.6 $2(\sqrt{2}-1)$ where $x = \sqrt{2}$ **Q.7** 1338 **Q.8** $a = 1 - \sqrt{2}$ or $5 + \sqrt{10}$ **Q.9** $k = 86$

Q.10 $(-\infty, -2] \cup [0, 1) \cup (2, 4) \cup (5, \infty)$

Q.11 $a \in \left(-\frac{1}{4}, 1\right)$ **Q.12** $2\sqrt{2} \leq a < \frac{11}{3}$ **Q.13** $\left(-\infty, -\frac{1}{4}\right) \cup \{2\} \cup (5, 6]$ **Q.14** $K \leq -1$

Q.15 $P(1) = 4$ **Q.16** $(-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$ **Q.17** $(0, 8]$ **Q.18** $[1, \infty)$

Q.19 (A) Q, R; (B) P, R; (C) S; (D) R **Q.20** (A) P, S ; (B) P, S ; (C) P, S ; (D) P, S

EXERCISE–III

Q.1 $x \in \left(\frac{1}{16}, \frac{1}{8}\right) \cup (8, 16)$ **Q.2** $\frac{1}{\sqrt{10}^{\,9}} \leq x \leq 10$ **Q.3** $-1 < x < \frac{1-\sqrt{5}}{2}$ or $\frac{1+\sqrt{5}}{2} < x < 2$

Q.4 $2^{-\sqrt{2}} < x < 2^{-1}$; $1 < x < 2^{\sqrt{2}}$ **Q.5** $(-\infty, -2.5) \cup (0, \infty)$

Q.6 $0 < x < 3^{1/1-\log 3}$ (where base of log is 2) **Q.7** $-2<x<-1, -1<x<0, 0<x<1, x>2$

Q.8 $x < -7$, $-5 < x \leq -2$, $x \geq 4$ **Q.9** $\frac{1}{2} < x < 1$ **Q.10** $x \leq -\frac{2}{3}$; $\frac{1}{2} \leq x \leq 2$

Q.11 $y_{max} = 216$ **Q.12** $K \in \left(0, \frac{1}{8}\right) \cup (2, 8) \cup (128, \infty)$ **Q.13** $a \geq \frac{5}{2}$

Q.14 $x \in (3, 5-\sqrt{3}) \cup (7, \infty)$ **Q.15** $(-\infty, -2\sqrt{2}) \cup \left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(0, \frac{1}{\sqrt{2}}\right) \cup (2\sqrt{2}, \infty)$

EXERCISE–IV

Q.1 (a, b) **Q.2 (a)** C, **(b)** B, **(c)** D

Q.3 $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ **Q.4** B **Q.5** $a > 1$

Q.6 (a) D ; (b) A **Q.7** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ **Q.8** **(a)** A, **(b)** 1210

Q.9 **(a)** D, **(b)** (A) P, R, S; (B) Q, S; (C) Q, S; (D) P, R, S **Q.10** B Q.11 2