ETOOS
Energy to Succeed!

Question Bank

Application of Derivability

MANOJ CHAUHAN SIR(IIT-DELHI)
EX. SR. FACULTY (BANSAL CLASSES)

[STRAIGHT OBJECTIVE TYPE]

Q.1 Point 'A' lies on the curve $y = e^{-x^2}$ and has the coordinate (x, e^{-x^2}) where $x > 0$. Point B has the coordinates (x, 0). If 'O' is the origin then the maximum area of the triangle AOB is

(A) $\frac{1}{\sqrt{2e}}$ (B) $\frac{1}{\sqrt{4e}}$ (C) $\frac{1}{\sqrt{e}}$ (D) $\frac{1}{\sqrt{8e}}$

Q.2 The angle at which the curve $y = Ke^{Kx}$ intersects the y-axis is :

(A) $\tan^{-1} k^2$ (B) $\cot^{-1}(k^2)$ (C) $\sec^{-1}\left(\sqrt{1+k^4}\right)$ (D) none

Q.3 The angle between the tangent lines to the graph of the function $f(x) = \int_2^x (2t-5)\,dt$ at the points where the graph cuts the x-axis is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q.4 The equation $\sin x + x\cos x = 0$ has at least one root in

(A) $\left(-\frac{\pi}{2}, 0\right)$ (B) $(0, \pi)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(0, \frac{\pi}{2}\right)$

Q.5 The minimum value of the function $f(x) = \frac{\tan\left(x + \frac{\pi}{6}\right)}{\tan x}$ is :

(A) 0 (B) 1/2 (C) 1 (D) 3

Q.6 If $a < b < c < d$ & $x \in R$ then the least value of the function,
$f(x) = |x-a| + |x-b| + |x-c| + |x-d|$ is

(A) $c - d + b - a$ (B) $c + d - b - a$ (C) $c + d - b + a$ (D) $c - d + b + a$

Q.7 If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, b on x and y axis respectively, then the value of a^2b is

(A) $27\,c^3$ (B) $\frac{4}{27}c^3$ (C) $\frac{27}{4}c^3$ (D) $\frac{4}{9}c^3$

Q.8 Let $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 3-x & 5-3x^2 & 3x^3-1 \\ 2x^2-1 & 3x^5-1 & 7x^8-1 \end{vmatrix}$. Then the equation $f(x) = 0$ has

(A) no real root
(B) atmost one real root
(C) atleast 2 real roots
(D) exactly one real root in (0,1) and no other real root.

Q.9 Difference between the greatest and the least values of the function
$f(x) = x(\ln x - 2)$ on $[1, e^2]$ is

(A) 2 (B) e (C) e^2 (D) 1

Q.10 The function $f: [a, \infty) \rightarrow R$ where R denotes the range corresponding to the given domain, with rule $f(x) = 2x^3 - 3x^2 + 6$, will have an inverse provided

(A) $a \geq 1$ (B) $a \geq 0$ (C) $a \leq 0$ (D) $a \leq 1$

Q.11 The graphs $y = 2x^3 - 4x + 2$ and $y = x^3 + 2x - 1$ intersect in exactly 3 distinct points. The slope of the line passing through two of these points

(A) is equal to 4 (B) is equal to 6 (C) is equal to 8 (D) is not unique

Q.12 In which of the following functions Rolle's theorem is applicable?

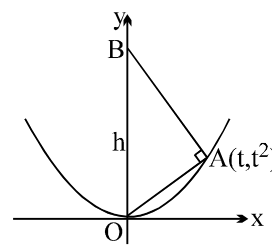
(A) $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & x = 1 \end{cases}$ on $[0, 1]$

(B) $f(x) = \begin{cases} \dfrac{\sin x}{x}, & -\pi \leq x < 0 \\ 0, & x = 0 \end{cases}$ on $[-\pi, 0]$

(C) $f(x) = \dfrac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$

(D) $f(x) = \begin{cases} \dfrac{x^3 - 2x^2 - 5x + 6}{x - 1} & \text{if } x \neq 1, \text{on } [-2,3] \\ -6 & \text{if } x = 1 \end{cases}$

Q.13 The figure shows a right triangle with its hypotenuse OB along the y-axis and its vertex A on the parabola $y = x^2$. Let h represents the length of the hypotenuse which depends on the x-coordinate of the point A. The value of $\lim\limits_{x \to 0} (h)$ equals

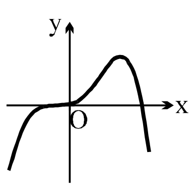


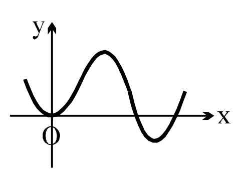
(A) 0 (B) 1/2 (C) 1 (D) 2

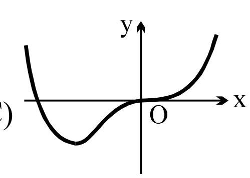
Q.14 Number of positive integral values of 'a' for which the curve $y = a^x$ intersects the line $y = x$ is

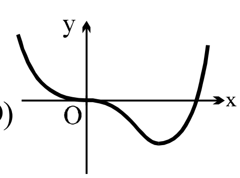
(A) 0 (B) 1 (C) 2 (D) More than 2

Q.15 Which one of the following can best represent the graph of the function $f(x) = 3x^4 - 4x^3$?

(A) 

(B) 

(C) 

(D) 

Q.16 The function $f(x) = \tan^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$ is

(A) increasing in its domain

(B) decreasing in its domain

(C) decreasing in $(-\infty, 0)$ and increasing in $(0, \infty)$

(D) increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$

Q.17 The tangent to the graph of the function $y = f(x)$ at the point with abscissa $x = a$ forms with the x-axis an angle of $\pi/3$ and at the point with abscissa $x = b$ at an angle of $\pi/4$, then the value of the integral, $\int_a^b f'(x) . f''(x)\, dx$ is equal to

(A) 1 (B) 0 (C) $-\sqrt{3}$ (D) -1

[assume $f''(x)$ to be continuous]

Q.18 Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A then K is equal to

(A) 4 (B) 2 (C) -2 (D) $\frac{1}{4}$

Q.19 Which one of the following statements does not hold good for the function

$f(x) = \cos^{-1}(2x^2 - 1)$?

(A) f is not differentiable at $x = 0$ (B) f is monotonic

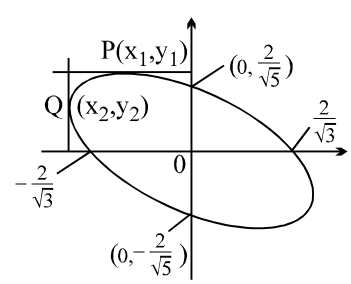
(C) f is even (D) f has an extremum

Q.20 The length of the shortest path that begins at the point (2, 5), touches the x-axis and then ends at a point on the circle

$x^2 + y^2 + 12x - 20y + 120 = 0$, is

(A) 13 (B) $4\sqrt{10}$ (C) 15 (D) $6 + \sqrt{89}$

Q.21 The lines $y = -\frac{3}{2}x$ and $y = -\frac{2}{5}x$ intersect the curve $3x^2 + 4xy + 5y^2 - 4 = 0$ at the points P and Q respectively. The tangents drawn to the curve at P and Q :



(A) intersect each other at angle of 45°

(B) are parallel to each other

(C) are perpendicular to each other

(D) none of these

Q.22 The bottom of the legs of a three legged table are the vertices of an isoceles triangle with sides 5, 5 and 6. The legs are to be braced at the bottom by three wires in the shape of a Y. The minimum length of the wire needed for this purpose, is

(A) $4 + 3\sqrt{3}$ (B) 10 (C) $3 + 4\sqrt{3}$ (D) $1 + 6\sqrt{2}$

Q.23 The least value of 'a' for which the equation,

$\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$ has atleast one solution on the interval $(0, \pi/2)$ is :

(A) 3 (B) 5 (C) 7 (D) 9

Q.24 If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} \text{Min}\ \{f(t) : 0 \le t \le x\} & ;\ 0 \le x \le 1 \\ 3 - x & ;\ 1 < x \le 2 \end{cases}$ then

$g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right)$ has the value equal to :

(A) $\frac{7}{4}$ (B) $\frac{9}{4}$ (C) $\frac{13}{4}$ (D) $\frac{5}{2}$

Q.25 Given: $f(x) = 4 - \left(\frac{1}{2} - x\right)^{2/3}$ $g(x) = \begin{cases} \frac{\tan[x]}{x} & , x \ne 0 \\ 1 & , x = 0 \end{cases}$

$h(x) = \{x\}$ $k(x) = 5^{\log_2(x+3)}$

then in $[0, 1]$, Lagranges Mean Value Theorem is NOT applicable to

(A) f, g, h (B) h, k (C) f, g (D) g, h, k

where [x] and {x} denotes the greatest integer and fraction part function.

Q.26 If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x then the value of b is equal to

(A) – 1 (B) 1 (C) 6 (D) – 6

Q.27 $f(x) = \int \left(2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}\right) dx$ then f is

(A) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$
(B) increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$
(C) increasing in $(-\infty, \infty)$
(D) decreasing in $(-\infty, \infty)$

Q.28 The lower corner of a leaf in a book is folded over so as to just reach the inner edge of the page. The fraction of width folded over if the area of the folded part is minimum is :

(A) 5/8 (B) 2/3 (C) 3/4 (D) 4/5

Q.29 A closed vessel tapers to a point both at its top E and its bottom F and is fixed with EF vertical when the depth of the liquid in it is x cm, the volume of the liquid in it is, $x^2(15 - x)$ cu. cm. The length EF is:

(A) 7.5 cm (B) 8 cm (C) 10 cm (D) 12 cm

Q.30 Coffee is draining from a conical filter, height and diameter both 15 cms into a cylinderical coffee pot diameter 15 cm. The rate at which coffee drains from the filter into the pot is 100 cu cm /min.
The rate in cms/min at which the level in the pot is rising at the instant when the coffee in the pot is 10 cm, is

(A) $\frac{9}{16\pi}$ (B) $\frac{25}{9\pi}$ (C) $\frac{5}{3\pi}$ (D) $\frac{16}{9\pi}$

Q.31 The true set of real values of x for which the function, $f(x) = x\ln x - x + 1$ is positive is
(A) $(1, \infty)$ (B) $(1/e, \infty)$ (C) $[e, \infty)$ (D) $(0, 1) \cup (1, \infty)$

Q.32 A horse runs along a circle with a speed of 20 km/hr. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts . The speed with which the shadow of the horse moves along the fence at the moment when it covers 1/8 of the circle in km/hr is
(A) 20 (B)40
(C) 30 (D) 60

Q.33 Give the correct order of initials **T** or **F** for following statements. Use **T** if statement is true and **F** if it is false.
Statement-1: If $f: R \to R$ and $c \in R$ is such that f is increasing in $(c - \delta, c)$ and f is decreasing in $(c, c + \delta)$ then f has a local maximum at c. Where δ is a sufficiently small positive quantity.
Statement-2 : Let $f: (a, b) \to R$, $c \in (a, b)$. Then f can not have both a local maximum and a point of inflection at $x = c$.
Statement-3 : The function $f(x) = x^2 |x|$ is twice differentiable at $x = 0$.
Statement-4 : Let $f: [c - 1, c + 1] \to [a, b]$ be bijective map such that f is differentiable at c then f^{-1} is also differentiable at $f(c)$.
(A) FFTF (B) TTFT (C) FTTF (D) TTTF

Q.34 A curve is represented by the equations, $x = \sec^2 t$ and $y = \cot t$ where t is a parameter. If the tangent at the point P on the curve where $t = \pi/4$ meets the curve again at the point Q then $|PQ|$ is equal to:
(A) $\frac{5\sqrt{3}}{2}$ (B) $\frac{5\sqrt{5}}{2}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{3\sqrt{5}}{2}$

Q.35 Water runs into an inverted conical tent at the rate of 20 cubic feet per minute and leaks out at the rate of 5 cubic feet per minute. The height of the water is three times the radius of the water's surface. The radius of the water surface is increasing when the radius is 5 feet, is
(A) $\frac{1}{5\pi}$ ft./min (B) $\frac{1}{10\pi}$ ft./min (C) $\frac{1}{15\pi}$ ft./min (D) none

Q.36 The set of values of p for which the equation $|\ln x| - px = 0$ possess three distinct roots is
(A) $\left(0, \frac{1}{e}\right)$ (B) (0, 1) (C) (1,e) (D) (0,e)

Q.37 The lateral edge of a regular rectangular pyramid is 'a' cm long . The lateral edge makes an angle α with the plane of the base. The value of α for which the volume of the pyramid is greatest, is
(A) $\frac{\pi}{4}$ (B) $\sin^{-1}\sqrt{\frac{2}{3}}$ (C) $\cot^{-1}\sqrt{2}$ (D) $\frac{\pi}{3}$

Q.38 In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is l. The altitude of the prism for which the volume is greatest :
(A) $\frac{\ell}{2}$ (B) $\frac{\ell}{\sqrt{3}}$ (C) $\frac{\ell}{3}$ (D) $\frac{\ell}{4}$

Q.39 Let $f(x) = \begin{cases} x^{3/5} & \text{if } x \le 1 \\ -(x-2)^3 & \text{if } x > 1 \end{cases}$
then the number of critical points on the graph of the function is
(A) 1 (B) 2 (C) 3 (D) 4

Q.40 Number of roots of the equation $x^2e^{2-|x|} = 1$ is :
(A) 2 (B) 4 (C) 6 (D) zero

Q.41 The point(s) at each of which the tangents to the curve $y = x^3 - 3x^2 - 7x + 6$ cut off on the positive semi axis OX a line segment half that on the negative semi axis OY then the co-ordinates the point(s) is/are given by :
(A) $(-1, 9)$ (B) $(3, -15)$ (C) $(1, -3)$ (D) none

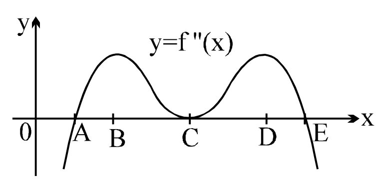
Q.42 A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0, 1) and also touches the x–axis at the point $(-1, 0)$ then the values of x for which the curve has a negative gradient are:
(A) $x > -1$ (B) $x < 1$ (C) $x < -1$ (D) $-1 \le x \le 1$

Q.43 Consider the function
$f(x) = x\cos x - \sin x$, then identify the statement which is correct .
(A) f is neither odd nor even (B) f is monotonic decreasing at $x = 0$
(C) f has a maxima at $x = \pi$ (D) f has a minima at $x = -\pi$

Q.44 Let $f(x) = x^3 - 3x^2 + 2x$. If the equation $f(x) = k$ has exactly one positive and one negative solution then the value of k equals
(A) $-\dfrac{2\sqrt{3}}{9}$ (B) $-\dfrac{2}{9}$ (C) $\dfrac{2}{3\sqrt{3}}$ (D) $\dfrac{1}{3\sqrt{3}}$

Q.45 The x-intercept of the tangent at any arbitrary point of the curve $\dfrac{a}{x^2} + \dfrac{b}{y^2} = 1$ is proportional to:
(A) square of the abscissa of the point of tangency
(B) square root of the abscissa of the point of tangency
(C) cube of the abscissa of the point of tangency
(D) cube root of the abscissa of the point of tangency .

Q.46 The graph of $y = f''(x)$ for a function f is shown. Number of points of inflection for $y = f(x)$ is



(A) 4 (B) 3
(C) 2 (D) 1

Q.47 Let h be a twice continuously differentiable positive function on an open interval J. Let
$g(x) = \ln(h(x))$ for each $x \in J$
Suppose $(h'(x))^2 > h''(x)\,h(x)$ for each $x \in J$. Then
(A) g is increasing on J (B) g is decreasing on J
(C) g is concave up on J (D) g is concave down on J

Q.48 If $f(x)$ is continuous and differentiable over [–2, 5] and $-4 \le f'(x) \le 3$ for all x in (–2, 5) then the greatest possible value of $f(5) - f(-2)$ is
(A) 7 (B) 9 (C) 15 (D) 21

Q.49 Let $f(x)$ and $g(x)$ be two continuous functions defined from $R \longrightarrow R$, such that $f(x_1) > f(x_2)$ and $g(x_1) < g(x_2)$, $\forall\ x_1 > x_2$, then solution set of $f\left(g(\alpha^2 - 2\alpha)\right) > f\left(g(3\alpha - 4)\right)$ is
(A) R (B) ϕ (C) (1, 4) (D) R – [1, 4]

Q.50 A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$ when $t \in R$ and $a > 0$. If the curve touches the axis of x at the point A, then the coordinates of the point A are
(A) (1, 0) (B) (1/e, 0) (C) (e, 0) (D) (2e, 0)

Q.51 Let $f(x) = x - \frac{1}{x}$ then which one of the following statement is true
(A) Function is invertible if defined from $R - \{0\} \to R$.
(B) $f(x_1) > f(x_2),\ \forall\ x_1 > x_2$ and $x_1, x_2 \neq 0$.
(C) Graph of the function has exactly one asymptote.
(D) Function is one-one in every continuous interval [a, b] defined on one side of origin.

Q.52 If $f(x) = \int\limits_{x}^{x^2} (t-1)\,dt,\ 1 \le x \le 2$, then global maximum value of f(x) is
(A) 1 (B) 2 (C) 4 (D) 5

Q.53 A right triangle is drawn in a semicircle of radius $\frac{1}{2}$ with one of its legs along the diameter. The maximum area of the triangle is
(A) $\frac{1}{4}$ (B) $\frac{3\sqrt{3}}{32}$ (C) $\frac{3\sqrt{3}}{16}$ (D) $\frac{1}{8}$

Q.54 At any two points of the curve represented parametrically by $x = a(2\cos t - \cos 2t)$; $y = a(2\sin t - \sin 2t)$ the tangents are parallel to the axis of x corresponding to the values of the parameter t differing from each other by :
(A) $2\pi/3$ (B) $3\pi/4$ (C) $\pi/2$ (D) $\pi/3$

Q.55 If the function $f(x) = \frac{t + 3x - x^2}{x - 4}$, where 't' is a parameter has a minimum and a maximum then the range of values of 't' is
(A) (0, 4) (B) $(0, \infty)$ (C) $(-\infty, 4)$ (D) $(4, \infty)$

Q.56 Let $F(x) = \int\limits_{\sin x}^{\cos x} e^{(1+\arcsin t)^2}\,dt$ on $\left[0, \frac{\pi}{2}\right]$ then
(A) $F''(c) = 0$ for all $c \in \left(0, \frac{\pi}{2}\right)$ (B) $F''(c) = 0$ for some $c \in \left(0, \frac{\pi}{2}\right)$
(C) $F'(c) = 0$ for some $c \in \left(0, \frac{\pi}{2}\right)$ (D) $F(c) \neq 0$ for all $c \in \left(0, \frac{\pi}{2}\right)$

Q.57 The least area of a circle circumscribing any right triangle of area S is :
(A) πS (B) $2\pi S$ (C) $\sqrt{2}\,\pi S$ (D) $4\pi S$

Q.58 Given $f'(1) = 1$ and $\frac{d}{dx}(f(2x)) = f'(x)\ \forall x > 0$. If $f'(x)$ is differentiable then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals
(A) – 1/4 (B) – 1/8 (C) 1/4 (D) 1/8

Q.59 A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa changes at slower rate than ordinate, is

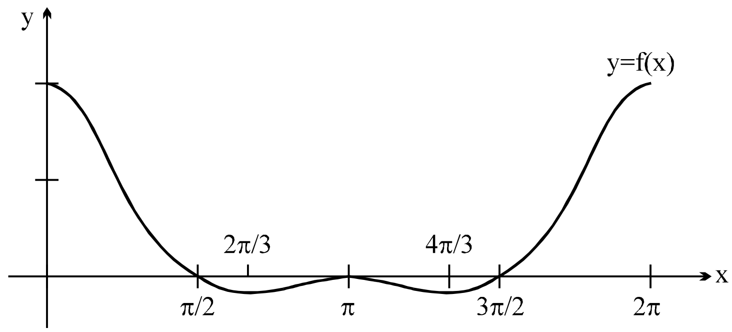
(A) $(-3, 3)$ (B) $(-\infty, \infty)$ (C) $(-1, 1)$ (D) $(-\infty, -3) \cup (3, \infty)$

Q.60 Let f(x) and g(x) are two function which are defined and differentiable for all $x \ge x_0$. If $f(x_0) = g(x_0)$ and $f'(x) > g'(x)$ for all $x > x_0$ then

(A) $f(x) < g(x)$ for some $x > x_0$ (B) $f(x) = g(x)$ for some $x > x_0$

(C) $f(x) > g(x)$ only for some $x > x_0$ (D) $f(x) > g(x)$ for all $x > x_0$

Q.61 The graph of $y = f(x)$ is shown. Let F(x) be an antiderivative of f(x). Then F(x) has



(A) points of inflexion at $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ and 2π, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

(B) points of inflexion at $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ and 2π, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

(C) point of inflexion at $x = \pi$, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

(D) point of inflexion at $x = \pi$, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

Q.62 P and Q are two points on a circle of centre C and radius α, the angle PCQ being 2θ then the radius of the circle inscribed in the triangle CPQ is maximum when

(A) $\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}$ (B) $\sin\theta = \frac{\sqrt{5}-1}{2}$ (C) $\sin\theta = \frac{\sqrt{5}+1}{2}$ (D) $\sin\theta = \frac{\sqrt{5}-1}{4}$

Q.63 Number of critical points of the function,

$$f(x) = \frac{2}{3}\sqrt{x^3} - \frac{x}{2} + \int_1^x \left(\frac{1}{2} + \frac{1}{2}\cos 2t - \sqrt{t}\right) dt$$

which lie in the interval $[-2\pi, 2\pi]$ is :

(A) 2 (B) 4 (C) 6 (D) 8

Q.64 Suppose that water is emptied from a spherical tank of radius 10 cm. If the depth of the water in the tank is 4 cm and is decreasing at the rate of 2 cm/sec, then the radius of the top surface of water is decreasing at the rate of

(A) 1 (B) 2/3 (C) 3/2 (D) 2

Q.65 The range of values of m for which the line $y = mx$ and the curve $y = \frac{x}{x^2+1}$ enclose a region, is

(A) $(-1, 1)$ (B) $(0, 1)$ (C) $[0, 1]$ (D) $(1, \infty)$

Q.66 For a steamer the consumption of petrol (per hour) varies as the cube of its speed (in km). If the speed of the current is steady at C km/hr then the most economical speed of the steamer going against the current will be

(A) 1.25 C (B) 1.5 C (C) 1.75C (D) 2 C

Q.67 Let f and g be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f[g(x)]$. If $h(0) = 0$, then $h(x) - h(1)$ is :

(A) always zero (B) strictly increasing
(C) always negative (D) always positive

Q.68 A function $y = f(x)$ is given by $x = \frac{1}{1+t^2}$ & $y = \frac{1}{t\left(1+t^2\right)}$ for all $t > 0$ then f is :

(A) increasing in $(0, 3/2)$ & decreasing in $(3/2, \infty)$
(B) increasing in $(0, 1)$
(C) increasing in $(0, \infty)$
(D) decreasing in $(0, 1)$

Q.69 If the function $f(x) = 2x^2 - kx + 5$ is increasing in $[1, 2]$, then 'k' lies in the interval

(A) $(-\infty, 4)$ (B) $(4, \infty)$ (C) $(-\infty, 8]$ (D) $(8, \infty)$

Q.70 The set of all values of 'a' for which the function,

$f(x) = (a^2 - 3a + 2)\left(\cos^2\frac{x}{4} - \sin^2\frac{x}{4}\right) + (a-1)x + \sin 1$ does not possess critical points is:

(A) $[1, \infty)$ (B) $(0, 1) \cup (1, 4)$ (C) $(-2, 4)$ (D) $(1, 3) \cup (3, 5)$

Q.71 The value of n for which the area of the triangle included between the co-ordinate axes and any tangent to the curve $xy^n = a^{n+1}$ is constant is

(A) -1 (B) 0 (C) 1 (D) a

Q.72 Read the following mathematical statements carefully:

I. A differentiable function 'f' with maximum at $x = c \Rightarrow f''(c) < 0$.

II. Antiderivative of a periodic function is also a periodic function.

III. If f has a period T then for any $a \in R$. $\int_0^T f(x)\,dx = \int_0^T f(x+a)\,dx$

IV. If $f(x)$ has a maxima at $x = c$, then 'f' is increasing in $(c-h, c)$ and decreasing in $(c, c+h)$ as $h \to 0$ for $h > 0$.

Now indicate the correct alternative.

(A) exactly one statement is correct. (B) exactly two statements are correct.
(C) exactly three statements are correct. (D) All the four statements are correct.

Q.73 Let a function f be defined as $f(x) = \begin{cases} \dfrac{|x-1|}{x^2+1} & \text{if} \quad x > -1 \\ x^2 & \text{if} \quad x \le -1 \end{cases}$

Then the number of critical point(s) on the graph of this function is/are :

(A) 4 (B) 3 (C) 2 (D) 1

Q.74 Two sides of a triangle are to have lengths 'a' cm & 'b' cm. If the triangle is to have the maximum area, then the length of the median from the vertex containing the sides 'a' and 'b' is

(A) $\frac{1}{2}\sqrt{a^2+b^2}$ (B) $\frac{2a+b}{3}$ (C) $\sqrt{\frac{a^2+b^2}{2}}$ (D) $\frac{a+2b}{3}$

Q.75 Let S be a square with sides of length x. If we approximate the change in size of the area of S by $h \cdot \left.\frac{dA}{dx}\right|_{x=x_0}$, when the sides are changed from x_0 to $x_0 + h$, then the absolute value of the error in our approximation, is

(A) h^2 (B) $2hx_0$ (C) x_0^2 (D) h

Q.76 Number of critical points on the graph of the function $f(x) = x^{\frac{1}{3}}(x-4)$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.77 A rectangle has one side on the positive y-axis and one side on the positive x - axis. The upper right hand vertex of the rectangle lies on the curve $y = \frac{\ell n x}{x^2}$. The maximum area of the rectangle is

(A) e^{-1} (B) $e^{-½}$ (C) 1 (D) $e^{½}$

Q.78 All roots of the cubic $x^3 + ax + b = 0$ $(a, b, \in R)$ are real and distinct, and $b > 0$ then

(A) $a = 0$ (B) $a < 0$ (C) $a > 0$ (D) $a \ge b$

Q.79 Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at $x = 0$, $f(x)$ has

(A) a maxima whenever $a > 0, b > 0$

(B) a maxima whenever $a > 0, b < 0$

(C) minima whenever $a > 0, b > 0$

(D) neither a maxima nor minima whenever $a > 0, b < 0$

Q.80 Consider $f(x) = |1-x| \quad 1 \le x \le 2$ and

$g(x) = f(x) + b \sin \frac{\pi}{2} x, \quad 1 \le x \le 2$

then which of the following is correct?

(A) Rolles theorem is applicable to both f, g and $b = \frac{3}{2}$

(B) LMVT is not applicable to f and Rolles theorem if applicable to g with $b = \frac{1}{2}$

(C) LMVT is applicable to f and Rolles theorem is applicable to g with $b = 1$

(D) Rolles theorem is not applicable to both f, g for any real b.

Q.81 Consider $f(x)=\int_1^x\left(t+\frac{1}{t}\right)dt$ and $g(x)=f'(x)$ for $x\in\left[\frac{1}{2},3\right]$

If P is a point on the curve $y=g(x)$ such that the tangent to this curve at P is parallel to a chord joining the points $\left(\frac{1}{2},\ g\left(\frac{1}{2}\right)\right)$ and $(3,\ g(3))$ of the curve, then the coordinates of the point P

(A) can't be found out (B) $\left(\frac{7}{4},\frac{65}{28}\right)$ (C) (1, 2) (D) $\left(\sqrt{\frac{3}{2}},\frac{5}{\sqrt{6}}\right)$

Q.82 The angle made by the tangent of the curve $x=a(t+\sin t\cos t)$; $y=a(1+\sin t)^2$ with the x-axis at any point on it is

(A) $\frac{1}{4}(\pi+2t)$ (B) $\frac{1-\sin t}{\cos t}$ (C) $\frac{1}{4}(2t-\pi)$ (D) $\frac{1+\sin t}{\cos 2t}$

Q.83 If the function $f(x)=2x^2+3x+5$ satisfies LMVT at $x=2$ on the closed interval [1, a] then the value of 'a' is equal to

(A) 3 (B) 4 (C) 6 (D) 1

Q.84 Given that $f(x)$ is continuously differentiable on $a\le x\le b$ where $a<b$, $f(a)<0$ and $f(b)>0$, which of the following are always true?

(i) $f(x)$ is bounded on $a\le x\le b$.

(ii) The equation $f(x)=0$ has at least one solution in $a<x<b$.

(iii) The maximum and minimum values of $f(x)$ on $a\le x\le b$ occur at points where $f'(c)=0$.

(iv) There is at least one point c with $a<c<b$ where $f'(c)>0$.

(v) There is at least one point d with $a<d<b$ where $f'(c)<0$.

(A) only (ii) and (iv) are true (B) all but (iii) are true

(C) all but (v) are true (D) only (i), (ii) and (iv) are true

Q.85 Consider the curve represented parametrically by the equation

$x=t^3-4t^2-3t$ and

$y=2t^2+3t-5$ where $t\in R$.

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then

(A) H = 2 and V = 1 (B) H = 1 and V = 2 (C) H = 2 and V = 2 (D) H = 1 and V = 1

Q.86 At the point $P(a, a^n)$ on the graph of $y=x^n$ $(n\in N)$ in the first quadrant a normal is drawn. The normal intersects the y-axis at the point (0, b). If $\lim_{a\to 0} b=\frac{1}{2}$, then n equals

(A) 1 (B) 3 (C) 2 (D) 4

Q.87 P is a point on positive x-axis, Q is a point on the positive y-axis and 'O' is the origin. If the line passing through P and Q is tangent to the curve $y=3-x^2$ then the minimum area of the triangle OPQ, is

(A) 2 (B) 4 (C) 8 (D) 9

Q.88 Suppose that f is differentiable for all x and that $f'(x) \le 2$ for all x. If $f(1) = 2$ and $f(4) = 8$ then $f(2)$ has the value equal to
(A) 3 (B) 4 (C) 6 (D) 8

Q.89 There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per additional tree drops by 10 apples. Number of trees that should be added to the existing orchard for maximising the output of the trees, is
(A) 5 (B) 10 (C) 15 (D) 20

Q.90 Range of the function $f(x) = \dfrac{\ell nx}{\sqrt{x}}$ is

(A) $(-\infty, e)$ (B) $(-\infty, e^2)$ (C) $\left(-\infty, \dfrac{2}{e}\right]$ (D) $\left(-\infty, \dfrac{1}{e}\right)$

Q.91 The curve $y = x^3 + x^2 - x$ has two horizontal tangents. The distance between these two horizontal lines, is

(A) $\dfrac{13}{9}$ (B) $\dfrac{11}{9}$ (C) $\dfrac{22}{27}$ (D) $\dfrac{32}{27}$

[REASONING TYPE]

Q.92 $f : R \to R$
Statement-1: $f(x) = 12x^5 - 15x^4 + 20x^3 - 30x^2 + 60x + 1$ is monotonic and surjective on R.
because
Statement-2: A continuous function defined on R, if strictly monotonic has its range R.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.93 Consider function $f(x) = \begin{cases} x\{x\}+1, & 0 \le x < 1 \\ 2-\{x\}, & 1 \le x \le 2 \end{cases}$ where $\{ x \}$: fractional part function of x.

Statement-1: Rolles Theorem is not applicable to $f(x)$ in $[0, 2]$
because
Statement-2: $f(0) \ne f(2)$
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.94 Let $f(x) = ln(2 + x) - \dfrac{2x+2}{x+3}$.
Statement-1: The equation $f(x) = 0$ has a unique solution in the domain of $f(x)$.
because
Statement-2: If $f(x)$ is continuous in $[a, b]$ and is strictly monotonic in (a, b) then f has a unique root in (a, b)
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.95 Consider the polynomial function $f(x) = \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x$

Statement-1: The equation $f(x) = 0$ can not have two or more roots.

because

Statement-2: Rolles theorem is not applicable for $y = f(x)$ on any interval $[a, b]$ where $a, b \in R$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.96 Consider the functions $f(x) = x - \sin x$ and $g(x) = x^4$.

Statement-1 : $f(x)$ is increasing for $x > 0$ as well as $x < 0$ and $g(x)$ is increasing for $x > 0$ and decreasing for $x < 0$.

because

Statement-2 : If an odd function is known to be increasing on the interval $x > 0$ then it is increasing for $x < 0$ also and if an even function is increasing for $x > 0$ then it is decreasing for $x < 0$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.97 Consider the function $f(x) = \begin{cases} x\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ in $[-1, 1]$

Statement-1: LMVT is not applicable to $f(x)$ in the indicated interval.

because

Statement-2: As $f(x)$ is neither continuous nor differentiable in $[-1, 1]$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.98 A function $y = f(x)$ is defined on $[0, 4]$ as

$$f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ (x-2)^2 & \text{if } 1 < x \le 3 \\ 1 & \text{if } 3 < x \le 4 \end{cases}$$

For the function $y = f(x)$

Statement-1: All the three conditions of Rolles Theorem are violated on $[0, 4]$ but still $f'(x)$ vanishes at a point in $(0, 4)$.

because

Statement-2: The conditions for Rolles Theorem are sufficient but not necessary.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.99 Consider the graph of the function $f(x) = x + \sqrt{|x|}$

Statement–1: The graph of $y = f(x)$ has only one critical point

because

Statement-2: $f'(x)$ vanishes only at one point

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.100 Consider the following statements

Statement-1: The function $f(x) = \ln x$ is increasing in (0, 10) and $g(x) = \frac{1}{x}$ is decreasing in (0,10)

because

Statement-2: If a differentiable function increases in the interval (a, b) then its derivative function decreases in the interval (a, b).

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

[COMPREHENSION TYPE]

Paragraph for question nos. 101 to 103

Let $f(x) = \left(1 + \frac{1}{x}\right)^x$ $(x > 0)$ and $g(x) = \begin{cases} x \ln\left(1 + (1/x)\right) & \text{if } 0 < x \le 1 \\ 0 & \text{if } x = 0 \end{cases}$

Q.101 $\lim_{x \to 0^+} g(x)$

(A) is equal to 0
(B) is equal to 1
(C) is equal to e
(D) is non existent

Q.102 The function f

(A) has a maxima but no minima
(B) has a minima but no maxima
(C) has both a maxima and minima
(D) is monotonic

Q.103 $\lim_{n \to \infty} \left\{ f\left(\frac{1}{n}\right) \cdot f\left(\frac{2}{n}\right) \cdot f\left(\frac{3}{n}\right) \ldots\ldots\ldots f\left(\frac{n}{n}\right) \right\}^{1/n}$ equals

(A) $\sqrt{2}\,e$
(B) $\sqrt{2e}$
(C) $2\sqrt{e}$
(D) $\sqrt{e}$

Paragraph for question nos. 104 to 106

Consider the function $f(x) = \dfrac{x^2}{x^2 - 1}$

Q.104 The interval in which f is increasing is

(A) $(-1, 1)$ (B) $(-\infty, -1) \cup (-1, 0)$

(C) $(-\infty, \infty) - \{-1, 1\}$ (C) $(0, 1) \cup (1, \infty)$

Q.105 If f is defined from $R - \{-1, 1\} \rightarrow R$ then f is

(A) injective but not surjective (B) surjective but not injective

(C) injective as well as surjective (D) neither injective nor surjective.

Q.106 f has

(A) local maxima but no local minima (B) local minima but no local maxima

(C) both local maxima and local minima (D) neither local maxima nor local minima.

Paragraph for question nos. 107 to 109

Consider the cubic $f(x) = 8x^3 + 4ax^2 + 2bx + a$ where $a, b \in R$.

Q.107 For a = 1 if $y = f(x)$ is strictly increasing $\forall x \in R$ then maximum range of values of b is

(A) $\left(-\infty, \frac{1}{3}\right]$ (B) $\left(\frac{1}{3}, \infty\right)$ (C) $\left[\frac{1}{3}, \infty\right)$ (D) $(-\infty, \infty)$

Q.108 For b = 1, if $y = f(x)$ is non monotonic then the sum of all the integral values of $a \in [1, 100]$, is

(A) 4950 (B) 5049 (C) 5050 (D) 5047

Q.109 If the sum of the base 2 logarithms of the roots of the cubic $f(x) = 0$ is 5 then the value of 'a' is

(A) – 64 (B) – 8 (C) – 128 (D) – 256

Paragraph for question nos. 110 to 112

Suppose you do not know the function f (x), however some information about $f(x)$ is listed below. Read the following carefully before attempting the questions

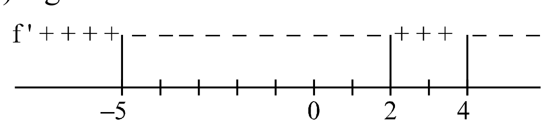
(i) $f(x)$ is continuous and defined for all real numbers

(ii) $f'(-5) = 0$; $f'(2)$ is not defined and $f'(4) = 0$

(iii) (–5, 12) is a point which lies on the graph of f (x)

(iv) $f''(2)$ is undefined, but $f''(x)$ is negative everywhere else.

(v) the signs of $f'(x)$ is given below



Q.110 On the possible graph of $y = f(x)$ we have

(A) x = – 5 is a point of relative minima.

(B) x = 2 is a point of relative maxima.

(C) x = 4 is a point of relative minima.

(D) graph of $y = f(x)$ must have a geometrical sharp corner.

Q.111 From the possible graph of $y = f(x)$, we can say that

(A) There is exactly one point of inflection on the curve.

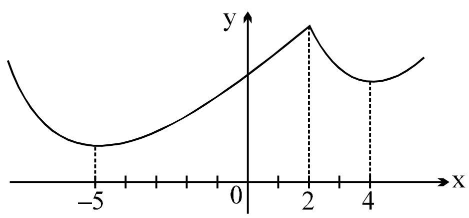
(B) $f(x)$ increases on $-5 < x < 2$ and $x > 4$ and decreases on $-\infty < x < -5$ and $2 < x < 4$.

(C) The curve is always concave down.

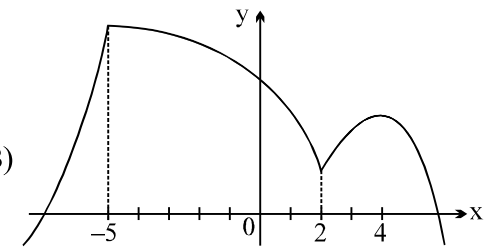
(D) Curve always concave up.

Q.112 Possible graph of $y = f(x)$ is

(A)



(B)



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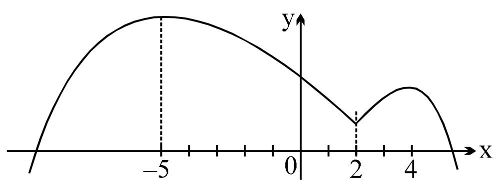
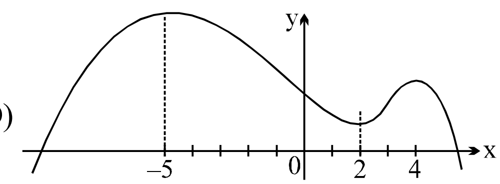


(D)



[MULTIPLE OBJECTIVE TYPE]

Q.113 The function $f(x) = x^{1/3}(x-1)$

(A) has 2 inflection points.

(B) is strictly increasing for $x > 1/4$ and strictly decreasing for $x < 1/4$.

(C) is concave down in $(-1/2,\ 0)$.

(D) Area enclosed by the curve lying in the fourth quadrant is $9/28$.

Q.114 If $\frac{x}{a} + \frac{y}{b} = 1$ is a tangent to the curve $x = Kt,\ y = \frac{K}{t},\ K > 0$ then :

(A) $a > 0,\ b > 0$ (B) $a > 0,\ b < 0$ (C) $a < 0,\ b > 0$ (D) $a < 0,\ b < 0$

Q.115 The function $f(x) = \int_0^x \sqrt{1-t^4}\ dt$ is such that :

(A) it is defined on the interval $[-1, 1]$ (B) it is an increasing function

(C) it is an odd function (D) the point $(0, 0)$ is the point of inflection

Q.116 The co-ordinates of the point(s) on the graph of the function, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$ where the tangent drawn cut off intercepts from the co-ordinate axes which are equal in magnitude but opposite in sign, is

(A) $(2, 8/3)$ (B) $(3, 7/2)$ (C) $(1, 5/6)$ (D) none

Q.117 If $f(x) = a^{\left\{a^{|x|}\operatorname{sgn} x\right\}}$; $g(x) = a^{\left[a^{|x|}\operatorname{sgn} x\right]}$ for $a > 0,\ a \neq 1$ and $x \in R$, where { } & [ ] denote the fractional part and integral part functions respectively, then which of the following statements can hold good for the function $h(x)$, where

$(\ln a)\, h(x) = (\ln f(x) + \ln g(x))$.

(A) 'h' is even and increasing (B) 'h' is odd and decreasing

(C) 'h' is even and decreasing (D) 'h' is odd and increasing.

Q.118 On which of the following intervals, the function $x^{100} + \sin x - 1$ is strictly increasing.
(A) $(-1, 1)$ (B) $(0, 1)$ (C) $(\pi/2, \pi)$ (D) $(0, \pi/2)$

Q.119 If $f(x) = \begin{cases} 3x^2 + 12x - 1 & , \ -1 \le x \le 2 \\ 37 - x & , \ 2 < x \le 3 \end{cases}$ then :
(A) $f(x)$ is increasing on $[-1, 2]$
(B) $f(x)$ is continuous on $[-1, 3]$
(C) $f'(2)$ does not exist
(D) $f(x)$ has the maximum value at $x = 2$.

Q.120 Consider the function $f(x) = x^2 - x \sin x - \cos x$ then the statements which holds good, are
(A) $f(x) = k$ has no solution for $k < -1$.
(B) f is increasing for $x < 0$ and decreasing for $x > 0$.
(C) $\lim_{x \to \pm\infty} f(x) \to \infty$
(D) The zeros of $f(x) = 0$ lie on the same side of the origin.

Q.121 Assume that inverse of the function f is denoted by g. Then which of the following statement hold good?
(A) If f is increasing then g is also increasing.
(B) If f is decreasing then g is increasing.
(C) The function f is injective.
(D) The function g is onto.

Q.122 For the function $f(x) = \ln(1 - \ln x)$ which of the following do not hold good?
(A) increasing in (0, 1) and decreasing in (1, e)
(B) decreasing in (0, 1) and increasing in (1, e)
(C) $x = 1$ is the critical number for $f(x)$.
(D) f has two asymptotes

Q.123 The function $f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ (x-2)^2 & \text{if } x \ge 1 \end{cases}$
(A) is continuous for all $x \in R$
(B) is continuous but not differentiable $\forall\, x \in R$
(C) is such that $f'(x)$ change its sign exactly twice
(D) has two local maxima and two local minima.

Q.124 Consider the function $f(x) = \left[\cos\left(\tan^{-1}\left(\sin(\cot^{-1} x)\right)\right)\right]^2$. Which of the following is correct?
(A) range of f is (0, 1)
(B) f is even
(C) $f'(0) = 0$
(D) the line $y = 1$ is asymptotes to the graph $y = f(x)$

Q.125 Equation of a line which is tangent to both the curves $y = x^2 + 1$ and $y = -x^2$ is
(A) $y = \sqrt{2}x + \frac{1}{2}$
(B) $y = \sqrt{2}x - \frac{1}{2}$
(C) $y = -\sqrt{2}x + \frac{1}{2}$
(D) $y = -\sqrt{2}x - \frac{1}{2}$

Q.126 A function f is defined by $f(x) = \int_0^{\pi} \cos t \cos(x - t)dt$, $0 \le x \le 2\pi$ then which of the following hold(s) good?

(A) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$

(B) Maximum value of f is π

(C) There exists atleast one $c \in (0, 2\pi)$ s.t. $f'(c) = 0$.

(D) Minimum value of f is $-\frac{\pi}{2}$.

Q.127 Which of the following inequalities always hold good in (0, 1)

(A) $x > \tan^{-1}x$

(B) $\cos x < 1 - \frac{x^2}{2}$

(C) $1 + x\, ln\left(x + \sqrt{1+x^2}\right) > \sqrt{1+x^2}$

(D) $x - \frac{x^2}{2} < ln(1 + x)$

Q.128 Let $f(x) = \frac{x-1}{x^2}$ then which of the following is correct.

(A) $f(x)$ has minima but no maxima.

(B) $f(x)$ increases in the interval $(0, 2)$ and decreases in the interval $(-\infty, 0) \cup (2, \infty)$.

(C) $f(x)$ is concave down in $(-\infty, 0) \cup (0, 3)$.

(D) $x = 3$ is the point of inflection.

Q.129 $f''(x) > 0$ for all $x \in [-3, 4]$, then which of the following are always true?

(A) $f(x)$ has a relative minimum on $(-3, 4)$

(B) $f(x)$ has a minimum on $[-3, 4]$

(C) $f(x)$ is concave upwards on $[-3, 4]$

(D) if $f(3) = f(4)$ then $f(x)$ has a critical point on $[-3, 4]$

Q.130 Which of the following statements is/are **TRUE**?

(A) If f has domain $[0, \infty)$ and has no horizontal asymptotes then $\lim\limits_{x\to\infty} f(x) = \infty$ or $\lim\limits_{x\to\infty} f(x) = -\infty$.

(B) If f is continuous on $[-1, 1]$ and $f(-1) = 4$ and $f(1) = 3$ then there exist a number r such that $|r| < 1$ and $f(r) = \pi$.

(C) $\lim\limits_{x\to\infty} \arcsin\left(\frac{x+1}{x}\right)$ does not exist.

(D) For all values of $m \in R$ the line $y - mx + m - 1 = 0$ cuts the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ orthogonally.

ANSWER KEY
[STRAIGHT OBJECTIVE TYPE]

Q.1	D	Q.2	B	Q.3	D	Q.4	B	Q.5	D	Q.6	B	Q.7	C
Q.8	C	Q.9	B	Q.10	A	Q.11	C	Q.12	D	Q.13	C	Q.14	B
Q.15	D	Q.16	D	Q.17	D	Q.18	A	Q.19	B	Q.20	A	Q.21	C
Q.22	A	Q.23	D	Q.24	D	Q.25	A	Q.26	D	Q.27	C	Q.28	B
Q.29	C	Q.30	D	Q.31	D	Q.32	B	Q.33	C	Q.34	D	Q.35	A
Q.36	A	Q.37	C	Q.38	B	Q.39	C	Q.40	B	Q.41	B	Q.42	C
Q.43	B	Q.44	A	Q.45	C	Q.46	C	Q.47	D	Q.48	D	Q.49	C
Q.50	D	Q.51	D	Q.52	C	Q.53	B	Q.54	A	Q.55	C	Q.56	B
Q.57	A	Q.58	B	Q.59	C	Q.60	D	Q.61	C	Q.62	B	Q.63	B
Q.64	C	Q.65	B	Q.66	B	Q.67	A	Q.68	B	Q.69	A	Q.70	B
Q.71	C	Q.72	A	Q.73	A	Q.74	A	Q.75	A	Q.76	C	Q.77	A
Q.78	B	Q.79	A	Q.80	C	Q.81	D	Q.82	A	Q.83	A	Q.84	D
Q.85	B	Q.86	C	Q.87	B	Q.88	B	Q.89	C	Q.90	C	Q.91	D

[REASONING TYPE]

Q.92	C	Q.93	A	Q.94	C	Q.95	A	Q.96	A	Q.97	C	Q.98	A
Q.99	D	Q.100	C										

[COMPREHENSION TYPE]

Q.101	A	Q.102	D	Q.103	D	Q.104	B	Q.105	D	Q.106	A	Q.107	C
Q.108	B	Q.109	D	Q.110	D	Q.111	C	Q.112	C				

[MULTIPLE OBJECTIVE TYPE]

Q.113	A, B, c, D	Q.114	A, D	Q.115	A, B, C, D	Q.116	A, B
Q.117	B, D	Q.118	B, C, D	Q.119	B, C, D	Q.120	A, C
Q.121	A, C, D	Q.122	A, B, C	Q.123	A, B, D	Q.124	B, C, D
Q.125	A, C	Q.126	C, D	Q.127	A, C, D	Q.128	B, C, D
Q.129	B, C, D	Q.130	B, C, D				