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QUESTION BANK

DEFINITE INTEGRATION

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[STRAIGHT OBJECTIVE TYPE]

Q.1 The value of the definite integral, $\int_{0}^{\sqrt{ln\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right)\cdot 2x\, e^{x^2} dx$ is

(A) 1 (B) $1 + (\sin 1)$ (C) $1 - (\sin 1)$ (D) $(\sin 1) - 1$

Q.2 The value of the definite integral $\int_{0}^{\pi/2} \sin\left|\, 2x - \alpha \,\right| dx$ where $\alpha \in [0, \pi]$

(A) 1 (B) $\cos\alpha$ (C) $\frac{1+\cos\alpha}{2}$ (D) $\frac{1-\cos\alpha}{2}$

Q.3 Value of the definite integral $\int_{-1/2}^{1/2} \left(\sin^{-1}(3x - 4x^3) - \cos^{-1}(4x^3 - 3x) \right) dx$

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{7\pi}{2}$ (D) $\frac{\pi}{2}$

Q.4 Let $f(x) = \int_{2}^{x} \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of g'(0) is

(A) 1 (B) 17 (C) $\sqrt{17}$ (D) none of these

Q.5 If a, b and c are real numbers then the value of $\lim_{t\to 0} ln\left(\frac{1}{t}\int_{0}^{t}(1 + a\sin bx)^{c/x}\, dx\right)$ equals

(A) abc (B) $\frac{ab}{c}$ (C) $\frac{bc}{a}$ (D) $\frac{ca}{b}$

Q.6 The value of the definite integral $\int_{0}^{\infty} \frac{dx}{(1+x^a)(1+x^2)}$ $(a > 0)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) some function of a.

Q.7 Let $a_n = \int_{0}^{\pi/2} (1 - \sin t)^n \sin 2t\, dt$ then $\lim_{n\to\infty} \sum_{n=1}^{n} \frac{a_n}{n}$ is equal to

(A) 1/2 (B) 1 (C) 4/3 (D) 3/2

Q.8 The value of the definite integral $\int_{0}^{3\pi/4} \left((1+x)\sin x + (1-x)\cos x\right) dx$, is

(A) $2\tan\frac{3\pi}{8}$ (B) $2\tan\frac{\pi}{4}$ (C) $2\tan\frac{\pi}{8}$ (D) 0

Q.9 Let $C_n = \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{\tan^{-1}(nx)}{\sin^{-1}(nx)} dx$ then $\lim_{n\to\infty} n^2 \cdot C_n$ equals

(A) 1 (B) 0 (C) – 1 (D) $\frac{1}{2}$

Q.10 If x satisfies the equation $\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right)x^2 - \left(\int_{-3}^{3} \frac{t^2\sin 2t}{t^2+1}dt\right)x - 2 = 0$ $(0 < \alpha < \pi)$, then the value x is

(A) $\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ (B) $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ (C) $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ (D) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

Q.11 If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t\,dt}{1+t^4}$ then $f'(2)$

(A) equals 2/17 (B) equals 0 (C) equals 1 (D) cannot be determined

Q.12 A function $f(x)$ satisfies $f(x) = \sin x + \int_0^x f'(t)(2\sin t - \sin^2 t)\,dt$ then $f(x)$ is

(A) $\frac{x}{1-\sin x}$ (B) $\frac{\sin x}{1-\sin x}$ (C) $\frac{1-\cos x}{\cos x}$ (D) $\frac{\tan x}{1-\sin x}$

Q.13 Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n$ $(n \in N)$ satisfies the equation $\int_{-1}^{1}(px+q)\,g_n(x)\,dx = 0$ for all linear functions $(px+q)$ then

(A) $a_n = b_n = 0$ (B) $b_n = 0;\ a_n = -\frac{3}{2n+3}$

(C) $a_n = 0;\ b_n = -\frac{3}{2n+3}$ (D) $a_n = \frac{3}{2n+3};\ b_n = -\frac{3}{2n+3}$

Q.14 The value of $\lim_{n\to\infty}\sum_{r=1}^{r=4n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to

(A) $\frac{1}{35}$ (B) $\frac{1}{14}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$

Q.15 If $F(x) = \int_1^x f(t)\,dt$ where $f(t) = \int_1^{t^2}\frac{\sqrt{1+u^4}}{u}du$ then the value of $F''(2)$ equals

(A) $\frac{7}{4\sqrt{17}}$ (B) $\frac{15}{\sqrt{17}}$ (C) $\sqrt{257}$ (D) $\frac{15\sqrt{17}}{68}$

Q.16 Let $f(x) = \int_{-1}^{x} e^{t^2}dt$ and $h(x) = f\left(1+g(x)\right)$, where g (x) is defined for all x, g'(x) exists for all x, and g (x) < 0 for x > 0. If $h'(1) = e$ and $g'(1) = 1$, then the possible values which g(1) can take

(A) 0 (B) – 1 (C) – 2 (D) – 4

Q.17 The value of x > 1 satisfying the equation $\int_1^x t \ln t \, dt = \frac{1}{4}$, is

(A) $\sqrt{e}$ (B) e (C) e^2 (D) $e - 1$

Q.18 Let f be a one-to-one continuous function such that $f(2) = 3$ and $f(5) = 7$. Given $\int_2^5 f(x)\,dx = 17$, then the value of the definite integral $\int_3^7 f^{-1}(x)\,dx$ equals

(A) 10 (B) 11 (C) 12 (D) 13

Q.19 Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g be the function satisfying $f(x) + g(x) = x^2$. The value of the integral $\int_0^1 f(x)g(x)\,dx$ is

(A) $e - \frac{1}{2}e^2 - \frac{5}{2}$ (B) $e - e^2 - 3$ (C) $\frac{1}{2}(e - 3)$ (D) $e - \frac{1}{2}e^2 - \frac{3}{2}$

Q.20 Let $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^2}}$ where $g(x) = \int_0^{\cos x} (1 + \sin t^2)\,dt$. Also $h(x) = e^{-|x|}$ and $f(x) = x^2 \sin\frac{1}{x}$ if $x \neq 0$ and $f(0) = 0$ then $f'\left(\frac{\pi}{2}\right)$ equals

(A) $l'(0)$ (B) $h'(0^-)$ (C) $h'(0^+)$ (D) $\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$

Q.21 $\lim_{t \to 0} \int_0^{2\pi} \frac{|\sin(x+t) - \sin x|}{|t|}\,dx$ equals

(A) 0 (B) 1 (C) 2 (D) 4

Q.22 The value of $\int_{-1}^{1} \frac{dx}{(2-x)\sqrt{1-x^2}}$ is

(A) 0 (B) $\frac{\pi}{\sqrt{3}}$ (C) $\frac{2\pi}{\sqrt{3}}$ (D) cannot be evaluated

Q.23 $\lim_{n \to \infty} \frac{\pi}{6n}\left[\sec^2\left(\frac{\pi}{6n}\right) + \sec^2\left(2 \cdot \frac{\pi}{6n}\right) + \ldots.. + \sec^2(n-1)\frac{\pi}{6n} + \frac{4}{3}\right]$ has the value equal to

(A) $\frac{\sqrt{3}}{3}$ (B) $\sqrt{3}$ (C) 2 (D) $\frac{2}{\sqrt{3}}$

Q.24 For $f(x) = x^4 + |x|$, let $I_1 = \int_0^{\pi} f(\cos x)\,dx$ and $I_2 = \int_0^{\pi/2} f(\sin x)\,dx$ then $\frac{I_1}{I_2}$ has the value equal to

(A) 1 (B) 1/2 (C) 2 (D) 4

Q.25 If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)\,g(\pi)$ (D) $[g(x)/g(\pi)]$

Q.26 $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} e^{-x/2} \frac{\sqrt{1-\sin x}}{1+\cos x}\,dx$ is

(A) $\left[e^{-\pi/2}\frac{2}{\sqrt{3}} - e^{-\pi/4}\sqrt{2}\right]$ (B) $2e^{-\pi/3}\left[\frac{e^{\pi/6}}{\sqrt{3}} - 1\right]$

(C) $2e^{-\pi/2}\left(\frac{e^{\pi/3}}{\sqrt{3}} - \sqrt{2}\,e^{\pi/4} + e^{\pi/6}\right)$ (D) $\left[2e^{-\pi/3} - \sqrt{2}\,e^{-\pi/4}\right]$

Q.27 Let f be a positive function. Let $I_1 = \int_{1-k}^{k} x\,f(x(1-x))dx$; $I_2 = \int_{1-k}^{k} f(x(1-x))dx$, where $2k - 1 > 0$. Then $\frac{I_2}{I_1}$ is

(A) k (B) 1/2 (C) 1 (D) 2

Q.28 If $\lim_{a\to\infty} \frac{1}{a}\int_0^{\infty} \frac{x^2+ax+1}{1+x^4}\cdot\tan^{-1}\left(\frac{1}{x}\right)dx$ is equal to $\frac{\pi^2}{k}$ where $k \in N$ equals

(A) 4 (B) 8 (C) 16 (D) 32

Q.29 Suppose that the quadratic function $f(x) = ax^2 + bx + c$ is non-negative on the interval [–1, 1]. Then the area under the graph of f over the interval [–1, 1] and the x-axis is given by the formula

(A) $A = f(-1) + f(1)$ (B) $A = f\left(-\frac{1}{2}\right) + f\left(\frac{1}{2}\right)$

(C) $A = \frac{1}{2}[f(-1) + 2f(0) + f(1)]$ (D) $A = \frac{1}{3}[f(-1) + 4f(0) + f(1)]$

Q.30 If $\int_0^{f(x)} t^2\,dt = x\cos\pi x$, then $f'(9)$

(A) is equal to $-\frac{1}{9}$ (B) is equal to $-\frac{1}{3}$ (C) is equal to $\frac{1}{3}$ (D) is non existent

Q.31 Let $I(a) = \int_0^{\pi}\left(\frac{x}{a} + a\sin x\right)^2 dx$ where 'a' is positive real. The value of 'a' for which I (a) attains its minimum value is

(A) $\sqrt{\pi\sqrt{\frac{2}{3}}}$ (B) $\sqrt{\pi\sqrt{\frac{3}{2}}}$ (C) $\sqrt{\frac{\pi}{16}}$ (D) $\sqrt{\frac{\pi}{13}}$

Q.32 Let $u = \int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx$ and $v = \int_0^{\pi/2} \cos\left(\frac{\pi}{3}\sin x\right)dx$, then the relation between u and v is

(A) $2u = v$ (B) $2u = 3v$ (C) $u = v$ (D) $u = 2v$

Q.33 $\int_0^1 \frac{\tan^{-1}x}{x}dx =$

(A) $\int_0^{\pi/4} \frac{\sin x}{x}dx$ (B) $\int_0^{\pi/2} \frac{x}{\sin x}dx$ (C) $\frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x}dx$ (D) $\frac{1}{2}\int_0^{\pi/4} \frac{x}{\sin x}dx$

Q.34 Let $f(x) = \int_3^x \frac{dt}{\sqrt{t^4+3t^2+13}}$. If g (x) is the inverse of $f(x)$ then g'(0) has the value equal to

(A) $\frac{1}{11}$ (B) 11 (C) $\sqrt{13}$ (D) $\frac{1}{\sqrt{13}}$

Q.35 Domain of definition of the function $f(x) = \int_0^x \frac{dt}{\sqrt{x^2+t^2}}$ is

(A) R (B) R^+ (C) $R^+ \cup \{0\}$ (D) $R - \{0\}$

Q.36 The set of values of 'a' which satisfy the equation $\int_0^2 (t - \log_2 a)\,dt = \log_2\left(\frac{4}{a^2}\right)$ is

(A) $a \in R$ (B) $a \in R^+$ (C) $a < 2$ (D) $a > 2$

Q.37 $\lim_{x\to\infty}\left(x^3 \int_{-1/x}^{1/x} \frac{\ln(1+t^2)}{1+e^t}dt\right)$ equals

(A) 1/3 (B) 2/3 (C) 1 (D) 0

Q.38 Variable x and y are related by equation $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$. The value of $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{y}{\sqrt{1+y^2}}$ (B) y (C) $\frac{2y}{\sqrt{1+y^2}}$ (D) 4y

Q.39 The value of the definite integral $\int_{-1}^1 \frac{dx}{(1+e^x)(1+x^2)}$ is

(A) $\pi/2$ (B) $\pi/4$ (C) $\pi/8$ (D) $\pi/16$

Q.40 If f & g are continuous functions in [0, a] satisfying f (x) = f (a – x) & g (x) + g (a – x) = 4 then $\int_0^a f(x).g(x)dx =$

(A) $\frac{1}{2}\int_0^a f(x)dx$ (B) $2\int_0^a f(x)dx$ (C) $\int_0^a f(x)dx$ (D) $4\int_0^a f(x)dx$

Q.41 If $\int_0^x f(t)\,dt = x + \int_x^1 t^2 \cdot f(t)\,dt$, then the value of the integral $\int_{-1}^1 f(x)\,dx$ is equal to

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Q.42 The value of the definite integral $\int_0^1 e^{e^x}(1 + x \cdot e^x)\,dx$ is equal to

(A) e^e (B) $e^e - e$ (C) $e^e - 1$ (D) e

Q.43 If the value of definite integral $\int_1^a x \cdot a^{-[\log_a x]}\,dx$ where $a > 1$, and [x] denotes the greatest integer, is $\frac{e-1}{2}$ then the value of 'a' equals

(A) $\sqrt{e}$ (B) e (C) $\sqrt{e+1}$ (D) $e - 1$

Q.44 $\int_{e^{e^e}}^{e^{e^{e^e}}} \frac{dx}{x\,\ln x \cdot \ln(\ln x) \cdot \ln(\ln(\ln x))}$ equals

(A) 1 (B) $\frac{1}{e}$ (C) $e - 1$ (D) $1 + e$

Q.45 Let f be a continuous functions satisfying $f'(\ln x) = \begin{cases} 1 & \text{for } 0 < x \le 1 \\ x & \text{for } x > 1 \end{cases}$ and $f(0) = 0$ then $f(x)$ can be defined as

(A) $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 1 - e^x & \text{if } x > 0 \end{cases}$ (B) $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

(C) $f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases}$ (D) $f(x) = \begin{cases} x & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

Q.46 The value of $\sqrt{\pi\left(\int_0^{2008} x\,|\sin \pi x|\,dx\right)}$ is equal to

(A) $\sqrt{2008}$ (B) $\pi\sqrt{2008}$ (C) 1004 (D) 2008

Q.47 $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2 x^2}$, $x > 0$ is equal to

(A) $x\tan^{-1}(x)$ (B) $\tan^{-1}(x)$ (C) $\frac{\tan^{-1}(x)}{x}$ (D) $\frac{\tan^{-1}(x)}{x^2}$

Q.48 The interval [0, 4] is divided into *n* equal sub-intervals by the points $x_0, x_1, x_2, \ldots, x_{n-1}, x_n$ where $0 = x_0 < x_1 < x_2 < x_3 \ldots < x_n = 4$. If $\delta x = x_i - x_{i-1}$ for i = 1, 2, 3, ... n then $\lim\limits_{\delta x \to 0} \sum_{i=1}^{n} x_i \delta x$ is equal to

(A) 4 (B) 8 (C) $\frac{32}{3}$ (D) 16

Q.49 The absolute value of $\int_{10}^{19} \frac{(\sin x)\,dx}{(1+x^8)}$ is less than

(A) 10^{-10} (B) 10^{-11} (C) 10^{-7} (D) 10^{-9}

Q.50 Let $a > 0$ and let $f(x)$ is monotonic increasing such that $f(0) = 0$ and $f(a) = b$ then $\int_0^a f(x)\,dx + \int_0^b f^{-1}(x)\,dx$ equals

(A) a + b (B) ab + b (C) ab + a (D) ab

Q.51 $\lim_{n\to\infty} \frac{n}{(n!)^{1/n}}$ is equal to

(A) e (B) $\frac{1}{e}$ (C) 1 (D) $\int_0^1 \ln x\,dx$

Q.52 The value of the limit, $\lim_{n\to\infty} \int_0^1 \frac{n \cdot x^{n-1}}{1+x}\,dx$ is equals

(A) 0 (B) 1/2 (C) 1 (D) non existent

Q.53 The value of the definite integral $\int_{19}^{37} \left(\{x\}^2 + 3(\sin 2\pi x)\right)dx$ where $\{x\}$ denotes the fractional part function.

(A) 0 (B) 6 (C) 9 (D) can not be determined

Q.54 If $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1}\frac{2x}{1+x^2}\,dx = k \int_0^{1/\sqrt{3}} \frac{x^4}{1-x^4}\,dx$ then 'k' equals

(A) π (B) 2π (C) 2 (D) 1

Q.55 $\int_0^\infty f\left(x + \frac{1}{x}\right) \cdot \frac{\ln x}{x}\,dx$

(A) is equal to zero (B) is equal to one (C) is equal to $\frac{1}{2}$ (D) can not be evaluated

Q.56 The value of the definite integral $\int_0^{\pi/2} \sqrt{\tan x}\,dx$, is

(A) $\sqrt{2}\,\pi$ (B) $\frac{\pi}{\sqrt{2}}$ (C) $2\sqrt{2}\,\pi$ (D) $\frac{\pi}{2\sqrt{2}}$

Q.57 Positive value of 'a' so that the definite integral $\int_a^{a^2} \frac{dx}{x+\sqrt{x}}$ achieves the smallest value is

(A) $\tan^2\left(\frac{\pi}{8}\right)$ (B) $\tan^2\left(\frac{3\pi}{8}\right)$ (C) $\tan^2\left(\frac{\pi}{12}\right)$ (D) 0

Q.58 The value of $\int_0^1 \left(\prod_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\frac{1}{x+k}\right)dx$ equals

(A) n (B) n ! (C) (n + 1) ! (D) n · n !

Q.59 The value of the function $f(x) = 1 + x + \int_1^x (ln^2 t + 2\, ln t)\, dt$ where $f'(x)$ vanishes is

(A) e^{-1} (B) 0 (C) $2\,e^{-1}$ (D) $1 + 2\,e^{-1}$

Q.60 $\lim_{\lambda \to 0}\left(\int_0^1 (1+x)^{\lambda}\,dx\right)^{1/\lambda}$ is equal to

(A) $2\,ln\,2$ (B) $\frac{4}{e}$ (C) $ln\,\frac{4}{e}$ (D) 4

Q.61 $\int_0^{\infty} x^{2n+1} \cdot e^{-x^2}\,dx$ is equal to $(n \in N)$.

(A) $(n-1)!$ (B) $n\,!$ (C) $\frac{n\,!}{2}$ (D) $\frac{(n+1)!}{4}$

Q.62 The true set of values of 'a' for which the inequality $\int_a^0 (3^{-2x} - 2.\,3^{-x})\,dx \ge 0$ is true is:

(A) [0 , 1] (B) $(-\infty, -1]$ (C) $[0, \infty)$ (D) $(-\infty, -1] \cup [0, \infty)$

Q.63 If the value of the integral $\int_1^2 e^{x^2}\,dx$ is α, then the value of $\int_e^{e^4} \sqrt{\ell n x}\,dx$ is :

(A) $e^4 - e - \alpha$ (B) $2\,e^4 - e - \alpha$ (C) $2\,(e^4 - e) - \alpha$ (D) $2\,e^4 - 1 - \alpha$

Q.64 If g (x) is the inverse of f (x) and f (x) has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then the values of $\int_1^5 f(x)\,dx + \int_2^{10} g(y)\,dy$ equals

(A) 48 (B) 64 (C) 71 (D) 52

Q.65 Which one of the following functions is not continuous on $(0,\pi)$?

(A) f(x)= cotx

(B) $g(x) = \int_0^x t\,\sin\frac{1}{t}\,dt$

(C) $h(x) = \begin{cases} 1 & 0 < x \le \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x & \frac{3\pi}{4} < x < \pi \end{cases}$

(D) $l(x) = \begin{cases} x\sin x\ , & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}\sin(x+\pi)\ , & \frac{\pi}{2} < x < \pi \end{cases}$

Q.66 If $f(x) = x\,\sin x^2$; $g(x) = x\,\cos x^2$ for $x \in [-1, 2]$

$A = \int_{-1}^{2} f(x)\,dx$; $B = \int_{-1}^{2} g(x)\,dx$ then

(A) A > 0 ; B < 0 (B) A < 0 ; B > 0 (C) A > 0 ; B > 0 (D) A < 0 ; B < 0

Q.67 The value of $\int_{-1}^{1} \frac{dx}{\sqrt{|x|}}$ is

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) undefined

Q.68 $\int_{0}^{1} x\, ln\left(1+\frac{x}{2}\right) dx =$

(A) $\frac{3}{4}\left(1-2ln\frac{3}{2}\right)$ (B) $\frac{3}{2}-\frac{7}{2}ln\frac{3}{2}$ (C) $\frac{3}{4}+\frac{1}{2}ln\frac{1}{54}$ (D) $\frac{1}{2}ln\frac{27}{2}-\frac{3}{4}$

Q.69 For $0 < x < \frac{\pi}{2}$, $\int_{1/2}^{\sqrt{3}/2} \ln(e^{\cos x}).\, d(\sin x)$ is equal to :

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{1}{4}\left[\left(\sqrt{3}-1\right)+\left(\sin\sqrt{3}-\sin 1\right)\right]$ (D) $\frac{1}{4}\left[\left(\sqrt{3}-1\right)-\left(\sin\sqrt{3}-\sin 1\right)\right]$

Q.70 The true solution set of the inequality, $\sqrt{5x-6-x^2}+\left(\frac{\pi}{2}\int_{0}^{x} dz\right) > x\int_{0}^{\pi} \sin^2 x\, dx$ is :

(A) R (B) (1, 6) (C) (– 6, 1) (D) (2, 3)

Q.71 The integral, $\int_{\pi/4}^{5\pi/4} (|\cos t|\sin t + |\sin t|\cos t)\, dt$ has the value equal to

(A) 0 (B) 1/2 (C) $1/\sqrt{2}$ (D) 1

Q.72 The value of the definite integral $\int_{0}^{\pi/2} \sin x \sin 2x \sin 3x\, dx$ is equal to :

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{6}$

Q.73 If the value of the definite integral $\int_{\pi/6}^{\pi/4} \frac{1+\cot x}{e^x \sin x} dx$, is equal to $ae^{-\pi/6} + be^{-\pi/4}$ then $(a+b)$ equals

(A) $2-\sqrt{2}$ (B) $2+\sqrt{2}$ (C) $2\sqrt{2}-2$ (D) $2\sqrt{3}-\sqrt{2}$

Q.74 For $U_n = \int_{0}^{1} x^n (2-x)^n\, dx$; $V_n = \int_{0}^{1} x^n (1-x)^n\, dx$ $n \in N$, which of the following statement(s) is/are ture ?

(A) $U_n = 2^n V_n$ (B) $U_n = 2^{-n} V_n$ (C) $U_n = 2^{2n} V_n$ (D) $U_n = 2^{-2n} V_n$

Q.75 Let $S(x) = \int_{x^2}^{x^3} \ln t\, dt$ $(x > 0)$ and $H(x) = \frac{S'(x)}{x}$. Then H(x) is :

(A) continuous but not derivable in its domain (B) derivable and continuous in its domain
(C) neither derivable nor continuous in its domain (D) derivable but not continuous in its domain.

Q.76 Let $f(x)=\frac{\sin x}{x}$, then $\int_0^{\pi/2} f(x)\, f\left(\frac{\pi}{2}-x\right)dx =$

(A) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ (B) $\int_0^{\pi} f(x)\,dx$ (C) $\pi\int_0^{\pi} f(x)\,dx$ (D) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$

[REASONING TYPE]

Q.77 **Statement-1 :** If $f(x)=\int_0^1 (x\,f(t)+1)\,dt$, then $\int_0^3 f(x)\,dx = 12$

because

Statement-2 : $f(x) = 3x + 1$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.78 Consider $I=\int_{-\pi/4}^{\pi/4}\frac{dx}{1-\sin x}$

Statement-1: $I = 0$

because

Statement-2: $\int_{-a}^{a} f(x)\,dx = 0$, wherever $f(x)$ is an odd function

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.79 **Statement-1:** The function $f(x)=\int_0^x \sqrt{1+t^2}\,dt$ is an odd function and $g(x)=f'(x)$ is an even function.

because

Statement-2: For a differentiable function $f(x)$ if $f'(x)$ is an even function then $f(x)$ is an odd function.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.80 Given $f(x) = \sin^3 x$ and $P(x)$ is a quadratic polynomial with leading coefficient unity.

Statement-1: $\int_0^{2\pi} P(x)\cdot f''(x)\,dx$ vanishes.

because

Statement-2: $\int_0^{2\pi} f(x)\,dx$ vanishes

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[COMPREHENSION TYPE]

Paragraph for Question Nos. 81 to 83

Suppose $\displaystyle \lim_{x\to 0}\frac{\int_0^x \frac{t^2\,dt}{(a+t^r)^{1/p}}}{bx-\sin x}=l$ where $p\in N, p\ge 2, a>0, r>0$ and $b\ne 0$.

Q.81 If l exists and is non zero then

(A) $b>1$ (B) $0<b<1$ (C) $b<0$ (D) $b=1$

Q.82 If $p=3$ and $l=1$ then the value of 'a' is equal to

(A) 8 (B) 3 (C) 6 (D) 3/2

Q.83 If $p=2$ and $a=9$ and l exists then the value of l is equal to

(A) 3/2 (B) 2/3 (C) 1/3 (D) 7/9

Paragraph for Question Nos. 84 to 86

Let the function f satisfies

$$f(x)\cdot f'(-x)=f(-x)\cdot f'(x) \text{ for all x and } f(0)=3.$$

Q.84 The value of $f(x)\cdot f(-x)$ for all x, is

(A) 4 (B) 9 (C) 12 (D) 16

Q.85 $\displaystyle\int_{-51}^{51}\frac{dx}{3+f(x)}$ has the value equal to

(A) 17 (B) 34 (C) 102 (D) 0

Q.86 Number of roots of $f(x)=0$ in $[-2, 2]$ is

(A) 0 (B) 1 (C) 2 (D) 4

Paragraph for Question Nos. 87 to 89

Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0\le x\le 1$.

Given $\displaystyle f(x)=\int_0^1 e^{x+t}\cdot f(t)\,dt$ and $\displaystyle g(x)=\int_0^1 e^{x+t}\cdot g(t)\,dt+x.$

Q.87 The value of $f(1)$ equals

(A) 0 (B) 1 (C) e^{-1} (D) e

Q.88 The value of $g(0)-f(0)$ equals

(A) $\frac{2}{3-e^2}$ (B) $\frac{3}{e^2-2}$ (C) $\frac{1}{e^2-1}$ (D) 0

Q.89 The value of $\frac{g(0)}{g(2)}$ equals

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{e^2}$ (D) $\frac{2}{e^2}$

Paragraph for Question Nos. 90 to 91

Consider the function defined on $[0, 1] \to R$

$$f(x) = \frac{\sin x - x\cos x}{x^2} \text{ if } x \neq 0 \text{ and } f(0) = 0$$

Q.90 $\int_0^1 f(x)\,dx$ equals

(A) $1 - \sin(1)$ (B) $\sin(1) - 1$ (C) $\sin(1)$ (D) $-\sin(1)$

Q.91 $\lim_{t\to 0} \frac{1}{t^2}\int_0^t f(x)\,dx$ equals

(A) 1/3 (B) 1/6 (C) 1/12 (D) 1/24

Paragraph for Question Nos. 92 to 94

Suppose a and b are positive real numbers such that $ab = 1$. Let for any real parameter t, the distance from the origin to the line $(ae^t)x + (be^{-t})y = 1$ be denoted by D(t) then

Q.92 The value of the definite integral $I = \int_0^1 \frac{dt}{(D(t))^2}$ is equal to

(A) $\frac{e^2-1}{2}\left(b^2 + \frac{a^2}{e^2}\right)$ (B) $\frac{e^2+1}{2}\left(a^2 + \frac{b^2}{e^2}\right)$

(C) $\frac{e^2-1}{2}\left(a^2 + \frac{b^2}{e^2}\right)$ (D) $\frac{e^2+1}{2}\left(b^2 + \frac{a^2}{e^2}\right)$ **[5]**

Q.93 The value of 'b' at which I is minimum, is

(A) e (B) $\frac{1}{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) $\sqrt{e}$ **[4]**

Q.94 Minimum value of I is

(A) $e - 1$ (B) $e - \frac{1}{e}$ (C) e (D) $e + \frac{1}{e}$ **[3]**

[MULTIPLE OBJECTIVE TYPE]

Q.95 Which of the following definite integral(s) vanishes

(A) $\int_0^{\pi/2} \ln(\cot x)\,dx$ (B) $\int_0^{2\pi} \sin^3 x\,dx$ (C) $\int_{1/e}^{e} \frac{dx}{x(\ln x)^{1/3}}$ (D) $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\,dx$

Q.96 The equation $10x^4 - 3x^2 - 1 = 0$ has

(A) at least one root in $(-1, 0)$ (B) at least one root in $(0, 1)$

(C) at least two roots in $(-1, 1)$ (D) no root in $(-1, 1)$

Q.97 Which of the following are true?

(A) $\int_{a}^{\pi-a} x \cdot f(\sin x)\, dx = \frac{\pi}{2} \cdot \int_{a}^{\pi-a} f(\sin x)\, dx$ (B) $\int_{-a}^{a} f(x)^2\, dx = 2. \int_{0}^{a} f(x)^2\, dx$

(C) $\int_{0}^{n\pi} f\left(\cos^2 x\right) dx = n. \int_{0}^{\pi} f\left(\cos^2 x\right) dx$ (D) $\int_{0}^{b-c} f(x+c)\, dx = \int_{c}^{b} f(x)\, dx$

Q.98 The value of $\int_{0}^{1} \frac{2x^2+3x+3}{(x+1)\left(x^2+2x+2\right)} dx$ is :

(A) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1} 2$ (B) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1}\frac{1}{3}$

(C) $2\ln 2 - \cot^{-1} 3$ (D) $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$

Q.99 Suppose $I_1 = \int_{0}^{\pi/2} \cos(\pi \sin^2 x)\, dx$; $I_2 = \int_{0}^{\pi/2} \cos(2\pi \sin^2 x)\, dx$ and $I_3 = \int_{0}^{\pi/2} \cos(\pi \sin x)\, dx$, then

(A) $I_1 = 0$ (B) $I_2 + I_3 = 0$ (C) $I_1 + I_2 + I_3 = 0$ (D) $I_2 = I_3$

Q.100 If $I_n = \int_{0}^{1} \frac{dx}{\left(1+x^2\right)^n}$; $n \in N$, then which of the following statements hold good ?

(A) $2n\, I_{n+1} = 2^{-n} + (2n-1)\, I_n$ (B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$

(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

Q.101 If $f(x) = \int_{1}^{x} \frac{\ell n\, t}{1+t} dt$ where $x > 0$ then the value(s) of x satisfying the equation,

$f(x) + f(1/x) = 2$ is :

(A) 2 (B) e (C) e^{-2} (D) e^2

Q.102 Let $f(x) = \int_{-1}^{1} (1-|t|)\cos(xt)\, dt$ then which of the following hold true?

(A) $f(0)$ is not defined (B) $\lim_{x\to 0} f(x)$ exists and equals 2

(C) $\lim_{x\to 0} f(x)$ exists and is equal to 1 (D) $f(x)$ is continuous at $x = 0$

Q.103 The function f is continuous and has the property

$$f(f(x)) = 1 - x \text{ for all } x \in [0, 1] \text{ and } J = \int_{0}^{1} f(x)\, dx \text{ then}$$

(A) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$ (B) the value of J equal to $\frac{1}{2}$

(C) $f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$ (D) $\int_{0}^{\pi/2} \frac{\sin x\, dx}{(\sin x + \cos x)^3}$ has the same value as J

Q.104 Let f(x) is a real valued function defined by :

$$f(x) = x^2 + x^2 \int_{-1}^{1} t \cdot f(t)dt + x^3 \int_{-1}^{1} f(t)dt$$

then which of the following hold(s) good ?

(A) $\int_{-1}^{1} t \cdot f(t)dt = \frac{10}{11}$ (B) $f(1) + f(-1) = \frac{30}{11}$

(C) $\int_{-1}^{1} t \cdot f(t)dt > \int_{-1}^{1} f(t)dt$ (D) $f(1) - f(-1) = \frac{20}{11}$

Q.105 Let f (x) and g (x) are differentiable function such that $f(x) + \int_{0}^{x} g(t)\,dt = \sin x\,(\cos x - \sin x)$, and $(f'(x))^2 + (g(x))^2 = 1$ then f (x) and g (x) respectively, can be

(A) $\frac{1}{2}\sin 2x, \sin 2x$ (B) $\frac{\cos 2x}{2}, \cos 2x$

(C) $\frac{1}{2}\sin 2x, -\sin 2x$ (D) $-\sin^2 x, \cos 2x$

Q.106 Let $f(x) = \int_{-x}^{x} (t \sin at + bt + c)dt$ where a,b, c are non zero real numbers, then $\lim_{x\to 0} \frac{f(x)}{x}$ is

(A) independent of a (B) independent of a and b and has the value equals to c.
(C) independent a, b and c. (D) dependent only on c.

Q.107 Let L = $\lim_{n\to\infty} \int_{a}^{\infty} \frac{n\,dx}{1+n^2x^2}$ where $a \in R$ then L can be

(A) π (B) $\frac{\pi}{2}$ (C) 0 (D) 1

[MATCH THE COLUMN]

Q.108

	Column I	Column II	
(A)	Suppose, $f(n) = \log_2(3) \cdot \log_3(4) \cdot \log_4(5)\ ....\ \log_{n-1}(n)$ then the sum $\sum_{k=2}^{100} f(2^k)$ equals	(P)	5010
(B	Let $f(x) = \sqrt{1 + x\sqrt{1 + (x+1)\sqrt{1 + (x+2)(x+4)}}}$ then $\int_{0}^{100} f(x)dx$ is	(Q)	5050
		(R)	5100
(C	In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is	(S)	5049
(D)	$\lim_{x\to 0} \frac{\prod_{r=1}^{100}(1+rx) - 1}{x}$ equals		

Q.109 Let $\lim\limits_{T\to\infty} \frac{1}{T}\int\limits_0^T (\sin x + \sin ax)^2\, dx = L$ then

	Column I	Column II	
(A)	for $a = 0$, the value of L is	(P)	0
(B)	for $a = 1$ the value of L is	(Q)	1/2
(C)	for $a = -1$ the value of L is	(R)	1
(D)	$\forall\ a \in R - \{-1, 0, 1\}$ the value of L is	(S)	2

Q.110

	Column I	Column II	
(A)	The function $f(x) = \dfrac{e^{x\cos x} - 1 - x}{\sin x^2}$ is not defined at $x = 0$. The value of $f(0)$ so that f is continuous at $x = 0$ is	(P)	-1
(B)	The value of the definite integral $\int\limits_0^1 \dfrac{dx}{\sqrt{x} + \sqrt[3]{x}}$ equals $a + b\, ln 2$ where a and b are integers then $(a + b)$ equals	(Q)	0
(C)	Given $e^n \int\limits_0^n \dfrac{\sec^2\theta - \tan\theta}{e^\theta} d\theta = 1$ then the value of $\tan(n)$ is equal to	(R)	1/2
(D)	Let $a_n = \int\limits_{\frac{1}{n+1}}^{\frac{1}{n}} \tan^{-1}(nx)\, dx$ and $b_n = \int\limits_{\frac{1}{n+1}}^{\frac{1}{n}} \sin^{-1}(nx)\, dx$ then $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n}$ has the value equal to	(S)	1

Q.111

	Column–I	Column–II	
(A)	If $f(x) = \int\limits_0^{g(x)} \dfrac{dt}{\sqrt{1+t^3}}$ where $g(x) = \int\limits_0^{\cos x} (1 + \sin t^2) dt$ then the value of $f'(\pi/2)$	(P)	3
(B)	If $f(x)$ is a non zero differentiable function such that	(Q)	2
	$\int\limits_0^x f(t) dt = (f(x))^2$ for all x, then $f(2)$ equals	(R)	1
(C)	If $\int\limits_a^b (2 + x - x^2)\, dx$ is maximum then $(a + b)$ is equal to	(S)	-1
(D)	If $\lim\limits_{x\to 0} \left(\dfrac{\sin 2x}{x^3} + a + \dfrac{b}{x^2}\right) = 0$ then $(3a + b)$ has the the value equal to		

Q.112 **Column-I** **Column-II**

(A) $\lim\limits_{x\to 0} \frac{1}{\sin x} \int\limits_0^{\ln(1+x)} (1-\tan 2y)^{1/y}\, dy$ equals — (P) 1

(B) $\lim\limits_{x\to\infty} (e^{2x} + e^x + x)^{1/x}$ equals — (Q) e

(C) Let $f(x) = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{n}{n^2 + k^2 x^2}$ then $\lim\limits_{x\to 0} f(x)$ equals — (R) e^2

(S) e^{-2}

Q.113 Let $f(\theta) = \int\limits_0^1 (x + \sin\theta)^2\, dx$ and $g(\theta) = \int\limits_0^1 (x + \cos\theta)^2\, dx$ where $\theta \in [0, 2\pi]$.

The quantity $f(\theta) - g(\theta)\ \forall\ \theta$ in the interval given in **column-I**, is

Column-I	**Column-II**
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$	(P) negative
(B) $\left(\frac{3\pi}{4}, \pi\right]$	(Q) positive
(C) $\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right)$	(R) non negative
(D) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$	(S) non positive

Q.114 **Column-I** **Column-II**

(A) $\int\limits_0^1 \left(1 + (2008)x^{2008}\right) e^{x^{2008}}\, dx$ equals — (P) e^{-1}

(B) The value of the definite integral $\int\limits_0^1 e^{-x^2} dx + \int\limits_1^{1/e} \sqrt{-\ln x}\, dx$ is equal to — (Q) $e^{-1/4}$

(C) $\lim\limits_{n\to\infty} \left(\frac{1^1 \cdot 2^2 \cdot 3^3 \ldots\ldots\ldots (n-1)^{n-1} \cdot n^n}{n^{1+2+3+\ldots\ldots.+n}} \right)^{\frac{1}{n^2}}$ equals — (R) $e^{1/2}$

(S) e

Q.115 **Column-I** **Column-II**

(A) $\int_0^{\pi} x\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx$ (P) π^2

(B) $\int_0^{\pi} \frac{x\,dx}{1+\sin^2 x}$ (Q) $\frac{\pi^2}{2}$

(C) $\int_0^{\pi^2/4} \left(2\sin\sqrt{x}+\sqrt{x}\cos\sqrt{x}\right)dx$ equals (R) $\frac{\pi^2}{4}$

(S) $\frac{\pi^2}{2\sqrt{2}}$

Q.116 **Column-I** **Column-II**

(A) Let $f(x)=\int x^{\sin x}\left(1+x\cos x\cdot \ln x+\sin x\right)dx$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$ then the value of $f(\pi)$ is (P) rational

(B) Let $g(x)=\int \frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0)=0$ (Q) irrational

then the value of $g\left(\frac{\pi}{2}\right)$ is (R) integral

(C) If real numbers x and y satisfy $(x+5)^2+(y-12)^2=(14)^2$ then the minimum value of $\sqrt{(x^2+y^2)}$ is (S) prime

(D) Let $k(x)=\int \frac{(x^2+1)\,dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1)=\frac{1}{\sqrt[3]{2}}$ then the value of $k(-2)$ is

ANSWERS

[SINGLE OBJECTIVE TYPE]

Q.1	C	Q.2	A	Q.3	B	Q.4	C	Q.5	A	Q.6	A	Q.7	A
Q.8	A	Q.9	D	Q.10	D	Q.11	A	Q.12	B	Q.13	B	Q.14	C
Q.15	C	Q.16	C	Q.17	A	Q.18	C	Q.19	D	Q.20	C	Q.21	D
Q.22	B	Q.23	A	Q.24	C	Q.25	A	Q.26	D	Q.27	D	Q.28	C
Q.29	D	Q.30	A	Q.31	A	Q.32	A	Q.33	C	Q.34	B	Q.35	D
Q.36	B	Q.37	A	Q.38	B	Q.39	B	Q.40	B	Q.41	C	Q.42	A
Q.43	A	Q.44	A	Q.45	D	Q.46	D	Q.47	C	Q.48	B	Q.49	C
Q.50	D	Q.51	A	Q.52	B	Q.53	B	Q.54	A	Q.55	A	Q.56	B
Q.57	A	Q.58	D	Q.59	D	Q.60	B	Q.61	C	Q.62	D	Q.63	B
Q.64	A	Q.65	D	Q.66	A	Q.67	C	Q.68	A	Q.69	A	Q.70	D
Q.71	A	Q.72	D	Q.73	A	Q.74	C	Q.75	B	Q.76	A	Q.77	C
Q.78	D	Q.79	C	Q.80	A	Q.81	D	Q.82	A	Q.83	B	Q.84	B
Q.85	A	Q.86	A	Q.87	A	Q.88	A	Q.89	B	Q.90	A	Q.91	B
Q.92	C	Q.93	D	Q.94	B								

[MULTIPLE OBJECTIVE TYPE]

Q.95	ABC	Q.96	ABC	Q.97	ABCD	Q.98	ACD	Q.99	ABC
Q.100	AB	Q.101	CD	Q.102	CD	Q.103	ABD	Q.104	BD
Q.105	CD	Q.106	AD	Q.107	ABC				

[MATCH THE COLUMN]

Q.108 (A) S; (B) R; (C) S; (D) Q

Q.109 (A) Q; (B) S; (C) P; (D) R

Q.110 (A) R; (B) P; (C) S; (D) R

Q.111 (A) S; (B) R; (C) R; (D) Q

Q.112 (A) S; (B) R; (C) P; (D) Q, R

Q.113 (A) Q; (B) R; (C) S; (D) P

Q.114 (A) S; (B) P; (C) Q

Q.115 (A) Q; (B) S; (C) Q

Q.116 (A) Q; (B) P; (C) P, R; (D) P, R, S