For More Study Material & Test Papers
Visit : www.mathsiit.com

ETOOS
Energy to Succeed!

QUESTION BANK

FUNCTION & ITF

MANOJ CHAUHAN SIR(IIT-DELHI)
EX. SR. FACULTY (BANSAL CLASSES)

[STRAIGHT OBJECTIVE TYPE]

Q.1 Let f be a real valued function such that

$$f(x) + 2f\left(\frac{2002}{x}\right) = 3x$$

for all $x > 0$. The value of $f(2)$, is
(A) 1000 (B) 2000 (C) 3000 (D) 4000

Q.2 The number k is such that $\tan\{\arctan(2) + \arctan(20k)\} = k$. The sum of all possible values of k is
(A) $-\frac{19}{40}$ (B) $-\frac{21}{40}$ (C) 0 (D) $\frac{1}{5}$

Q.3 Let $f_1(x) = \begin{bmatrix} x & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x > 1 \\ 0 & \text{otherwise} \end{bmatrix}$ and $f_2(x) = f_1(-x)$ for all x

$f_3(x) = f_2(x)$ for all x
$f_4(x) = f_3(-x)$ for all x
Which of the following is necessarily true?
(A) $f_4(x) = -f_1(x)$ for all x (B) $f_1(x) = -f_3(-x)$ for all x
(C) $f_2(-x) = f_4(x)$ for all x (D) $f_1(x) + f_3(x) = 0$ for all x

Q.4 Domain of definition of the function $f(x) = \log\left(\sqrt{10 \cdot 3^{x-2} - 9^{x-1} - 1}\right) + \sqrt{\cos^{-1}(1-x)}$ is
(A) [0, 1] (B) [1, 2] (C) (0, 2) (D) (0, 1)

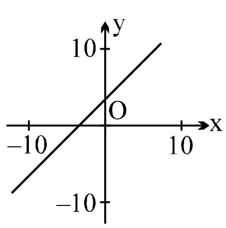
Q.5 If $\text{Sin}\theta = \frac{12}{13}, \cos\theta = -\frac{5}{13}$, $0 < \theta < 2\pi$. Consider the following statements.

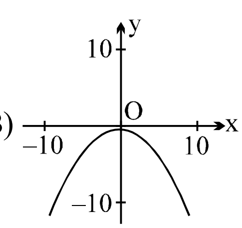
I. $\theta = \cos^{-1}\left(-\frac{5}{13}\right)$ **II.** $\theta = \sin^{-1}\left(\frac{12}{13}\right)$ **III.** $\theta = \pi - \sin^{-1}\left(\frac{12}{13}\right)$

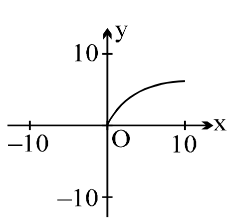
IV. $\theta = \tan^{-1}\left(-\frac{12}{5}\right)$ **V.** $\theta = \pi - \tan^{-1}\left(\frac{12}{5}\right)$

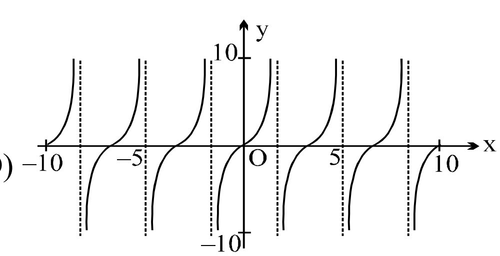
then which of the folloing statements are true ?
(A) I, II and IV only (B) III and V only (C) I and III only (D) I, III and V only

Q.6 Which one of the following depicts the graph of an odd function?

(A)


(B)


(C)


(D)


Q.7 The sum $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{3}{n^2+n-1}\right)$ is equal to

(A) $\frac{3\pi}{4}+\cot^{-1}2$ (B) $\frac{\pi}{2}+\cot^{-1}3$ (C) π (D) $\frac{\pi}{2}+\tan^{-1}2$

Q.8 The value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)$ for $0 < A < (\pi/4)$ is

(A) $4\tan^{-1}(1)$ (B) $2\tan^{-1}(2)$ (C) 0 (D) None

Q.9 Given $f(x)=\sqrt{\frac{8}{1-x}+\frac{8}{1+x}}$ and $g(x)=\frac{4}{f(\sin x)}+\frac{4}{f(\cos x)}$ then g(x) is

(A) periodic with period $\pi/2$ (B) periodic with period π
(C) periodic with period 2π (D) aperiodic

Q.10 $\alpha=\sin^{-1}(\cos(\sin^{-1}x))$ and $\beta=\cos^{-1}(\sin(\cos^{-1}x))$, then :

(A) $\tan\alpha=\cot\beta$ (B) $\tan\alpha=-\cot\beta$ (C) $\tan\alpha=\tan\beta$ (D) $\tan\alpha=-\tan\beta$

Q.11 The period of the function $f(x)=\frac{|\sin x|+|\cos x|}{|\sin x-\cos x|}$

(A) $\pi/2$ (B) $\pi/4$ (C) π (D) 2π

Q.12 The value of $\left[\tan\left\{\frac{\pi}{4}+\frac{1}{2}\sin^{-1}\left(\frac{a}{b}\right)\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\sin^{-1}\left(\frac{a}{b}\right)\right\}\right]^{-1}$, where $(0<a<b)$, is

(A) $\frac{b}{2a}$ (B) $\frac{a}{2b}$ (C) $\frac{\sqrt{b^2-a^2}}{2b}$ (D) $\frac{\sqrt{b^2-a^2}}{2a}$

Q.13 The sides of a triangle are 9, 12, 15. The area of the triangle formed by its income, centroid and orthocentre, is

(A) 2 (B) 3 (C) 3/2 (D) 5

Q.14 $f(x)=\sin^{23}x-\cos^{22}x$ and $g(x)=1+\frac{1}{2}\tan^{-1}|x|$, then the number of values of x in interval $[-10\pi, 20\pi]$ satisfying the equation $f(x)=\mathrm{sgn}(g(x))$, is

(A) 6 (B) 10 (C) 15 (D) 20

Q.15 Number of solutions of the equation $2\cot^{-1}2+\cos^{-1}(3/5)=\mathrm{cosec}^{-1}x$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

Q.16 $f(x)=\frac{x}{l\,nx}$ and $g(x)=\frac{l\,nx}{x}$. Then identify the CORRECT statement

(A) $\frac{1}{g(x)}$ and $f(x)$ are identical functions (B) $\frac{1}{f(x)}$ and g(x) are identical functions

(C) $f(x).g(x)=1 \quad \forall\ x>0$ (D) $\frac{1}{f(x).g(x)}=1 \quad \forall\ x>0$

Q.17 The number of solutions of the equation $\tan^{-1}\left(\frac{x}{3}\right)+\tanh^{-1}\left(\frac{x}{2}\right)=\tan^{-1}x$ is
(A) 3 (B) 2 (C) 1 (D) 0

Q.18 Let $f(x)=\sin^2x+\cos^4x+2$ and $g(x)=\cos(\cos x)+\cos(\sin x)$. Also let period of $f(x)$ and $g(x)$ be T_1 and T_2 respectively then
(A) $T_1=2T_2$ (B) $2T_1=T_2$ (C) $T_1=T_2$ (D) $T_1=4T_2$

Q.19 Which of the following is the solution set of the equation $2\cos^{-1}(x)=\cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$?
(A) (0, 1) (B) (–1, 1) – {0} (C) (–1, 0) (D) [–1, 1]

Q.20 Let $f(x)=e^{\left\{e^{|x|}\operatorname{sgn}x\right\}}$ and $g(x)=e^{\left[e^{|x|}\operatorname{sgn}x\right]}$, $x\in R$ where { x } and [ ] denotes the fractional part and integral part functions respectively. Also $h(x)=ln(f(x))+ln(g(x))$ then for all real x, h (x) is
(A) an odd function (B) an even function
(C) neither an odd nor an even function (D) both odd as well as even function

Q.21 Find the range of the function $f(x)=\cot^{-1}x+\sec^{-1}+\operatorname{cosec}^{-1}x$.
(A) $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ (B) $\left(\frac{\pi}{2},\frac{3\pi}{4}\right]\cup\left[\frac{5\pi}{4},\frac{3\pi}{2}\right)$
(C) $\left[\frac{\pi}{2},\pi\right)\cup\left(\pi,\frac{3\pi}{2}\right]$ (D) $\left(\frac{\pi}{2},\pi\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

Q.22 Which of the following function is surjective but not injective
(A) $f:R\to R$ $f(x)=x^4+2x^3-x^2+1$ (B) $f:R\to R$ $f(x)=x^3+x+1$
(C) $f:R\to R^+$ $f(x)=\sqrt{1+x^2}$ (D) $f:R\to R$ $f(x)=x^3+2x^2-x+1$

Q.23 $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{7\pi}{5}-\sin\frac{2\pi}{5}\right)\right\}$ is equal to
(A) $\frac{23\pi}{20}$ (B) $\frac{23\pi}{20}$ (C) $\frac{3\pi}{20}$ (D) $\frac{17\pi}{20}$

Q.24 Let $f(x)=\frac{2}{x+1}$; $g(x)=\cos x$ and $h(x)=\sqrt{x+3}$ then tha range of the composite function *fogoh,* is
(A) R^+ (B) $R-\{0\}$ (C) $[1,\infty)$ (D) $R^+-\{1\}$

Q.25 There exists a positive real number x satisfying $\cos(\tan^{-1}x)=x$. The value of $\cos^{-1}\left(\frac{x^2}{2}\right)$ is
(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{5}$ (C) $\frac{2\pi}{5}$ (D) $\frac{4\pi}{5}$

Q.26 If $f(x,y)=(\max(x,y))^{\min(x,y)}$ and $g(x,y)=\max(x,y)-\min(x,y)$, then
$f\left(g\left(-1,-\frac{3}{2}\right),g(-4,-1.75)\right)$ equals
(A) - 0.5 (B) 0.5 (C) 1 (D) 1.5

Q.27 The number of solutions of the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is
(A) 0 (B) 1 (C) 2 (D) 3

Q.28 If the solution set for $f(x) < 3$ is $(0, \infty)$ and the solution set for $f(x) > -2$ is $(-\infty, 5)$, then the true solution set for $(f(x))^2 \geq f(x) + 6$, is

(A) $(-\infty, +\infty)$ (B) $(-\infty, 0]$ (C) $[0, 5]$ (D) $(-\infty, 0] \cup [5, \infty)$

Q.29 The range of the value of p for which the equation $\sin\cos^{-1}\left(\cos(\tan^{-1}x)\right) = p$ has a solution is :

(A) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ (B) $[0, 1)$ (C) $\left[\frac{1}{\sqrt{2}}, 1\right)$ (D) $(-1, 1)$

Q.30 Let $f(x) = (x+1)^2 - 1, (x \geq -1)$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ is

(A) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$ (B) $\{0, 1, -1\}$

(C) $\{0, -1\}$ (D) empty

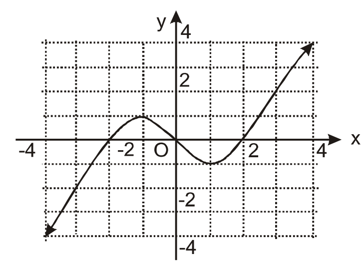
Q.31 If $(x-1)^2 + (y-2)^2 = \lambda(x+y-3)^2$ is a parabola, then $\lambda =$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{4}$

Q.32 $2\cot\left(\cot^{-1}(3) + \cot^{-1}(7) + \cot^{-1}(13) + \cot^{-1}(21)\right)$ has the value equal to

(A) 1 (B) 2 (C) 3 (D) 4

Q.33 The graph of the function $y = g(x)$ is shown.



The number of solutions of the equation $\left|\,|g(x)| - 1\,\right| = \frac{1}{2}$, is

(A) 4 (B) 5
(C) 6 (D) 8

Q.34 Let $f(x) = \frac{x}{1+x}$ and let $g(x) = \frac{rx}{1-x}$. Let S be the set off all real numbers r such that $f(g(x)) = g(f(x))$ for infinitely many real number x. The number of elements in set S is

(A) 1 (B) (C) 3 (D) 5

Q.35 Number of natural solution(s) of the equation $\sin^{-1}(\sin x) = \cos^{-1}(\cos x)$ in $[0, 5\pi]$ is

(A) 2 (B) 3 (C) 4 (D) infinite

Q.36 Range of the function $f(x) = \tan^{-1}\sqrt{[x]+[-x]} + \sqrt{2-|x|} + \frac{1}{x^2}$ is
where [*] is the greatest integer function.

(A) $\left[\frac{1}{4}, \infty\right)$ (B) $\left\{\frac{1}{4}\right\} \cup [2, \infty)$ (C) $\left\{\frac{1}{4}, 2\right\}$ (D) $\left[\frac{1}{4}, 2\right]$

Q.37 The set of values of ox, satisfying the equation $\tan^2(\sin^{-1}x) > 1$ is

(A) $[-1, 1]$ (B) $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

(C) $(-1, 1) - \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ (D) $[-1, 1] - \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Q.38 Let $f(x)=\begin{cases}0 & \text{if } x \text{ is irrational} \\ x & \text{if } x \text{ is rational}\end{cases}$ and $g(x)=\begin{cases}0 & \text{if } x \text{ is irrational} \\ x & \text{if } x \text{ is rational}\end{cases}$

Then the function $(f-g)x$ is

(A) odd (B) even (C) neither odd nor even (D) odd as well as even

Q.39 The solution set of the equation $\sin^{-1}\sqrt{1-x^2}+\cos^{-1}x=\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)-\sin^{-1}x$

(A) $[-1,1]-\{0\}$ (B) $(0,1]\cup\{-1\}$ (C) $[-1,0)\cup\{1\}$ (D) $[-1, 1]$

Q.40 The period of the function $\cos\sqrt{2}\,x+\cos 2x$ is :

(A) π (B) $\pi\sqrt{2}$ (C) 2π (D) none of these

Q.41 The value of the angle $\tan^{-1}\left(\tan 65^0-2\tan 40^0\right)$ in degrees is equal to

(A) -20^0 (B) 20^0 (C) 25^0 (D) 40^0

Q.42 Range of the function $f(x)=\frac{\{x\}}{1+\{x\}}$ where $\{x\}$ denotes the fractional part function is

(A) $[0,1)$ (B) $\left[0,\frac{1}{2}\right]$ (C) $\left[0,\frac{1}{2}\right)$ (D) $\left(0,\frac{1}{2}\right)$

Q.43 Consider the function g (x) defined as

$g(x)\cdot\left(x^{(2^{2008}-1)}-1\right)=(x+1)(x^2+1)(x^4+1)\ldots\ldots\left(x^{2^{2007}}+1\right)-1.$

the value of g (2) equals

(A) 1 (B) $2^{2008}-1$ (C) 2^{2008} (D) 2

Q.44 The range of the function, $f(x)=(1+\sec^{-1}x)(1+\cos^{-1}x)$ is

(A) $(-\infty,\infty)$ (B) $(-\infty,0]\cup[4,\infty)$ (C) $\{1,(1+\pi)^2\}$ (D) $[0,(1+\pi)^2]$

Q.45 Which of the following is true for a real valued function y = f (x), defined on [-a, a]?

(A) f (x) can be expressed as a sum or a difference of two even function.

(B) f (x) can be expressed as a sum or a difference of two odd function.

(C) f (x) can be expressed as a sum or a difference of an odd and an even function.

(D) f (x) can never be expressed as a sum or a difference of an odd and an even function.

Q.46 Which of the following represents an odd function?

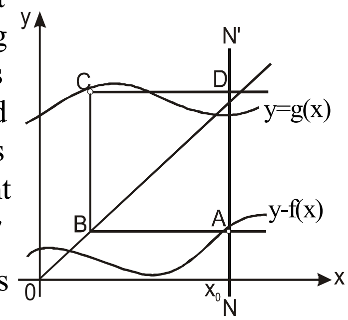
(A) $f(x)=\frac{(1+e^x)^2}{e^x}$ (B) $g(x)+\sec^{-1}(\sec x)$

(C) $h(x)=\cos(\cos^{-1}x)$ (D) $k(x)=\cot^{-1}(\cot x)$

Q.47 Let f be a real valued function defined by $f(x)=\sin^{-1}\left(\frac{1-|x|}{3}\right)+\cos^{-1}\left(\frac{|x|-3}{5}\right)$. Then domain of f(x) is given by :

(A) $[-4,4]$ (B) $[0,4]$ (C) $[-3,3]$ (D) $[-5,5]$

Q.48 Given the graphs of the two functions, $y = f(x)$ & $y = g(x)$. In the adjacent figure from point A on the graph of the function $y = f(x)$ corresponding to the given value of the independent variable (say x_0), a straight line is drawn parallel to the X-axis to intersect the bisector of the first and the third quadrants at point B. From the point B a straight line parallel to the Y-axis is drawn to intersect the graph of the function $y = g(x)$ at C. Again a straight line is drawn from the point C parallel to the X-axis, to intersect the line NN' at D. If the straight line NN' is parallel to Y-axis, then the co-ordinates of the point D are



(A) $f(x_0), g(f(x_0))$ (B) $x_0, g(x_0)$ (C) $x_0, g(f(x_0))$ (D) $f(x_0), f(g(x_0))$

Q.49 If $x = \frac{1}{2}$ and $(x+1)(y+1) = 2$ the the radian measue of $\cot^{-1} x + \cot^{-1} y$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Q.50 Given f(x) is a polynomial function of x, satisfying $f(x) . f(y) = f(x) + f(y) + f(xy) - 2$ and that $f(2) = 5$. then $f(3)$ is equal to

(A) 10 (B) 24 (C) 15 (D) none

Q.51 Let $\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$. If x satisfies the cubic $ax^3 + bx^2 + cs - 1 = 0$, then $(a+b+c)$ has the value equal to

(A) 24 (B) 25 (C) 26 (D) 27

Q.52 Let $f(x) = \begin{bmatrix} x|x| & \text{if} & x \le -1 \\ [1+x]+[1-x] & \text{if} & -1 < x < 1 \\ -x|x| & \text{if} & x \ge 1 \end{bmatrix}$

where [x] denotes the greatest integer function then F(x) is

(A) even (B) odd

(C) neither odd nor even (D) even as well as odd

Q.53 The domain of the function $f(x) = \frac{\text{arc cot } x}{\sqrt{x^2 - [x^2]}}$, where [x] denotes the greatest integer not greater than x, is :

(A) R (B) R - {0}

(C) $R - \{\pm\sqrt{n} : n \in I^+ \cup \{0\}\}$ (D) $R - \{n : n \in I\}$

Q.54 If $\alpha = \tan^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$, then which one of the following can not be equal to α

(A) $\sqrt{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (B) $\cot^{-1} 2 + \cot^{-1} 3$

(C) $\pi - (\tan^{-1} 2 + \tan^{-1} 3)$ (D) $\tan^{-1} 3 - \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Q.55 The range of the function, $f(x)=\cot^{-1}\log_{0.5}\left(x^4-2x^2+3\right)$ is:

(A) $(0,\pi)$ (B) $\left(0,\frac{3\pi}{4}\right]$ (C) $\left[\frac{3\pi}{4},\pi\right)$ (D) $\left[\frac{\pi}{2},\frac{3\pi}{4}\right]$

Q.56 The value of $\cos\left[\cos^{-1}\left(\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)\right)-\cos^{-1}\left(\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\right)\right]$ equals

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) 0

Q.57 If $f(x+ay, x-ay)=axy$ then $f(x,y)$ is equal to :

(A) $\frac{x^2-y^2}{4}$ (B) $\frac{x^2+y^2}{4}$ (C) $4xy$ (D) none

Q.58 If $f(x)=px+q$ and $f(f(f(x)))=8x+21$, where p and q are real number, then $p+q$ equals

(A) 3 (B) 5 (C) 7 (D) 11

Q.59 The value of x satisfying the equation $\sin(\tan^{-1}x)=\cos\left(\cot^{-1}(x+1)\right)$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\sqrt{2}-1$ (D) no finite value

Q.60 If $f(x)=2\tan 3x+5\sqrt{1-\cos 6x}$; $g(x)$ is a function having the same period as that of f(x), then which of the following can be g(x).

(A) $(\sec^2 3x+\text{cosec}^2 3x)\tan^2 3x$ (B) $2\sin 3x+3\cos 3x$

(C) $2\sqrt{1-\cos^2 3x}+\text{cosec}\,3x$ (D) $3\,\text{cosec}\,3x+2\tan 3x$

Q.61 Let

$$f(x) = \max.\{\sin t : 0\le t\le x\}$$
$$g(x) = \min.\{\sin t : 0\le t\le x\}$$
$$\text{and } h(x) = [f(x)-g(x)]$$

where [ ] denotes greatest integer function, then the range of h(x) is

(A) {0, 1} (B) {1, 2} (C) {0, 1, 2} (D) {-3, -2, -1, 0, 1, 2, 3}

[COMPREHENSION TYPE]

Paragraph for question nos. 62 to 64

Consider a function $y=f(x)$ satisfying the equation $\tan^{-1}y=\tan^{-1}x+C$ where $y=1$ when $x=0$.

Q.62 The domain of the explicit form of the function is

(A) $(-\infty,1)$ (B) $R-\{-1,1\}$ (C) $(-1,1)$ (D) $[0,\infty)$

Q.63 Range of the function is

(A) $R-\{-1\}$ (B) $(-1,\infty)$ (C) $[1,\infty)$ (D) $(-\infty,1]$

Q.64 For the function $y=f(x)$ which one of the following does not hold good ?

(A) $f(x)$ is injective (B) $f(x)$ is neither odd nor even

(C) $f(x)$ is aperiodic (D) explicit form of $f(x)$ is $\frac{x+1}{x-1}$

Paragraph for question nos. 65 to 68

Let $f(x) = x^2 - 2x - 1 \;\; \forall\; x \in R$. Let $f : (-\infty, a] \to [b, \infty)$, where 'a' is the largest real number for which $f(x)$ is bijective.

Q.65 The value of $(a + b)$ is equal to

(A) - 2 (B) - 1 (C) 0 (D) 1

Q.66 Let $f : R \to R, g(x) = f(x) + 3x - 1$, then the least value of function $y = g(|x|)$ is

(A) - 9/4 (B) - 5/4 (C) - 2 (D) - 1

Q.67 Let $f : [a, \infty) \to [b, \infty)$, then $f^{-1}(x)$ is given by

(A) $1 + \sqrt{x+2}$ (B) $1 - \sqrt{x+3}$ (C) $1 - \sqrt{x+2}$ (D) $1 + \sqrt{x+3}$

Q.68 Let $f : R \to R$, then range of values of k for which equation $f(|x|) = k$ has 4 distinct real roots is

(A) (- 2, - 1) (B) (- 2, 0) (C) (- 1, 0) (D) (0, 1)

[MULTIPLE OBJECTIVE TYPE]

Q.69 Which of the following function (s) is/are Transcendental ?

(A) $f(x) = 5 \sin \sqrt{x}$ (B) $f(x) = \dfrac{2 \sin 3x}{x^2 + 2x - 1}$

(C) $f(x) = \sqrt{x^2 + 2x + 1}$ (D) $f(x) = (x^2 + 3).2^x$

Q.70 $\sin^{-1}(\sin 3) + \sin^{-1}(\sin 4) + \sin^{-1}(\sin 5)$ when simplified reduces to

(A) an irrational number (B) a rational number

(C) an even prime (D) a negative integer

Q.71 The functions which are aperiodic are :

(A) $y = [x + 1]$ (B) $y = \sin x^2$ (C) $y = \sin^2 x$ (D) $y = \sin^{-1} x$

where [x] denotes greatest integer function

Q.72 Which of the following pairs of functions are identical ?

(A) $f(x) = e^{\ell n \sec^{-1}}$ and $g(x) = \sec^{-1} x$ (B) $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{1} x)$

(C) $f(x) = \text{sgn}(x)$ and $g(x) = \text{sgn}(\text{sgn}(x))$ (D) $f(x) = \cot^2 x . \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$

Q.73 Which of the functions defined below are one-one function(s) ?

(A) $f(x) = (x + 1), (x \ge -1)$ (B) $g(x) = x + (1/x) \;\; (x > 0)$

(C) $h(x) = x^2 + 4x - 5, (x > 0)$ (D) $f(x) = e^{-x}, (x \ge 0)$

Q.74 The value of $\cos\left[\dfrac{1}{2}\cos^{-1}\left(\cos\left(-\dfrac{14\pi}{5}\right)\right)\right]$ is :

(A) $\cos\left(-\dfrac{7\pi}{5}\right)$ (B) $\sin\left(\dfrac{\pi}{10}\right)$ (C) $\cos\left(\dfrac{2\pi}{5}\right)$ (D) $-\cos\left(\dfrac{3\pi}{5}\right)$

Q.75 If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then

(A) $x^2 + y^2 + z^2 + 2xyz = 1$

(B) $2(\sin^{-1} x + \sin^{-1} y + \sin^{-1} z) = \cos^{-1} x + \cos^{-1} y + \cos^{-1} z$

(C) $xy + yz + zx = x + y + z - 1$

(D) $\left(x + \dfrac{1}{x}\right) + \left(y + \dfrac{1}{y}\right) + \left(z + \dfrac{1}{z}\right) \ge 6$

Q.76 Which of the following functions are homogeneous ?

(A) $x \sin y + y \sin x$ (B) $x e^{y/x} + y e^{x/y}$ (C) $x^2 - xy$ (D) arc sin xy

Q.77 Let $\tan^{-1}(f(x)) = \cos^{-1} x$ then which of the following do/does not hold good ?

(A) Domain of $f(x)$ is [-1, 1] (B) Range of $f(x)$ is $(-\infty, \infty)$

(C) $f(x)$ is bounded (D) $f(x)$ is odd

Q.78 Suppose $f(x) = ax + b$ and $g(x) = bx + a$, where a and b are positive integers. If $f(g(50)) - g(f(50)) = 28$ then the product (ab) can have the value equal to

(A) 12 (B) 48 (C) 180 (D) 210

Q.79 Which of the following function(s) have the same domain and range ?

(A) $f(x) = \sqrt{1-x^2}$ (B) $g(x) = \frac{1}{x}$ (C) $h(x) = \sqrt{x}$ (D) $l(x) = \sqrt{4-x}$

Q.80 $2\tan\left(\tan^{-1}(x) + \tan^{-1}(x^3)\right)$ where $x \in R - \{-1, 1\}$ is equal to

l

(A) $\frac{2x}{1-x^2}$ (B) $\tan\left(2\tan^{-1} x\right)$

(C) $\tan\left(\cot^{-1}(-x) - \cot^{-1}(x)\right)$ (D) $\tan\left(2\cot^{-1} x\right)$

Q.81 Which pair(s) of function(s) is/are equal ?

(A) $f(x) = \cos(2\tan^{-1} x); g(x) = \frac{1-x^2}{1+x^2}$ (B) $f(x) = \frac{2x}{1+x^2}; g(x) = \sin\left(2\cot^{-1} x\right)$

(C) $f(x) = e^{\ell n(\operatorname{sgn} \cot^{-1} x)}$; $g(x) = e^{\ell n[1+\{x\}]}$ (D) $f(x) = \sqrt[x]{a}, a > 0$; $g(x) = a^{\frac{1}{x}}, a > 0$

where $\{x\}$ and $[x]$ denotes the fractional part and integral part functions.

Q.82 Which of the following function(s) would represent a non singular mapping.

f

(A) $f : R \to R$ $f(x) = |x| \operatorname{Sgn} x$ where Sgn denotes Signum function

(B) $g : R \to R$ $g(x) = x^{3/5}$

(C) $h : R \to R$ $h(x) = x^4 + 3x^2 + 1$

(D) $k : R \to R$ $k(x) = \frac{3x^2 - 7x + 6}{x - x^2 - 2}$

Q.83 Let x_1, x_2, x_3, x_4 be four non zero numbers satisfying the equation

$$\tan^{-1}\frac{a}{x} + \tan^{-1}\frac{b}{x} + \tan^{-1}\frac{c}{x} + \tan^{-1}\frac{d}{x} = \frac{\pi}{2}$$

(A) $\sum_{i=1}^{4} x_i = a + b + c + d$

(B) $\sum_{i=1}^{4} \frac{1}{x_i} = 0$

(C) $\prod_{i=1}^{4} x_i = abcd$

(D) $(x_1 + x_2 + x_3)(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2) = abcd$

Q.84 If the function $f(x) = ax + b$ has its own inverse then the ordered pair (a, b) can be

(A) (1, 0) (B) (-1, 0) (C) (-1, 1) (D) (1, 1)

Q.85 Let $y=\sqrt{(\sin x+\sin 2x+\sin 3x)^2+(\cos x+\cos 2x+\cos 3x)^2}$ then which of the following is correct ?

(A) $\frac{dy}{dx}$ when $x=\frac{\pi}{2}$ is -2

(B) value of y when $x=\frac{\pi}{5}$ is $\frac{3+\sqrt{5}}{2}$

(C) value of y when $x=\frac{\pi}{12}$ is $\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}}{2}$

(D) y simplifies to $(1+2\cos x)$ in $[0,\pi]$

Q.86 Which of the following trigonometric equation(s) has/have no solution $\forall\ x\in R$?

(A) $2\cos^2\frac{x}{2}\sin^2 x=x^2+\frac{1}{x^2}$

(B) $\sin^4 x-2\sin^2 x-1=0$

(C) $\cos e^x=5^x+5^{-x}$

(D) $\sin 2x=[1+\sin 2x]+[1-\cos 2x]$
where [ ] denotes greatest integer function.

[MATCH THE COLUMN]

Q.87 **Column-I** **Column-II**

(A) The period of the function $f(x)=\sin\left(\cos\frac{x}{2}\right)+\cos(\sin x)$ equals $k\pi$ then k is equal to — (P) 2

(B) The integral value(s) in the domain of definition of the function, — (Q) 3

$f(x)=\text{arc}\cos\left[\frac{3x^2-7x+8}{1+x^2}\right]$

where [*] denotes the greatest integer functin, is

(C) Let $f(x)=\sin\sqrt{[a]}\ x$. If f is periodic with fundamental period π, then the possible integral vlaue(s) of 'a' is/are
(where [ ] denotes the greatest integer function) — (R) 4

(D) If the values of x satisfying the equation $[x]^2-5[x]+6=0$, then integral value of x, is/are
(where [ ] denotes the greatest integer function) — (S) 5

Q.88 **Column-I** **Column-II**

(A) $\underset{n\to\infty}{\text{Lim}}\sum_{n=1}^{n}\frac{{}^nC_2}{2^n}$ equals — (P) 0

(B) Let the roots of $f(x)=0$ are 2, 3, 5, 7 and 9 and the roots of $g(x)=0$ are -1, 3, 5, 7 and 8. — (Q) 1

Number of solutions of the euqation $\frac{f(x)}{g(x)}=0$ is

(C) Let $y=\frac{\sin^3 x}{\cos x}+\frac{\cos^3 x}{\sin x}$ where $0<x<\frac{\pi}{2}$, then the minimum value of y is — (R) 3/2

(D) A circle passes through vertex D of the square ABCD, and is tangent to the sides AB and BC. If AB = 1, the radius of the circle can be expressed as $p+q\sqrt{2}$, then p + q has the value equal to — (S) 2

Q.89 **Column-I** **Column-II**

(A) $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x,\ 0 \le x \le 2\pi$, is

(B) $4\sin^2 x - 8\sin\ x + \le 0, 0 \le x \le 2\pi$, is

(C) $|\tan x| \le 1$ and $x \in [-\pi, \pi]$, is

(D) $\cos x - \sin x \ge 1$ and $0 \le x \le 2\pi$

(P) $\left[-\pi, \frac{3}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

(Q) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$

(R) $\left(0, \frac{\pi}{4}\right)$

(S) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Q.90 Let: $f : R \to [\alpha, \infty), f(x) = x^2 + 3ax + b, g(x) = \sin^{-1}\frac{x}{4} (\alpha \in R)$.

Column-I **Column-II**

(A) The possible integral values of 'a' for which f(x) is many one in interval [-3, 5] is/are

(B) Let a = - 1 and gof(x) is defined for $x \in [-1, 1]$ then possible integral values of b can be

(C) Let $a = 2, \alpha = -8$ the value(s) of which f(x) is surjective is/are

(D) If a = 1, b = 2, then integers in the range of fog(x) is/are

(P) - 2

(Q) - 1

(R) 0

(S) 1

Q.91 **Column-I** **Column-II**

(A) $\cot^{-1}(\tan(-37^0))$

(B) $\cos^{-1}(\cos(-233^0))$

(C) $\sin\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{9}\right)\right)$

(D) $\cos\left(\frac{1}{2}\text{arc}\cos\left(\frac{1}{8}\right)\right)$

(P) 143^0

(Q) 127^0

(R) $\frac{3}{4}$

(S) $\frac{2}{3}$

Q.92 Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

Match the composite function given in Column-I with their respective domains given in Column-II.

Column-I **Column-II**

(A) fog

(B) gof

(C) fof

(D) gog

(P) R - {-2, -5/3}

(Q) R - {-1, 0}

(R) R - {0}

(S) R - {-2, -1}

Q.93 **Column-I** **Column-II**

(A) $f(x) = \sin(\sin^{-1} 2x) + \text{cosec}(\text{cosec}^{-1} 2x) + \tan(\tan^{-1} 2x)$

(B) $g(x) = \sin^{-1}\{x\}$, where {x} denotes fractional part function

(C) $h(x) = \frac{2}{\pi}\text{cosec}^{-1}(\text{sgn } x)$

(D) $k(x) = \cos^{-1}(|\sin x + \cos x|)$

(P) odd function

(Q) injective mapping

(R) *range contian two integer only*

(S) aperiodic

[SUJBECTIVE]

Q.94 Find the value of x satisfying the equation,

$$\log_{10}\sqrt{5\cot^{-1}x-1}+\frac{1}{2}\log_{10}\left(2\cot^{-1}x+3\right)+\log_{10}\sqrt{5}=1.$$

Q.95 Let the straight line $L:\tan(\cot^{-1}2)\,x-y=4$ be rotated through an angle $\cot^{-1}3$ about the point M(0, - 4) in anticlockwise sense. After rotation the line become tangent to the circle which lies in 4^{th} quadrant and also touches coordinate axes. Find the sum of radii of all possible circles.

ANSWER KEY

Ques.	Ans.	Ques.	Ans.	Ques.	Ans.	Ques.	Ans.	Ques.	Ans.
1	B	21	B	41	C	61	C	81	A,B,C
2	A	22	D	42	C	62	A	82	A,B
3	B	23	D	43	D	63	B	83	B,C,D
4	C	24	C	44	C	64	D	84	A,B,C
5	D	25	C	45	C	65	B	85	A,B
6	D	26	D	46	C	66	C	86	A,B,C,D
7	A	27	B	47	A	67	A	87	(A) R; (B) P, Q, R, S; (C) R; (D) P,Q
8	A	28	D	48	C	68	A	88	(A) S; (B) S; (C) Q; (D) Q
9	A	29	B	49	D	69	A,B,D	89	(A) R; (B) S; (C) P; (D) Q
10	A	30	C	50	A	70	B,D	90	(A) P,Q,R,S (B) P,Q,R (C) S (D) R,S
11	C	31	B	51	C	71	A,B,D	91	(A) Q; (B) Q; (C) S; (D) R
12	C	32	C	52	A	72	B,C,D	92	(A) S; (B) Q; (C) R; (D) P
13	C	33	D	53	C	73	A,C,D	93	(A) P,Q,R,S; (B) R,S; (C) P,R,S; (D) R
14	C	34	B	54	A	74	B,C,D	94	x = cot 1
15	A	35	C	55	C	75	A,B	95	0008
16	A	36	C	56	C	76	B,C		
17	A	37	C	57	A	77	A,B,C,D		
18	C	38	A	58	B	78	A,D		
19	A	39	C	59	D	79	B,C		
20	A	40	D	60	A	80	A,B,C		