QUESTION BANK

LIMIT, CONTINUITY, DIFFERENTIABILITY OF FUNCTION

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[STRAIGHT OBJECTIVE TYPE]

Q.1 If both f (x) & g(x) are differentiable functions at $x = x_0$, then the function defined as, h(x) = Maximum {f(x), g(x)}
(A) is always differentiable at $x = x_0$
(B) is never differentiable at $x = x_0$
(C) is differentiable at $x = x_0$ when $f(x_0) \neq g(x_0)$
(D) cannot be differentiable at $x = x_0$ if $f(x_0) = g(x_0)$.

Q.2 If $\lim\limits_{x\to 0} (x^{-3} \sin 3x + ax^{-2} + b)$ exists and is equal to zero then
(A) $a = -3$ & $b = 9/2$ (B) $a = 3$ & $b = 9/2$
(C) $a = -3$ & $b = -9/2$ (D) $a = 3$ & $b = -9/2$

Q.3 Let $l = \lim\limits_{x\to\infty} \left(\frac{x+1}{x-1}\right)^x$ then $\{l\}$ where $\{x\}$ denotes the fractional part function is
(A) $8 - e^2$ (B) $7 - e^2$ (C) $e^2 - 6$ (D) $e^2 - 7$

Q.4 For $x > 0$, let $h(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ *where p & q > 0 are relatively prime integers*
then which one does not hold good?
(A) h(x) is discontinuous for all x in $(0, \infty)$
(B) h(x) is continuous for each irrational in $(0, \infty)$
(C) h(x) is discontinuous for each rational in $(0, \infty)$
(D) h(x) is not derivable for all x in $(0, \infty)$.

Q.5 For a certain value of c, $\lim\limits_{x\to -\infty} [(x^5 + 7x^4 + 2)^C - x]$ is finite & non zero. The value of c and the value of the limit is
(A) 1/5, 7/5 (B) 0, 1 (C) 1, 7/5 (D) none

Q.6 If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $\lim\limits_{x\to\alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ equals
(A) 0 (B) $\frac{1}{2}(\alpha - \beta)^2$ (C) $\frac{a^2}{2}(\alpha - \beta)^2$ (D) $-\frac{a^2}{2}(\alpha - \beta)^2$

Q.7 $\lim\limits_{x\to\infty} \left(\sqrt[3]{(x+a)(x+b)(x+c)} - x\right) =$
(A) $\sqrt{abc}$ (B) $\frac{a+b+c}{3}$ (C) abc (D) $(abc)^{1/3}$

Q.8 $\lim\limits_{x\to\infty} x\left(\tan^{-1}\frac{x+1}{x+2} - \cot^{-1}\frac{x+2}{x}\right)$ is
(A) -1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) non existent

Q.9 Given $f(x) = \dfrac{e^x - \cos 2x - x}{x^2}$ for $x \in R - \{0\}$

$$g(x) = f(\{x\}) \quad \text{for } n < x < n + \frac{1}{2}$$

$$= f(1 - \{x\}) \quad \text{for } n + \frac{1}{2} \le x < n + 1,\ n \in I \quad \left\{\begin{array}{l}\text{where } \{x\} \text{ denotes} \\ \text{fractional part} \\ \text{function}\end{array}\right.$$

$$= \frac{5}{2} \quad \text{otherwise}$$

then g (x) is

(A) discontinuous at all integral values of x only

(B) continuous everywhere except for x = 0

(C) discontinuous at $x = n + \frac{1}{2}$; $n \in I$ and at some $x \in I$

(D) continuous everywhere

Q.10 Let the function f, g and h be defined as follows

$$f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 & \text{for } \quad x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 & \text{for } \quad x = 0 \end{cases}$$

$$h(x) = |x|^3 \quad \text{for } -1 \le x \le 1$$

Which of these functions are differentiable at x = 0?

(A) f and g only (B) f and h only (C) g and h only (D) none

Q.11 $\lim\limits_{n\to\infty}\left(\left(\dfrac{n}{n+1}\right)^{\alpha} + \sin\dfrac{1}{n}\right)^{n}$ when $\alpha \in Q$ is equal to

(A) $e^{-\alpha}$ (B) $-\alpha$ (C) $e^{1-\alpha}$ (D) $e^{1+\alpha}$

Q.12 Let $f(x) = \dfrac{g(x)}{h(x)}$, where g and h are cotinuous functions on the open interval (a, b). Which of the following statements is true for a < x < b?

(A) f is continuous at all x for which x is not zero.

(B) f is continuous at all x for which $g(x) = 0$

(C) f is continuous at all x for which $g(x)$ is not equal to zero.

(D) f is continuous at all x for which $h(x)$ is not equal to zero.

Q.13 $f(x) = \dfrac{2\cos x - \sin 2x}{(\pi - 2x)^2}$; $g(x) = \dfrac{e^{-\cos x} - 1}{8x - 4\pi}$

h (x) $= f(x)$ for $x < \pi/2$
$= g(x)$ for $x > \pi/2$

then which of the following holds?

(A) h is continuous at $x = \pi/2$
(B) h has an irremovable discontinuity at $x = \pi/2$
(C) h has a removable discontinuity at $x = \pi/2$
(D) $f\left(\dfrac{\pi}{2}^{+}\right) = g\left(\dfrac{\pi}{2}^{-}\right)$

Q.14 If $f(x) = \dfrac{x - e^x + \cos 2x}{x^2}$, $x \neq 0$ is continuous at x = 0, then

(A) $f(0) = \dfrac{5}{2}$
(B) $[f(0)] = -2$
(C) $\{f(0)\} = -0.5$
(D) $[f(0)] . \{f(0)\} = -1.5$

where [x] and {x} denotes greatest integer and fractional part function

Q.15 The function $g(x) = \begin{cases} x + b, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$ can be made differentiable at x = 0.

(A) if b is equal to zero
(B) if b is not equal to zero
(C) if b takes any real value
(D) for no value of b

Q.16 If $f(x) = \sin^{-1}(\sin x)$; $x \in R$ then f is

(A) continuous and differentiable for all x
(B) continuous for all x but not differentiable for all $x = (2k + 1)\dfrac{\pi}{2}$, $k \in I$
(C) neither continuous nor differentiable for $x = (2k - 1)\dfrac{\pi}{2}$; $k \in I$
(D) neither continuous nor differentiable for $x \in R - [-1, 1]$

Q.17 $\displaystyle \underset{x \to \frac{\pi}{2}}{\text{Limit}} \frac{\sin x}{\cos^{-1}\left[\frac{1}{4}(3\sin x - \sin 3x)\right]}$ where [ ] denotes greatest integer function , is

(A) $\dfrac{2}{\pi}$
(B) 1
(C) $\dfrac{4}{\pi}$
(D) does not exist

Q.18 $\displaystyle \underset{x \to 0}{\text{Lim}} \frac{\left(\sqrt{(1 - \cos x) + \sqrt{(1 - \cos x) + \sqrt{(1 - \cos x) + ...\infty}}}\right) - 1}{x^2}$ equals

(A) 0
(B) $\dfrac{1}{2}$
(C) 1
(D) 2

Q.19 Consider the function $f(x) = \begin{cases} x\{x\} + 1 & 0 \leq x < 1 \\ 2 - \{x\} & 1 \leq x \leq 2 \end{cases}$ where {x} denotes the fractional part function. Which one of the following statements is NOT correct?

(A) $\underset{x \to 1}{\text{Lim}} f(x)$ exists
(B) $f(0) \neq f(2)$
(C) f (x) is continuous in [0, 2]
(D) Rolles theorem is not applicable to f (x) in [0, 2]

Q.20 The function $f(x) = \lim\limits_{n\to\infty} \dfrac{x^{2n}-1}{x^{2n}+1}$ is identical with the function

(A) g (x) = sgn(x – 1) (B) h (x) = sgn $(\tan^{-1}x)$
(C) u (x) = sgn(| x | – 1) (D) v (x) = sgn $(\cot^{-1}x)$

Q.21 Which one of the following statement is true?

(A) If $\lim\limits_{x\to c} f(x)\cdot g(x)$ and $\lim\limits_{x\to c} f(x)$ exist, then $\lim\limits_{x\to c} g(x)$ exists.

(B) If $\lim\limits_{x\to c} f(x)\cdot g(x)$ exists, then $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ exist.

(C) If $\lim\limits_{x\to c} \left(f(x)+g(x)\right)$ and $\lim\limits_{x\to c} f(x)$ exist, then $\lim\limits_{x\to c} g(x)$ exist.

(D) If $\lim\limits_{x\to c} \left(f(x)+g(x)\right)$ exists, then $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ exist.

Q.22 The functions defined by $f(x) = \max\{x^2, (x-1)^2, 2x(1-x)\}$, $0 \le x \le 1$
(A) is differentiable for all x
(B) is differentiable for all x excetp at one point
(C) is differentiable for all x except at two points
(D) is not differentiable at more than two points.

Q.23 Which one of the following functions is continuous everywhere in its domain but has atleast one point where it is not differentiable?

(A) $f(x) = x^{1/3}$ (B) $f(x) = \dfrac{|x|}{x}$ (C) $f(x) = e^{-x}$ (D) f (x) = tan x

Q.24 The limiting value of the function $f(x) = \dfrac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$ when $x \to \dfrac{\pi}{4}$ is

(A) $\sqrt{2}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $3\sqrt{2}$ (D) $\dfrac{3}{\sqrt{2}}$

Q.25 Let $f(x) = \begin{cases} \dfrac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} & \text{if } x > 2 \\[2ex] \dfrac{x^2-4}{x-\sqrt{3x-2}} & \text{if } x < 2 \end{cases}$ then

(A) f (2) = 8 $\Rightarrow$ f is continuous at x = 2 (B) f (2) = 16 $\Rightarrow$ f is continuous at x = 2
(C) $f(2^-) \ne f(2^+)$ $\Rightarrow$ f is discontinuous (D) f has a removable discontinuity at x = 2

Q.26 If $\lim\limits_{x\to a}[f(x)+g(x)] = 2$ and $\lim\limits_{x\to a}[f(x)-g(x)] = 1$, then $\lim\limits_{x\to a} f(x)g(x)$

(A) need not exist (B) exist and is $\dfrac{3}{4}$ (C) exists and is $-\dfrac{3}{4}$ (D) exists and is $\dfrac{4}{3}$

Q.27 $\lim\limits_{x\to 1} \dfrac{\sin^2(x^3+x^2+x-3)}{1-\cos(x^2-4x+3)}$ has the value equal to

(A) 18 (B) 9/2 (C) 9 (D) none

Q.28 The graph of function f contains the point P (1, 2) and Q(s, r). The equation of the secant line through P and Q is $y = \left(\frac{s^2+2s-3}{s-1}\right)x - 1 - s$. The value of $f'(1)$, is

(A) 2 (B) 3 (C) 4 (D) non existent

Q.29 Consider $f(x) = \left[\frac{2\left(\sin x - \sin^3 x\right) + \left|\sin x - \sin^3 x\right|}{2\left(\sin x - \sin^3 x\right) - \left|\sin x - \sin^3 x\right|}\right]$, $x \neq \frac{\pi}{2}$ for $x \in (0, \pi)$

$f(\pi/2) = 3$ where [ ] denotes the greatest integer function then,

(A) f is continuous & differentiable at $x = \pi/2$

(B) f is continuous but not differentiable at $x = \pi/2$

(C) f is neither continuous nor differentiable at $x = \pi/2$

(D) none of these

Q.30 Let [x] denote the integral part of $x \in R$. $g(x) = x - [x]$. Let f(x) be any continuous function with $f(0) = f(1)$ then the function $h(x) = f(g(x))$

(A) has finitely many discontinuities (B) is discontinuous at some x = c

(C) is continuous on R (D) is a constant function.

Q.31 If $f(x + y) = f(x) + f(y) + |x|y + xy^2$, $\forall$ x, y $\in$ R and $f'(0) = 0$, then

(A) f need not be differentiable at every non zero x

(B) f is differentiable for all x $\in$ R

(C) f is twice differentiable at x = 0

(D) none

Q.32 $\lim\limits_{x \to e^+} (\ln x)^{\frac{1}{x-e}}$ is

(A) $e^{\frac{1}{e}}$ (B) e^e (C) e^{-2} (D) e^{-1}

Q.33 $\lim\limits_{n \to \infty} \frac{1^2 n + 2^2(n-1) + 3^2(n-2) + \ldots.. + n^2.1}{1^3 + 2^3 + 3^3 + \ldots\ldots + n^3}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

Q.34 Let f be a differentiable function on the open interval (a, b). Which of the following statements must be true?

I. f is continuous on the closed interval [a, b]

II. f is bounded on the open interval (a, b)

III. If $a<a_1<b_1<b$, and $f(a_1)<0<f(b_1)$, then there is a number c such that $a_1<c<b_1$ and $f(c)=0$

(A) I and II only (B) I and III only (C) II and III only (D) only III

Q.35 The value of $\underset{x \to \infty}{\text{Limit}} \frac{\cot^{-1}\left(x^{-a} \log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}$ (a > 1) is equal to

(A) 1 (B) 0 (C) $\pi/2$ (D) does not exist

Q.36 Let $f(x) = \max. \left\{ \left| x^2 - 2|x| \right|, |x| \right\}$ and $g(x) = \min. \left\{ \left| x^2 - 2|x| \right|, |x| \right\}$ then

(A) both f (x) and g (x) are non differentiable at 5 points.
(B) f (x) is not differentiable at 5 points whether g (x) is non differentiable at 7 points.
(C) number of points of non differentibility for f (x) and g (x) are 7 and 5 respectively.
(D) both f (x) and g (x) are non differentiable at 3 and 5 points respectively.

Q.37 If $\underset{x\to 0}{\text{Limit}} \dfrac{\sin nx\,[(a-n).nx - \tan x]}{x^2} = 0 \quad (n > 0)$ then the value of 'a' is equal to

(A) $\dfrac{1}{n}$ (B) $n^2 + 1$ (C) $\dfrac{n^2+1}{n}$ (D) none

Q.38 Let $g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \ge 1 \end{cases}$.

If g (x) is the continuous and differentiable for all numbers in its domain then
(A) a = b = 4 (B) a = b = – 4
(C) a = 4 and b = – 4 (D) a = – 4 and b = 4

Q.39 Let $f(x) = \begin{cases} a\sin^{2n} x & \text{for } x \ge 0 \text{ and } n \to \infty \\ b(\cos^{2m} x) - 1 & \text{for } x < 0 \text{ and } m \to \infty \end{cases}$ then

(A) $f(0^-) \ne f(0^+)$ (B) $f(0^+) \ne f(0)$
(C) $f(0^-) = f(0)$ (D) f is continuous at x = 0

Q.40 Let $f(x)$ be continuous and differentiable function for all reals.

$f(x + y) = f(x) - 3xy + f(y)$. If $\underset{h\to 0}{\text{Lim}} \dfrac{f(h)}{h} = 7$, then the value of $f'(x)$ is

(A) – 3x (B) 7 (C) – 3x + 7 (D) $2f(x) + 7$

Q.41 Let $a = \min [x^2 + 2x + 3,\ x \in R]$ and $b = \underset{x\to 0}{\text{Lim}} \dfrac{\sin x \cos x}{e^x - e^{-x}}$. Then the value of $\sum_{r=0}^{n} a^r b^{n-r}$ is

(A) $\dfrac{2^{n+1}+1}{3\cdot 2^n}$ (B) $\dfrac{2^{n+1}-1}{3\cdot 2^n}$ (C) $\dfrac{2^n-1}{3\cdot 2^n}$ (D) $\dfrac{4^{n+1}-1}{3\cdot 2^n}$

Q.42 Given $l_1 = \underset{x\to \pi/4}{\text{Lim}} \cos^{-1}\left[\sec\left(x - \dfrac{\pi}{4}\right)\right]$; $l_2 = \underset{x\to \pi/4}{\text{Lim}} \sin^{-1}\left[\text{cosec}\left(x + \dfrac{\pi}{4}\right)\right]$;

$l_3 = \underset{x\to \pi/4}{\text{Lim}} \tan^{-1}\left[\cot\left(x + \dfrac{\pi}{4}\right)\right]$; $l_4 = \underset{x\to \pi/4}{\text{Lim}} \cot^{-1}\left[\tan\left(x - \dfrac{\pi}{4}\right)\right]$

where [x] denotes greatest integer function then which of the following limits exist?
(A) l_1 and l_2 only (B) l_1 and l_3 only (C) l_1 and l_4 only (D) All of them

Q.43 Suppose that a and b $(b \neq a)$ are real positive numbers then the value of

$\lim\limits_{t \to 0} \left(\dfrac{b^{t+1} - a^{t+1}}{b - a} \right)^{1/t}$ has the value equals to

(A) $\dfrac{a \ln b - b \ln a}{b - a}$ (B) $\dfrac{b \ln b - a \ln a}{b - a}$ (C) $b \ln b - a \ln a$ (D) $\left(\dfrac{b^b}{a^a}\right)^{\frac{1}{b-a}}$

Q.44 Which of the following functions defined below are NOT differentiable at the indicated point?

(A) $f(x) = \begin{cases} x^2 & \text{if } -1 \le x < 0 \\ -x^2 & \text{if } 0 \le x \le 1 \end{cases}$ at $x = 0$ (B) $g(x) = \begin{cases} x & \text{if } -1 \le x < 0 \\ \tan x & \text{if } 0 \le x \le \frac{\pi}{4} \end{cases}$ at $x = 0$

(C) $h(x) = \begin{cases} \sin 2x & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$ at $x = 0$ (D) $k(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } 1 < x \le 2 \end{cases}$ at $x = 1$

Q.45 If $f(x) = \cos x,\ x = n\pi,\ n = 0, 1, 2, 3, \ldots..$
$= 3$, otherwise and

$\phi(x) = \begin{cases} x^2 + 1 & \text{when } x \neq 3, x \neq 0 \\ 3 & \text{when } x = 0 \\ 5 & \text{when } x = 3 \end{cases}$ then $\underset{x \to 0}{\text{Limit}}\ f(\phi(x)) =$

(A) 1 (B) 3 (C) 5 (D) none

Q.46 Let $[x]$ be the greatest integer function and $f(x) = \dfrac{\sin \frac{1}{4} \pi [x]}{[x]}$. Then which one of the following does not hold good?

(A) not continuous at any point (B) continuous at 3/2
(C) discontinuous at 2 (D) differentiable at 4/3

Q.47 Number of points where the function $f(x) = (x^2 - 1)\,|x^2 - x - 2| + \sin(|x|)$ is not differentiable, is
(A) 0 (B) 1 (C) 2 (D) 3

Q.48 $\underset{x \to 0}{\text{Limit}} \left(\dfrac{3}{1 + \sqrt{4 + x}} \right)^{\operatorname{cosec} x}$ has the value equal to :

(A) $e^{-1/12}$ (B) $e^{-1/6}$ (C) $e^{-1/4}$ (D) $e^{-1/3}$

Q.49 $\underset{x \to \infty}{\text{Limit}} \dfrac{\cot^{-1}\left(\sqrt{x+1} - \sqrt{x}\right)}{\sec^{-1}\left\{\left(\dfrac{2x+1}{x-1}\right)^x\right\}}$ is equal to

(A) 1 (B) 0 (C) $\pi/2$ (D) non existent

Q.50 Consider function $f : R - \{-1, 1\} \to R$. $f(x) = \dfrac{x}{1 - |x|}$. Then the incorrect statement is

(A) it is continuous at the origin. (B) it is not derivable at the origin.
(C) the range of the function is R. (D) f is continuous and derivable in its domain

Q.51 Given $f(x) = b([x]^2 + [x]) + 1$ for $x \geq -1$
$= \sin(\pi(x+a))$ for $x < -1$
where $[x]$ denotes the integral part of x, then for what values of a, b the function is continuous at $x = -1$?
(A) $a = 2n + (3/2)$; $b \in R$; $n \in I$
(B) $a = 4n + 2$; $b \in R$; $n \in I$
(C) $a = 4n + (3/2)$; $b \in R^+$; $n \in I$
(D) $a = 4n + 1$; $b \in R^+$; $n \in I$

Q.52 $\lim\limits_{n\to\infty} \cos\left(\pi\sqrt{n^2+n}\right)$ when n is an integer :
(A) is equal to 1 (B) is equal to – 1 (C) is equal to zero (D) does not exist

Q.53 $\text{Limit}_{x\to 0} \left(1 + \log^2_{\cos\frac{x}{2}} \cos x\right)^2$
(A) is equal to 4 (B) is equal to 9 (C) is equal to 289 (D) is non existent

Q.54 The value of $\text{Limit}_{x\to 0} \dfrac{\left(\tan\left(\{x\}-1\right)\right)\sin\{x\}}{\{x\}\left(\{x\}-1\right)}$ where $\{x\}$ denotes the fractional part function:
(A) is 1 (B) is tan 1 (C) is sin 1 (D) is non existent

Q.55 $\text{Limit}_{x\to 0^-} \left(-\ell n\left(\{x\}+|[x]|\right)\right)^{\{x\}}$
(A) is 0 (B) is 1 (C) $\ln 2$ (D) is $\ln\frac{1}{2}$
where [ ] is the greatest integer function and { } is the fractional part .

Q.56 If $f(x) = \dfrac{\ell n\left(e^{x^2} + 2\sqrt{x}\right)}{\tan\sqrt{x}}$ is continuous at $x = 0$, then $f(0)$ must be equal to :
(A) 0 (B) 1 (C) e^2 (D) 2

Q.57 $\lim\limits_{x\to\infty} \dfrac{2+2x+\sin 2x}{(2x+\sin 2x)e^{\sin x}}$ is :
(A) equal to zero (B) equal to 1 (C) equal to – 1 (D) non existent

Q.58 The value of $\lim\limits_{x\to 0} (\cos ax)^{\operatorname{cosec}^2 bx}$ is
(A) $e^{\left(-\frac{8b^2}{a^2}\right)}$ (B) $e^{\left(-\frac{8a^2}{b^2}\right)}$ (C) $e^{\left(-\frac{a^2}{2b^2}\right)}$ (D) $e^{\left(-\frac{b^2}{2a^2}\right)}$

Q.59 If $f(x + y) = f(x) + f(y) + c$, for all real x and y and f(x) is continuous at x = 0 and $f'(0) = 1$ then $f'(x)$ equals to
(A) c (B) –1 (C) 0 (D) 1

Q.60 If x is a real number in [0, 1] then the value of $\text{Limit}_{m\to\infty} \text{Limit}_{n\to\infty} [1 + \cos^{2m}(n!\,\pi x)]$ is given by
(A) 1 or 2 according as x is rational or irrational
(B) 2 or 1 according as x is rational or irrational
(C) 1 for all x
(D) 2 for all x

Q.61 $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2+n+r}$ equals

(A) 0 (B) 1/3 (C) 1/2 (D) 1

Q.62 $\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\left[\sqrt{n^2+n+1}\right]\right)$ where [ ] denotes the greatest integer function is

(A) 0 (B) 1/2 (C) 2/3 (D) 1/4

Q.63 $\lim_{n\to\infty}\left(\frac{\sqrt[n]{p}+\sqrt[n]{q}}{2}\right)^n$, p, q > 0 equals

(A) 1 (B) $\sqrt{pq}$ (C) pq (D) $\frac{pq}{2}$

Q.64 Let f (x) be the continuous function such that $f(x) = \frac{1-e^x}{x}$ for $x \neq 0$ then

(A) $f'(0^+) = \frac{1}{2}$ and $f'(0^-) = -\frac{1}{2}$ (B) $f'(0^+) = -\frac{1}{2}$ and $f'(0^-) = \frac{1}{2}$

(C) $f'(0^+) = f'(0^-) = \frac{1}{2}$ (D) $f'(0^+) = f'(0^-) = -\frac{1}{2}$

[COMPREHENSION TYPE]

Paragraph for question nos 65 to 67

Let $f(x) = \begin{cases} e^{\{x^2\}}-1, & x>0 \\ \dfrac{\sin x - \tan x + \cos x - 1}{2x^2 + \ln(2+x) + \tan x}, & x<0 \\ 0, & x=0 \end{cases}$

where { } represents fractional part function. Suppose lines L_1 and L_2 represent tangent and normal to curve $y = f(x)$ at x = 0. Consider the family of circles touching both the lines L_1 and L_2.

Q.65 Ratio of radii of two circles belonging to this family cutting each other orthogonally is

(A) $2+\sqrt{3}$ (B) $\sqrt{3}$ (C) $2+\sqrt{2}$ (D) $2-\sqrt{2}$

Q.66 A circle having radius unity is inscribed in the triangle formed by L_1 and L_2 and a tangent to it. Then the minimum area of the triangle possible is

(A) $3+\sqrt{2}$ (B) $2+\sqrt{3}$ (C) $3+2\sqrt{2}$ (D) $3-2\sqrt{2}$

Q.67 If centers of circles belonging to family having equal radii 'r' are joined, the area of figure formed is

(A) $2r^2$ (B) $4r^2$ (C) $8r^2$ (D) r^2

Paragraph for Question Nos. 68 to 70

Let f(x) is a function continuous for all $x \in R$ except at $x = 0$. Such that $f'(x) < 0 \ \forall \ x \in (-\infty, 0)$ and $f'(x) > 0 \ \forall x \in (0, \infty)$. Let $\lim\limits_{x \to 0^+} f(x) = 2$, $\lim\limits_{x \to 0^-} f(x) = 3$ and $f(0) = 4$.

Q.68 The value of λ for which $2\left(\lim\limits_{x \to 0} f(x^3 - x^2)\right) = \lambda\left(\lim\limits_{x \to 0} f(2x^4 - x^5)\right)$ is

(A) $\frac{4}{3}$ (B) 2 (C) 3 (D) 5

Q.69 The values of $\lim\limits_{x \to 0^+} \frac{f(-x)\,x^2}{\left\{\frac{1-\cos x}{[f(x)]}\right\}}$ where [·] denote greatest integer function and { · } denote fraction part function.

(A) 6 (B) 12 (C) 18 (D) 24

Q.70 $\lim\limits_{x \to 0^-} \left(\left[3f\left(\frac{x^3 - \sin^3 x}{x^4}\right)\right] - f\left(\left[\frac{\sin x^3}{x}\right]\right)\right)$ where [·] denote greatest integer function.

(A) 3 (B) 5 (C) 7 (D) 9

[REASONING TYPE]

Q.71 Let $h(x) = f_1(x) + f_2(x) + f_3(x) + \ldots\ldots + f_n(x)$ where $f_1(x), f_2(x), f_3(x), \ldots\ldots, f_n(x)$ are real valued functions of x.

Statement-1: $f(x) = \left|\cos|x|\right| + \cos^{-1}(\operatorname{sgn} x) + \left|\ln x\right|$ is not differentiable at 3 points in $(0, 2\pi)$

because

Statement-2: Exactly one function $f_i(x)$, $i = 1, 2, \ldots\ldots, n$ not differentiable and the rest of the function differentiable at $x = a$ makes h (x) not differentiable at $x = a$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.72 **Statement-1 :** $f(x) = |x| \sin x$ is differentiable at $x = 0$

because

Statement-2 : If g (x) is not differentiable at x= a and h (x) is differentiable at x = a then $g(x) \cdot h(x)$ can not be differentiable at x = a.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.73 Statement-1: $f(x) = |\cos x|$ is not deviable at $x = \frac{\pi}{2}$.

because

Statement-2: If g (x) is differentiable at x = a and g (a) = 0 then | g(x) | is non-derivable at x = a.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.74 Let $f(x) = x - x^2$ and $g(x) = \{x\}\ \forall\ x \in R$. Where $\{\cdot\}$ denotes fractional part function.

Statement-1: $f(g(x))$ will be continuous $\forall\ x \in R$.

because

Statement-2: $f(0) = f(1)$ and $g(x)$ is periodic with period 1.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.75 Let $f(x) = \begin{cases} -ax^2 - b|x| - c & -\alpha \le x < 0 \\ ax^2 + b|x| + c & 0 \le x \le \alpha \end{cases}$ where a, b, c are positive and $\alpha > 0$, then

Statement-1: The equation $f(x) = 0$ has atleast one real root for $x \in [-\alpha, \alpha]$

because

Statement-2: Values of $f(-\alpha)$ and $f(\alpha)$ are opposite in sign.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

[MULTIPLE OBJECTIVE TYPE]

Q.76 $\lim\limits_{x \to c} f(x)$ does not exist when

(A) $f(x) = [[x]] - [2x - 1],\ c = 3$
(B) $f(x) = [x] - x,\ c = 1$
(C) $f(x) = \{x\}^2 - \{-x\}^2,\ c = 0$
(D) $f(x) = \dfrac{\tan(\text{sgn}\, x)}{\text{sgn}\, x},\ c = 0$.

where [x] denotes step up function & {x} fractional part function.

Q.77 Let $f(x) = \begin{cases} \dfrac{\tan^2\{x\}}{x^2 - [x]^2} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{\{x\}\cot\{x\}} & \text{for } x < 0 \end{cases}$ where $[x]$ is the step up function and $\{x\}$ is the fractional part function of x, then :

(A) $\lim\limits_{x \to 0^+} f(x) = 1$
(B) $\lim\limits_{x \to 0^-} f(x) = 1$
(C) $\cot^{-1}\left(\lim\limits_{x \to 0^-} f(x)\right)^2 = 1$
(D) f is continuous at $x = 1$.

Q.78 If $f(x) = \begin{cases} \dfrac{x \cdot \ell n(\cos x)}{\ell n(1 + x^2)} & x \ne 0 \\ 0 & x = 0 \end{cases}$ then :

(A) f is continuous at $x = 0$
(B) f is continuous at $x = 0$ but not differentiable at $x = 0$
(C) f is differentiable at $x = 0$
(D) f is not continuous at $x = 0$.

Q.79 Which of the following limits vanish?

(A) $\underset{x \to \infty}{\text{Limit}}\ x^{\frac{1}{4}} \sin \frac{1}{\sqrt{x}}$ (B) $\underset{x \to \pi/2}{\text{Limit}}\ (1 - \sin x) . \tan x$

(C) $\underset{x \to \infty}{\text{Limit}}\ \frac{2x^2 + 3}{x^2 + x - 5} . \text{sgn}(x)$ (D) $\underset{x \to 3^+}{\text{Limit}}\ \frac{[x]^2 - 9}{x^2 - 9}$

where [ ] denotes greatest integer function

Q.80 Which of the following function(s) not defined at $x = 0$ has/have non-removable discontinuity at the point $x = 0$?

(A) $f(x) = \frac{1}{1 + 2^{\frac{1}{x}}}$ (B) $f(x) = \text{arc} \tan \frac{1}{x}$ (C) $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ (D) $f(x) = \frac{1}{\ell n|x|}$

Q.81 Which of the following function(s) not defined at x = 0 has/have removable discontinuity at x=0?

(A) $f(x) = \frac{1}{1 + 2^{\cot x}}$ (B) $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$ (C) $f(x) = x \sin \frac{\pi}{x}$ (D) $f(x) = \frac{1}{\ell n|x|}$

Q.82 The function $1 + |\sin x|$ is

(A) continuous everywhere (B) differentiable nowhere

(C) not differentiable at $x = 0$ (D) not differentiable at infinite no. of points

Q.83 Let $[x]$ denote the greatest integer less than or equal to x . If $f(x) = [x \text{ Sin } \pi x]$, then f(x) is:

(A) continuous at $x = 0$ (B) continuous in $(-1, 0)$

(C) differentiable at $x = 1$ (D) differentiable in $(-1, 1)$

Q.84 The function, $f(x) = [\,|x|\,] - |[x]|$ where $[x]$ denotes greatest integer function

(A) is continuous for all positive integers

(B) is discontinuous for all non positive integers

(C) has finite number of elements in its range

(D) is such that its graph does not lie above the x – axis.

Q.85 Let $f(x + y) = f(x) + f(y)$ for all $x, y \in R$. Then

(A) $f(x)$ must be continuous $\forall x \in R$ (B) $f(x)$ may be continuous $\forall x \in R$

(C) $f(x)$ must be discontinuous $\forall x \in R$ (D) $f(x)$ may be discontinuous $\forall x \in R$

Q.86 Given that the derivative $f'(a)$ exists. Indicate which of the following statement(s) is/are always True

(A) $f'(a) = \underset{h \to a}{\text{Limit}} \frac{f(h) - f(a)}{h - a}$ (B) $f'(a) = \underset{h \to 0}{\text{Limit}} \frac{f(a) - f(a - h)}{h}$

(C) $f'(a) = \underset{t \to 0}{\text{Limit}} \frac{f(a + 2t) - f(a)}{t}$ (D) $f'(a) = \underset{t \to 0}{\text{Limit}} \frac{f(a + 2t) - f(a + t)}{2t}$

Q.87 The function $f(x) = \sqrt{1-\sqrt{1-x^2}}$

(A) has its domain $-1 \le x \le 1$.

(B) has finite one sided derivates at the point x = 0.

(C) is continuous and differentiable at x = 0.

(D) is continuous but not differentiable at x = 0.

Q.88 Consider the function $f(x) = |x^3 + 1|$ then

(A) Domain of f $x \in R$

(B) Range of f is R^+

(C) f has no inverse.

(D) f is continuous and differentiable for every $x \in R$.

Q.89 Select the correct statements.

(A) The function f defined by $f(x) = \begin{cases} 2x^2+3 & \text{for } x \le 1 \\ 3x+2 & \text{for } x > 1 \end{cases}$ is neither differentiable nor continuous at $x = 1$.

(B) The function $f(x) = x^2\,|x|$ is twice differentiable at x = 0.

(C) If f is continuous at x = 5 and $f(5) = 2$ then $\lim\limits_{x\to 2} f(4x^2-11)$ exists.

(D) If $\lim\limits_{x\to a}\left(f(x)+g(x)\right) = 2$ and $\lim\limits_{x\to a}\left(f(x)-g(x)\right) = 1$ then $\lim\limits_{x\to a} f(x)\cdot g(x)$ need not exist.

Q.90 Assume that $\lim\limits_{\theta\to -1} f(\theta)$ exists and $\dfrac{\theta^2+\theta-2}{\theta+3} \le \dfrac{f(\theta)}{\theta^2} \le \dfrac{\theta^2+2\theta-1}{\theta+3}$ holds for certain interval containing the point θ = – 1 then $\lim\limits_{\theta\to -1} f(\theta)$

(A) is equal to $f(-1)$

(B) is equal to 1

(C) is non existent

(D) is equal to – 1

Q.91 f is a continous function in [a, b]; g is a continuous functin in [b, c]

A function h (x) is defined as

$h(x) = f(x)$ for $x \in [a, b)$

$= g(x)$ for $x \in (b, c]$

if f (b) = g (b), then

(A) h(x) has a removable discontinuity at x=b.

(B) h(x) may or may not be continuous in [a, c]

(C) $h(b^-) = g(b^+)$ and $h(b^+) = f(b^-)$

(D) $h(b^+) = g(b^-)$ and $h(b^-) = f(b^+)$

Q.92 Which of the following function(s) has/have the same range?

(A) $f(x) = \dfrac{1}{1+x}$

(B) $f(x) = \dfrac{1}{1+x^2}$

(C) $f(x) = \dfrac{1}{1+\sqrt{x}}$

(D) $f(x) = \dfrac{1}{\sqrt{3-x}}$

[MATCH THE COLUMN]

Q.93 **Column–I** **Column-II**

(A) $\displaystyle\lim_{x\to 1}\frac{\ln x}{x^4-1}$ (P) 2

(B) $\displaystyle\lim_{x\to 0}\frac{3e^x-x^3-3x-3}{\tan^2 x}$ (Q) $\frac{2}{3}$

(C) $\displaystyle\lim_{x\to \infty}\frac{\pi-2\tan^{-1}x}{\ln\left(1+\frac{1}{x}\right)}$ (R) $\frac{3}{2}$

(D) $\displaystyle\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x(\cos x-\cos 2x)}$ (S) $\frac{1}{4}$

(E) $\displaystyle\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}$

Q.94 **Column-I** contains 4 functions and **column-II** contains comments w.r.t their continuity and differentiability at x = 0. Note that **column-I** may have more than one matching options in **column-II**.

Column-I **Column-II**

(A) $f(x) = [x] + |1-x|$ (P) continuous

[ ] denotes the greatest integer function

(B) $g(x) = |x-2| + |x|$ (Q) differentiable

(C) $h(x) = [\tan^2 x]$ (R) discontinuous

[ ] denotes the greatest integer function

(D) $l(x) = \begin{cases} \dfrac{x(3e^{1/x}+4)}{(2-e^{1/x})} & x \neq 0 \\ 0 & x = 0 \end{cases}$ (S) non derivable

Q.95 **Column I** **Column II**

(A) $\displaystyle\lim_{x\to \infty}\left(\frac{x}{1+x}\right)^x$ equals (P) e^2

(B) $\displaystyle\lim_{x\to \infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x$ (Q) $e^{-1/2}$

(C) $\displaystyle\lim_{x\to 0}(\cos x)^{\cot^2 x}$ (R) e

(D) $\displaystyle\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ (S) e^{-1}

Q.96 **Column-I** **Column-II**

(A) $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ \cos x & \text{if } x \ge 0 \end{cases}$ at x = 0 is — (P) continuous

(B) For every x ∈ R the function — (Q) differentiability

$g(x) = \dfrac{\sin(\pi[x-\pi])}{1+[x]^2}$ — (R) discontinuous

where [x] denotes the greatest integer function is — (S) non derivable

(C) $h(x) = \sqrt{\{x\}^2}$ where {x} denotes fractional part function for all x ∈ I, is

(D) $k(x) = \begin{cases} x^{\frac{1}{\ln x}} & \text{if } x \ne 1 \\ e & \text{if } x = 1 \end{cases}$ at x = 1 is

Q.97 **Column-I** **Column-II**

(A) $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}\right)$ equals — (P) -2

(B) The value of the limit, $\lim\limits_{x\to 0}\dfrac{\sin 2x - 2\tan x}{\ln(1+x^3)}$ is — (Q) -1

(C) $\lim\limits_{x\to 0^+}\left(\ln\sin^3 x - \ln(x^4+ex^3)\right)$ equals — (R) 0

(D) Let $\tan(2\pi\,|\sin\theta|) = \cot(2\pi\,|\cos\theta|)$, where $\theta \in R$ — (S) 1

and $f(x) = (|\sin\theta| + |\cos\theta|)^x$. The value of $\lim\limits_{x\to\infty}\left[\dfrac{2}{f(x)}\right]$ equals

(Here [ ] represents greatest integer function)

Q.98 **Column-I** **Column-II**

(A) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{e^{x^2}-e^x+x}$ equals — (P) 1

(B) If the value of $\lim\limits_{x\to 0^+}\left(\dfrac{(3/x)+1}{(3/x)-1}\right)^{1/x}$ can be expressed in the — (Q) 2

form of $e^{p/q}$, where p and q are relative prime then (p + q) is equal to

(C) $\lim\limits_{x\to 0}\dfrac{\tan^3 x - \tan x^3}{x^5}$ equals — (R) 4

(D) $\lim\limits_{x\to 0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x - x + 1}}$ — (S) 5

Q.99 **Column-I** **Column-II**

(A) $\lim_{n\to\infty} \cos^2\left(\pi\left(\sqrt[3]{n^3+n^2+2n}\right)\right)$ where *n* is an integer, equals (P) $\frac{1}{2}$

(B) $\lim_{n\to\infty} n\sin\left(2\pi\sqrt{1+n^2}\right)$ $(n \in N)$ equals (Q) $\frac{1}{4}$

(C) $\lim_{n\to\infty} (-1)^n \sin\left(\pi\sqrt{n^2+0.5n+1}\right)\left(\sin\frac{(n+1)\pi}{4n}\right)$ is (where $n \in N$) (R) π

(D) If $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x = e$ where 'a' is some real constant then the value of 'a' is equal to (S) non existent

Q.100 **Column-I** **Column-II**

(A) $\lim_{x\to\infty}\left(e^{\sqrt{x^4+1}} - e^{(x^2+1)}\right)$ is (P) e

(B) For a > 0 let $f(x) = \begin{cases} \dfrac{a^x + a^{-x} - 2}{x^2} & \text{if } x > 0 \\ 3\ln(a-x) - 2, & \text{if } x \le 0 \end{cases}$ (Q) e^2

If f is continuous at x = 0 then 'a' equals (R) 1/e

(C) Let $L = \lim_{x\to a}\frac{x^x - a^a}{x-a}$ and $M = \lim_{x\to a}\frac{x^x - a^x}{x-a}$ $(a > 0)$.
If L = 2M then the value of 'a' is equal to (S) non existent

ANSWER KEY

[STRAIGHT OBJECTIVE TYPE]

Q.1	C	Q.2	A	Q.3	D	Q.4	A	Q.5	A	Q.6	C	Q.7	B
Q.8	B	Q.9	D	Q.10	C	Q.11	C	Q.12	D	Q.13	B	Q.14	D
Q.15	D	Q.16	B	Q.17	A	Q.18	B	Q.19	C	Q.20	C	Q.21	C
Q.22	C	Q.23	A	Q.24	D	Q.25	C	Q.26	B	Q.27	A	Q.28	C
Q.29	A	Q.30	C	Q.31	B	Q.32	A	Q.33	A	Q.34	D	Q.35	A
Q.36	B	Q.37	C	Q.38	C	Q.39	A	Q.40	C	Q.41	D	Q.42	A
Q.43	D	Q.44	D	Q.45	B	Q.46	A	Q.47	C	Q.48	A	Q.49	A
Q.50	B	Q.51	A	Q.52	C	Q.53	C	Q.54	D	Q.55	D	Q.56	D
Q.57	D	Q.58	C	Q.59	D	Q.60	B	Q.61	C	Q.62	B	Q.63	B
Q.64	D	Q.65	A	Q.66	C	Q.67	B	Q.68	C	Q.69	B	Q.70	B
Q.71	A	Q.72	C	Q.73	C	Q.74	A	Q.75	D				

[MULTIPLE OBJECTIVE TYPE]

Q.76	B,C	Q.77	A,C	Q.78	A,C	Q.79	A,B,D
Q.80	A,B,C	Q.81	B,C,D	Q.82	A, C, D	Q.83	A, B, D
Q.84	A,B,C,D	Q.85	B,D	Q.86	A,B	Q.87	A,B,D
Q.88	A,C	Q.89	B,C	Q.90	A,D	Q.91	A,C
Q.92	B,C						

[MATCH THE COLUMN]

Q.93	(A) S; (B) R; (C) P; (D) Q ; (E) P	Q.94	(A) R, S; (B) P, S; (C) P, Q; (D) P, S
Q.95	(A) S; (B) R; (C) Q; (D) P	Q.96	(A) P, S; (B) P, Q; (C) R, S; (D) P, Q
Q.97	(A) S; (B) P ; (C) Q; (D) R	Q.98	(A) R; (B) S; (C) P; (D) Q
Q.99	(A) Q; (B) R; (C) P; (D) P	Q.100	(A) S ; (B) P, Q ; (C) P