QUESTION BANK

MOD & INDEFINITE INTEGRATION

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EX. SR. FACULTY (BANSAL CLASSES)

[STRAIGHT OBJECTIVE TYPE]

Q.1 If $y = \tan^{-1}\left(\dfrac{\ell n \frac{e}{x^2}}{\ell n e x^2}\right) + \tan^{-1}\dfrac{3 + 2\ell n x}{1 - 6\ell n x}$ then $\dfrac{d^2y}{dx^2} =$

(A) 2 (B) 1 (C) 0 (D) -1

Q.2 Let $u(x)$ and $v(x)$ are differentiable functions such that $\dfrac{u(x)}{v(x)} = 7$. If $\dfrac{u'(x)}{v'(x)} = p$ and $\left(\dfrac{u(x)}{v(x)}\right)' = q$, then $\dfrac{p+q}{p-q}$ has the value equal to

(A) 1 (B) 0 (C) 7 (D) -7

Q.3 Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$ where $f(x)$ is continuously differentiable function with $f'(x) \neq 0$ and satisfies $f(0) = 1$ and $f'(0) = 2$ then $f(x)$ is

(A) $x^2 + 2x + 1$ (B) $2e^x - 1$ (C) e^{2x} (D) $4e^{x/2} - 3$

Q.4 If $y = f\left(\dfrac{3x+4}{5x+6}\right)$ & $f'(x) = \tan x^2$ then $\dfrac{dy}{dx} =$

(A) $\tan x^3$

(B) $-2\tan\left[\dfrac{3x+4}{5x+6}\right]^2 \cdot \dfrac{1}{(5x+6)^2}$

(C) $f\left(\dfrac{3\tan x^2 + 4}{5\tan x^2 + 6}\right)\tan x^2$

(D) none

Q.5 If $x = t^3 + t + 5$ & $y = \sin t$ then $\dfrac{d^2y}{dx^2} =$

(A) $-\dfrac{(3t^2+1)\sin t + 6t\cos t}{(3t^2+1)^3}$

(B) $\dfrac{(3t^2+1)\sin t + 6t\cos t}{(3t^2+1)^2}$

(C) $-\dfrac{(3t^2+1)\sin t + 6t\cos t}{(3t^2+1)^2}$

(D) $\dfrac{\cos t}{3t^2+1}$

Q.6 Let g is the inverse function of f & $f'(x) = \dfrac{x^{10}}{(1+x^2)}$. If $g(2) = a$ then $g'(2)$ is equal to

(A) $\dfrac{5}{2^{10}}$ (B) $\dfrac{1+a^2}{a^{10}}$ (C) $\dfrac{a^{10}}{1+a^2}$ (D) $\dfrac{1+a^{10}}{a^2}$

Q.7 $\displaystyle\int \dfrac{\cot^{-1}(e^x)}{e^x}\,dx$ is equal to :

(A) $\dfrac{1}{2}\ln(e^{2x}+1) - \dfrac{\cot^{-1}(e^x)}{e^x} + x + c$

(B) $\dfrac{1}{2}\ln(e^{2x}+1) + \dfrac{\cot^{-1}(e^x)}{e^x} + x + c$

(C) $\dfrac{1}{2}\ln(e^{2x}+1) - \dfrac{\cot^{-1}(e^x)}{e^x} - x + c$

(D) $\dfrac{1}{2}\ln(e^{2x}+1) + \dfrac{\cot^{-1}(e^x)}{e^x} - x + c$

Q.8 If $y = \frac{1}{2x^2 + 3x + 1}$ then $\frac{d^2y}{dx^2}$ at $x = -2$ is :

(A) $\frac{38}{27}$ (B) $-\frac{38}{27}$ (C) $\frac{27}{38}$ (D) none

Q.9 The function $f(x) = \frac{1 - \cos x(\cos 2x)^{1/2}(\cos 3x)^{1/3}}{x^2}$ is not defined at x = 0. If $f(x)$ is continuous at x = 0 then $f(0)$ equals

(A) 1 (B) 3 (C) 6 (D) – 6

Q.10 $\int \frac{1 - x^7}{x(1 + x^7)}\, dx$ equals :

(A) $\ln x + \frac{2}{7} \ln(1 + x^7) + c$ (B) $\ln x - \frac{2}{7} \ln(1 - x^7) + c$

(C) $\ln x - \frac{2}{7} \ln(1 + x^7) + c$ (D) $\ln x + \frac{2}{7} \ln(1 - x^7) + c$

Q.11 If $f(x) = \frac{a + \sqrt{a^2 - x^2} + x}{\sqrt{a^2 - x^2} + a - x}$ where a > 0 and x < a, then $f'(0)$ has the value equal to

(A) $\sqrt{a}$ (B) a (C) $\frac{1}{\sqrt{a}}$ (D) $\frac{1}{a}$

Q.12 Suppose that f (0) = 0 and f ' (0) = 2, and let g (x) = $f(-x + f(f(x)))$. The value of g ' (0) is equal to

(A) 0 (B) 1 (C) 6 (D) 8

Q.13 $\int \frac{x\,dx}{\sqrt{1 + x^2 + \sqrt{(1 + x^2)^3}}}$ is equal to :

(A) $\frac{1}{2} \ln\left(1 + \sqrt{1 + x^2}\right) + c$ (B) $2\sqrt{1 + \sqrt{1 + x^2}} + c$

(C) $2\left(1 + \sqrt{1 + x^2}\right) + c$ (D) none of these

Q.14 If $\frac{x + a}{2} = b \cot^{-1}(b \ln y)$, b > 0 then, value of yy'' + yy' ln y equals

(A) y' (B) y'^2 (C) 0 (D) 1

Q.15 If $y^2 = P(x)$, is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^3 \cdot \frac{d^2y}{dx^2}\right)$ equals

(A) $P'''(x) + P'(x)$ (B) $P''(x) \cdot P'''(x)$ (C) $P(x) \cdot P'''(x)$ (D) a constant

Q.16 Let F(x) be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ w.r.t. x. If $F(10) = 60$ then the value of $F(13)$, is

(A) 66 (B) 132 (C) 248 (D) 264

Q.17 If $f(x) = |x-2|$ & $g(x) = f[f(x)]$ then for $x > 20$, $g'(x) =$

(A) 1 (B) -1 (C) 0 (D) none

Q.18 Let $f(x) = \begin{cases} g(x) \cdot \cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ where g(x) is an even function differentiable at x = 0, passing through the origin. Then $f'(0)$

(A) is equal to 1 (B) is equal to 0 (C) is equal to 2 (D) does not exist

Q.19 If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x}\,dx = \ln(f(x)) + g(x) + C$ where C is the constant of integration and $f(x)$ is positive, then $f(x) + g(x)$ has the value equal to

(A) $e^x + \sin x + 2x$ (B) $e^x + \sin x$ (C) $e^x - \sin x$ (D) $e^x + \sin x + x$

Q.20 Let $f(x) = \begin{cases} \frac{3x^2 + 2x - 1}{6x^2 - 5x + 1} & \text{for } x \neq \frac{1}{3} \\ -4 & \text{for } x = \frac{1}{3} \end{cases}$ then $f'\left(\frac{1}{3}\right)$:

(A) is equal to -9 (B) is equal to -27 (C) is equal to 27 (D) does not exist

Q.21 If $y = \frac{1}{1 + x^{n-m} + x^{p-m}} + \frac{1}{1 + x^{m-n} + x^{p-n}} + \frac{1}{1 + x^{m-p} + x^{n-p}}$ then $\frac{dy}{dx}$ at $e^{m^{n^p}}$ is equal to:

(A) e^{mnp} (B) $e^{mn/p}$ (C) $e^{np/m}$ (D) none

Q.22 If f is differentiable in (0, 6) & $f'(4) = 5$ then $\lim_{x \to 2} \frac{f(4) - f(x^2)}{2 - x} =$

(A) 5 (B) 5/4 (C) 10 (D) 20

Q.23 Integral of $\sqrt{1 + 2\cot x(\cot x + \operatorname{cosec} x)}$ w.r.t. x is :

(A) $2\ln\cos\frac{x}{2} + c$ (B) $2\ln\sin\frac{x}{2} + c$

(C) $\frac{1}{2}\ln\cos\frac{x}{2} + c$ (D) $\ln\sin x - \ln(\operatorname{cosec} x - \cot x) + c$

Q.24 Let $f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then $f'\left(\frac{\pi}{2}\right) =$

(A) 0 (B) – 12 (C) 4 (D) 12

Q.25 People living at Mars, instead of the usual definition of derivative D f(x), define a new kind of derivative, D*f(x) by the formula

$D^*f(x) = \underset{h\to 0}{\text{Limit}} \frac{f^2(x+h) - f^2(x)}{h}$ where f^2 (x) means $[f(x)]^2$. If f(x) = x *l*nx then

$D^*f(x)\big|_{x=e}$ has the value

(A) e (B) 2e (C) 4e (D) 8e

Q.26 $\int x . \frac{ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$ equals :

(A) $\sqrt{1+x^2}\ ln\left(x+\sqrt{1+x^2}\right) - x + c$ (B) $\frac{x}{2} . ln^2\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + c$

(C) $\frac{x}{2} . ln^2\left(x+\sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + c$ (D) $\sqrt{1+x^2}\ ln\left(x+\sqrt{1+x^2}\right) + x + c$

Q.27 If $\phi(x) = x . \sin x$ then $\underset{x\to\pi/2}{\text{Limit}} \frac{\phi(x) - \phi\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}} =$

(A) 1 (B) 2 (C) 0 (D) none

Q.28 Let $f(x) = x + \sin x$. Suppose g denotes the inverse function of f. The value of $g'\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ has the value equal to

(A) $\sqrt{2} - 1$ (B) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (C) $2 - \sqrt{2}$ (D) $\sqrt{2} + 1$

Q.29 A differentiable function satisfies

$3f^2(x)f'(x) = 2x$. Given $f(2) = 1$ then the value of $f(3)$ is

(A) $\sqrt[3]{24}$ (B) $\sqrt[3]{6}$ (C) 6 (D) 2

Q.30 If $y = x + e^x$ then $\frac{d^2x}{dy^2}$ is :

(A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{-1}{(1+e^x)^3}$

Q.31 Primitive of $f(x) = x \cdot 2^{ln(x^2+1)}$ w.r.t. x is

(A) $\frac{2^{ln(x^2+1)}}{2(x^2+1)} + C$ (B) $\frac{(x^2+1)2^{ln(x^2+1)}}{ln2+1} + C$

(C) $\frac{(x^2+1)^{ln2+1}}{2(ln2+1)} + C$ (D) $\frac{(x^2+1)^{ln2}}{2(ln\,2+1)} + C$

Q.32 Let y = ln $(1 + \cos x)^2$ then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals

(A) 0 (B) $\frac{2}{1+\cos x}$ (C) $\frac{4}{(1+\cos x)}$ (D) $\frac{-4}{(1+\cos x)^2}$

Q.33 Let $g(x)$ be an antiderivative for $f(x)$. Then $ln\left(1+(g(x))^2\right)$ is an antiderivative for

(A) $\frac{2f(x)g(x)}{1+(f(x))^2}$ (B) $\frac{2f(x)g(x)}{1+(g(x))^2}$ (C) $\frac{2f(x)}{1+(f(x))^2}$ (D) none

Q.34 If f is twice differentiable such that $f''(x) = -f(x),\ f'(x) = g(x)$

$h'(x) = [f(x)]^2 + [g(x)]^2$ and

$h(0) = 2,\ h(1) = 4$

then the equation $y = h(x)$ represents :

(A) a curve of degree 2 (B) a curve passing through the origin

(C) a straight line with slope 2 (D) a straight line with y intercept equal to -2 .

Q.35 If f(x) is a twice differentiable function, then between two consecutive roots of the equation $f'(x) = 0$, there exists :

(A) atleast one root of $f(x) = 0$ (B) atmost one root of $f(x) = 0$

(C) exactly one root of $f(x) = 0$ (D) atmost one root of $f''(x) = 0$

Q.36 A function $y = f(x)$ satisfies $f''(x) = -\frac{1}{x^2} - \pi^2 \sin(\pi x)$; $f'(2) = \pi + \frac{1}{2}$ and $f(1)=0$. The value of $f\left(\frac{1}{2}\right)$ is

(A) ln 2 (B) 1 (C) $\frac{\pi}{2} - ln\,2$ (D) $1 - ln\,2$

Q.37 Let a, b, c are non zero constant number then $\lim_{r\to\infty} \frac{\cos\frac{a}{r} - \cos\frac{b}{r}\cos\frac{c}{r}}{\sin\frac{b}{r}\sin\frac{c}{r}}$ equals

(A) $\frac{a^2+b^2-c^2}{2bc}$ (B) $\frac{c^2+a^2-b^2}{2bc}$ (C) $\frac{b^2+c^2-a^2}{2bc}$ (D) independent of a, b and c

Q.38 $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}\, dx$

(A) $\sin x - 6 \tan^{-1} (\sin x) + c$
(B) $\sin x - 2 \sin^{-1} x + c$
(C) $\sin x - 2 (\sin x)^{-1} - 6 \tan^{-1} (\sin x) + c$
(D) $\sin x - 2 (\sin x)^{-1} + 5 \tan^{-1} (\sin x) + c$

Q.39 If $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$, then the value of $10 f'(102^+)$

(A) is – 1
(B) is 0
(C) is 1
(D) does not exist

Q.40 Which one of the following is TRUE.

(A) $x.\int \frac{dx}{x} = x\, ln|x| + C$
(B) $x.\int \frac{dx}{x} = x\, ln|x| + Cx$
(C) $\frac{1}{\cos x}.\int \cos x\, dx = \tan x + C$
(D) $\frac{1}{\cos x}.\int \cos x\, dx = x + C$

Q.41 The derivative of the function,

$$f(x) = \cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x - \sin x)\right\} + \sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x + 3\sin x)\right\} \text{ w.r.t.} \sqrt{1+x^2} \text{ at } x = \frac{3}{4} \text{ is}$$

(A) $\frac{3}{2}$
(B) $\frac{5}{2}$
(C) $\frac{10}{3}$
(D) 0

Q.42 Let f (x) be a polynomial function of second degree. If f (1) = f (–1) and a, b, c are in A.P., then f '(a), f '(b) and f '(c) are in

(A) G.P.
(B) H.P.
(C) A.G.P.
(D) A.P.

Q.43 $\int \frac{(2x+1)}{(x^2+4x+1)^{3/2}}\, dx$

(A) $\frac{x^3}{(x^2+4x+1)^{1/2}} + C$
(B) $\frac{x}{(x^2+4x+1)^{1/2}} + C$
(C) $\frac{x^2}{(x^2+4x+1)^{1/2}} + C$
(D) $\frac{1}{(x^2+4x+1)^{1/2}} + C$

Q.44 If $x^2 + y^2 = R^2$ $(R > 0)$ then $k = \frac{y''}{\sqrt{(1+y'^2)^3}}$ where k in terms of R alone is equal to

(A) $-\frac{1}{R^2}$
(B) $-\frac{1}{R}$
(C) $\frac{2}{R}$
(D) $-\frac{2}{R^2}$

Q.45 $\int \left(\sin(101x)\cdot\sin^{99}x\right)dx$ equals

(A) $\frac{\sin(100x)(\sin x)^{100}}{100}+C$ (B) $\frac{\cos(100x)(\sin x)^{100}}{100}+C$

(C) $\frac{\cos(100x)(\cos x)^{100}}{100}+C$ (D) $\frac{\sin(100x)(\sin x)^{101}}{101}+C$

Q.46 If f & g are differentiable functions such that $g'(a) = 2$ & $g(a) = b$ and if fog is an identity function then $f'(b)$ has the value equal to :

(A) 2/3 (B) 1 (C) 0 (D) 1/2

Q.47 Given $f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a \cdot \sin 2a - 5 \arcsin (a^2 - 8a + 17)$ then :

(A) f(x) is not defined at x = sin 8 (B) $f'(\sin 8) > 0$

(C) $f'(x)$ is not defined at x = sin 8 (D) $f'(\sin 8) < 0$

Q.48 The evaluation of $\int \frac{PX^{p+2q-1} - q\ x^{q-1}}{X^{2p+2q}+2x^{p+q}+1}\ dx$ is

(A) $-\frac{x^p}{x^{p+q}+1}+C$ (B) $\frac{x^q}{x^{p+q}+1}+C$ (C) $-\frac{x^q}{x^{p+q}+1}+C$ (D) $\frac{x^p}{x^{p+q}+1}+C$

Q.49 Given: $f(x) = 4x^3 - 6x^2 \cos 2a + 3x \sin 2a \cdot \sin 6a + \sqrt{\ell n\left(2a - a^2\right)}$ then

(A) f(x) is not defined at x = 1/2 (B) $f'(1/2) < 0$

(C) $f'(x)$ is not defined at x = 1/2 (D) $f'(1/2) > 0$

Q.50 If $y = (A + Bx)\, e^{mx} + (m - 1)^{-2} e^x$ then $\frac{d^2y}{dx^2} - 2m\frac{dy}{dx} + m^2y$ is equal to :

(A) e^x (B) e^{mx} (C) e^{-mx} (D) $e^{(1-m)x}$

Q.51 If $I_n = \int (\sin x)^n\, dx \quad n \in N$

Then $5I_4 - 6I_6$ is equal to

(A) $\sin x \cdot (\cos x)^5 + C$ (B) $\sin 2x \cdot \cos 2x + C$

(C) $\frac{\sin 2x}{8}[\cos^2 2x + 1 - 2\cos 2x] + C$ (D) $\frac{\sin 2x}{8}[\cos^2 2x + 1 + 2\cos 2x] + C$

Q.52 Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the product *ab* is equal to

(A) 25 (B) 9 (C) – 15 (D) – 9

Q.53 Let h (x) be differentiable for all x and let $f(x) = (kx + e^x)\,h(x)$ where k is some constant. If h (0) = 5, h ' (0) = – 2 and f ' (0) = 18 then the value of k is equal to
(A) 5 (B) 4 (C) 3 (D) 2.2

Q.54 $\int \frac{e^{\tan^{-1} x}}{(1+x^2)}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2 + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] dx \quad (x > 0)$

(A) $e^{\tan^{-1} x}.\tan^{-1} x + C$ (B) $\frac{e^{\tan^{-1} x}.\left(\tan^{-1} x\right)^2}{2} + C$

(C) $e^{\tan^{-1} x}.\left(\sec^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$ (D) $e^{\tan^{-1} x}.\left(\operatorname{cosec}^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

Q.55 Let $f(x) = x^n$, n being a non-negative integer. The number of values of n for which $f'(p + q) = f'(p) + f'(q)$ is valid for all p, q > 0 is :
(A) 0 (B) 1 (C) 2 (D) none of these

Q.56 Let $e^{f(x)} = ln\, x$. If g(x) is the inverse function of f(x) then $g'(x)$ equals to :
(A) e^x (B) $e^x + x$ (C) $e^{(x + e^x)}$ (D) $e^{(x + ln\, x)}$

Q.57 $\int \frac{(x^2 - 1)\, dx}{(x^4 + 3x^2 + 1)\, \tan^{-1}\left(\frac{x^2+1}{x}\right)} = ln\,|\,f(x)\,| + C$ then f (x) is

(A) $ln\left(x + \frac{1}{x}\right)$ (B) $\tan^{-1}\left(x + \frac{1}{x}\right)$ (C) $\cot^{-1}\left(x + \frac{1}{x}\right)$ (D) $ln\left(\tan^{-1}\left(x + \frac{1}{x}\right)\right)$

Q.58 A non zero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$. The leading coefficient of $f(x)$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{9}$ (C) $\frac{1}{12}$ (D) $\frac{1}{18}$

Q.59 Let $f(x) = \frac{2\sin^2 x - 1}{\cos x} + \frac{\cos x(2\sin x + 1)}{1 + \sin x}$ then

$\int e^x\left(f(x) + f'(x)\right)dx$ where c is the constant of integeration)
(A) $e^x \tan x + c$ (B) $e^x \cot x + c$ (C) $e^x \operatorname{cosec}^2 x + c$ (D) $e^x \sec^2 x + c$

Q.60 The function $f(x) = e^x + x$, being differentiable and one to one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point $f(\ell n2)$ is

(A) $\frac{1}{\ell n2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) none

Q.61 The ends A and B of a rod of length $\sqrt{5}$ are sliding along the curve $y = 2x^2$. Let x_A and x_B be the x-coordinate of the ends. At the moment when A is at (0, 0) and B is at (1, 2) the derivative $\frac{dx_B}{dx_A}$ has the value(s) equal to

(A) 1/3 (B) 1/5 (C) 1/8 (D) 1/9

Q.62 If $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{\sqrt{a-x} + \sqrt{x-b}}$ then $\frac{dy}{dx}$ wherever it is defined is equal to :

(A) $\frac{x + (a+b)}{\sqrt{(a-x)(x-b)}}$ (B) $\frac{2x - (a+b)}{2\sqrt{(a-x)(x-b)}}$ (C) $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$ (D) $\frac{2x + (a+b)}{2\sqrt{(a-x)(x-b)}}$

Q.63 If $I_n = \int \cot^n x \, dx$, then $I_0 + I_1 + 2(I_2 + I_3 + ... + I_8) + I_9 + I_{10}$ equals to :
(where $u = \cot x$)

(A) $u + \frac{u^2}{2} + ... + \frac{u^9}{9}$ (B) $-\left(u + \frac{u^2}{2} + ... + \frac{u^9}{9}\right)$

(C) $-\left(u + \frac{u^2}{2!} + ... + \frac{u^9}{9!}\right)$ (D) $\frac{u}{2} + \frac{2u^2}{3} + ... + \frac{9u^9}{10}$

Q.64 For the curve represented implicitly as $3^x - 2^y = 1$, the value of $\lim_{x\to\infty}\left(\frac{dy}{dx}\right)$ is

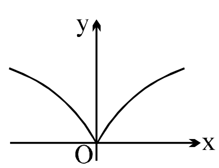
(A) equal to 1 (B) equal to 0 (C) equal to $\log_2 3$ (D) non existent

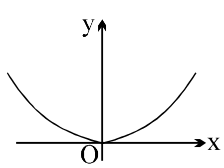
Q.65 If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = K$ then the value of K is equal to

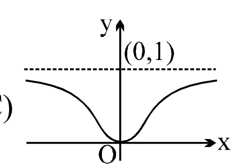
(A) 1 (B) –1 (C) 2 (D) 0

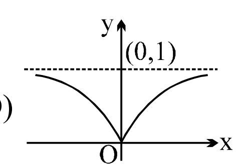
Q.66 Let $y = f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Then which of the following can best represent the graph of y = f(x)?

(A) 

(B) 

(C) 

(D) 

Q.67 Let $f(x) = \sin^3 x + \sin^3\left(x + \frac{2\pi}{3}\right) + \sin^3\left(x + \frac{4\pi}{3}\right)$ then the primitive of $f(x)$ w.r.t. x is

(A) $-\frac{3\sin 3x}{4} + C$ (B) $-\frac{3\cos 3x}{4} + C$ (C) $\frac{\sin 3x}{4} + C$ (D) $\frac{\cos 3x}{4} + C$

where C is an arbitrary constant.

Q.68 Differential coefficient of $\left(x^{\frac{\ell+m}{m-n}}\right)^{\frac{1}{n-\ell}} \cdot \left(x^{\frac{m+n}{n-\ell}}\right)^{\frac{1}{\ell-m}} \cdot \left(x^{\frac{n+\ell}{\ell-m}}\right)^{\frac{1}{m-n}}$ w.r.t. x is

(A) 1 (B) 0 (C) – 1 (D) $x^{\ell mn}$

Q.69 The integral $\int \sqrt{\cot x}\, e^{\sqrt{\sin x}} \sqrt{\cos x}\, dx$ equals

(A) $\dfrac{\sqrt{\tan x}\, e^{\sqrt{\sin x}}}{\sqrt{\cos x}} + C$ (B) $2e^{\sqrt{\sin x}} + C$

(C) $-\dfrac{1}{2} e^{\sqrt{\sin x}} + C$ (D) $\dfrac{\sqrt{\cot x}\, e^{\sqrt{\sin x}}}{2\sqrt{\cos x}} + C$

Q.70 If $y = at^2 + 2bt + c$ and $t = ax^2 + 2bx + c$, then $\dfrac{d^3y}{dx^3}$ equals

(A) $24\, a^2\, (at + b)$ (B) $24\, a\, (ax + b)^2$ (C) $24\, a\, (at + b)^2$ (D) $24\, a^2\, (ax + b)$

Q.71 $\int \dfrac{x^2(1 - \ln x)}{\ln^4 x - x^4} dx$ equals

(A) $\dfrac{1}{2} \ln\left(\dfrac{x}{\ln x}\right) - \dfrac{1}{4} \ln\left(\ln^2 x - x^2\right) + C$ (B) $\dfrac{1}{4} \ln\left(\dfrac{\ln x - x}{\ln x + x}\right) - \dfrac{1}{2} \tan^{-1}\left(\dfrac{\ln x}{x}\right) + C$

(C) $\dfrac{1}{4} \ln\left(\dfrac{\ln x + x}{\ln x - x}\right) + \dfrac{1}{2} \tan^{-1}\left(\dfrac{\ln x}{x}\right) + C$ (D) $\dfrac{1}{4}\left(\ln\left(\dfrac{\ln x - x}{\ln x + x}\right) + \tan^{-1}\left(\dfrac{\ln x}{x}\right)\right) + C$

Q.72 $\displaystyle\operatorname*{Limit}_{x \to 0^+} \frac{1}{x\sqrt{x}}\left(a \arctan\frac{\sqrt{x}}{a} - b \arctan\frac{\sqrt{x}}{b}\right)$ has the value equal to

(A) $\dfrac{a-b}{3}$ (B) 0 (C) $\dfrac{(a^2 - b^2)}{6a^2b^2}$ (D) $\dfrac{a^2 - b^2}{3a^2b^2}$

Q.73 If $\int \dfrac{(2x+3)\, dx}{x(x+1)(x+2)(x+3)+1} = C - \dfrac{1}{f(x)}$ where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals

(A) 4 (B) 5 (C) 6 (D) none

Q.74 Suppose $A = \dfrac{dy}{dx}$ of $x^2 + y^2 = 4$ at $(\sqrt{2}, \sqrt{2})$, $B = \dfrac{dy}{dx}$ of $\sin y + \sin x = \sin x \cdot \sin y$ at (π, π) and $C = \dfrac{dy}{dx}$ of $2e^{xy} + e^x e^y - e^x - e^y = e^{xy+1}$ at (1, 1), then (A + B + C) has the value equal to

(A) – 1 (B) e (C) – 3 (D) 0

Q.75 A function is represented parametrically by the equations $x = \frac{1+t}{t^3}$; $y = \frac{3}{2t^2} + \frac{2}{t}$ then $\frac{dy}{dx} - x.\left(\frac{dy}{dx}\right)^3$ has the value equal to
(A) 2 (B) 0 (C) –1 (D) –2

Q.76 Suppose $A = \int \frac{dx}{x^2 + 6x + 25}$ and $B = \int \frac{dx}{x^2 - 6x - 27}$.
If $12(A + B) = \lambda \cdot \tan^{-1}\left(\frac{x+3}{4}\right) + \mu \cdot \ln\left|\frac{x-9}{x+3}\right| + C$, then the value of $(\lambda + \mu)$ is
(A) 3 (B) 4 (C) 5 (D) 6

Q.77 Suppose the function $f(x) - f(2x)$ has the derivative 5 at x = 1 and derivative 7 at x = 2. The derivative of the function $f(x) - f(4x)$ at x = 1, has the value equal to
(A) 19 (B) 9 (C) 17 (D) 14

Q.78 If $x + y = 3e^2$ then $D(x^y)$ vanishes when x equals to
(A) e (B) e^2 (C) e^e (D) $2e^2$

Q.79 Let $\int \frac{dx}{x^{2008} + x} = \frac{1}{p} \ln\left(\frac{x^q}{1+x^r}\right) + C$
where p, q, $r \in N$ and need not be distinct, then the value of (p + q + r) equals
(A) 6024 (B) 6022 (C) 6021 (D) 6020

[COMPREHENSION TYPE]

Paragraph for Question Nos. 80 to 82

A curve is represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then

Q.80 The relation between the parameter 't' and the angle α between the tangent to the given curve and the x-axis is given by, '*t*' equals
(A) $\frac{\pi}{2} - \alpha$ (B) $\frac{\pi}{4} + \alpha$ (C) $\alpha - \frac{\pi}{4}$ (D) $\frac{\pi}{4} - \alpha$

Q.81 The value of $\frac{d^2y}{dx^2}$ at the point where t = 0 is
(A) 1 (B) 2 (C) – 2 (D) 3

Q.82 If $F(t) = \int (x + y)\, dt$ then the value of $F\left(\frac{\pi}{2}\right) - F(0)$ is
(A) 1 (B) – 1 (C) $e^{\pi/2}$ (D) 0

[REASONING TYPE]

Q.83 Consider the following statements
Statement-1: $f(x) = x\, e^x$ and $g(x) = e^x(x + 1)$ are both aperiodic function.
because

Statement-2: Derivative of a differentiable aperiodic function is an aperiodic function.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.84 Statement-1: The function $F(x) = \int \frac{x}{(x-1)(x^2+1)} dx$ is discontinuous at x = 1

because

Statement-2: If $F(x) = \int f(x)\,dx$ and $f(x)$ is discontinuous at x = a then F (x) is also discontinuous at x = a.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[MULTIPLE OBJECTIVE TYPE]

Q.85 If $\sqrt{y+x} + \sqrt{y-x} = c$ (where $c \neq 0$), then $\frac{dy}{dx}$ has the value equal to

(A) $\frac{2x}{c^2}$ (B) $\frac{x}{y+\sqrt{y^2-x^2}}$ (C) $\frac{y-\sqrt{y^2-x^2}}{x}$ (D) $\frac{c^2}{2y}$

Q.86 If $y = \tan x \tan 2x \tan 3x$ then $\frac{dy}{dx}$ has the value equal to
(A) $3 \sec^2 3x \tan x \tan 2x + \sec^2 x \tan 2x \tan 3x + 2 \sec^2 2x \tan 3x \tan x$
(B) $2y(\text{cosec } 2x + 2 \text{ cosec } 4x + 3 \text{ cosec } 6x)$
(C) $3 \sec^2 3x - 2 \sec^2 2x - \sec^2 x$
(D) $\sec^2 x + 2 \sec^2 2x + 3 \sec^2 3x$

Q.87 $\int \frac{\ln(\tan x)}{\sin x \cos x} dx$ equal:

(A) $\frac{1}{2} \ln^2(\cot x) + c$ (B) $\frac{1}{2} \ln^2(\sec x) + c$

(C) $\frac{1}{2} \ln^2(\sin x \sec x) + c$ (D) $\frac{1}{2} \ln^2(\cos x \text{ cosec } x) + c$

Q.88 If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ has the value equal to

(A) $-\frac{2^y}{2^x}$ (B) $\frac{1}{1-2^x}$ (C) $1 - 2^y$ (D) $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

Q.89 For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds?

(A) if $f(x) > 0$ for all real x $\not\Rightarrow$ $f'(x) > 0$ (B) if $f(x) > 0$ for all real x $\Rightarrow$ $f'(x) > 0$

(C) if $f'(x) > 0$ for all real x $\Rightarrow$ $f(x) > 0$ (D) if $f'(x) > 0$ for all real x $\not\Rightarrow$ $f(x) > 0$

Q.90 If $\int e^u \cdot \sin 2x \; dx$ can be found in terms of known functions of x then u can be:

(A) x (B) sin x (C) cos x (D) cos 2x

Q.91 Let $f(x) = \dfrac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1} \cdot x$ then

(A) $f'(10) = 1$ (B) $f'(3/2) = -1$

(C) domain of f(x) is $x \geq 1$ (D) none

Q.92 Let $f'(x) = 3x^2 \sin \dfrac{1}{x} - x \cos \dfrac{1}{x}$, if $x \neq 0$; $f(0) = 0$ and $f(1/\pi) = 0$ then :

(A) f(x) is continuous at x = 0 (B) f(x) is non derivable at x = 0

(C) $f'(x)$ is continuous at x = 0 (D) $f'(x)$ is non derivable at x = 0

Q.93 If $y = x^{(\ell n x)^{\ell n(\ell n x)}}$, then $\dfrac{dy}{dx}$ is equal to :

(A) $\dfrac{y}{x}\left(\ell n x^{\ell n x - 1} + 2 \ell n x \, \ell n\left(\ell n x\right)\right)$ (B) $\dfrac{y}{x} (\ln x)^{\ln(\ln x)} (2 \ln(\ln x) + 1)$

(C) $\dfrac{y}{x \, \ell n x}\left((\ln x)^2 + 2 \ln(\ln x)\right)$ (D) $\dfrac{y \, \ell n y}{x \, \ell n x} (2 \ln(\ln x) + 1)$

Q.94 Which of the following functions are not derivable at x = 0?

(A) $f(x) = \sin^{-1} 2x \sqrt{1 - x^2}$ (B) $g(x) = \sin^{-1}\left(\dfrac{2^{x+1}}{1 + 4^x}\right)$

(C) $h(x) = \sin^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$ (D) $k(x) = \sin^{-1}(\cos x)$

Q.95 Suppose $J = \int \dfrac{\sin^2 x + \sin x}{1 + \sin x + \cos x} dx$ and $K = \int \dfrac{\cos^2 x + \cos x}{1 + \sin x + \cos x} dx$. If C is an arbitrary constant of integration then which of the following is/are correct?

(A) $J = \dfrac{1}{2}(x - \sin x + \cos x) + C$ (B) $J = K - (\sin x + \cos x) + C$

(C) $J = x - K + C$ (D) $K = \dfrac{1}{2}(x - \sin x + \cos x) + C$

ANSWER KEY

[STRAIGHT OBJECTIVE TYPE]

Q.1	C	Q.2	A	Q.3	C	Q.4	B	Q.5	A	Q.6	B
Q.7	C	Q.8	A	Q.9	B	Q.10	C	Q.11	D	Q.12	C
Q.13	B	Q.14	B	Q.15	C	Q.16	B	Q.17	A	Q.18	B
Q.19	B	Q.20	B	Q.21	D	Q.22	D	Q.23	B	Q.24	C
Q.25	C	Q.26	A	Q.27	A	Q.28	C	Q.29	B	Q.30	B
Q.31	C	Q.32	A	Q.33	B	Q.34	C	Q.35	B	Q.36	D
Q.37	C	Q.38	C	Q.39	C	Q.40	B	Q.41	C	Q.42	D
Q.43	B	Q.44	B	Q.45	A	Q.46	D	Q.47	D	Q.48	C
Q.49	D	Q.50	A	Q.51	C	Q.52	C	Q.53	C	Q.54	C
Q.55	C	Q.56	C	Q.57	B	Q.58	D	Q.59	A	Q.60	B
Q.61	D	Q.62	B	Q.63	B	Q.64	C	Q.65	D	Q.66	C
Q.67	D	Q.68	B	Q.69	B	Q.70	D	Q.71	B	Q.72	D
Q.73	B	Q.74	C	Q.75	C	Q.76	B	Q.77	A	Q.78	B
Q.79	C	Q.80	C	Q.81	B	Q.82	C	Q.83	C	Q.84	C

[MULTIPLE OBJECTIVE TYPE]

Q.85	A, B, C	Q.86	A, B, C	Q.87	A, C, D	Q.88	A, B, C, D
Q.89	A, C	Q.90	A, B, C, D	Q.91	A, B	Q.92	A, C, D
Q.93	B, D	Q.94	B, C, D	Q.95	B, C		