Energy to Succeed!
ETOOS

SPECIAL DPP's OF COMPLEX NUMBER

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DPP - 1

Q.1 The sequence $S = i + 2i^2 + 3i^3 + ...$ upto 100 terms simplifies to where $i = \sqrt{-1}$:
(A) $50(1-i)$ (B) $25i$ (C) $25(1+i)$ (D) $100(1-i)$

Q.2 If $z + z^3 = 0$ then which of the following must be true on the complex plane?
(A) $Re(z) < 0$ (B) $Re(z) = 0$ (C) $Im(z) = 0$ (D) $z^4 = 1$

Q.3 Number of integral values of n for which the quantity $(n + i)^4$ where $i^2 = -1$, is an integer is
(A) 1 (B) 2 (C) 3 (D) 4

Q.4 Let $i = \sqrt{-1}$. The product of the real part of the roots of $z^2 - z = 5 - 5i$ is
(A) -25 (B) -6 (C) -5 (D) 25

Q.5 There is only one way to choose real numbers M and N such that when the polynomial $5x^4 + 4x^3 + 3x^2 + Mx + N$ is divided by the polynomial $x^2 + 1$, the remainder is 0. If M and N assume these unique values, then $M - N$ is
(A) -6 (B) -2 (C) 6 (D) 2

Q.6 In the quadratic equation $x^2 + (p + iq)x + 3i = 0$, p & q are real. If the sum of the squares of the roots is 8 then
(A) $p = 3,\ q = -1$ (B) $p = -3, q = -1$ (C) $p = \pm 3,\ q = \pm 1$ (D) $p = -3,\ q = 1$

Q.7 The complex number z satisfying $z + |z| = 1 + 7i$ then the value of $|z|^2$ equals
(A) 625 (B) 169 (C) 49 (D) 25

Q.8 The figure formed by four points $1 + 0i$; $-1 + 0i$; $3 + 4i$ & $\frac{25}{-3-4i}$ on the argand plane is :
(A) a parallelogram but not a rectangle (B) a trapezium which is not equilateral
(C) a cyclic quadrilateral (D) none of these

Q.9 If $z = (3 + 7i)(p + iq)$ where $p, q \in I - \{0\}$, is purely imaginary then minimum value of $|z|^2$ is
(A) 0 (B) 58 (C) $\frac{3364}{3}$ (D) 3364

Q.10 Number of values of z (real or complex) simultaneously satisfying the system of equations
$1 + z + z^2 + z^3 + ... + z^{17} = 0$ and $1 + z + z^2 + z^3 + ... + z^{13} = 0$ is
(A) 1 (B) 2 (C) 3 (D) 4

Q.11 If $\frac{x-3}{3+i} + \frac{y-3}{3-i} = i$ where $x, y \in R$ then
(A) $x = 2$ & $y = -8$ (B) $x = -2$ & $y = 8$ (C) $x = -2$ & $y = -6$ (D) $x = 2$ & $y = 8$

Q.12 Number of complex numbers z satisfying $z^3 = \bar{z}$ is
(A) 1 (B) 2 (C) 4 (D) 5

Q.13 If $x = 9^{1/3}\ 9^{1/9}\ 9^{1/27}$ ......ad inf

$y = 4^{1/3}\ 4^{-1/9}\ 4^{1/27}$ ...... ad inf and $z = \sum_{r=1}^{\infty} (1+i)^{-r}$

then, the argument of the complex number $w = x + yz$ is

(A) 0 (B) $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (C) $-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Q.14 Let $z = 9 + bi$ where b is non zero real and $i^2 = -1$. If the imaginary part of z^2 and z^3 are equal, then b^2 equals

(A) 261 (B) 225 (C) 125 (D) 361

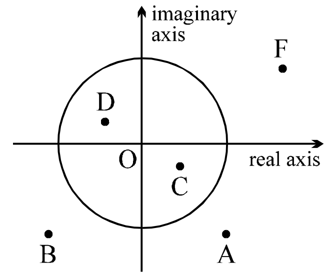
One or more than one is/are correct:

Q.15 If the expression $(1 + ir)^3$ is of the form of $s(1 + i)$ for some real 's' where 'r' is also real and $i = \sqrt{-1}$, then the value of 'r' can be

(A) $\cot\frac{\pi}{8}$ (B) $\sec \pi$ (C) $\tan\frac{\pi}{12}$ (D) $\tan\frac{5\pi}{12}$

DPP - 2

Q.1 The digram shows several numbers in the complex plane. The circle is the unit circle centered at the origin. One of these numbers is the reciprocal of F, which is



(A) A (B) B
(C) C (D) D

Q.2 If $z = x + iy$ & $\omega = \dfrac{1 - iz}{z - i}$ then $|\omega| = 1$ implies that, in the complex plane :
(A) z lies on the imaginary axis (B) z lies on the real axis
(C) z lies on the unit circle (D) none

Q.3 On the complex plane locus of a point z satisfying the inequality
$2 \le |z - 1| < 3$ denotes
(A) region between the concentric circles of radii 3 and 1 centered at (1, 0)
(B) region between the concentric circles of radii 3 and 2 centered at (1, 0) excluding the inner and outer boundaries.
(C) region between the concentric circles of radii 3 and 2 centered at (1, 0) including the inner and outer boundaries.
(D) region between the concentric circles of radii 3 and 2 centered at (1, 0) including the inner boundary and excluding the outer boundary.

Q.4 The complex number z satisfies $z + |z| = 2 + 8i$. The value of $|z|$ is
(A) 10 (B) 13 (C) 17 (D) 23

Q.5 Let $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ are two complex numbers. If $Z_1 \cdot Z_2 = a + ib$ where $a, b \in R$ then the largest value of $(a + b)\ \forall\ \theta \in R$, is
(A) 75 (B) 100 (C) 125 (D) 130

Q.6 The locus of z, for $\arg z = -\pi/3$ is
(A) same as the locus of z for $\arg z = 2\pi/3$
(B) same as the locus of z for $\arg z = \pi/3$
(C) the part of the straight line $\sqrt{3}\,x + y = 0$ with $(y < 0, x > 0)$
(D) the part of the straight line $\sqrt{3}\,x + y = 0$ with $(y > 0, x < 0)$

Q.7 If z_1 & $\bar{z}_1$ represent adjacent vertices of a regular polygon of n sides with centre at the origin & if $\dfrac{\operatorname{Im} z_1}{\operatorname{Re} z_1} = \sqrt{2} - 1$ then the value of n is equal to :
(A) 8 (B) 12 (C) 16 (D) 24

Q.8 If z_1, z_2 are two complex numbers & a, b are two real numbers then, $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$
(A) $(a+b)^2\left[|z_1|^2 + |z_2|^2\right]$ (B) $(a+b)\left[|z_1|^2 + |z_2|^2\right]$
(C) $\left(a^2 - b^2\right)\left[|z_1|^2 + |z_2|^2\right]$ (D) $\left(a^2 + b^2\right)\left[|z_1|^2 + |z_2|^2\right]$

Q.9 The value of $e\left(\text{CiS}(-i)-\text{CiS}(i)\right)$ is equal to

(A) 0 (B) $1-e$ (C) $e-\frac{1}{e}$ (D) e^2-1

Q.10 All real numbers *x* which satisfy the inequality $\left|1+4i-2^{-x}\right| \le 5$ where $i=\sqrt{-1}$, $x \in R$ are

(A) $[-2, \infty)$ (B) $(-\infty, 2]$ (C) $[0, \infty)$ (D) $[-2, 0]$

Q.11 For $Z_1=\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$; $Z_2=\sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$; $Z_3=\sqrt[6]{\frac{1+i}{\sqrt{3}-i}}$ which of the following holds good?

(A) $\sum|Z_1|^2=\frac{3}{2}$ (B) $|Z_1|^4+|Z_2|^4=|Z_3|^{-8}$

(C) $\sum|Z_1|^3+|Z_2|^3=|Z_3|^{-6}$ (D) $|Z_1|^4+|Z_2|^4=|Z_3|^8$

Q.12 Number of real or purely imaginary solution of the equation, $z^3+iz-1=0$ is :

(A) zero (B) one (C) two (D) three

Q.13 A point 'z' moves on the curve $|z-4-3i|=2$ in an argand plane. The maximum and minimum values of $|z|$ are :

(A) 2, 1 (B) 6, 5 (C) 4, 3 (D) 7, 3

Q.14 If z is a complex number satisfying the equation $|z+i|+|z-i|=8$, on the complex plane then maximum value of $|z|$ is

(A) 2 (B) 4 (C) 6 (D) 8

DPP - 3

Q.1 If z_1 & z_2 are two non-zero complex numbers such that $|z_1+z_2| = |z_1| + |z_2|$, then $\text{Arg}\, z_1 - \text{Arg}\, z_2$ is equal to:

(A) $-\pi$ (B) $-\pi/2$ (C) 0 (D) $\pi/2$

Q.2 Let Z be a complex number satisfying the equation

$(Z^3+3)^2 = -16$ then $|Z|$ has the value equal to

(A) $5^{1/2}$ (B) $5^{1/3}$ (C) $5^{2/3}$ (D) 5

Q.3 Let $i = \sqrt{-1}$. Define a sequence of complex number by $z_1 = 0$, $z_{n+1} = z_n^2 + i$ for $n \ge 1$. In the complex plane, how far from the origin is z_{111}?

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) $\sqrt{110}$

Q.4 The points representing the complex number z for which $|z+5|^2 - |z-5|^2 = 10$ lie on

(A) a straight line (B) a circle

(C) a parabola (D) the bisector of the line joining (5 , 0) & (– 5 , 0)

Q.5 If $x = \dfrac{1+\sqrt{3}\,i}{2}$ then the value of the expression, $y = x^4 - x^2 + 6x - 4$, equals

(A) $-1 + 2\sqrt{3}\,i$ (B) $2 - 2\sqrt{3}\,i$ (C) $2 + 2\sqrt{3}\,i$ (D) none

Q.6 Consider two complex numbers α and β as

$\alpha = \left(\dfrac{a+bi}{a-bi}\right)^2 + \left(\dfrac{a-bi}{a+bi}\right)^2$, where a, b $\in$ R and $\beta = \dfrac{z-1}{z+1}$, where $|z| = 1$, then

(A) Both α and β are purely real (B) Both α and β are purely imaginary

(C) α is purely real and β is purely imaginary (D) β is purely real and α is purely imaginary

Q.7 Let Z is complex satisfying the equation

$z^2 - (3+i)z + m + 2i = 0$, where m $\in$ R. Suppose the equation has a real root.

The additive inverse of non real root, is

(A) $1 - i$ (B) $1 + i$ (C) $-1 - i$ (D) -2

Q.8 The minimum value of $|1+z| + |1-z|$ where z is a complex number is :

(A) 2 (B) 3/2 (C) 1 (D) 0

Q.9 If $i = \sqrt{-1}$, then $4 + 5\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$ (C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

Q.10 Let $|z - 5 + 12i| \le 1$ and the least and greatest values of $|z|$ are *m* and *n* and if *l* be the least positive value of $\dfrac{x^2+24x+1}{x}$ $(x > 0)$, then *l* is

(A) $\dfrac{m+n}{2}$ (B) $m + n$ (C) m (D) n

Q.11 The system of equations $\left.\begin{matrix}|z+1-i| & = 2\\ \text{Re}\, z & \ge 1\end{matrix}\right\}$ where z is a complex number has :

(A) no solution (B) exactly one solution

(C) two distinct solutions (D) infinite solution

Q.12 Let C_1 and C_2 are concentric circles of radius 1 and 8/3 respectively having centre at (3, 0) on the argand plane. If the complex number z satisfies the inequality, $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$ then :

(A) z lies outside C_1 but inside C_2 (B) z lies inside of both C_1 and C_2
(C) z lies outside both of C_1 and C_2 (D) none of these

Q.13 Identify the incorrect statement.
(A) no non zero complex number z satisfies the equation, $\bar{z} = -4z$
(B) $\bar{z} = z$ implies that z is purely real
(C) $\bar{z} = -z$ implies that z is purely imaginary
(D) if z_1, z_2 are the roots of the quadratic equation $az^2 + bz + c = 0$ such that $\text{Im}(z_1 z_2) \neq 0$ then a, b, c must be real numbers .

Q.14 The equation of the radical axis of the two circles represented by the equations, $|z-2| = 3$ and $|z-2-3i| = 4$ on the complex plane is :
(A) $3y + 1 = 0$ (B) $3y - 1 = 0$ (C) $2y - 1 = 0$ (D) none

Q.15 If $z_1 = -3 + 5i$; $z_2 = -5 - 3i$ and z is a complex number lying on the line segment joining z_1 & z_2 then arg z can be :
(A) $-\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

Q.16 Given $z = f(x) + i\,g(x)$ where $f, g : (0, 1) \to (0, 1)$ are real valued functions then, which of the following holds good?

(A) $z = \frac{1}{1-ix} + i\left(\frac{1}{1+ix}\right)$ (B) $z = \frac{1}{1+ix} + i\left(\frac{1}{1-ix}\right)$

(C) $z = \frac{1}{1+ix} + i\left(\frac{1}{1+ix}\right)$ (D) $z = \frac{1}{1-ix} + i\left(\frac{1}{1-ix}\right)$

Q.17 $z_1 = \frac{a}{1-i}$; $z_2 = \frac{b}{2+i}$; $z_3 = a - bi$ for $a, b \in R$
if $z_1 - z_2 = 1$ then the centroid of the triangle formed by the points z_1, z_2, z_3 in the argand's plane is given by
(A) $\frac{1}{9}(1+7i)$ (B) $\frac{1}{3}(1+7i)$ (C) $\frac{1}{3}(1-3i)$ (D) $\frac{1}{9}(1-3i)$

Q.18 Consider the equation $10z^2 - 3iz - k = 0$, where z is a complex variable and $i^2 = -1$. Which of the following statements is True?
(A) For all real positive numbers k, both roots are pure imaginary.
(B) For negative real numbers k, both roots are pure imaginary.
(C) For all pure imaginary numbers k, both roots are real and irrational.
(D) For all complex numbers k, neither root is real.

Q.19 Number of complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is
(A) 4 (B) 6 (C) 8 (D) more than 8

Q.20 Number of ordered pairs(s) (a, b) of real numbers such that $(a + ib)^{2008} = a - ib$ holds good, is
(A) 2008 (B) 2009 (C) 2010 (D) 1

DPP - 4

Q.1 Consider $az^2 + bz + c = 0$, where $a, b, c \in R$ and $4ac > b^2$.

(i) If z_1 and z_2 are the roots of the equation given above, then which of the following complex numbers is purely real?

(A) $z_1\bar{z}_2$ (B) $\bar{z}_1 z_2$ (C) $z_1 - z_2$ (D) $(z_1 - z_2)i$

(ii) In the argand's plane, if A is the point representing z_1, B is the point representing z_2 and $z = \frac{\overrightarrow{OA}}{\overrightarrow{OB}}$ then

(A) z is purely real (B) z is purely imaginary

(C) $|z| = 1$ (D) ΔAOB is a scalene triangle.

Q.2 Let z be a complex number having the argument θ, $0 < \theta < \pi/2$ and satisfying the equality $|z - 3i| = 3$.

Then $\cot\theta - \frac{6}{z}$ is equal to :

(A) 1 (B) -1 (C) i (D) $-i$

Q.3 If the complex number z satisfies the condition $|z| \geq 3$, then the least value of $\left|z + \frac{1}{z}\right|$ is equal to :

(A) 5/3 (B) 8/3 (C) 11/3 (D) none of these

Q.4 Given $z_p = \cos\left(\frac{\pi}{2^P}\right) + i\sin\left(\frac{\pi}{2^P}\right)$, then $\lim_{n\to\infty} (z_1 z_2 z_3 .... z_n) =$

(A) 1 (B) -1 (C) i (D) $-i$

Q.5 The maximum & minimum values of $|z+1|$ when $|z+3| \leq 3$ are :

(A) (5 , 0) (B) (6 , 0) (C) (7 , 1) (D) (5 , 1)

Q.6 If $z^3 + (3 + 2i)z + (-1 + ia) = 0$ has one real root, then the value of 'a' lies in the interval ($a \in R$)

(A) $(-2, -1)$ (B) $(-1, 0)$ (C) $(0, 1)$ (D) $(1, 2)$

Q.7 If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 11i$ where $i = \sqrt{-1}$, then $(a + b)$ equal to

(A) 2 (B) 3 (C) 4 (D) 5

Q.8 If $\text{Arg}(z + a) = \frac{\pi}{6}$ and $\text{Arg}(z - a) = \frac{2\pi}{3}$; $a \in R^+$, then

(A) z is independent of a (B) $|a| = |z + a|$

(C) $z = a \text{ Cis } \frac{\pi}{6}$ (D) $z = a \text{ Cis } \frac{\pi}{3}$

Q.9 If z_1, z_2, z_3 are the vertices of the ΔABC on the complex plane which are also the roots of the equation, $z^3 - 3\alpha z^2 + 3\beta z + x = 0$, then the condition for the ΔABC to be equilateral triangle is

(A) $\alpha^2 = \beta$ (B) $\alpha = \beta^2$ (C) $\alpha^2 = 3\beta$ (D) $\alpha = 3\beta^2$

Q.10 The locus represented by the equation, $|z-1| + |z+1| = 2$ is :

(A) an ellipse with focii $(1, 0)$; $(-1, 0)$

(B) one of the family of circles passing through the points of intersection of the circles $|z-1| = 1$ and $|z+1| = 1$

(C) the radical axis of the circles $|z-1| = 1$ and $|z+1| = 1$

(D) the portion of the real axis between the points $(1, 0)$; $(-1, 0)$ including both.

Q.11 The points $z_1 = 3 + \sqrt{3}\,i$ and $z_2 = 2\sqrt{3} + 6i$ are given on a complex plane. The complex number lying on the bisector of the angle formed by the vectors z_1 and z_2 is :

(A) $z = \frac{(3 + 2\sqrt{3})}{2} + \frac{\sqrt{3} + 2}{2} i$ (B) $z = 5 + 5i$

(C) $z = -1 - i$ (D) none

Q.12 Let z_1 & z_2 be non zero complex numbers satisfying the equation, $z_1^2 - 2z_1z_2 + 2z_2^2 = 0$. The geometrical nature of the triangle whose vertices are the origin and the points representing z_1 & z_2 is :

(A) an isosceles right angled triangle

(B) a right angled triangle which is not isosceles

(C) an equilateral triangle

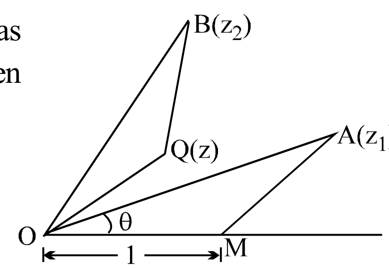
(D) an isosceles triangle which is not right angled .

Q.13 Let P denotes a complex number z on the Argand's plane, and Q denotes a complex number $\sqrt{2|z|^2}\ \text{CiS}\left(\frac{\pi}{4} + \theta\right)$ where $\theta = \text{amp } z$. If 'O' is the origin, then the ΔOPQ is :

(A) isosceles but not right angled (B) right angled but not isosceles

(C) right isosceles (D) equilateral .

Q.14 On the Argand plane point 'A' denotes a complex number z_1. A triangle OBQ is made directily similiar to the triangle OAM, where OM = 1 as shown in the figure. If the point B denotes the complex number z_2, then the complex number corresponding to the point 'Q' is



(A) $z_1 z_2$ (B) $\frac{z_1}{z_2}$

(C) $\frac{z_2}{z_1}$ (D) $\frac{z_1 + z_2}{z_2}$

Q.15 z_1 & z_2 are two distinct points in an argand plane. If $a|z_1| = b|z_2|$, (where a, b $\in$ R) then the point $\frac{a z_1}{b z_2} + \frac{b z_2}{a z_1}$ is a point on the :

(A) line segment $[-2, 2]$ of the real axis (B) line segment $[-2, 2]$ of the imaginary axis

(C) unit circle $|z| = 1$ (D) the line with $\arg z = \tan^{-1} 2$.

Q.16 When the polynomial $5x^3 + Mx + N$ is divided by $x^2 + x + 1$ the remainder is 0. The value of $(M + N)$ is equal to

(A) – 3 (B) 5 (C) – 5 (D) 15

Q.17 If $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1-\sqrt{\pi}\,i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}\,i}\right)$ then $\left(\frac{|z|}{\text{amp}\,z}\right)$ equals

(A) 1 (B) π (C) 3π (D) 4

Q.18 $\left(\sqrt[3]{3} + \left(3^{5/6}\right)i\right)^3$ is an integer where $i = \sqrt{-1}$. The value of the integer is equal to

(A) 24 (B) – 24 (C) – 22 (D) – 21

One ore more than one is/are correct:

Q.19 If $z \in C$, which of the following relation(s) represents a circle on an Argand diagram?

(A) $|z - 1| + |z + 1| = 3$
(B) $(z - 3 + i)(\bar{z} - 3 - i) = 5$
(C) $3|z - 2 + i| = 7$
(D) $|z - 3| = 2$

Q.20 Let z_1, z_2, z_3 be three complex number such that

$$|z_1| = |z_2| = |z_3| = 1 \text{ and } \frac{z_1^2}{z_2z_3} + \frac{z_2^2}{z_1z_3} + \frac{z_3^2}{z_1z_2} + 1 = 0$$

then $|z_1 + z_2 + z_3|$ can take the value equal to

(A) 1 (B) 2 (C) 3 (D) 4

DPP - 5

Q.1 A root of unity is a complex number that is a solution to the equation, $z^n = 1$ for some positive integer *n*. Number of roots of unity that are also the roots of the equation $z^2 + az + b = 0$, for some integer *a* and *b* is

(A) 6 (B) 8 (C) 9 (D) 10

Q.2 z is a complex number such that $z + \frac{1}{z} = 2\cos 3°$, then the value of $z^{2000} + \frac{1}{z^{2000}} + 1$ is equal to

(A) 0 (B) – 1 (C) $\sqrt{3}+1$ (D) $1-\sqrt{3}$

Q.3 The complex number ω satisfying the equation $\omega^3 = 8i$ and lying in the second quadrant on the complex plane is

(A) $-\sqrt{3}+i$ (B) $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$ (C) $-2\sqrt{3}+i$ (D) $-\sqrt{3}+2i$

Q.4 If $z^4 + 1 = \sqrt{3}\,i$

(A) z^3 is purely real
(B) z represents the vertices of a square of side $2^{1/4}$
(C) z^9 is purely imaginary
(D) z represents the vertices of a square of side $2^{3/4}$.

Q.5 The complex number z satisfies the condition $\left|z - \frac{25}{z}\right| = 24$. The maximum distance from the origin of co-ordinates to the point z is :

(A) 25 (B) 30 (C) 32 (D) none of these

Q.6 If the expression $x^{2m} + x^m + 1$ is divisible by $x^2 + x + 1$, then :

(A) m is any odd integer
(B) m is divisible by 3
(C) m is not divisible by 3
(D) none of these

Q.7 If $z_1 = 2 + 3i$, $z_2 = 3 - 2i$ and $z_3 = -1 - 2\sqrt{3}\,i$ then which of the following is true?

(A) $\arg\left(\frac{z_3}{z_2}\right) = \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$
(B) $\arg\left(\frac{z_3}{z_2}\right) = \arg\left(\frac{z_2}{z_1}\right)$
(C) $\arg\left(\frac{z_3}{z_2}\right) = 2\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$
(D) $\arg\left(\frac{z_3}{z_2}\right) = \frac{1}{2}\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

Q.8 If ***m*** and ***n*** are the smallest positive integers satisfying the relation

$\left(2\text{Cis}\frac{\pi}{6}\right)^m = \left(4\text{Cis}\frac{\pi}{4}\right)^n$, then $(\boldsymbol{m}+\boldsymbol{n})$ has the value equal to

(A) 120 (B) 96 (C) 72 (D) 60

Q.9 If z is a complex number satisfying the equation

$$Z^6 + Z^3 + 1 = 0.$$

If this equation has a root $re^{i\theta}$ with $90° < \theta < 180°$ then the value of 'θ' is

(A) 100° (B) 110° (C) 160° (D) 170°

Q.10 Least positive argument of the 4^{th} root of the complex number $2 - i\sqrt{12}$ is

(A) $\pi/6$ (B) $5\pi/12$ (C) $7\pi/12$ (D) $11\pi/12$

Q.11 P(z) is the point moving in the Argand's plane satisfying $\arg(z-1) - \arg(z+i) = \pi$ then, P is

(A) a real number, hence lies on the real axis.

(B) an imaginary number, hence lies on the imaginary axis.

(C) a point on the hypotenuse of the right angled triangle OAB formed by $O \equiv (0, 0)$; $A \equiv (1, 0)$; $B \equiv (0, -1)$.

(D) a point on an arc of the circle passing through $A \equiv (1, 0)$; $B \equiv (0, -1)$.

Q.12 Number of ordered pair(s) (z, ω) of the complex numbers z and ω satisfying the system of equations, $z^3 + \overline{\omega}^7 = 0$ and $z^5 . \omega^{11} = 1$ is :

(A) 7 (B) 5 (C) 3 (D) 2

Q.13 If $p = a + b\omega + c\omega^2$; $q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where a, b, c $\neq 0$ and ω is the complex cube root of unity, then :

(A) $p + q + r = a + b + c$ (B) $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

(C) $p^2 + q^2 + r^2 = 2(pq + qr + rp)$ (D) none of these

Q.14 If A and B be two complex numbers satisfying $\frac{A}{B} + \frac{B}{A} = 1$. Then the two points represented by A and B and the origin form the vertices of

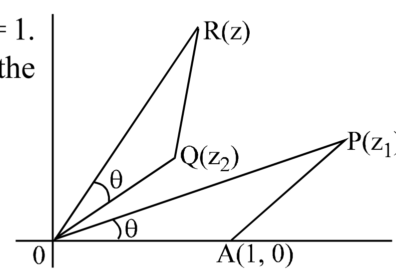
(A) an equilateral triangle

(B) an isosceles triangle which is not equilateral

(C) an isosceles triangle which is not right angled

(D) a right angled triangle

Q.15 On the complex plane triangles OAP & OQR are similiar and $l(OA) = 1$. If the points P and Q denotes the complex numbers z_1 & z_2 then the complex number 'z' denoted by the point R is given by :



(A) $z_1 z_2$ (B) $\frac{z_1}{z_2}$

(C) $\frac{z_2}{z_1}$ (D) $\frac{z_1 + z_2}{z_2}$

Paragraph for question nos. 16 to 18

For the complex number $w = \frac{4z - 5i}{2z + 1}$

Q.16 The locus of z, when w is a real number other than 2, is

(A) a point circle

(B) a straight line with slope $-\frac{5}{2}$ and y-intercept $\frac{5}{4}$

(C) a straight line with slope $\frac{5}{2}$ and y-intercept $\frac{5}{4}$

(D) a straight line passing through the origin

Q.17 The locus of z, when w is a purely imaginary number is

(A) a circle with centre $\left(\frac{1}{2}, -\frac{5}{4}\right)$ passing through origin.

(B) a circle with centre $\left(-\frac{1}{4}, \frac{5}{8}\right)$ passing through origin.

(C) a circle with centre $\left(\frac{1}{4}, -\frac{5}{8}\right)$ and radius $\frac{\sqrt{29}}{8}$

(D) any other circle

Q.18 The locus of z, when $|w| = 1$ is

(A) a circle with centre $\left(-\frac{5}{8}, \frac{1}{4}\right)$ and radius $\frac{1}{2}$ (B) a circle with centre $\left(\frac{1}{4}, -\frac{5}{8}\right)$ and radius $\frac{1}{2}$

(C) a circle with centre $\left(\frac{5}{8}, -\frac{1}{4}\right)$ and radius $\frac{1}{2}$ (D) any other circle

Paragraph of questions nos. 19 to 21

Consider the two complex numbers z and w such that $w = \frac{z-1}{z+2} = a + bi$, where $a, b \in R$.

Q.19 If $z = \text{CiS}\,\theta$ then, which of the following does hold good?

(A) $\cos\theta = \frac{1-5a}{1+4a}$

(B) $\sin\theta = \frac{9b}{1-4a}$

(C) $(1 + 5a)^2 + (3b)^2 = (1 - 4a)^2$

(D) All of these

Q.20 Which of the following is the value of $-\frac{b}{a}$, whenever it exists?

(A) $3\tan\left(\frac{\theta}{2}\right)$ (B) $\frac{1}{3}\tan\left(\frac{\theta}{2}\right)$ (C) $-\frac{1}{3}\cot\theta$ (D) $3\cot\left(\frac{\theta}{2}\right)$

Q.21 Which of the following equals $|z|$?

(A) $|w|$ (B) $(a + 1)^2 + b^2$ (C) $a^2 + (b + 2)^2$ (D) $(a + 1)^2 + (b + 1)^2$

DPP - 5

Q.1 If the six solutions of $x^6 = -64$ are written in the form $a + bi$, where *a* and *b* are real, then the product of those solutions with $a > 0$, is
(A) 4 (B) 8 (C) 16 (D) 64

Q.2 Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z}\, 2^{1-|z|}$ is
(A) 0 (B) 1 (C) 2 (D) 3

Q.3 If z_1 & z_2 are two complex numbers & if $\arg \dfrac{z_1 + z_2}{z_1 - z_2} = \dfrac{\pi}{2}$ but $|z_1 + z_2| \neq |z_1 - z_2|$ then the figure formed by the points represented by $0, z_1, z_2$ & $z_1 + z_2$ is :
(A) a parallelogram but not a rectangle or a rhombous
(B) a rectangle but not a square
(C) a rhombous but not a square (D) a square

Q.4 If $z_n = \cos \dfrac{\pi}{(2n+1)(2n+3)} + i \sin \dfrac{\pi}{(2n+1)(2n+3)}$, then $\underset{n\to\infty}{\text{Limit}} (z_1 . z_2 . z_3 . ... z_n) =$
(A) $\cos \dfrac{\pi}{3} + i \sin \dfrac{\pi}{3}$ (B) $\cos \dfrac{\pi}{6} + i \sin \dfrac{\pi}{6}$ (C) $\cos \dfrac{5\pi}{6} + i \sin \dfrac{5\pi}{6}$ (D) $\cos \dfrac{3\pi}{2} + i \sin \dfrac{3\pi}{2}$

Q.5 The straight line $(1 + 2i)z + (2i - 1)\bar{z} = 10i$ on the complex plane, has intercept on the imaginary axis equal to
(A) 5 (B) $\dfrac{5}{2}$ (C) $-\dfrac{5}{2}$ (D) -5

Q.6 If $\cos\theta + i \sin\theta$ is a root of the equation $x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ then the value of $\sum\limits_{r=1}^{n} a_r \cos r\theta$ equals (where all coefficient are real)
(A) 0 (B) 1 (C) -1 (D) none

Q.7 Let $A(z_1)$ and $B(z_2)$ represent two complex numbers on the complex plane. Suppose the complex slope of the line joining A and B is defined as $\dfrac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$. Then the lines l_1 with complex slope ω_1 and l_2 with complex slope ω_2 on the complex plane will be perpendicular to each other if
(A) $\omega_1 + \omega_2 = 0$ (B) $\omega_1 - \omega_2 = 0$ (C) $\omega_1\omega_2 = -1$ (D) $\omega_1 \omega_2 = 1$

Q.8 If the equation, $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$, where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root then the expression $\dfrac{a_3}{a_1 a_2} + \dfrac{a_1 a_4}{a_2 a_3}$ has the value equal to:
(A) 0 (B) 1 (C) -2 (D) 2

Q.9 Suppose A is a complex number & $n \in N$, such that $A^n = (A + 1)^n = 1$, then the least value of *n* is
(A) 3 (B) 6 (C) 9 (D) 12

Q.10 Intercept made by the circle $z\bar{z} + \bar{\alpha}z + \alpha\bar{z} + r = 0$ on the real axis on complex plane, is
(A) $\sqrt{(\alpha + \bar{\alpha}) - r}$ (B) $\sqrt{(\alpha + \bar{\alpha})^2 - 2r}$ (C) $\sqrt{(\alpha + \bar{\alpha})^2 + r}$ (D) $\sqrt{(\alpha + \bar{\alpha})^2 - 4r}$

Q.11 If Z_r ; $r = 1, 2, 3,..., 50$ are the roots of the equation $\sum\limits_{r=0}^{50} (Z)^r = 0$, then the value of $\sum\limits_{r=1}^{50} \dfrac{1}{Z_r - 1}$ is
(A) -85 (B) -25 (C) 25 (D) 75

Q.12 All roots of the equation, $(1+z)^6 + z^6 = 0$:
(A) lie on a unit circle with centre at the origin
(B) lie on a unit circle with centre at (- 1, 0)
(C) lie on the vertices of a regular polygon with centre at the origin
(D) are collinear

Q.13 If z & w are two complex numbers simultaneously satisfying the equations,
$z^3 + w^5 = 0$ and $z^2 . \overline{w}^4 = 1$, then :
(A) z and w both are purely real (B) z is purely real and w is purely imaginary
(C) w is purely real and z is purely imaginarly (D) z and w both are imaginary.

Q.14 A function f is defined by $f(z) = (4+i)z^2 + \alpha z + \gamma$ for all complex numbers z, where α and γ are complex numbers. If f(1) and f(i) are both real then the smallest possible value of $|\alpha| + |\gamma|$ equals
(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Q.15 Given f(z) = the real part of a complex number z. For example, $f(3-4i) = 3$. If $a \in N$, $n \in N$ then the value of $\sum_{n=1}^{6a} \log_2 \left| f\left(\left(1+i\sqrt{3}\right)^n \right) \right|$ has the value equal to
(A) $18a^2 + 9a$ (B) $18a^2 + 7a$ (C) $18a^2 - 3a$ (D) $18a^2 - a$

Q.16 It is given that complex numbers z_1 and z_2 satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is 60° then $\left| \frac{z_1 + z_2}{z_1 - z_2} \right|$ can be expressed as $\frac{\sqrt{N}}{7}$ where N is natural number then N equals
(A) 126 (B) 119 (C) 133 (D) 19

Q.17 Let $f(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial with real coefficients and all real roots. Also $|f(i)| = 1$ where $i = \sqrt{-1}$
Statement-1: All 3 roots of f(x) = 0 are zero

because

Statement-2: $a + b + c = 0$
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.18 All complex numbers 'z' which satisfy the relation $\left|z - |z+1|\right| = \left|z + |z-1|\right|$ on the complex plane lie on the
(A) line $y = 0$
(B) line $x = 0$
(C) circle $x^2 + y^2 = 1$
(D) line $x = 0$ or on a line segment joining (–1, 0) to (1, 0)

One ore more than one is/are correct:

Q.19 Let A and B be two distinct points denoting the complex numbers α and β respectively. A complex number z lies between A and B where $z \neq \alpha, z \neq \beta$. Which of the following relation(s) hold good?
(A) $|\alpha - z| + |z - \beta| = |\alpha - \beta|$
(B) $\exists$ a positive real number 't' such that $z = (1 - t)\alpha + t\beta$

(C) $\begin{vmatrix} z-\alpha & \bar{z}-\bar{\alpha} \\ \beta-\alpha & \bar{\beta}-\bar{\alpha} \end{vmatrix} = 0$ (D) $\begin{vmatrix} z & \bar{z} & 1 \\ \alpha & \bar{\alpha} & 1 \\ \beta & \bar{\beta} & 1 \end{vmatrix} = 0$

Q.20 Equation of a straight line on the complex plane passing through a point P denoting the complex number α and perpendicular to the vector $\overrightarrow{OP}$ where 'O' in the origin can be written as

(A) $\text{Im}\left(\frac{z-\alpha}{\alpha}\right) = 0$ (B) $\text{Re}\left(\frac{z-\alpha}{\alpha}\right) = 0$ (C) $\text{Re}(\bar{\alpha} z) = 0$ (D) $\bar{\alpha} z + \alpha\bar{z} - 2|\alpha|^2 = 0$

Q.21 Which of the following represents a point on an argands' plane, equidistant from the roots of the equation $(z + 1)^4 = 16z^4$?

(A) (0, 0) (B) $\left(-\frac{1}{3}, 0\right)$ (C) $\left(\frac{1}{3}, 0\right)$ (D) $\left(0, \frac{2}{\sqrt{5}}\right)$

Q.22 If z is a complex number which simultaneously satisfies the equations
$3|z - 12| = 5|z - 8i|$ and $|z - 4| = |z - 8|$ then the Im(z) can be
(A) 15 (B) 16 (C) 17 (D) 8

Q.23 Let z_1, z_2, z_3 are the coordinates of the vertices of the triangle $A_1A_2A_3$. Which of the following statements are equivalent.
(A) $A_1A_2A_3$ is an equilateral triangle.
(B) $(z_1 + \omega z_2 + \omega^2 z_3)(z_1 + \omega^2 z_2 + \omega z_3) = 0$, where ω is the cube root of unity.

(C) $\frac{z_2 - z_1}{z_3 - z_2} = \frac{z_3 - z_2}{z_1 - z_3}$ (D) $\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_2 & z_3 & z_1 \end{vmatrix} = 0$

Q.24 If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_{n-1}$ are the imaginary n^{th} roots of unity then the product $\prod_{r=1}^{n-1}(i - \alpha_r)$ (where $i = \sqrt{-1}$) can take the value equal to
(A) 0 (B) 1 (C) i (D) (1 + i)

[MATCH THE COLUMN]

Q.25 Match the equation in z, in **Column-I** with the corresponding values of arg(z) in **Column-II**.

Column-I (equations in z)	**Column-II** (principal value of arg (z))
(A) $z^2 - z + 1 = 0$	(P) $-2\pi/3$
(B) $z^2 + z + 1 = 0$	(Q) $-\pi/3$
(C) $2z^2 + 1 + i\sqrt{3} = 0$	(R) $\pi/3$
(D) $2z^2 + 1 - i\sqrt{3} = 0$	(S) $2\pi/3$

ANSWER KEY

DPP-1

Q.1	A	Q.2	B	Q.3	C	Q.4	B	Q.5	C	Q.6	C	Q.7	A
Q.8	C	Q.9	D	Q.10	A	Q.11	B	Q.12	D	Q.13	C	Q.14	B
Q.15	B, C, D												

DPP-2

Q.1	C	Q.2	B	Q.3	D	Q.4	C	Q.5	C	Q.6	C	Q.7	A
Q.8	D	Q.9	D	Q.10	A	Q.11	B	Q.12	A	Q.13	D	Q.14	B

DPP-3

Q.1	C	Q.2	B	Q.3	B	Q.4	A	Q.5	A	Q.6	C	Q.7	C
Q.8	A	Q.9	C	Q.10	B	Q.11	B	Q.12	A	Q.13	D	Q.14	B
Q.15	D	Q.16	B	Q.17	A	Q.18	B	Q.19	C	Q.20	C		

DPP-4

Q.1	(i)D (ii)C	Q.2	C	Q.3	B	Q.4	B	Q.5	A	Q.6	B	Q.7	B
Q.8	D	Q.9	A	Q.10	D	Q.11	B	Q.12	A	Q.13	C	Q.14	C
Q.15	A	Q.16	C	Q.17	D	Q.18	B	Q.19	B, C, D	Q.20	A, B		

DPP-5

Q.1	B	Q.2	A	Q.3	A	Q.4	D	Q.5	A	Q.6	C	Q.7	C
Q.8	C	Q.9	C	Q.10	B	Q.11	C	Q.12	D	Q.13	C	Q.14	A
Q.15	A	Q.16	C	Q.17	B	Q.18	D	Q.19	C	Q.20	D	Q.21	B

DPP-6

Q.1	A	Q.2	C	Q.3	C	Q.4	B	Q.5	A	Q.6	C	Q.7	A
Q.8	B	Q.9	B	Q.10	D	Q.11	B	Q.12	D	Q.13	A	Q.14	B
Q.15	D	Q.16	C	Q.17	B	Q.18	D	Q.19	A, B, C, D			Q.20	B, D
Q.21	C	Q.22	C, D	Q.23	A, B, C, D			Q.24	A, B, C, D				

Q.25 (A) Q, R; (B) P, S; (C) Q, S; (D) P, R