STRAIGHT LINE

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KEY CONCEPTS (STRAIGHT LINE)

1. DISTANCE FORMULA :

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

2. SECTION FORMULA :

If $P(x, y)$ divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then ;

$$x = \frac{mx_2+nx_1}{m+n} \quad ; \quad y = \frac{my_2+ny_1}{m+n} .$$

If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external .

Note : If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically ; $\frac{2}{AB}=\frac{1}{AP}+\frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

3. CENTROID AND INCENTRE :

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are : $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

& the coordinates of the incentre are : $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

Note that incentre divides the angle bisectors in the ratio
$(b + c) : a$; $(c + a) : b$ & $(a + b) : c$.

REMEMBER :

(i) Orthocentre , Centroid & circumcentre are always collinear & centroid divides the line joining orthocentre & cercumcentre in the ratio 2 : 1 .

(ii) In an isosceles triangle G, O, I & C lie on the same line .

4. SLOPE FORMULA :

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis.
If $\theta = 0$, then $m = 0$ & the line is parallel to the x–axis.
If $A(x_1, y_1)$ & $B(x_2, y_2)$, $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by:

$$m = \left(\frac{y_1-y_2}{x_1-x_2}\right).$$

5. CONDITION OF COLLINEARITY OF THREE POINTS – (SLOPE FORM) :

Points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are collinear if $\left(\frac{y_1-y_2}{x_1-x_2}\right) = \left(\frac{y_2-y_3}{x_2-x_3}\right)$.

6. EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS :

(i) **Slope – intercept form:** $y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis .

(ii) **Slope one point form:** $y - y_1 = m (x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

(iii) **Parametric form :** The equation of the line in parametric form is given by $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ (say). Where 'r' is the distance of any point (x , y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1).

(iv) **Two point form :** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

(v) **Intercept form :** $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

(vi) **Perpendicular form :** $x\cos\alpha + y\sin\alpha = p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x–axis.

(vii) **General Form :** $ax + by + c = 0$ is the equation of a straight line in the general form

7. POSITION OF THE POINT (x_1, y_1) RELATIVE TO THE LINE $ax + by + c = 0$:

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax + by + c = 0$.

8. THE RATIO IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS :

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A & B are on the same side of the given line then $\frac{m}{n}$ is negative but if A & B are on opposite sides of the given line, then $\frac{m}{n}$ is positive

9. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE :

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

10. ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES :

If m_1 & m_2 are the slopes of two intersecting straight lines $(m_1 m_2 \neq -1)$ & θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Note : Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the ΔABC found by these lines are given by,

$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}$; $\tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$ & $\tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$

11. PARALLEL LINES :

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Note that the coefficients of x & y in both the equations must be same.

(iii) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1, y = m_1x + c_2$ and $y = m_2x + d_1, y = m_2x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

12. PERPENDICULAR LINES :

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slopes is -1, i.e. $m_1 m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

(ii) Straight lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ are at right angles if & only if $aa' + bb' = 0$.

13. Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are:
$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

14. CONDITION OF CONCURRENCY :

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$. **Alternatively :** If three constants A, B & C can be found such that

$A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

15. AREA OF A TRIANGLE :

If $(x_i, y_i), i = 1, 2, 3$ are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a $(-)$ ve area if the vertices $(x_i, y_i), i = 1, 2, 3$ are placed in the clockwise sense.

16. CONDITION OF COLLINEARITY OF THREE POINTS–(AREA FORM):

The points $(x_i, y_i), i = 1, 2, 3$ are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. THE EQUATION OF A FAMILY OF STRAIGHT LINES PASSING THROUGH THE POINTS OF INTERSECTION OF TWO GIVEN LINES:

The equation of a family of lines passing through the point of intersection of $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note: If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$
then, $u_1 = 0$; $u_2 = 0$; $u_3 = 0$; $u_4 = 0$ form a parallelogram.
$u_2 u_3 - u_1 u_4 = 0$ represents the diagonal BD.

Proof : Since it is the first degree equation in x & y it is a straight line. Secondly point B satisfies the equation because the co–ordinates of B satisfy $u_2 = 0$ and $u_1 = 0$.
Similarly for the point D. Hence the result.
On the similar lines $u_1u_2 - u_3u_4 = 0$ represents the diagonal AC.

Note: The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some λ and μ.
[For getting the values of λ & μ compare the coefficients of x, y & the constant terms] .

18. BISECTORS OF THE ANGLES BETWEEN TWO LINES :

(i) Equations of the bisectors of angles between the lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = \pm \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

(ii) To discriminate between the acute angle bisector & the obtuse angle bisector

If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.
If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.
If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equations, $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that the constant terms c, c' are positive. Then;

$\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing the origin

& $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector & obtuse angle bisector proceed as follows Write $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that constant terms are positive.

If $aa' + bb' < 0$, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

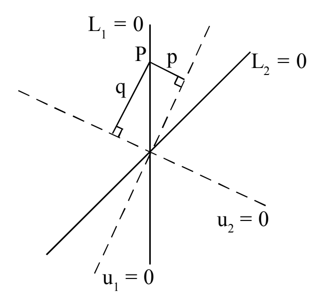
therefore $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ is the equation of other bisector.

If, however, $aa' + bb' > 0$, then the angle between the lines that contains the origin is obtuse & the equation of the bisector of this obtuse angle is:

$\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$; therefore $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

is the equation of other bisector.

(v) Another way of identifying an acute and obtuse angle bisector is as follows :
Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown. If,



$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.
$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.
$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

19. A PAIR OF STRAIGHT LINES THROUGH ORIGIN :

(i) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin & if :

(a) $h^2 > ab \Rightarrow$ lines are real & distinct.
(b) $h^2 = ab \Rightarrow$ lines are coincident.
(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$$m_1 + m_2 = -\frac{2h}{b} \quad \& \quad m_1 m_2 = \frac{a}{b}.$$

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then; $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$.

The condition that these lines are:

(a) At right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 + coefficient of $y^2 = 0$.

(b) Coincident is $h^2 = ab$.

(c) Equally inclined to the axis of x is $h = 0$. i.e. coeff. of $xy = 0$.

Note: A homogeneous equation of degree n represents n straight lines passing through origin.

20. GENERAL EQUATION OF SECOND DEGREE REPRESENTING A PAIR OF STRAIGHT LINES:

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if:

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0, \text{ i.e. if } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only .

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by $lx + my + n = 0$ .... **(i)** &
the 2nd degree curve : $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ .... **(ii)**

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$ ...... **(iii)**

(iii) is obtained by homogenizing (ii) with the help of **(i)**, by writing **(i)** in the form: $\left(\frac{lx+my}{-n}\right) = 1$.

22. The equation to the straight lines bisecting the angle between the straight lines,

$ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

23. The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 + 2hxy + by^2 = 0$ is $\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$.

24. Any second degree curve through the four point of intersection of $f(x\,y) = 0$ & $xy = 0$ is given by $f(x\,y) + \lambda\, xy = 0$ where $f(xy) = 0$ is also a second degree curve.

EXERCISE–I

Q.1 Line $\frac{x}{6}+\frac{y}{8}=1$ intersects the x and y axes at M and N respectively. If the coordinates of the point P lying inside the triangle OMN (where 'O' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find
(a) the coordinates of the point P and
(b) the radius of the circle escribed opposite to the angle N.

Q.2 Two vertices of a triangle are $(4, -3)$ & $(-2, 5)$. If the orthocentre of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

Q.3 The point A divides the join of $P(-5, 1)$ & $Q(3, 5)$ in the ratio $K : 1$. Find the two values of K for which the area of triangle ABC, where B is $(1, 5)$ & C is $(7, -2)$, is equal to 2 units in magnitude.

Q.4 Determine the ratio in which the point $P(3, 5)$ divides the join of $A(1, 3)$ & $B(7, 9)$. Find the harmonic conjugate of P w.r.t. A & B.

Q.5 A line is such that its segment between the straight lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain the equation.

Q.6 A line through the point $P(2, -3)$ meets the lines $x - 2y + 7 = 0$ and $x + 3y - 3 = 0$ at the points A and B respectively. If P divides AB externally in the ratio 3 : 2 then find the equation of the line AB.

Q.7 The area of a triangle is 5. Two of its vertices are $(2, 1)$ & $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex.

Q.8 A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ & $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

Q.9 In the xy plane, the line 'l_1' passes through the point $(1, 1)$ and the line 'l_2' passes through the point $(-1, 1)$. If the difference of the slopes of the lines is 2. Find the locus of the point of intersection of the lines l_1 and l_2.

Q.10 Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$, find the equation to the other diagonal.

Q.11 The line $3x + 2y = 24$ meets the y–axis at A & the x–axis at B. The perpendicular bisector of AB meets the line through $(0, -1)$ parallel to x–axis at C. Find the area of the triangle ABC.

Q.12 If the straight line drawn through the point $P(\sqrt{3}, 2)$ & inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}\,x - 4y + 8 = 0$ at Q. Find the length PQ.

Q.13 Find the area of the triangle formed by the straight lines whose equations are $x + 2y - 5 = 0$; $2x + y - 7 = 0$ and $x - y + 1 = 0$. Also compute the tangent of the interior angles of the triangle and hence comment upon the nature of triangle.

Q.14 A triangle has side lengths 18, 24 and 30. Find the area of the triangle whose vertices are the incentre, circumcentre and centroid of the triangle.

Q.15 The points $(1, 3)$ & $(5, 1)$ are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c & the remaining vertices.

Q.16 A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

Q.17 The triangle ABC, right angled at C, has median AD, BE and CF. AD lies along the line $y = x + 3$, BE lies along the line $y = 2x + 4$. If the length of the hypotenuse is 60, find the area of the triangle ABC.

Q.18 Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point $(1, -10)$. Determine the equation of the third side.

Q.19 The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is $(1, -2)$ find the equation of the line BC.

Q.20 If $(x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2$
$(x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2$
and $(x_3 - x_1)^2 + (y_3 - y_1)^2 = c^2$

then $\lambda \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = (a + b + c)(b + c - a)(c + a - b)(a + b - c)$. Find the value of λ.

Q.21 Given vertices A (1, 1), B (4, –2) & C (5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

Q.22 Triangle ABC lies in the Cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respectively and the coordinates of A are (p, q). The line containing the median to the side BC has slope –5. Find the largest possible value of $(p + q)$.

Q.23 Determine the range of values of $\theta \in [0, 2\pi]$ for which the point $(\cos\theta, \sin\theta)$ lies inside the triangle formed by the lines $x + y = 2$; $x - y = 1$ & $6x + 2y - \sqrt{10} = 0$.

Q.24 The points (–6, 1), (6, 10), (9, 6) and (–3, –3) are the vertices of a rectangle. If the area of the portion of this rectangle that lies above the x axis is a/b, find the value of $(a + b)$, given a and b are coprime.

Q.25 Let ABC be a triangle such that the coordinates of A are $(-3, 1)$. Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$. Then match the entries of column-I with their corresponding correct entries of column-II.

	Column-I		Column-II
(A)	Equation of the line AB is	(P)	$2x + y - 3 = 0$
(B)	Equation of the line BC is	(Q)	$2x - 3y + 9 = 0$
(C)	Equation of CA is	(R)	$4x + 7y + 5 = 0$
		(S)	$18x - y - 49 = 0$

EXERCISE–II

Q.1 Consider a triangle ABC with sides AB and AC having the equations $L_1 = 0$ and $L_2 = 0$. Let the centroid, orthocentre and circumcentre of the ΔABC are G, H and S respectively. $L = 0$ denotes the equation of side BC.

(a) If $L_1 : 2x - y = 0$ and $L_2 : x + y = 3$ and G (2, 3) then find the slope of the line $L = 0$.

(b) If $L_1 : 2x + y = 0$ and $L_2 : x - y + 2 = 0$ and H (2, 3) then find the y-intercept of $L = 0$.

(c) If $L_1 : x + y - 1 = 0$ and $L_2 : 2x - y + 4 = 0$ and S(2, 1) then find the x-intercept of the line $L = 0$.

Q.2 The equations of perpendiculars of the sides AB & AC of triangle ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.

Q.3 The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are $A(-8, 5)$; $B(-15, -19)$ and $C(1, -7)$ has the equation $ax + 2y + c = 0$. Find 'a' and 'c'.

Q.4 Find the equation of the straight lines passing through $(-2, -7)$ & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

Q.5 Two sides of a rhombous ABCD are parallel to the lines $y = x + 2$ & $y = 7x + 3$. If the diagonals of the rhombous intersect at the point $(1, 2)$ & the vertex A is on the y-axis, find the possible coordinates of A.

Q.6 A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y + 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equaion of the bisector.

Q.7 A point P is such that its perpendicular distance from the line $y - 2x + 1 = 0$ is equal to its distance from the origin. Find the equation of the locus of the point P. Prove that the line $y = 2x$ meets the locus in two points Q & R, such that the origin is the mid point of QR.

Q.8 Find the equations of the sides of a triangle having $(4, -1)$ as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

Q.9 P is the point $(-1, 2)$, a variable line through P cuts the x & y axes at A & B respectively Q is the point on AB such that PA, PQ, PB are H.P. Show that the locus of Q is the line $y = 2x$.

Q.10 The equations of the altitudes AD, BE, CF of a triangle ABC are $x + y = 0$, $x - 4y = 0$ and $2x - y = 0$ respectively. The coordinates of A are $(t, -t)$. Find coordinates of B & C. Prove that if t varies the locus of the centroid of the triangle ABC is $x + 5y = 0$.

Q.11 The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ; find the combined equation of these straight lines.

Q.12 Consider a Δ ABC whose sides AB, BC and CA are represented by the straight lines $2x + y = 0$, $x + py = q$ and $x - y = 3$ respectively. The point P is $(2, 3)$.

(a) If P is the centroid, then find the value of $(p + q)$.

(b) If P is the orthocentre, then find the value of $(p + q)$.

(c) If P is the circumcentre, then find the value of $(p + q)$.

Q.13 Consider a line pair $2x^2 + 3xy - 2y^2 - 10x + 15y - 28 = 0$ and another line L passing through origin with gradient 3. The line pair and line L form a triangle whose vertices are A, B and C.

(a) Find the sum of the contangents of the interior angles of the triangle ABC.

(b) Find the area of triangle ABC

(c) Find the radius of the circle touching all the 3 sides of the triangle.

Q.14 Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrency & if not, give reasons.

Q.15 A straight line is drawn from the point $(1, 0)$ to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

Q.16 The two line pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon find **(i)** area of the polygon; **(ii)** length of its diagonals.

Q.17 Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.

EXERCISE–III

Q.1(a) The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is :

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

(b) Let PS be the median of the triangle with vertices, P (2, 2), Q (6, - 1) and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is

(A) $2x - 9y - 7 = 0$ (B) $2x - 9y - 11 = 0$

(C) $2x + 9y - 11 = 0$ (D) $2x + 9y + 7 = 0$ [JEE 2000 (Scr.)1+1out of 35]

(c) For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance d(P, Q) is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let O = (0, 0) and A = (3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram.

[JEE 2000 (Mains) 10 out of 100]

Q.2 Find the position of point (4, 1) after it undergoes the following transformations successively.

(i) Reflection about the line, $y = x - 1$

(ii) Translation by one unit along x – axis in the positive direction .

(iii) Rotation through an angle $\pi/4$ about the origin in the anti–clockwise direction.

[REE 2000 (Mains) 3 out of 100]

Q.3(a) Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

(b) The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

(A) 2 (B) 0 (C) 4 (D) 1

[JEE 2001 (Screening)]

Q.4(a) Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

(b) A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio

(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

(c) The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

[JEE 2002 (Screening)]

(d) A straight line L through the origin meets the line $x + y = 1$ and $x + y = 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. [JEE 2002 (Mains)]

(e) A straight line L with negative slope passes through the point (8,2) and cuts the positive coordinates axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin.

[JEE 2002 Mains, 5 out of 60]

Q.5 The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is
(A) 2 (B) 3 (C) 4 (D) 6
[JEE 2004 (Screening)]

Q.6 The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P (h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. [JEE 2005, Mains, 2]

Q.7(a) Let O(0, 0), P (3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are
(A) $(4/3, 3)$ (B) $(3, 2/3)$ (C) $(3, 4/3)$ (D) $(4/3, 2/3)$

(b) Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
Statement-1: The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$
because
Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.
(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. [JEE 2007, 3+3]

Q.8 Consider the lines given by
$L_1 = x + 3y - 5 = 0$
$L_2 = 3x - ky - 1 = 0$
$L_3 = 5x + 2y - 12 = 0$
Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in OMR.

	Column-I		**Column-II**
(A)	L_1, L_2, L_3 are concurrent, if	(P)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(R)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(S)	$k = 5$

[JEE 2008, 6]

ANSWER SHEET
STRAIGHT LINE

EXERCISE–I

Q.1 (a) $\left(2, \frac{8}{3}\right)$; (b) 4 **Q.2** (33, 26) **Q.3** $K = 7$ or $\frac{31}{9}$

Q.4 1 : 2 ; Q(–5, –3) **Q.5** $83x - 35y + 92 = 0$ **Q.6** $2x + y - 1 = 0$

Q.7 $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ **Q.9** $y = x^2$ and $y = 2 - x^2$ **Q.10** $x - y = 0$ **Q.11** 91 sq.units

Q.12 6 units **Q.13** $\frac{3}{2}$ sq. units, $\left(3, 3, \frac{3}{4}\right)$, isosceles

Q.14 3 units **Q.15** $c = -4$; B(2 , 0); D(4 , 4)

Q.16 $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$ **Q.17** 400 sq. units

Q.18 $x - 3y - 31 = 0$ or $3x + y + 7 = 0$

Q.19 $14x + 23y = 40$ **Q.20** 4 **Q.21** $x - 5 = 0$ **Q.22** 47

Q.23 $0 < \theta < \frac{5\pi}{6} - \tan^{-1} 3$ **Q.24** 533 **Q.25** (A) R; (B) S ; (C) Q

EXERCISE–II

Q.1 (a) 5 ; (b) 2 ; (c) $\frac{3}{2}$ **Q.2** $x + 4y = 4$; $5x + 2y = 8$ **Q.3** $a = 11$, $c = 78$

Q.4 $7x + 24y + 182 = 0$ or $x = -2$ **Q.5** (0 , 0) or $\left(0, \frac{5}{2}\right)$

Q.6 $3x + 6y - 16 = 0$; $8x + 8y + 7 = 0$; $12x + 6y - 11 = 0$ **Q.7** $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

Q.8 $2x - y + 3 = 0$, $2x + y - 7 = 0$, $x - 2y - 6 = 0$

Q.10 $B\left(-\frac{2t}{3}, -\frac{t}{6}\right)$, $C\left(\frac{t}{2}, t\right)$ **Q.11** $(y_1^2 - \delta^2)x^2 - 2x_1y_1xy + (x_1^2 - \delta^2)y^2 = 0$

Q.12 (a) 74 ; (b) 50; (c) 47 **Q.13** (a) $\frac{50}{7}$; (b) $\frac{63}{10}$; (c) $\frac{3}{10}\left(8\sqrt{5} - 5\sqrt{10}\right)$

Q.14 (1, –2) , yes $\left(\frac{1}{3}, -\frac{2}{3}\right)$

Q.15 $x + y = 1$; $x + 9y = 1$ **Q.16** (i) area = 6 sq. units, (ii) diagonals are $\sqrt{5}$ & $\sqrt{53}$

Q.17 $6x^2 - xy - y^2 - x - 12y - 35 = 0$

EXERCISE–III

Q.1 **(a)** D; **(b)** D **Q.2** $(4, 1) \to (2, 3) \to (3, 3) \to \left(0, 3\sqrt{2}\right)$

Q.3 (a) D; (b) A **Q.4** **(a)** C; **(b)** B; **(c)** B; **(d)** $x - 3y + 5 = 0$; **(e)** 18

Q.5 A **Q.6** $y = 2x + 1$, $y = -2x + 1$

Q.7 (a) C; (b) C **Q.8** (A) S; (B) P,Q; (C) R ; (D) P,Q,S