MC Sir

Straight Line

1. Basic Geometry, H/G/O/I
2. Distance & Section formula, Area of triangle, co linearity
3. Locus, Straight line
4. Different forms of straight line equation
5. Examples based on different form of straight line equation
6. Position of point with respect to line, Length of perpendicular, Angle between two straight lines

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7. Parametric Form of Line
8. Family of Lines
9. Shifting of origin, Rotation of axes
10. Angle bisector with examples
11. Pair of straight line, Homogenization

Straight Line

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No. of Questions				
2008	2009	2010	2011	2012
2	1	1	2	--

Basic Concepts

Determinants

Array of No.

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Value of x and y
In Determinant form

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Method of Solving
2 × 2 Determinant

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2 × 2 Determinant

$$\begin{vmatrix} \overset{+}{b_1} & \overset{-}{c_1} \\ b_2 & c_2 \end{vmatrix} = b_1 c_2 - b_2 c_1$$

Method of Solving 3 × 3 Determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Definition of Minor

$[M_{ij}]$

Minor of an element is defined as minor determinant obtained by deleting a particular row or column in which that element lies.

Example

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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$$\text{Minor of } a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

Cofactor

$$C_{ij} = (-1)^{i + j} M_{ij}$$

Example

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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$$C_{12} = (-1)^{1+2} M_{12} = -M_{12}$$

Value of Determinant in term of Minor and Cofactor

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$$D = a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13}$$

Value of Determinant in term of Minor and Cofactor

$$D = a_{11}\,M_{11} - a_{12}\,M_{12} + a_{13}\,M_{13}$$

$$D = a_{11}\,C_{11} + a_{12}\,C_{12} + a_{13}\,C_{13}$$

Note

A determinant of order 3 will have 9 minors each minor will be a minor of order 2

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A determinant of order 4 will have 16 minors each minor will be a minor of order 3

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A determinant of order 4 will have 16 minors each minor will be a minor of order 3

We can expand a determinant in 6 ways (for 3×3 determinant)

Examples

Q. $$\begin{vmatrix} 2 & 3 & 0 \\ -2 & 1 & 2 \\ 0 & 5 & -1 \end{vmatrix}$$

Examples

Q. $$\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 8 \end{vmatrix}$$

Examples

Q. $$\begin{vmatrix} k+3 & 1 & -2 \\ 3 & -2 & 1 \\ -k & -3 & 3 \end{vmatrix} = 0, \text{ Find } k$$

Examples

Q. $$\begin{vmatrix} x & -6 & 1 \\ 2x & -3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 0, \text{Find } x$$

Properties of Determinant

P-1 Property

The value of determinant remains same if row and column are interchanged

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$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad D^T = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$$D = D^T$$

Skew Symmetric Determinant

$$D^T = - D$$

Value of skew symmetric determinant is zero

P-2 Property

If any two rows or column be interchanged the value of determinant is changed in sign only.

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If any two rows or column be interchanged the value of determinant is changed in sign only.

Example :

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; -D = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P-3 Property

If a determinant has any two row or column same then its value is zero.

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If a determinant has any two row or column same then its value is zero.

Example :

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

P-4 Property

If all element of any row or column be multiplied by same number than determinant is multiplied by that number

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$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Rightarrow \quad Dk = \begin{vmatrix} a_1 & b_1 & c_1 \\ ka_2 & kb_2 & kc_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P-5 Property

If each element of any row or column can be expressed as sum of two terms then determinant can be expressed as sum of two determinants.

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If each element of any row or column can be expressed as sum of two terms then determinant can be expressed as sum of two determinants.

Example :

$$D = \begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Result can be generalized.

Example

Q. If $D_r = \begin{vmatrix} r & x & n(n+1)/2 \\ 2r-1 & y & n^2 \\ 3r-2 & z & n(3n-1)/2 \end{vmatrix}$

Find the value of $\sum_{r=1}^{n} D_r$

P-6 Property

The value of determinant is not changed by adding to the element of any row or column the same multiples of the corresponding elements of any other row or column.

Example

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad R_1 \to R_1 + \lambda R_2 + \mu R_3$$

Example

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad R_1 \longrightarrow R_1 + \lambda R_2 + \mu R_3$$

$$D' = \begin{vmatrix} a_1 + \lambda a_2 + \mu a_3 & b_1 + \lambda b_2 + \mu c_2 & c_1 + \lambda c_2 + \mu c_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D = D'$$

Remainder Theorem

Any polynomial P(x) when divided by $(x - \alpha)$ then remainder will be $P(\alpha)$

If $P(\alpha) = 0 \Rightarrow x - \alpha$ is factor of P (x)

P-7 Property

If by putting $x = a$ the value of determinant vanishes then $(x-a)$ is a factor of the determinant.

Method

(i) Create zeros

Method

(i) Create zeros

(ii) Take common out of rows and columns

Method

(i) Create zeros

(ii) Take common out of rows and columns

(iii) Switch over between variable to create identical row or column.

Example

Q. Show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$$

Q. Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$= (a^2 + b^2 + c^2)\,(a + b + c)\,(a – b)\,(b – c)\,(c – a)$

Q. Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Q. $$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

Find f(100) (JEE 99)

Q. $$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

Q.

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Q. $$\begin{vmatrix} 23 & 66 & 11 \\ 36 & 55 & 26 \\ 63 & 143 & 37 \end{vmatrix} = 0$$

Q. $$D = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = 0$$

Q.
$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}=0$$

System of Equation

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2 x + b_2 y + c_2 = 0$$

System Consistant

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow unique\ solution$$

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$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow no\ solution$$

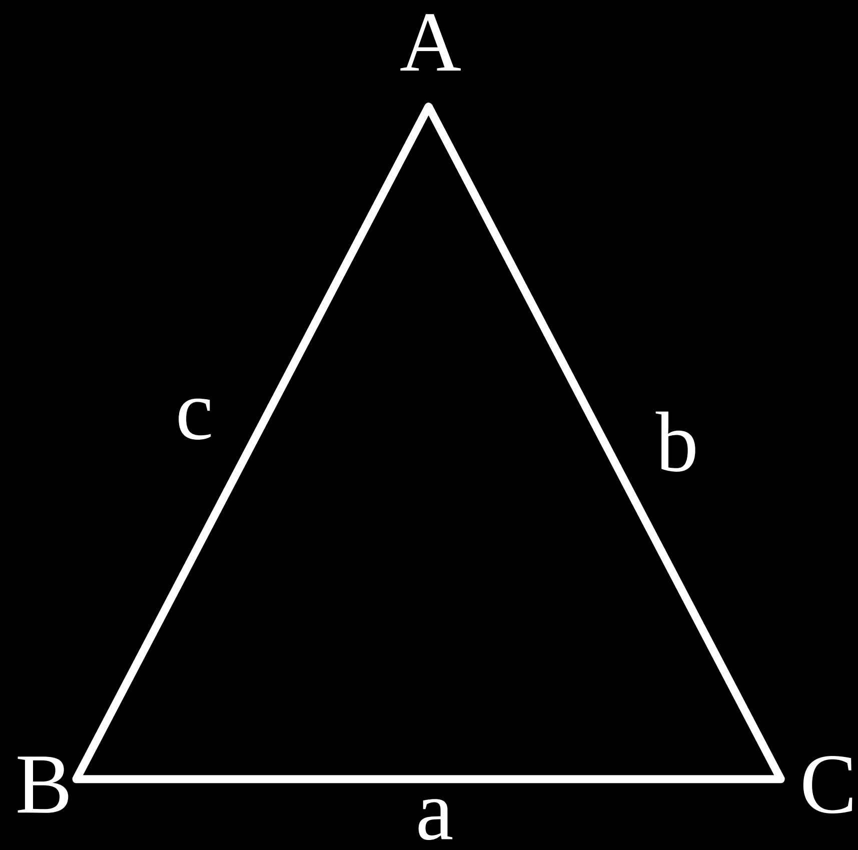
System Consistant

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \textit{unique solution}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \textit{no solution}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \textit{infinite solution}$$

Triangle



Triangle

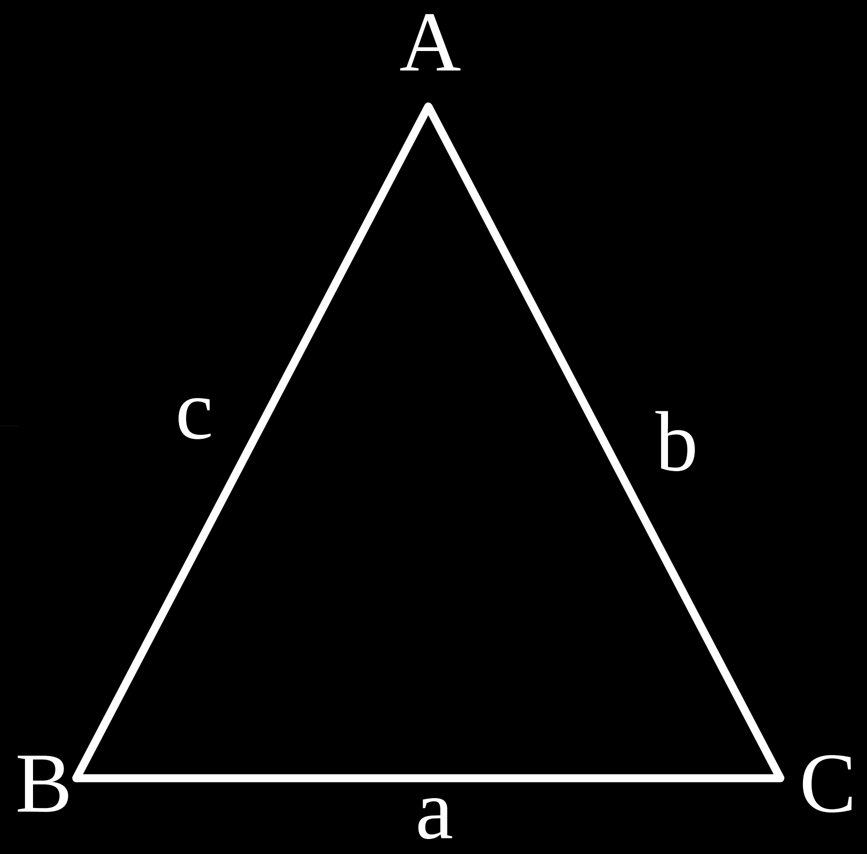
$\angle B > \angle C$

$b > c$

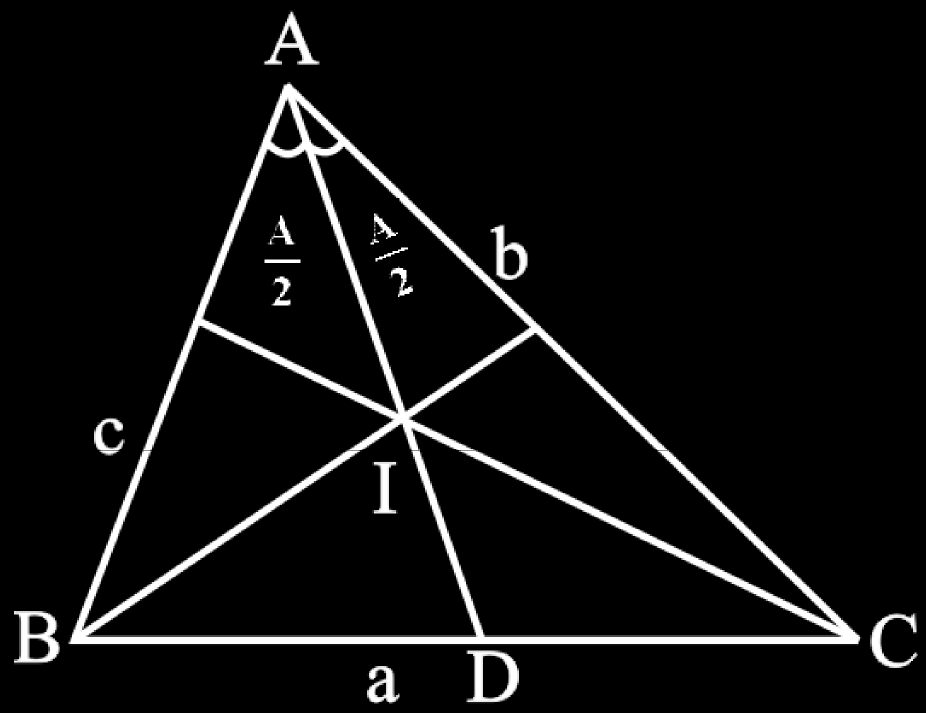
$\angle A + \angle B + \angle C = \pi$

$a + b > c$

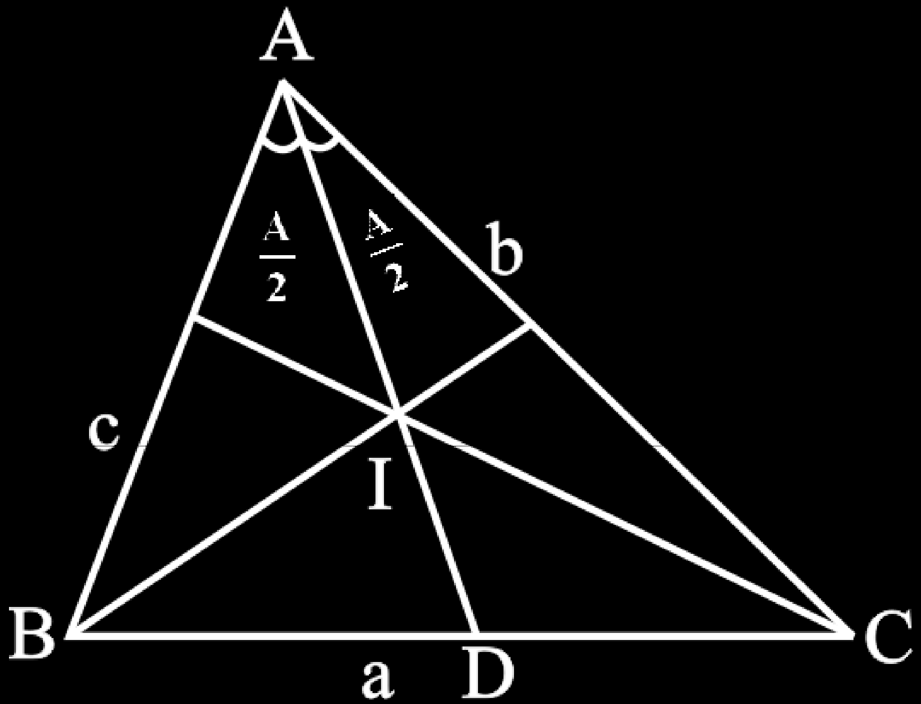
$|a - b| < c$



Angle bisector

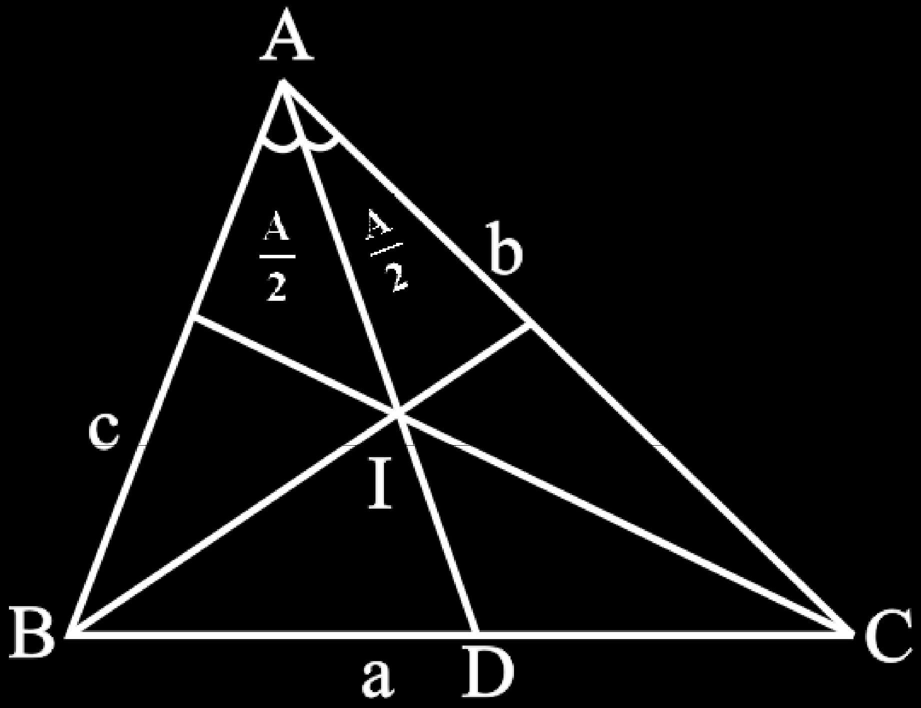


Angle bisector



$$AD = \frac{2bc}{b+c}\cos\left(A/2\right)$$

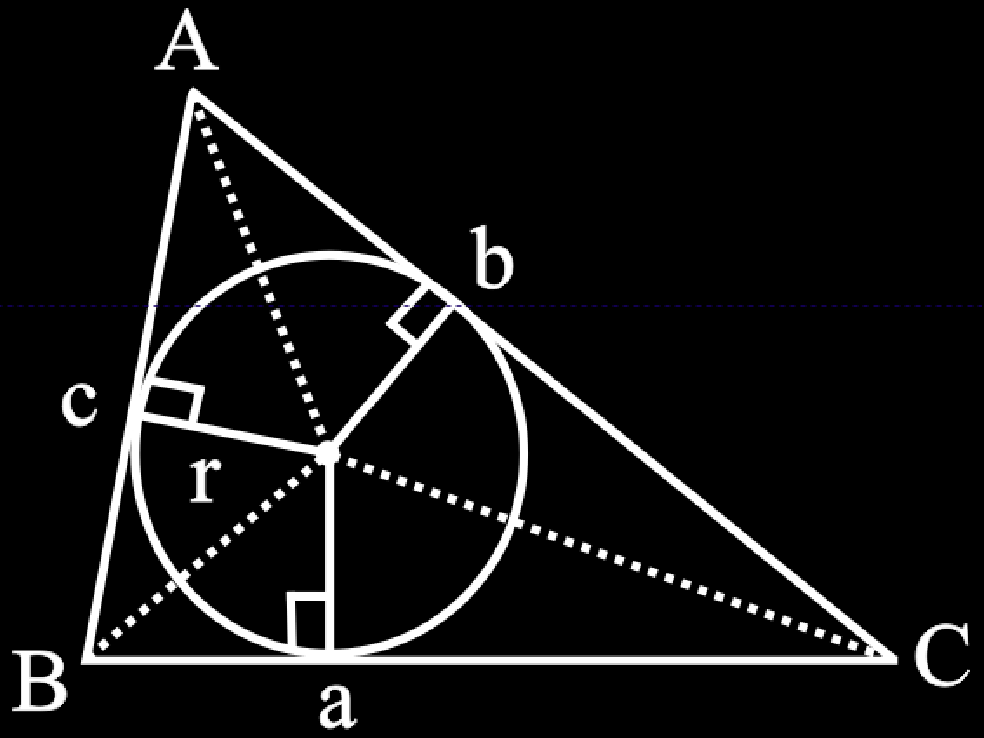
Angle bisector



$$AD = \frac{2bc}{b+c}\cos(A/2)$$

I is called Incentre (Point of concurrency of internal angle bisector)

Incircle



Circle who touches sides of triangle is called incircle, $r = \frac{\Delta}{S}$

Altitude

Perpendicular from vertex to opposite side

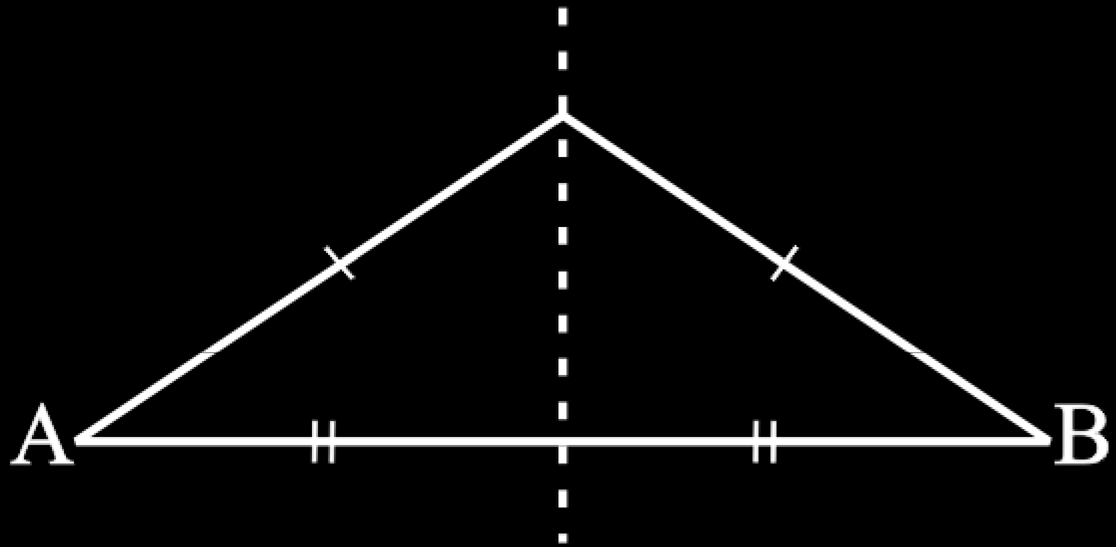
(Orthocenter)

Median

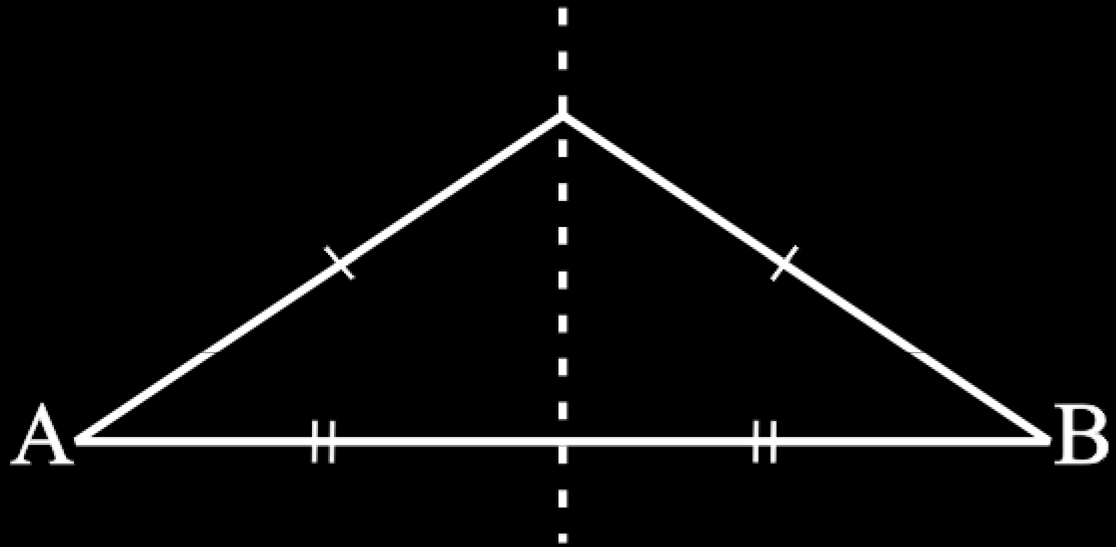
Line joining vertex to mid point of opposite sides

(Centroid)

Perpendicular bisector



Perpendicular bisector

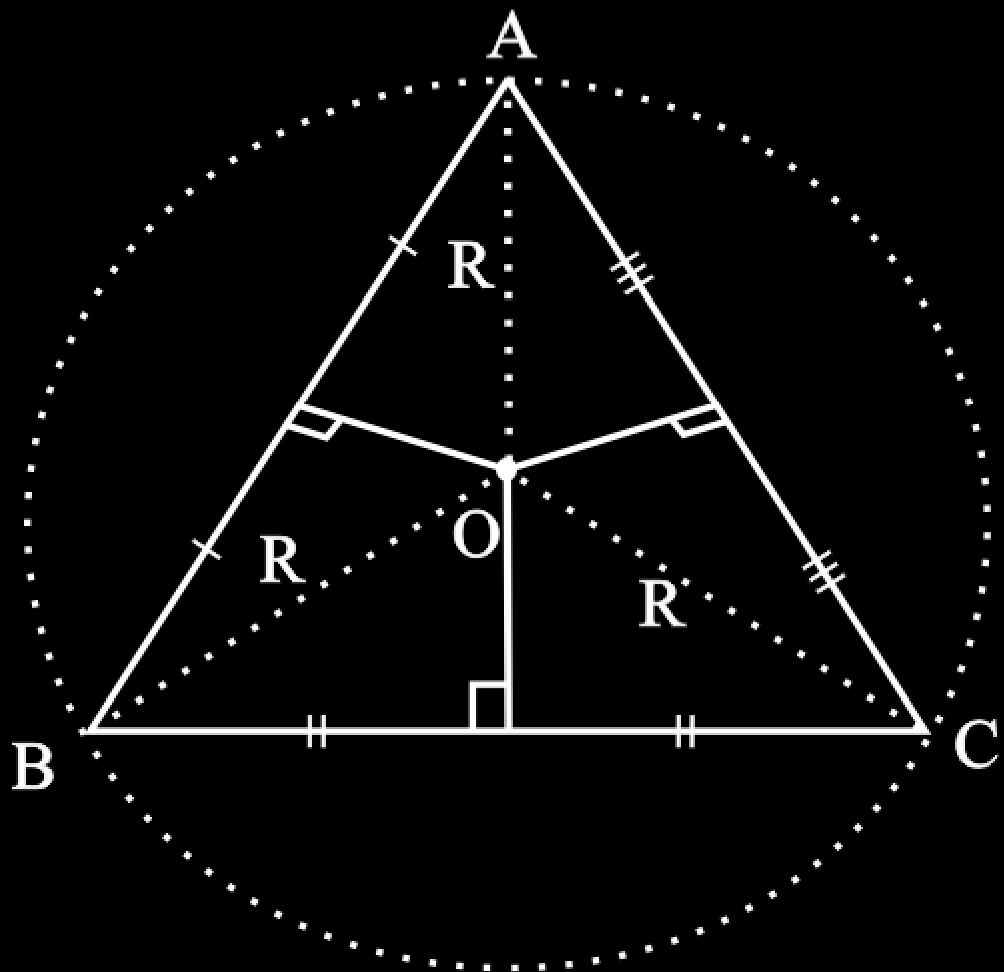


Any point on perpendicular bisector is at equal distance from A & B

Circumcircle

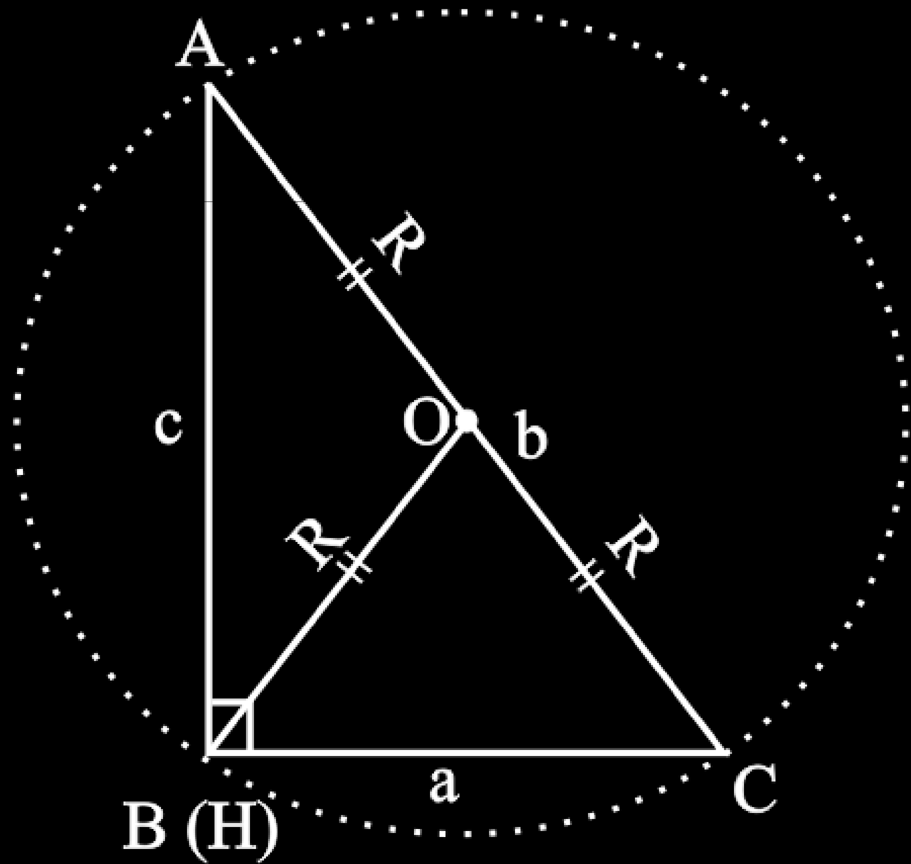
O is circumcentre

R is circumradius



Note

In Right angle triangle



Note

G (centroid) & I (Incentre) always lies in interior of triangle whereas H (Orthocenter) & O (Circumcentre) lies inside, outside or periphery depending upon triangle being acute, obtuse or right angle.

H G O

2 : 1

Quadrilaterals

Note

Sum of all interior angles of n sided figure is

$$= (n - 2)\,\pi$$

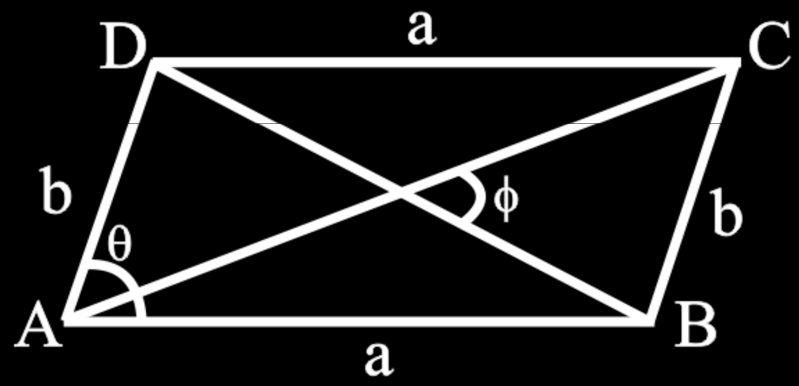
Parallelogram

(i) Opposite sides are parallel & equal

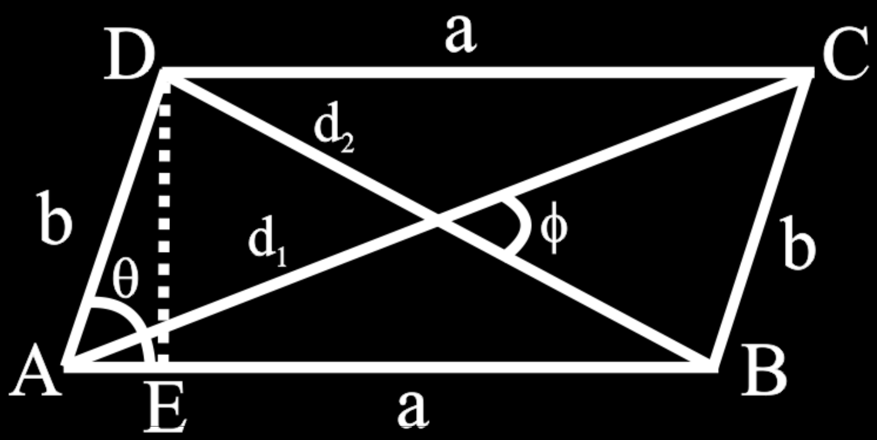
(ii) adjacent angles are supple-

-mentary

(iii) Diagonals are bisected.



Parallelogram



Area parallelogram = $\frac{1}{2} d_1\ d_2 \sin \phi$

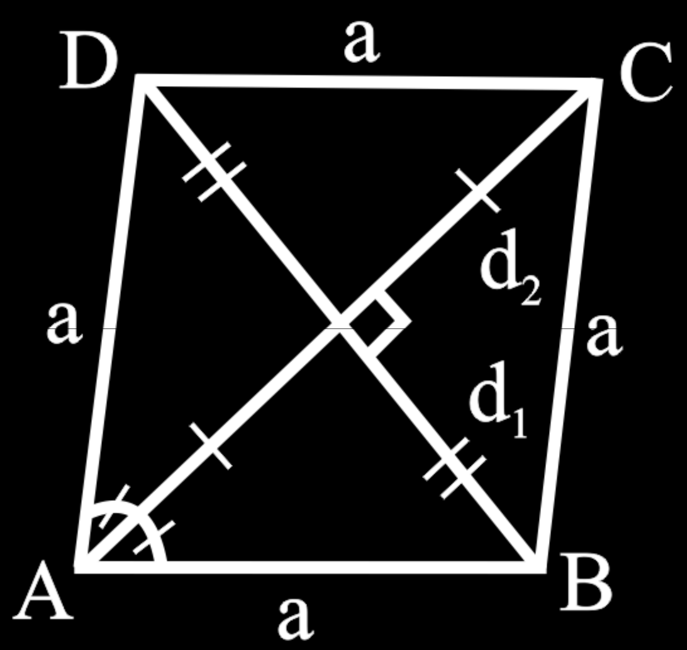
DE = b sin θ

Area of parallelogram = ab sin θ

Rhombus

Parallelogram will be Rhombus

If



(i) Diagonal are perpendicular

(ii) Sides equal

(iii) Diagonal bisects angle of parallelogram

(iv) Area of Rhombus = $\frac{1}{2} d_1 d_2$

Rectangle

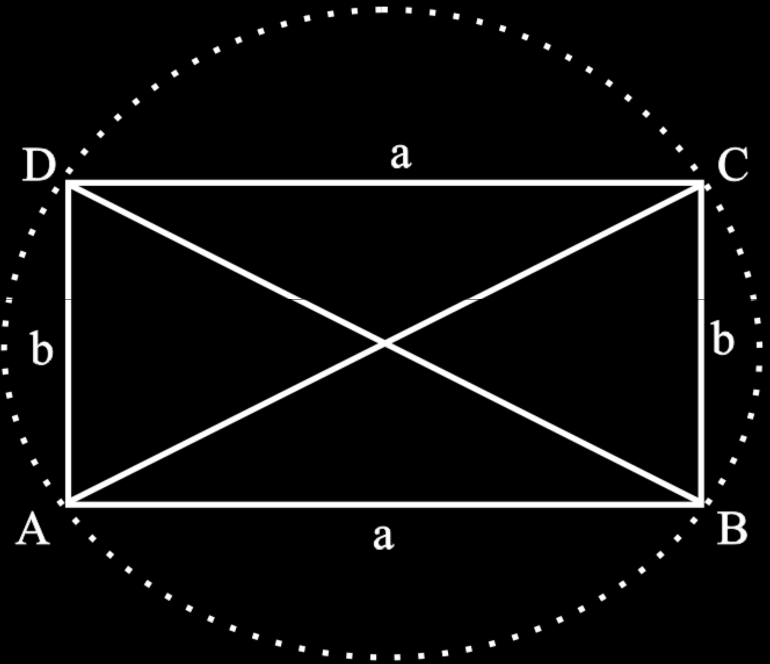
Parallelogram will be Rectangle

If

(1) Angle 90°

(2) Diagonals are equal

(3) $a^2 + b^2 = c^2$



Rectangle is cyclic quadrilateral

Square

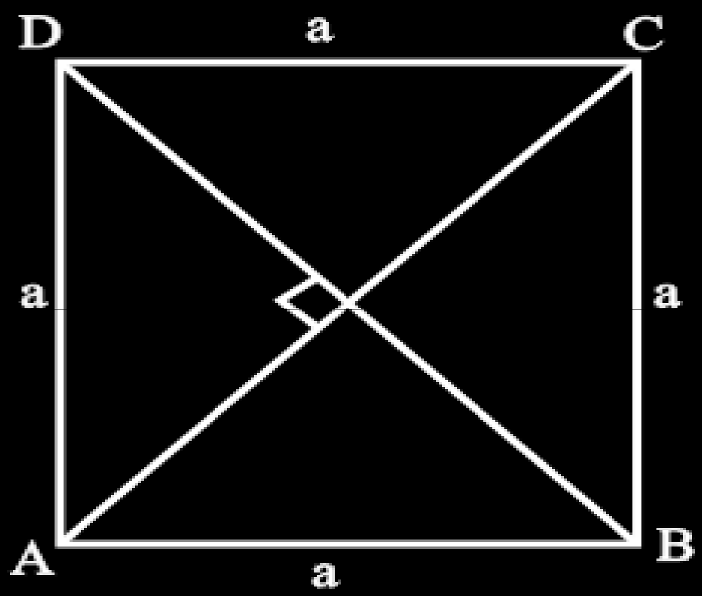
A Rectangle will be square

If

(1) Sides equal

(2) Diagonal are perpendicular

(3) Diagonal are angle bisector



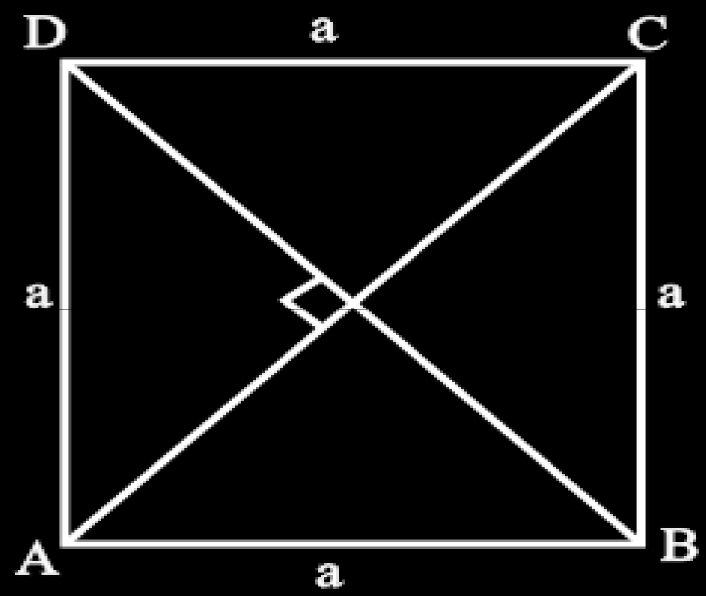
Square

A Rhombus will be square

If

(1) Diagonal are equal

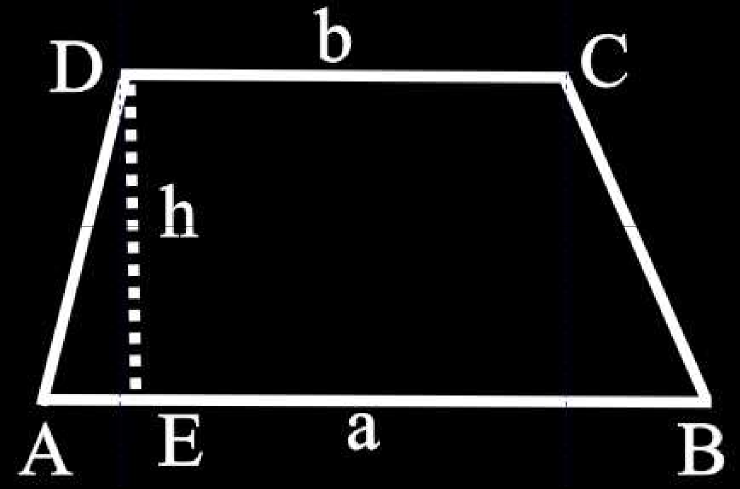
(2) Angle 90°



Trapezium

(1) One pair of opposite sides are parallel

$$\text{area} = \frac{1}{2} \times (a + b)h$$

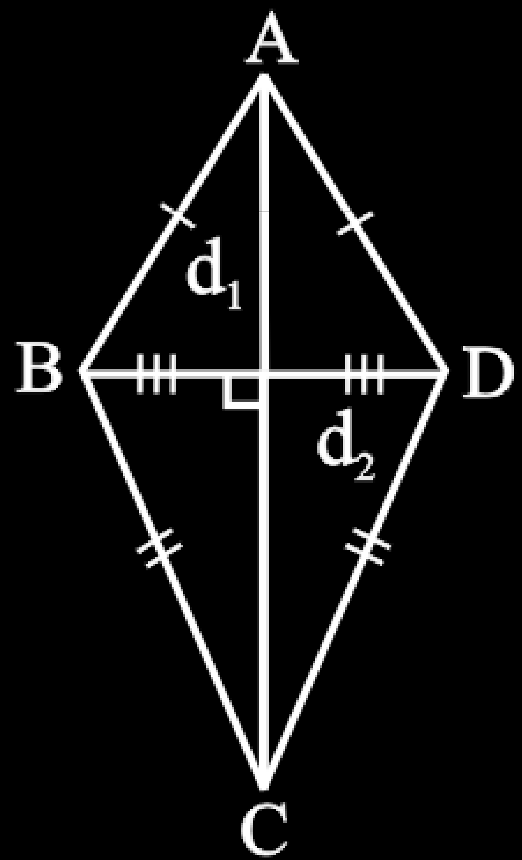


Kite

(1) One diagonal divide figure into two congruent part

(2) Diagonal are perpendicular

(3) Area = $\frac{1}{2}$ d_1 d_2

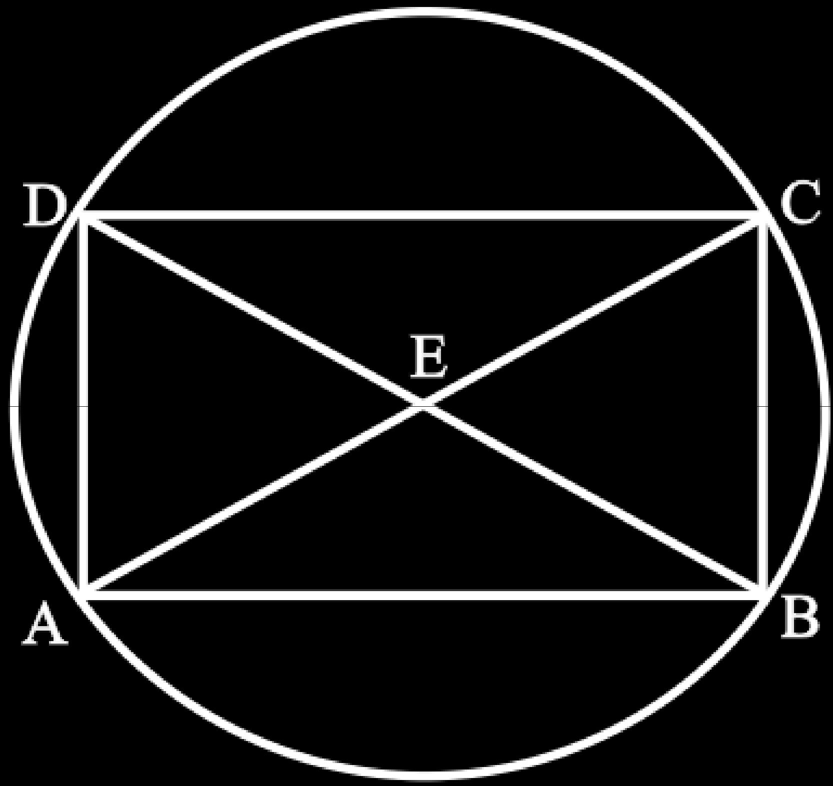


Cyclic Quadrilateral

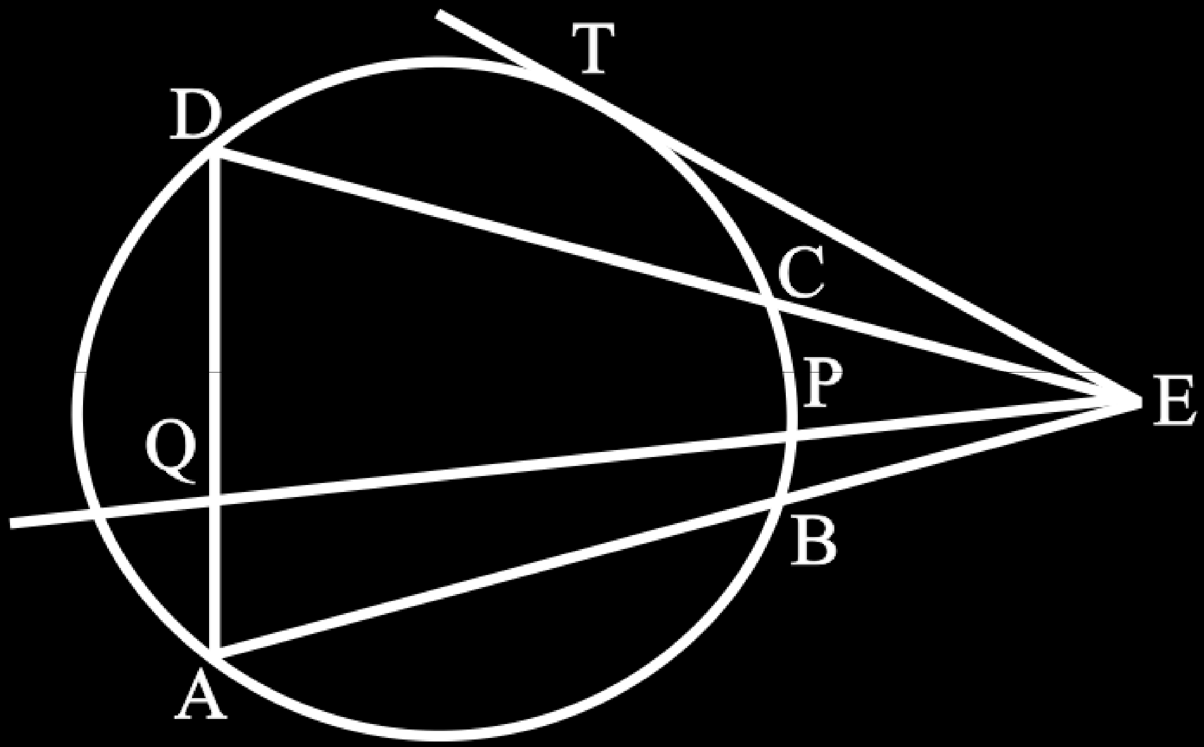
i. Vertices lie on circle

ii. $A + C = \pi = B + D$

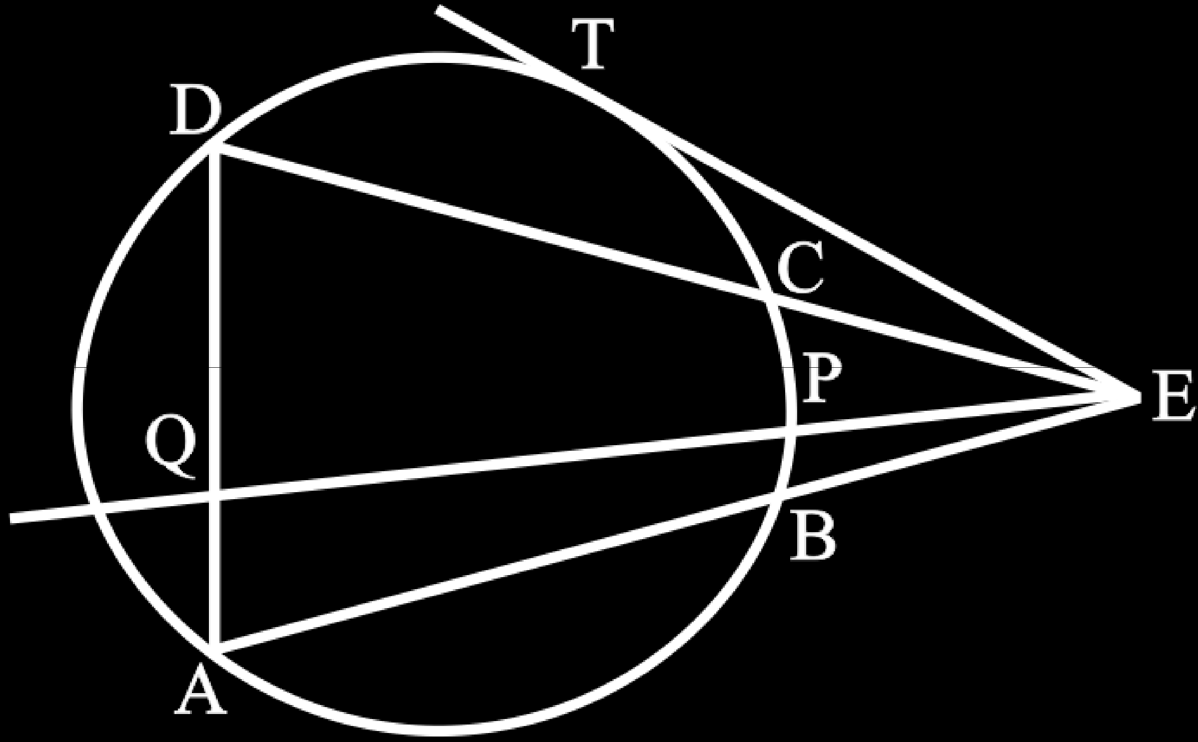
iii. $AE \times EC = BE \times DE$



Note



Note



$$(EB)\,(EA) = (EC)\,(ED) = (ET)^2 = (EP)\,(EQ)$$

PTolmey's Theorem

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Sum of product of opposite side = Product of diagonals

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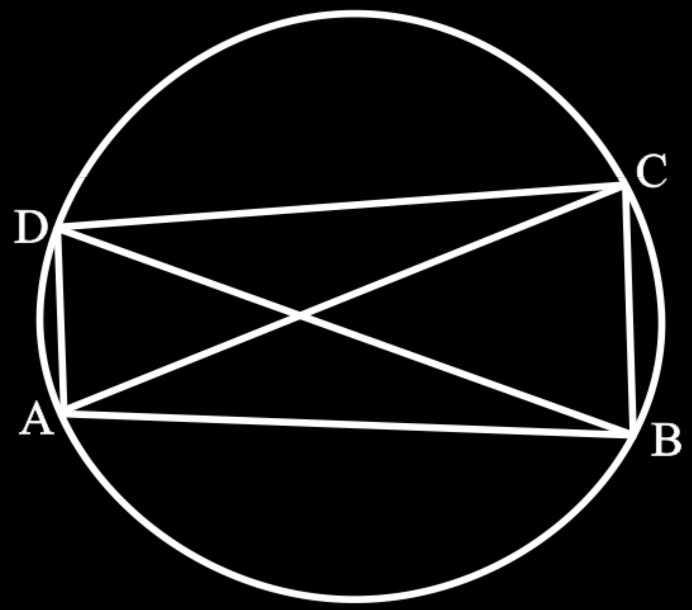
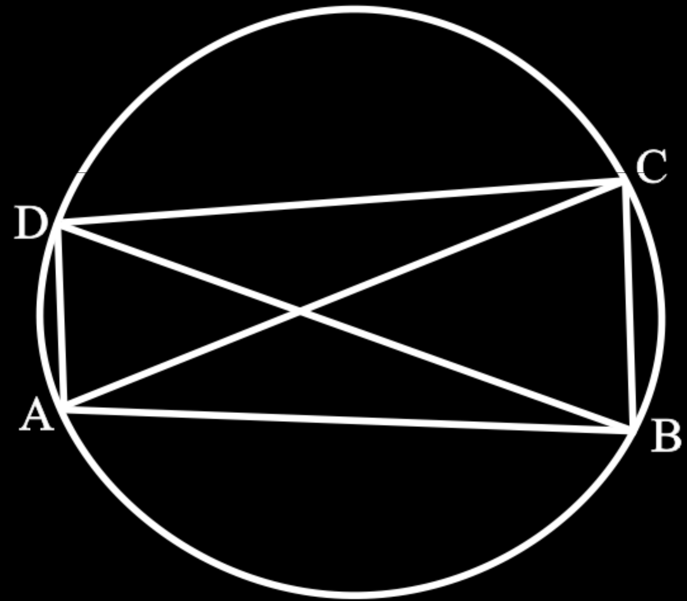
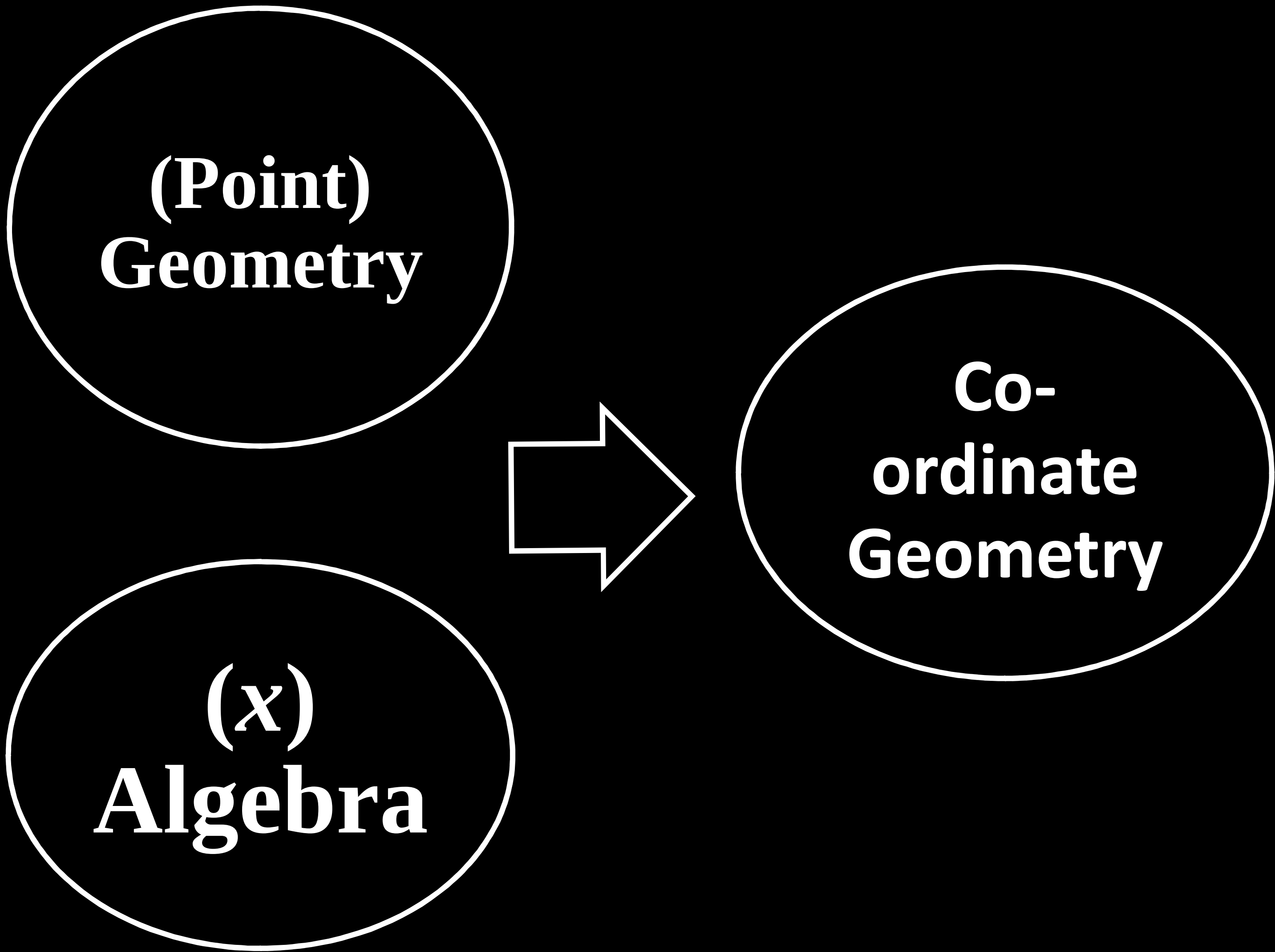


PTolmey's Theorem

Sum of product of opposite side = Product of diagonals



(AB) (CD) + (BC) (AD) = (AC) (BD)


(Point)
Geometry
(x)
Algebra
Co-
ordinate
Geometry

Distance Formulae

Distance Formulae

Distance Formulae

$A(x_1, y_1)$ $\qquad$ $B(x_2, y_2)$

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example

Find distance between following points :-

Q.1 (1, 3), (4, -1)

Find distance between following points :-

Q.2 (0, 0), (-5, -12)

Find distance between following points :-

Q.3 (1,1), (16, 9)

Find distance between following points :-

Q.4 (0, 0), (40, 9)

Find distance between following points :-

Q.5 (0, 0) $(2\cos\theta, 2\sin\theta)$

Section Formulae (Internal Division)

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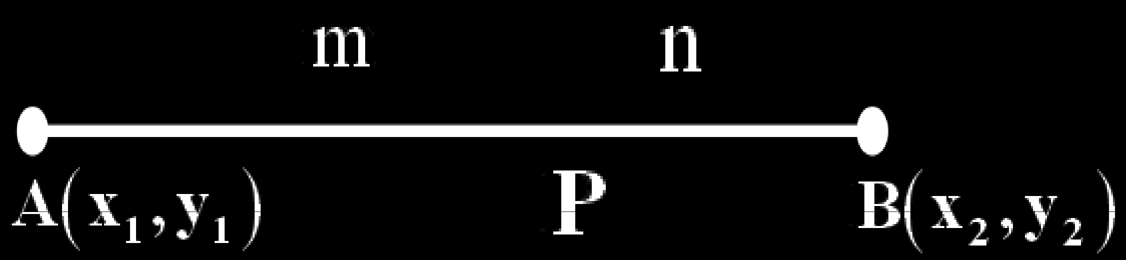
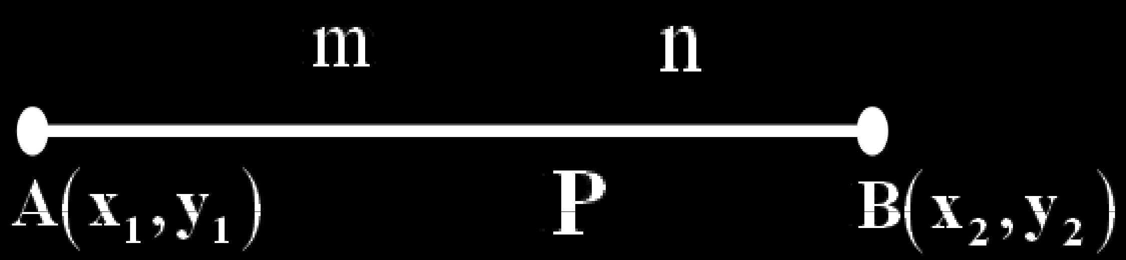


Section Formulae (Internal Division)



$$P \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Coordinate of mid point of A (x_1, y_1), B (x_2, y_2)

Coordinate of mid point of A (x_1, y_1), B (x_2, y_2)

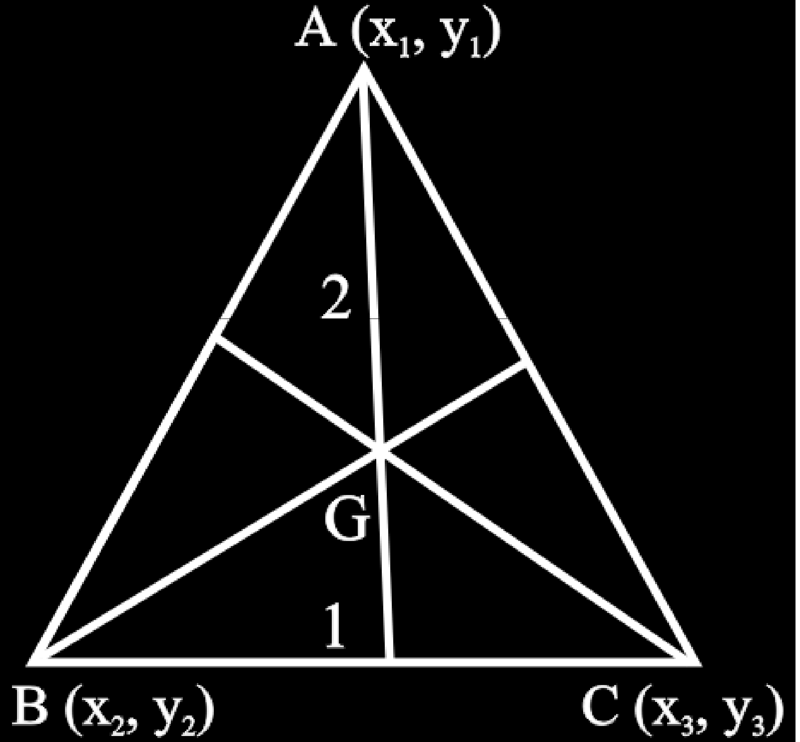
$$P \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example

Q. Find points of trisection of (1, 1) & (10, 13)

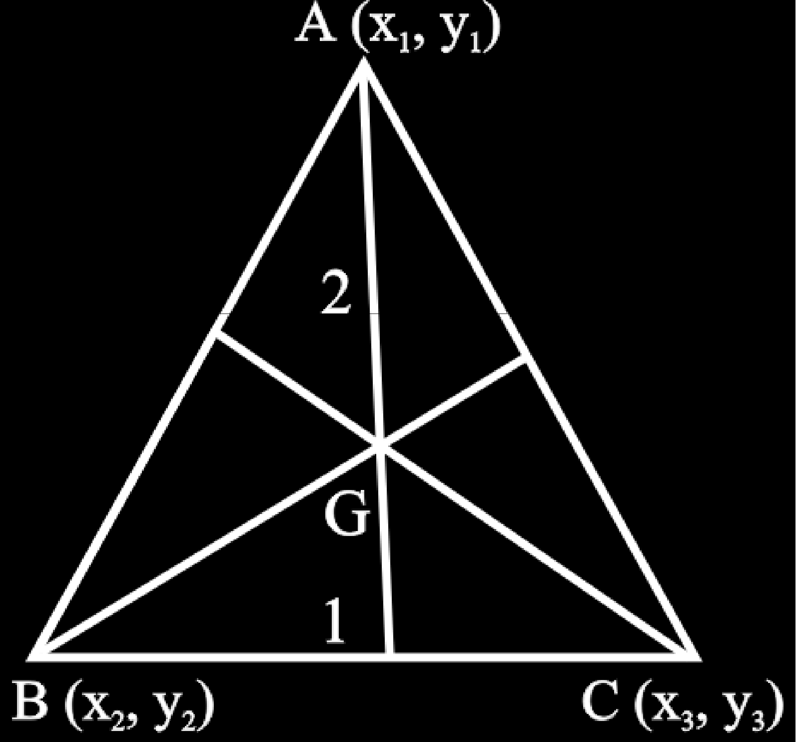
Co-ordinate of G

Co-ordinate of G



Co-ordinate of G

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$



Example

Q. Find mid points of sides of Δ if vertices are given (0, 0), (2, 3), (4, 0). Also find coordinate of G

Q. Find the ratio in which point on x axis divides the two points. (1,1), (3, -1) internally.

Section Formulae
(External Division)

Section Formulae (External Division)

Section Formulae (External Division)

m n

A B P

$$P \equiv \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n} \right)$$

Example

Q. Find the point dividing (2, 3), (7, 9) externally in the ratio 2 : 3

Harmonic Conjugate

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If a point P divides AB internally in the ratio a : b and point Q divides AB externally in the ratio a : b, then P & Q are said to be harmonic conjugate of each other w.r.t. AB

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$$\frac{AP}{BP} = \frac{\lambda}{1} = \frac{AQ}{BQ}$$

Harmonic Mean

Harmonic Mean

$$AB = \frac{2\,AP\;\;AQ}{AP + AQ}$$

(i) Internal & external bisector of an angle of a Δ divide base harmonically

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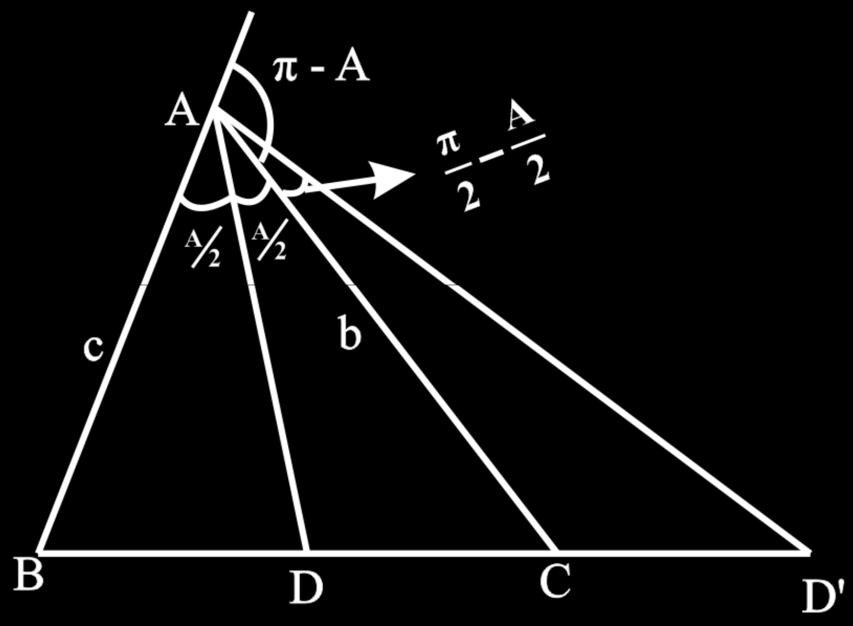
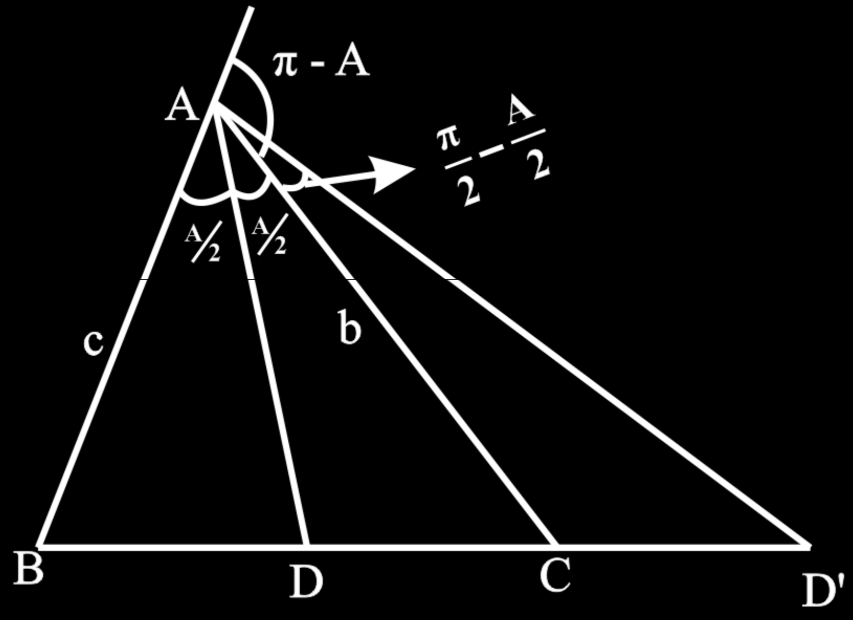


(i) Internal & external bisector of an angle of a Δ divide base harmonically



D & D' are harmonic conjugate of each other

Example

Q. If coordinate of A & B is (0, 0) & (9, 0) find point which divide AB externally in the ratio 1 : 2 find its harmonic conjugate.

External & Internal common tangents divides line joining centre of two circles externally & internally at the ratio of radii

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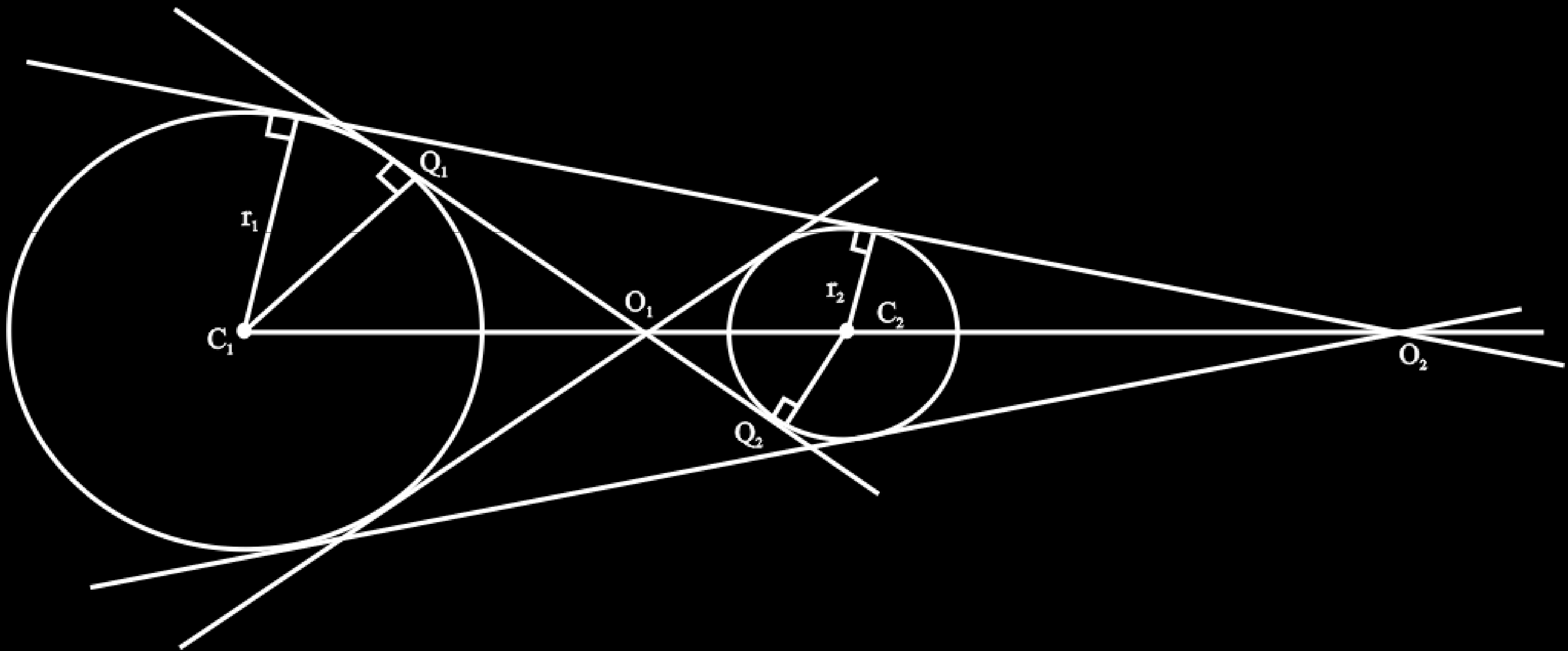
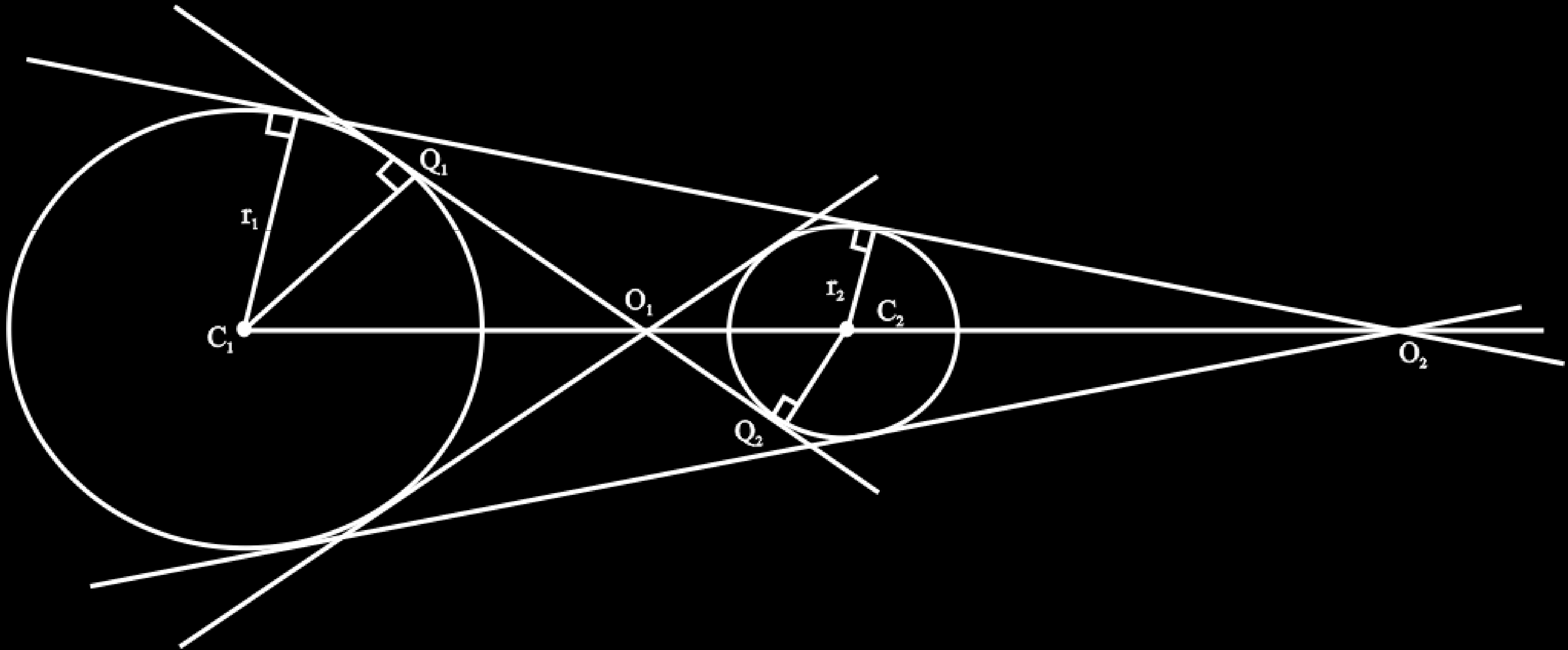


External & Internal common tangents divides line joining centre of two circles externally & internally at the ratio of radii



O_1 and O_2 are harmonic conjugate each other.

Co-ordinates of Incentre (I)

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$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + ay_2 + ay_3}{a+b+c} \right)$$

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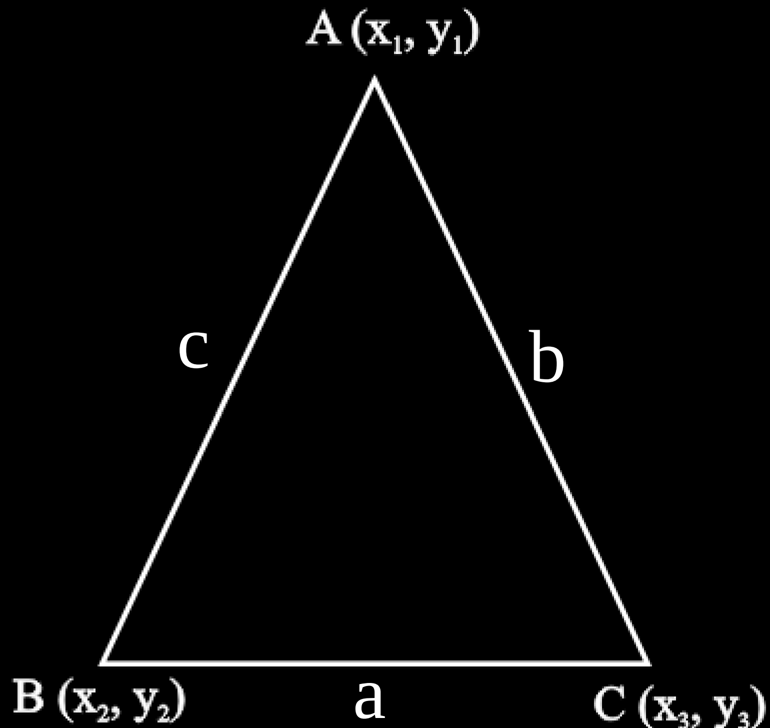


Q.1 If P (1, 2), Q (4, 6), R (5, 7) and S (a, b) are the vertices of parallelogram PQRS then

(A) a = 2, b = 4 (B) a = 3, b = 4

(C) a = 2, b = 3 (D) a = 1 or b = -1

[IIT-JEE 1998]

Q.2 The incentre of triangle with vertices $\left(1, \sqrt{3}\right)$, (0, 0) and (2, 0) is

(A) $\left(1, \sqrt{3}/2\right)$ (B) $\left(2/3, 1/\sqrt{3}\right)$

(C) $\left(2/3, \sqrt{3}/2\right)$ (D) $\left(1, 1/\sqrt{3}\right)$

[IIT-JEE 2000]

S.L. Loney

Assignment - 1

Find the distance between the following pairs of points

Q.1 (2, 3) and (5, 7)

Q.2 (4, -7) and (-1, 5)

Q.3 (a, 0) and (0, b)

Q.4 (b + c, c + a) and (c + a, a + b)

Q.5 $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$

Q.6 $(am_1^2, 2am_1)$ and $(am_2^2, 2am_2)$

Q.7 Lay down in a figure the position of the points (1, -3) and (-2, 1), and prove that the distance between them is 5.

Q.8 Find the value of x_1 if the distance between the points $(x_1, 2)$ and (3, 4) be 8.

Q.9 A line is of length 10 and one end is at the point (2, -3); if the abscissa of the other end be 10, prove that its ordinate must be 3 or -9.

Q.10 Prove that the points (2a, 4a), (2a, 6a) and $\left(2a+\sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle whose side is 2a.

Q.11 Prove that the points (2, -1), (1, 0), (4, 3), and (1, 2) are at the vertices of a parallelogram.

Q.12 Prove that the points (2, -2), (8,4), (5,7) and (-1,1) are at the angular points of a rectangle.

Q.13 Prove that the point $\left(-\frac{1}{14}, \frac{39}{14}\right)$ is the centre of the circle circumscribing the triangle whose angular points are (1, 1), (2, 3), and (-2, 2).

Find the coordinates of the point which

Q.14 Divide the line joining the points (1, 3) and (2, 7) in the ratio 3 : 4.

Q.15 Divides the same line in the ratio 3 : -4.

Q.16 Divides, internally and externally, the line joining (-1, 2) to (4, -5) in the ratio 2 : 3.

Q.17 Divide, internally and externally, the line joining (-3, -4) to (-8, 7) in the ratio 7 : 5

Q.18 The line joining the point (1, -2) and (-3, 4) is trisected; find the coordinate of the points of trisection.

Q.19 The line joining the points (-6, 8) and (8, -6) is divided into four equal pats; find the coordinates of the points of section.

Q.20 Find the coordinates of the points which divide, internally and externally, the line joining the point (a + b, a – b) to the point (a – b, a + b) in the ratio a : b.

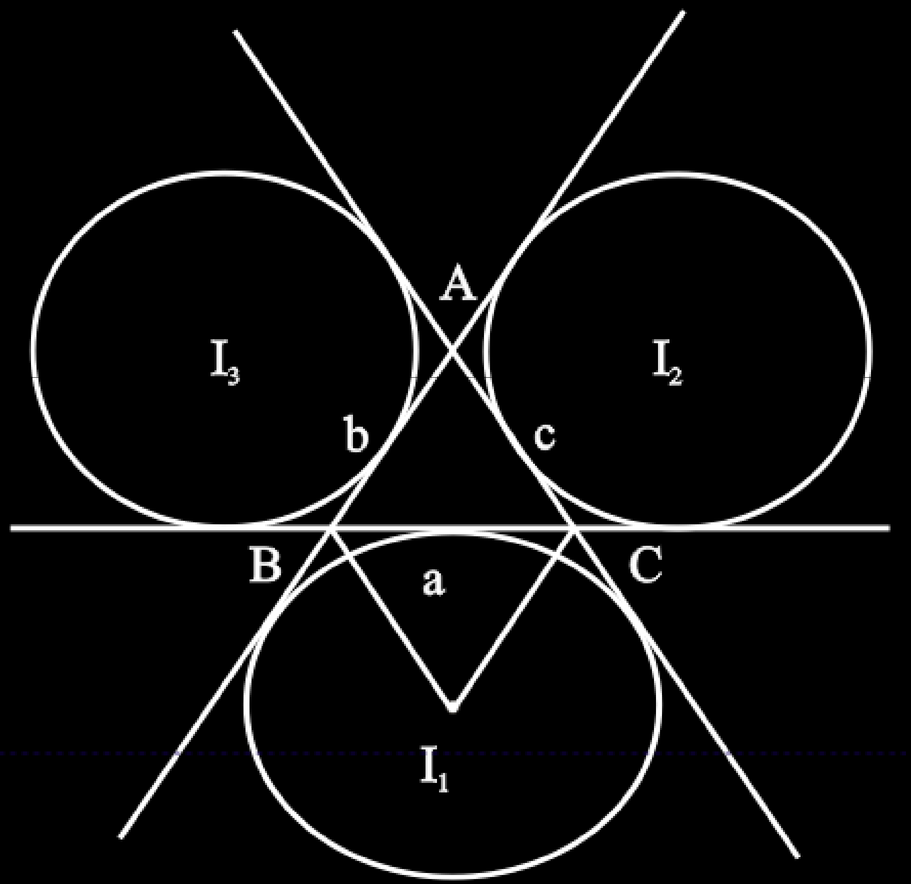
Q.21 The coordinates of the vertices of a triangle are $(x_1\ y_1)$, (x_2, y_2) and (x_3, y_3). The line joining the first two is divided in the ratio l : k, and the line joining this point of division to the opposite angular point is then divided in the ratio m : k + l. Find the coordinate of the latter point of section

Q.22 Prove that the coordinate, x and y of the middle point of the line joining the point (2, 3) to the points (3, 4) satisfy the equation,

$$x - y + 1 = 0$$

Coordinates of I_1, I_2 & I_3

Coordinates of I_1, I_2 & I_3



Coordinates of I_1, I_2 & I_3

$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + ay_2 + ay_3}{-a+b+c} \right)$$

Coordinates of I_1, I_2 & I_3

$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + ay_2 + ay_3}{-a+b+c} \right)$$

$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right)$$

Coordinates of I_1, I_2 & I_3

$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + ay_2 + ay_3}{-a+b+c} \right)$$

$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right)$$

$$I_3 \equiv \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c} \right)$$

Examples

Q.1 Mid point of sides of triangle are (1, 2), (0, -1) and (2, -1). Find coordinate of vertices

Q.2 Co-ordinate A, B, C are (4, 1), (5, -2) and (3, 7)
Find D so that A, B, C, D is $||^{gm}$

Q.3 Line $3x + 4y = 12$, $x = 0$, $y = 0$ form a Δ. Find the centre and radius of circles touching the line & the co-ordinate axis.

Q.4 Orthocenter and circumcenter of a ΔABC are (a, b), (c, d). If the co-ordinate of the vertex A are (x_1, y_1) then find co-ordinate of middle point of BC.

Q.5 Vertices of a triangle are (2, -2), (-2, 1), (5, 2). Find distance between circumcentre & centroid.

Area of equilateral triangle

$$= \frac{\sqrt{3}}{4}\left(\text{side}\right)^2$$

Area of Triangle

Area of Triangle

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

Where (x_1, y_1) (x_2, y_2), (x_3, y_3) are coordinates of vertices of triangle

Condition of collinearity of $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

Condition of collinearity of $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

$$\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = 0$$

Area of n sided figure

Area of n sided figure

$$\frac{1}{2}\left|\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix} + \dots\dots\dots\dots \begin{vmatrix} x_n & y_1 \\ y_n & y_1 \end{vmatrix}\right|$$

Example

Q.1 Find k for which points $(k + 1, 2 - k)$, $(1 - k, -k)$ $(2 + K, 3 - K)$ are collinear.

Q.2 If points (a, 0), (0, b) and (1, 1) are collinear then prove that $\frac{1}{a}+\frac{1}{b}=1$

Q.3 Find relation between x & y if x, y lies on line joining the points $(2, -3)$ and $(1, 4)$

Q.4 Show that (b, c + a) (c, a + b) and (a, b + c) are collinear.

Q.5 If the area of Δ formed by points (1, 2), (2, 3) and (x, 4) is 40 sq. unit. Find x.

Q.6 Find area of quadrilateral A (1, 1); B (3, 4); C (5, -2) and D (4, -7) in order are the vertices of a quadrilateral.

Q.7 Find co-ordinate of point P if PA = PB and area of $\Delta PAB = 10$ if coordinates of A and B are (3, 0) and (7,0) respectively.

Q.8 Find the area of the Δ if the coordinate of vertices of triangle are $\left(at_1^2, 2at_1\right)\left(at_2^2, 2at_2\right), \left(at_3^2, 2at_3\right)$

Assignment - 2

Find the areas of the triangles the coordinate of whose angular points are respectively.

Q.1 (1, 3), (-7, 6) and (5, -1)

Q.2 (0, 4), (3, 6) and (-8, -2).

Q.3 (5, 2), (-9, -3) and (-3, -5)

Q.4 (a, b + c), (a, b – c) and (-a, c)

Q.5 (a, c + a), (a, c) and (-a, c – a)

Prove (by shewing that the area of the triangle formed by them is zero that the following sets of three points are in a straight line :

Q.6 (1, 4), (3, -2) and (-3, 16)

Q.7 $\left(-\frac{1}{2}, 3\right)$, (-5, 6) and (-8, 8).

Q.8 (a, b + c), (b, c + a), and (c, a + b)

Find the area of the quadrilaterals the coordinates of whose angular points, taken in order, are :

Q.9 (1, 1), (3, 4), (5, -2) and (4, -7)

Q.10 (-1, 6), (-3, -9), (5, -8) and (3, 9)

LOCUS

To Find Locus

(1) Write geometrical condition & convert them in algebraic

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(2) Eliminate variable

(3) Get relation between h and k.

(4) To get equation of locus replace h by x & k by y

Example

Q.1 Find locus of curve / point which is equidistant from point (0, 0) and (2, 0)

Q.2 If A (0, 0), B (2, 0) find locus of point P such that $\angle APB = 90°$

Q.3 If A (0, 0), B (2, 0) find locus of point P such that area (Δ APB) = 4

Q.4 If A & B are variable point on x and y axis such that length (AB) = 4. Find :

(i) Locus of mid point of AB

Q.4 If A & B are variable point on x and y axis such that length (AB) = 4. Find :

(ii) Locus of circumcentre of ΔAOB

Q.4 If A & B are variable point on x and y axis such that length (AB) = 4. Find :

(iii) Locus of G of ΔAOB

Q.4 If A & B are variable point on x and y axis such that length (AB) = 4. Find :

(iv) Find locus of point which divides segment AB internally in the ratio 1 : 2, 1 from x axis

Q.5 A(1, 2) is a fixed point. A variable point B lies on a curve whose equation is $x^2+y^2 = 4$. Find the locus of the mid point of AB.

Parametric point

Example

Q.1 Find equation of curve represented parametrically by $x = \cos\theta$, $y = \sin\theta$

Q.2 Find equation of curve if $x = 2\cos\theta$, $y = \sin\theta$

Q.3 Find equation of curve if $x = \sec\theta$, $y = 2\tan\theta$

Q.4 Find equation of curve if $x = at^2$, $y = 2at$

Q.5 Find locus of point P such that ;

$PF_1 + PF_2 = 2a$ & $F_1 \equiv (c, 0)$ & $F_2 \equiv (-c, 0)$

Q.6 Find locus of point P such that $|PA - PB| = 2a$ & coordinates of A, B are $(c, 0)$ & $(-c, 0)$

Assignment - 3

Sketch the loci of the following equations :

Q.1 $2x + 3y = 10$

Q.2 $4x - y = 7$

Q.3 $x^2 - 2ax + y^2 = 0$

Q.4 $x^2 - 4ax + y^2 + 3a^2 = 0$

Q.5 $y^2 = x$

Q.6 $3x = y^2 - 9$

A and B being the fixed points (a, 0) and (-a, 0) respectively, obtain the equations giving the locus of P, when :

Q.7 $PA^2 - PB^2$ = a constant quantity = $2k^2$

Q.8 PA = nPB, n being constant.

Q.9 $PB^2 + PC^2 = 2PA^2$, C being the point (c, 0)

Q.10 Find the locus of a point whose distance from the point (1, 2) is equal to its distance from the axis of y.

Find the equation to the locus of a point which is always equidistant from the points whose coordinate are

Q.11 (1, 0) and (0, -2)

Q.12 (2, 3) and (4, 5)

Q.13 (a + b, a – c) and (a – b, a + b)

Find the equation to the locus of a point which moves so that

Q.14 Its distance from the axis of x is three times its distance from the axis of y.

Q.15 Its distance from the point (a, 0) is always four times its distance from the axis of y.

Q.16 The sum of the squares of its distances from the axes is equal to 3.

Q.17 The square of its distance from the point (0, 2) is equal to 4.

Q.18 Its distance from the point (3, 0) is three times its distance from (0, 2)

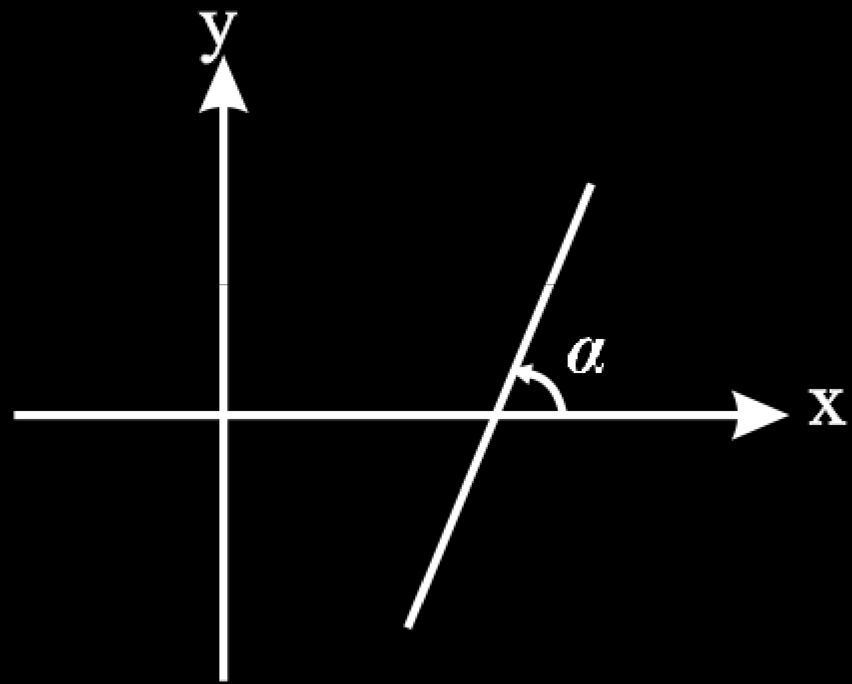
Q.19 Its distance from the axis of x is always one half its distance from the origin.

Straight Line

Locus of point such that if any two point of this locus are joined they define a unique direction.

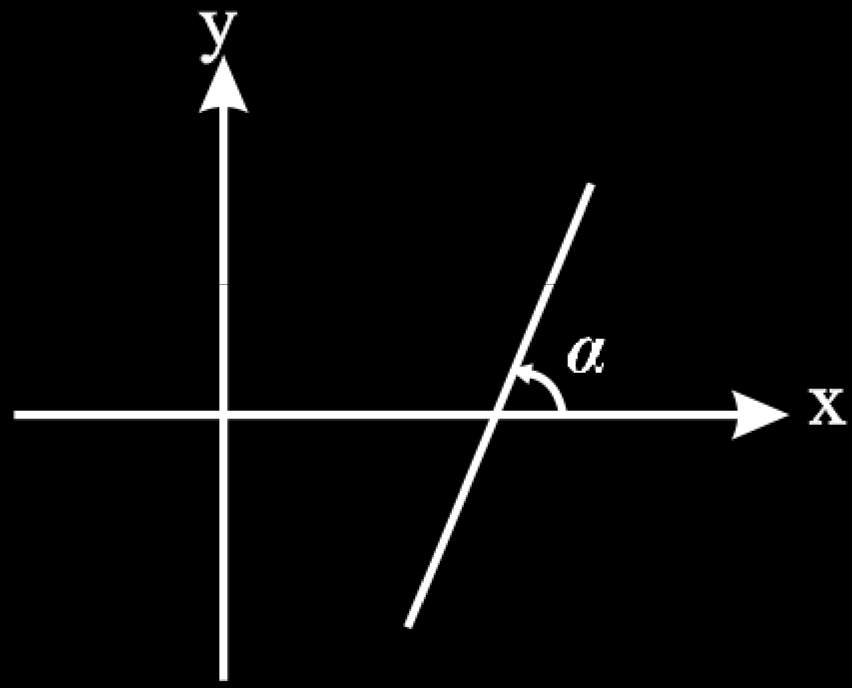
Inclination of Line

Inclination of Line



Inclination of Line

$$\alpha \in [0, \pi)$$



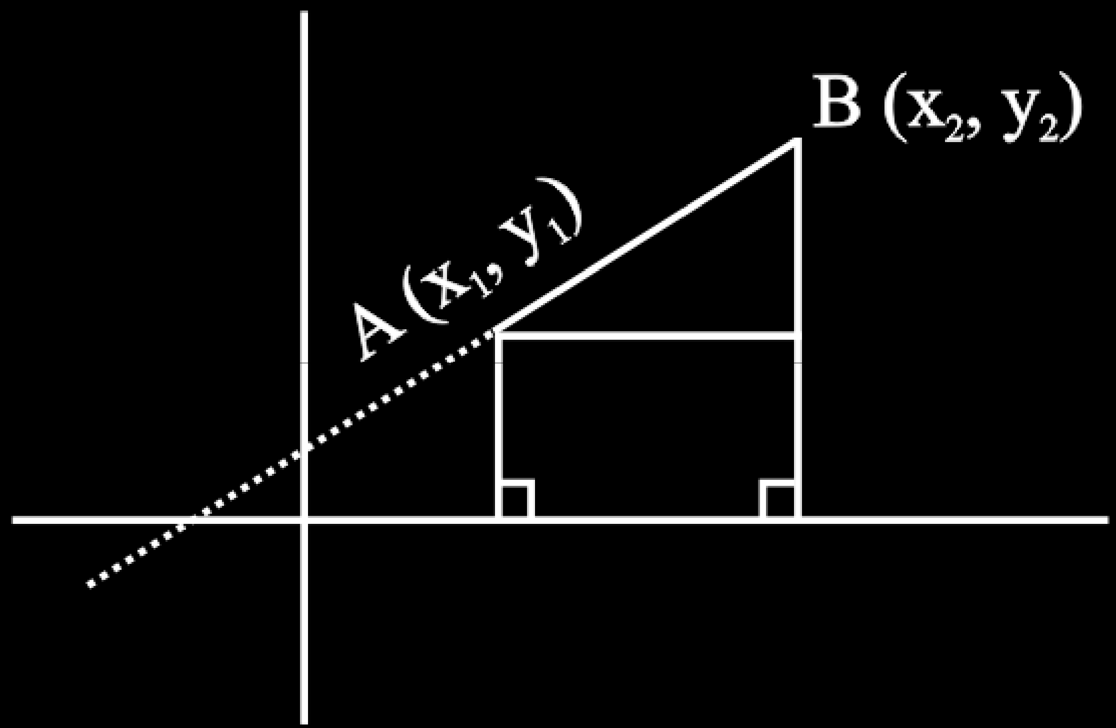
Slope / Gradient (m)

Slope / Gradient (m)

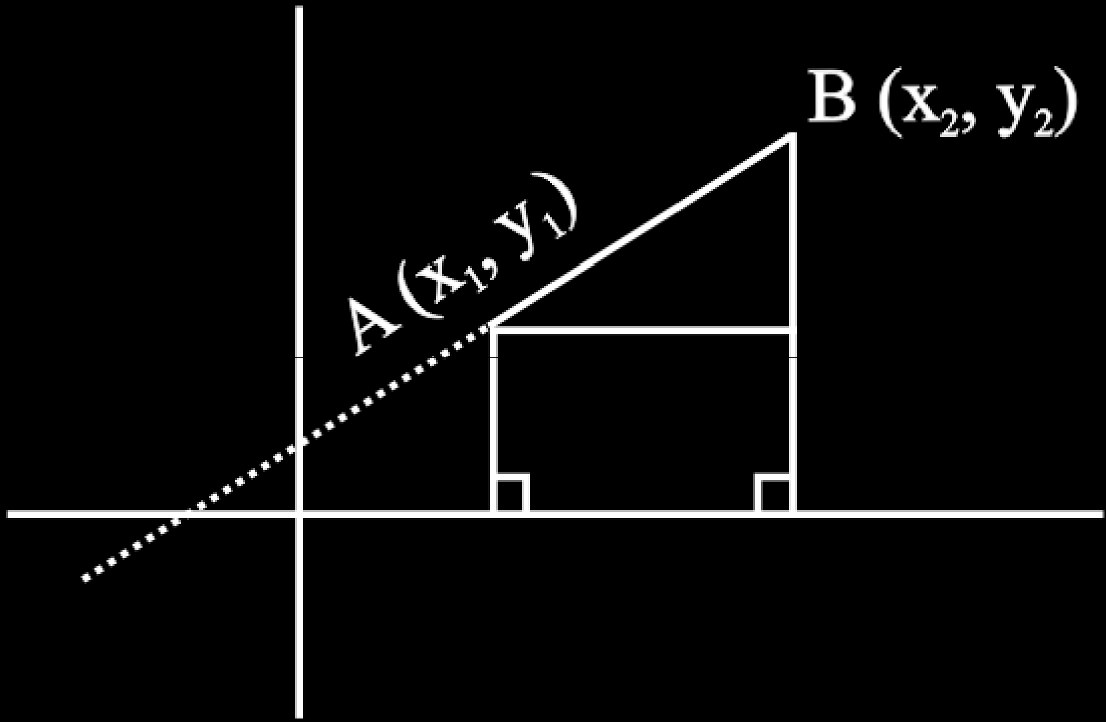
$$m = \tan\alpha \qquad ; \qquad \alpha \neq \pi/2$$

Slide of line joining two points

Slope of line joining two points



Slope of line joining two points



$$m = tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Q.1 Find slope of joining points (1, 1) & (100, 100)

Q.2 Find slope of joining points (1, 0) & (2, 0)

Q.3 Find slope of joining points (1, 9) & (7, 0)

Equation of Line in Various Form

General Form

$$ax + by + c = 0$$

Point Slope Form

$$(y - y_1) = m\,(x - x_1)$$

Example

Q.1 Find equation of line having slope 2 and passing through point (1, 3)

Q.2 Find equation line having slope $\sqrt{3}$ and passing through point (1, 7)

Q.3 Line passing through $(1, 0)$ and $(2, 1)$ is rotated about point $(1,0)$ by an angle $15°$ in clockwise direction. Find equation of line in new position.

Two Point Form

Two Point Form

$$\left(y - y_1\right) = \frac{y_2 - y_1}{x_2 - x_1}\left(x - x_1\right)$$

Example

Q.1 Find equation of line joining (1, 1), (3, 4)

Determinant Form

Determinant Form

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix} = 0$$

Intercept Slop Form

Intercept Slop Form

$y = mx + c$

Intercept Slop Form

$y = mx + c$

$y = mx$ [if line passes through origin]

Example

Q.1 Find slope, x intercept, y intercept of lines

(i) $2y = 3x + 7$

Example

Q.1 Find slope, x intercept, y intercept of lines

(ii) $2x + 7y - 3 = 0$

Example

Q.1 Find slope, x intercept, y intercept of lines

(iii) line joining point (1,1), (9, 3)

Double Intercept Form

Double Intercept Form

$$\frac{x}{a}+\frac{y}{b}=1$$

Example

Q.1 Find equation of straight line through (1, 2) & if its x intercept is twice the y intercept.

Q.2 Find equation of line passing through (2,3) and having intercept of y axis twice its intercept on x axis

Normal Form

Normal Form

$x \cos\alpha + y\sin\alpha = p \qquad ; \qquad \alpha \in [0, 2\pi)$

Example

Q.1 If equation of line is $3x - 4y + 5 = 0$ convert line in

(i) Intercept form

Example

Q.1 If equation of line is $3x - 4y + 5 = 0$ convert line in

(ii) Double intercept form

Example

Q.1 If equation of line is $3x - 4y + 5 = 0$ convert line in

(iii) Normal form

Note

Note

(1) Line having equal intercept $\Rightarrow$ $m = -1$

Note

(1) Line having equal intercept $\Rightarrow m = -1$

(2) Line equally inclined with coordinate axis $\Rightarrow m = \pm 1$

Example

Q.1 Find equation medians of ΔABC where coordinate of vertices are (0,0), (6,0), (3,8)

Q.2 If p is perpendicular distance from origin upon line whose intercept on coordinate axis are a & b prove that

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$$

Q.3 Find locus of middle point of AB where A is x intercept and B is Y intercept of a variable line always passing through point (2,3)

Q.4 Find number of lines passing through (2,4) & forming a triangle of area 16 units with the coordinate axis.

Q.5 Find equation of line

(i) Cuts off intercept 4 on x axis and passing through (2, -3)

Q.5 Find equation of line

(ii) Cuts off equal intercept on coordinate axis and passes through (2, 5)

Q.5 Find equation of line

(iii) Makes an angle 135° with positive x axis and cuts y axis at a distance 8 from the origin

Q.5 Find equation of line

(iv) Passing through (4,1) and making with the axes in the first quadrant a triangle whose area is 8

Q.6 Find equation of the two lines which join origin and points of trisection of the portion of line $x + 3y - 12 = 0$ intercepted between co-ordinate axis.

Line in Determinant Form

Equation of Median Through $A(x_1, y_1)$ in ΔABC

Equation of Median Through A (x_1, y_1) in Δ ABC

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Equation of Internal Angle Bisector

Equation of Internal Angle Bisector

$$b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Equation of External Angle Bisector

Equation of External Angle Bisector

$$b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Equation of the Altitude

Equation of the Altitude

$$b\cos C\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\cos B\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Assignment - 4

Find the equation to the straight line

Q.1 Cutting off an intercept unity from the positive direction of the axis of y and inclined at 45° to the axis of x.

Q.2 Cutting off an intercept -5 from the axis of y and being equally inclined to the axes.

Q.3 Cutting of an intercept 2 from the negative direction of the axis of y and inclined at 30° to OX.

Q.4 Cutting off an intercept -3 from the axis of y and inclined at an angle $\tan^{-1}\frac{3}{5}$ to the axis of x.

Find the equation to the straight line

Q.5 Cutting off intercepts 3 and 2 from the axes.

Q.6 Cutting off intercepts -5 and 6 from the axes.

Q.7 Find the equation to the straight line which passes through the points (5, 6) and has intercepts on the axes.

(i) equal in magnitude and both positive.

(ii) equal in magnitude but opposite in sign.

Q.8 Find the equations to the straight lines which passes through the point (1, -2) and cut off equal distance from the two axes.

Q.9 Find the equation to the straight line which passes through the given point (x', y') and is such that the given point bisects the part intercepted between the axes.

Q.10 Find the equation to the straight line which passes through the point (-4, 3) and is such that the portion of it between the axes is divided by the point of the ratio 5 : 3.

Trace the straight line whose equation are

Q.11 $x + 2y + 3 = 0$

Q.12 $5x - 7y - 9 = 0$

Q.13 $3x + 7y = 0$

Q.14 $2x - 3y + 4 = 0$

Find the equations to the straight lines passing through the following pairs of points

Q.15 (0, 0) and (2, -2)

Q.16 (3, 4) and (5, 6)

Q.17 (-1, 3) and (6, -7)

Q.18 (0, -a) and (b, 0)

Q.19 (a, b) and (a + b, a – b)

Find the equation to the sides of the triangles the coordinate of whose angular points are respectively.

Q.20 (1, 4), (2, -3), and (-1, -2)

Q.21 (0, 1), (2, 0), and (-1, -2)

Q.22 Find the equation to the diagonals of the rectangle the equations of whose sides are

$x = a$, $x = a'$, $y = b$, and $y = b'$

Q.23 Find the equation to the straight line which bisects the distance between the points (a, b) and (a′, b′) and also bisects the distance between the points (-a, b) and (a′, -b′).

Q.24 Find the equations to the straight lines which go through the origin and trisect the portion of the straight line $3x+y = 12$ which is intercepted between the axes of coordinates.

Angle Between Two Lines

Angle Between Two Lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Condition of lines being ||

Condition of lines being ||

$$m_1 = m_2$$

Condition of lines being Perpendicular

Condition of lines being Perpendicular

$$m_1 \, m_2 = -1$$

Equation of line parallel to $ax + by + c = 0$

Equation of line parallel to $ax + by + c = 0$

$$ax + by + \lambda = 0$$

Angle of line Perpendicular to $ax + by + c = 0$

Angle of line Perpendicular to $ax + by + c = 0$

$$bx - ay + \lambda = 0$$

Inclination of lines are complementary

Inclination of lines are complementary

$$m_1 \, m_2 = 1$$

Example

Q.1 Find equation of line parallel and perpendicular to $y = 3$ and passing through $(2, 7)$

Q.2 Find the equation of line parallel and $\perp$ to $x = 1$ and passing through $(-9, -3)$

Q.3 Find equation of line parallel and perpendicular to $2x + 3y = 7$ and passing through $(2, -3)$

Q.4 Line $2x+3y=7$ intersects coordinate axis in A&B. Find perpendicular bisector of AB

Q.5 A(0, 8), B(2, 4) & C(6,8) find equation of altitudes, ⊥ bisectors and Coordinates of Orthocenter and Circumcenter

To Find Tangent of Interior Angles of Triangle

To Find Tangent of Interior Angles of Triangle

$$\operatorname{Tan} A = \frac{m_1 - m_2}{1 + m_1 m_2}$$

To Find Tangent of Interior Angles of Triangle

$$\operatorname{Tan} A = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\operatorname{Tan} B = \frac{m_2 - m_3}{1 + m_2 m_3}$$

To Find Tangent of Interior Angles of Triangle

$$\operatorname{Tan} A = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\operatorname{Tan} B = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$\operatorname{Tan} C = \frac{m_3 - m_1}{1 + m_1 m_3}$$

Example

Q.1 If a Δ ABC is formed by the lines $2x + y - 3 = 0$, $x - y + 5 = 0$ and $3x - y + 1 = 0$ then obtain tangents of the interior angles of the triangle

Q.2 Equation of line passing through (1, 2) making an angle of 45^0 with the line $2x + 3y = 10$

Assignment - 5

Find the angles between the pairs of straight lines :

Q.1 $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$

Q.2 $x - 4y = 4$ and $6x - y \ 11$

Q.3 $y = 3x + 7$ and $3y - x = 8$

Q.4 $y = \left(2 - \sqrt{3}\right)x + 5$ and $y = \left(2 + \sqrt{3}\right)x - 7$

Q.5 $(m^2 - mn)y = (mn + n^2)x + n^3$ and $(mn + m^2)$

$= (mn - n^2)x + m^3$

Q.6 Find the tangent of the angle between the lines whose intercepts on the axes are respectively a, -b and b, -a.

Q.7 Prove that the points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the angular points of a parallelogram and find the angle between its diagonals.

Find the equation to the straight line

Q.8 Passing through the point (2, 3) and perpendicular to the straight line $4x - 3y = 10$

Q.9 Passing through the point (-6, 10) and perpendicular to the straight lines $7x + 8y = 5$.

Q.10 Passing through the point (2, -3) and perpendicular to the straight line joining the points (5, 7) and (-6, 3)

Q.11 Passing through the point (-4, -3) and perpendicular to the straight lines joining (1, 3) and (2, 7)

Q.12 Find the equation to the straight line drawn at right angles to the straight line $\frac{x}{a}-\frac{y}{b}=1$ through the point where it meets the axis of x.

Q.13 Find the equation to the straight line which bisects, and is perpendicular to the straight line joining the points (a, b) and (a′, b′).

Q.14 Prove that the equation to the straight line which passes through the point ($a \cos^3\theta$, $a \sin^3\theta$) and is perpendicular to the straight line $x \sec \theta + y \operatorname{cosec} \theta = a$ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.

Q.15 Find the equations to the straight lines which divide, internally and externally, the line joining (-3, 7) to (5, -4) in the ratio 4 : 7 and which are perpendicular to this line.

Q.16 Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line x – y = 2. Find their equations and find also the area included by the three lines.

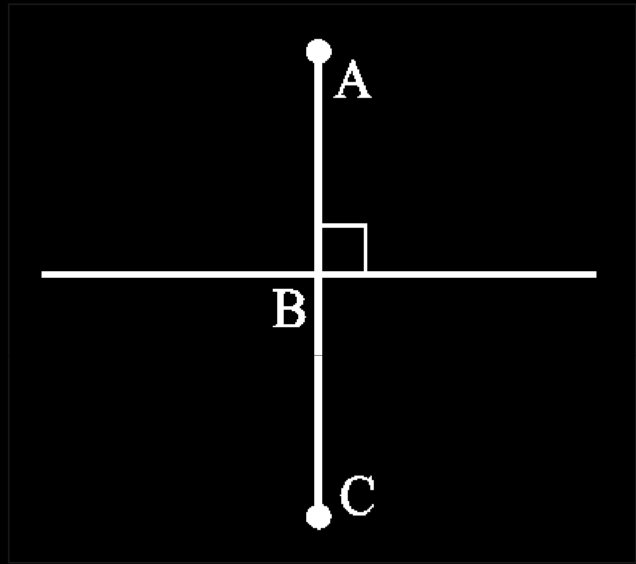
Q.17 Show that the equation to the straight line passing through the point (3, -2) and inclined $\sqrt{3}x + y = 1$ are $y + 2 = 0$ and $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Q.18 Find the equations to the straight lines which pass through the origin and are inclined at 75° to the straight line $x+y+\sqrt{3}(y-x)=a$

Q.19 Find the equations to the straight lines which pass through the point (h, k) and are inclined at an angle $\tan^{-1}m$ to the straight line $y = mx + c$.

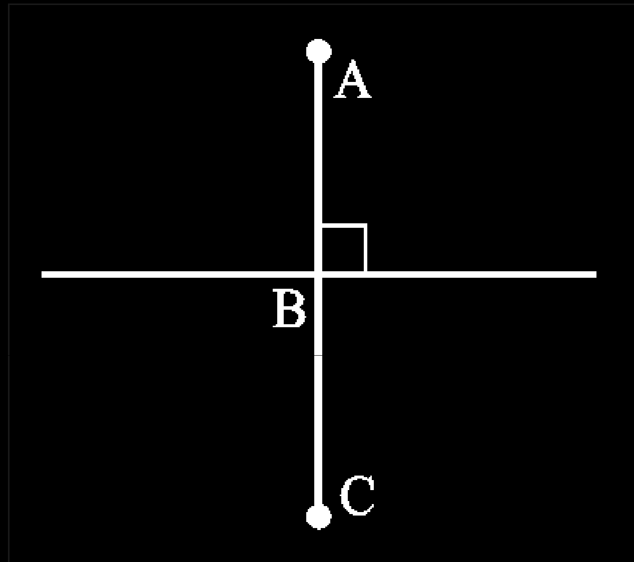
Reflection of a Point about a line

Reflection of a Point about a line



Reflection of a Point about a line

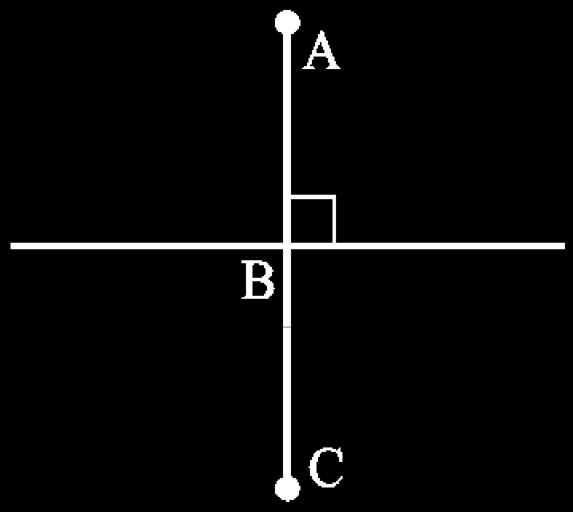
i. A B = B C



Reflection of a Point about a line

i. A B = B C

ii. Angle 90^o



Example

Q.1 Find equation of line passing through (-2, -7) making an angle of $\tan^{-1} \frac{3}{4}$ with the line $4x + 3y = 3$

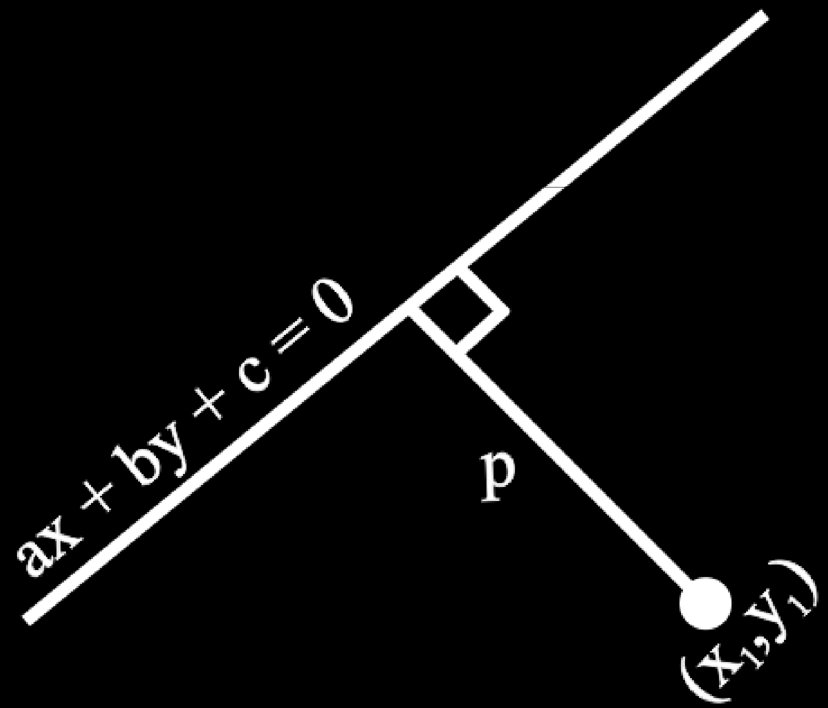
Q.2 Find reflection of point (1, -2) about the line

$x - y + 5 = 0$

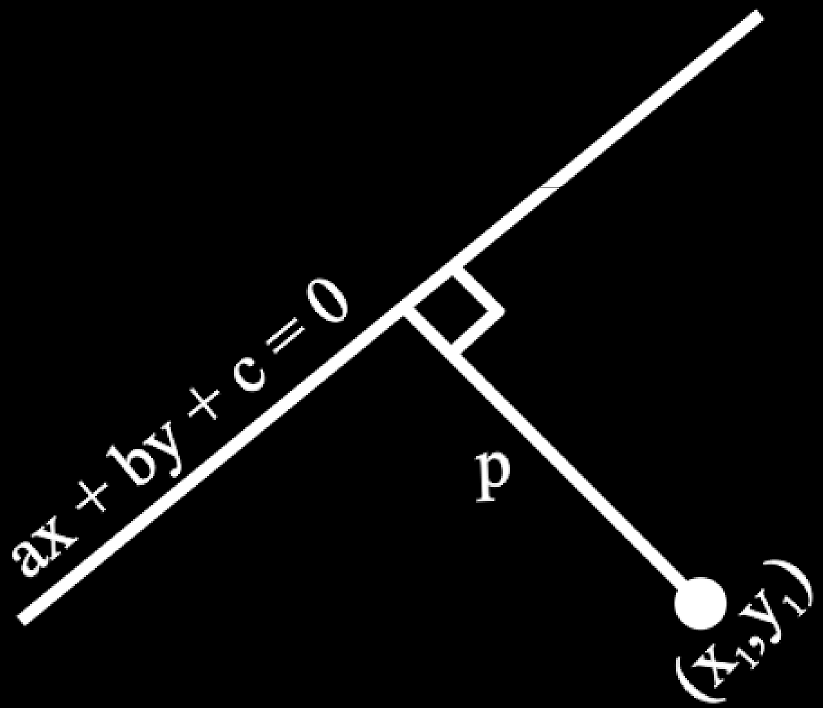
Q.3 Find reflection of point (1, -2) about the line $x + 2y = 0$

Length of $\perp$ from (x_1, y_1) on $ax + by + c = 0$

Length of $\perp$ from (x_1, y_1) on $ax + by + c = 0$



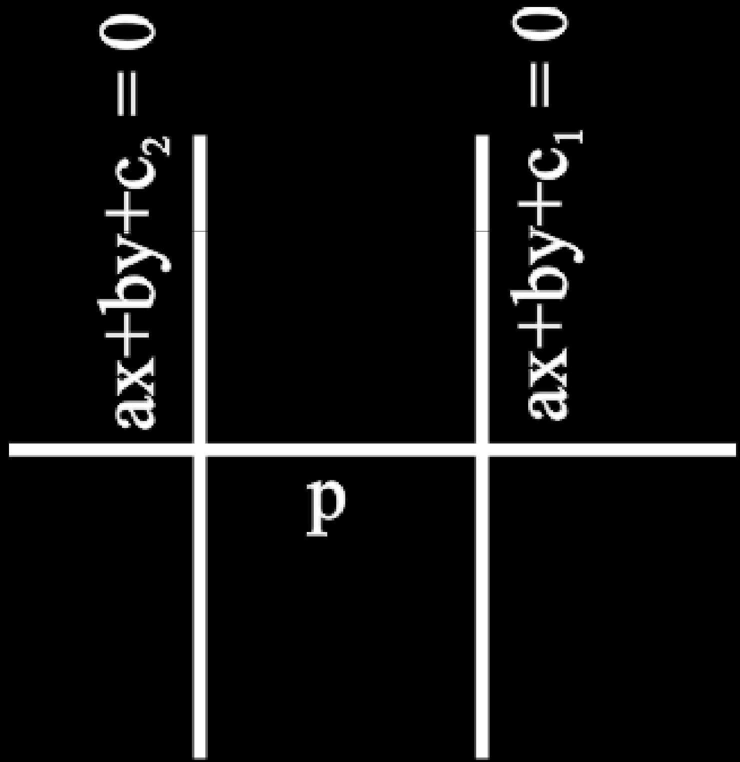
Length of ⊥ from (x_1, y_1) on $ax + by + c = 0$



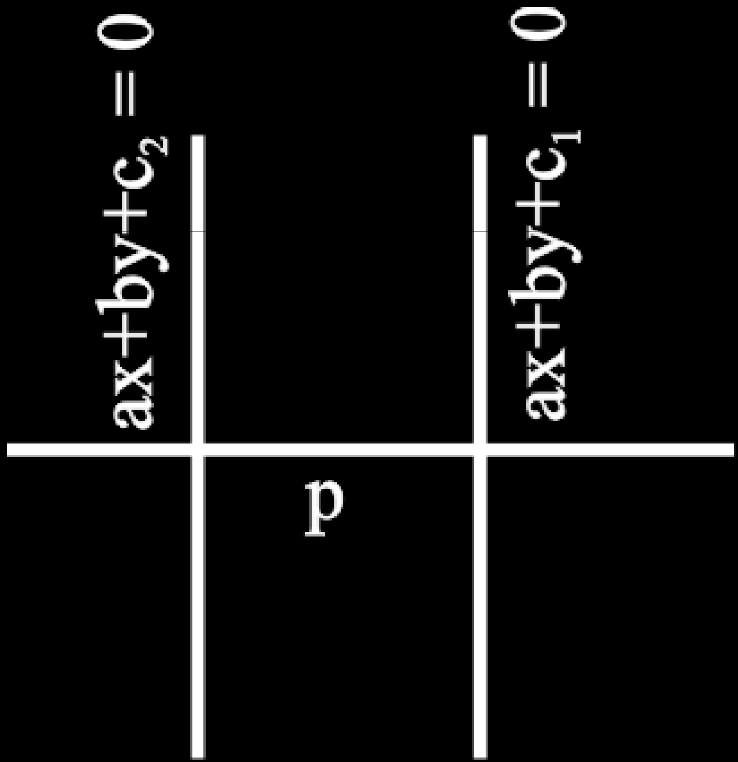
$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Distance Between Two Parallel Lines

Distance Between Two Parallel Lines



Distance Between Two Parallel Lines



$$p = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

Example

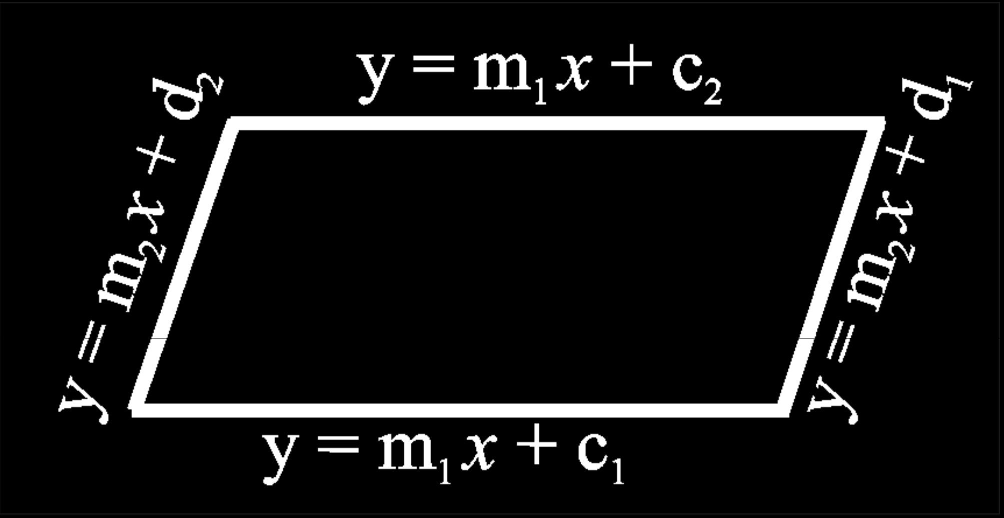
1. Find distance between point (1, 2) and line $3x - 4y + 1 = 0$

2. Find distance between point (0, 0) and line $12x - 5y + 7 = 0$

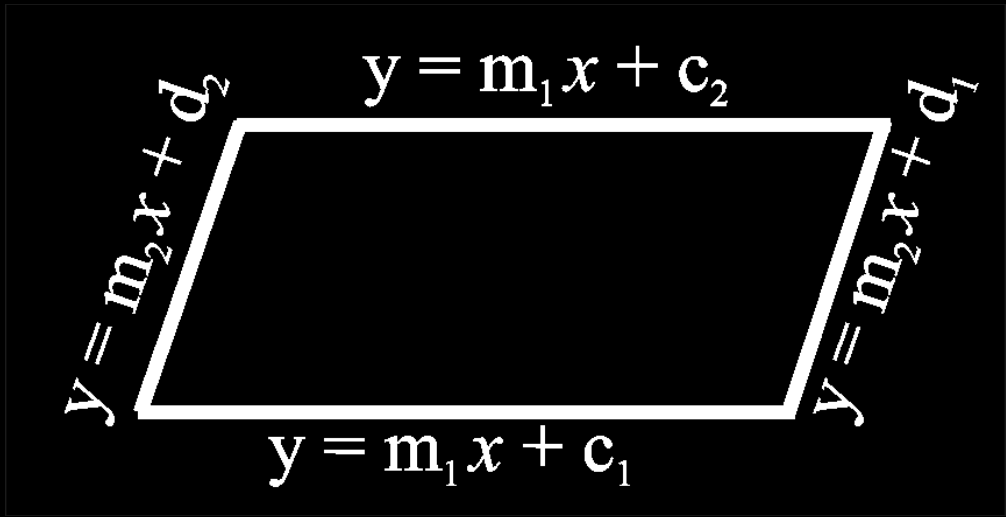
3. Find distance between line $3x + 4y + 7 = 0$ & $6x + 8y - 17 = 0$

Area of Parallelogram

Area of Parallelogram



Area of Parallelogram



$$\text{area} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{(m_1 - m_2)} \right|$$

Note

Two parallel lines are tangent to same circle.
Distance between them is diameter of the circle

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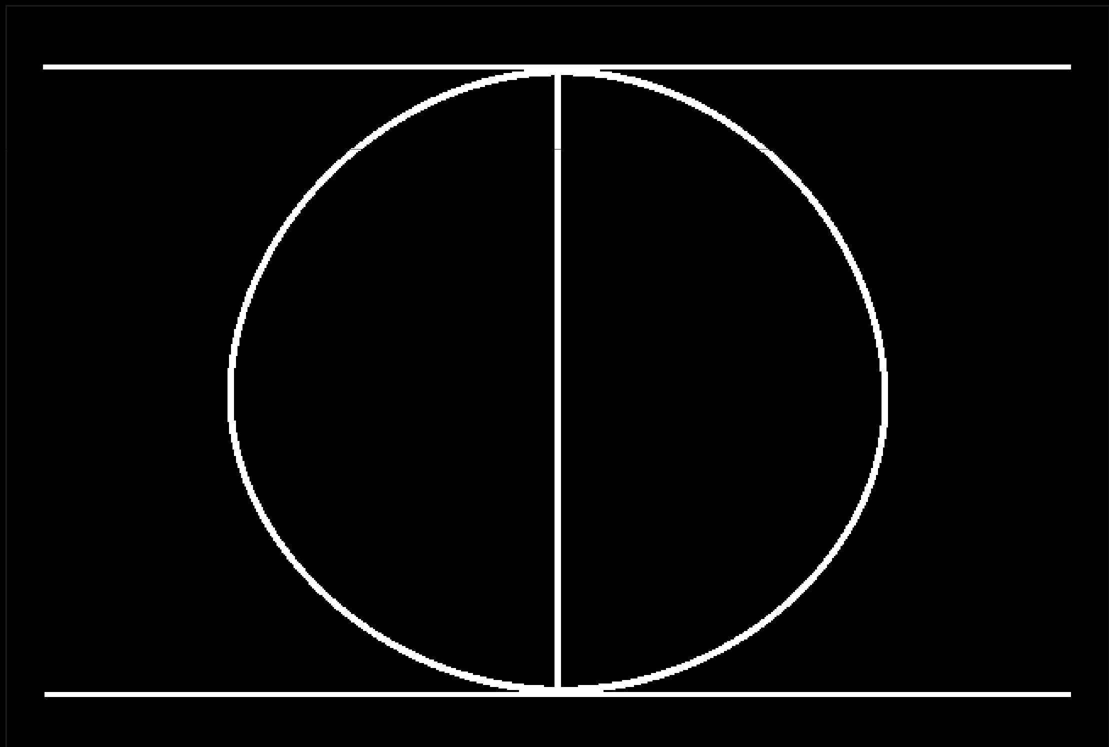
Two parallel lines are tangent to same circle.
Distance between them is diameter of the circle



Note

Equation of diameter parallel to tangent

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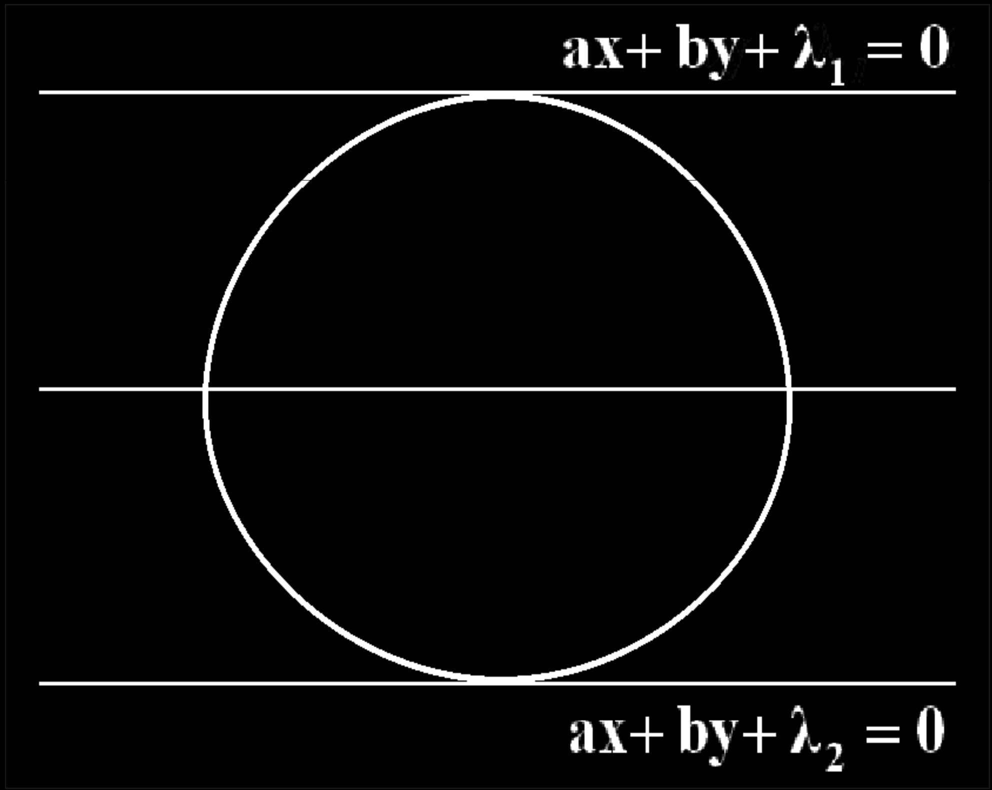
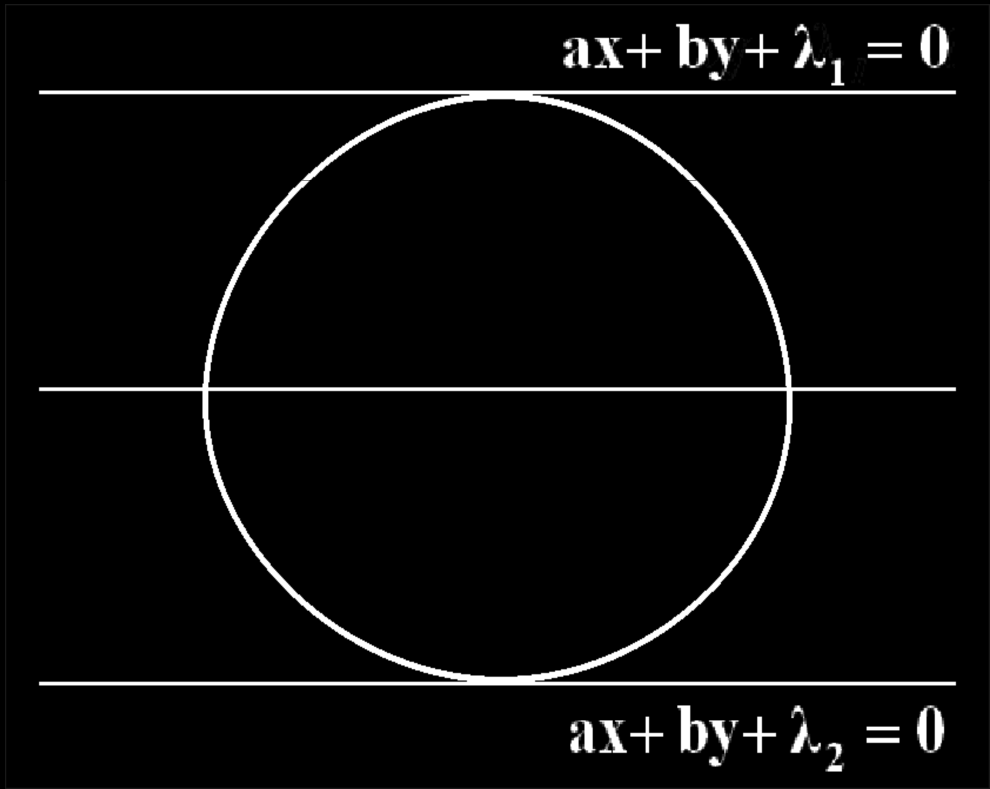
Equation of diameter parallel to tangent



Note

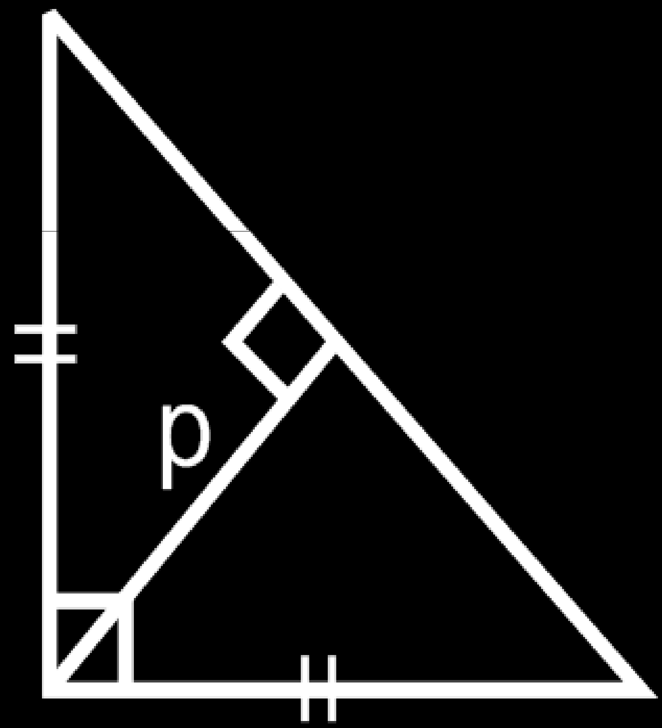
Equation of diameter parallel to tangent

$$ax + by + \left(\frac{\lambda_1 + \lambda_2}{2}\right) = 0$$



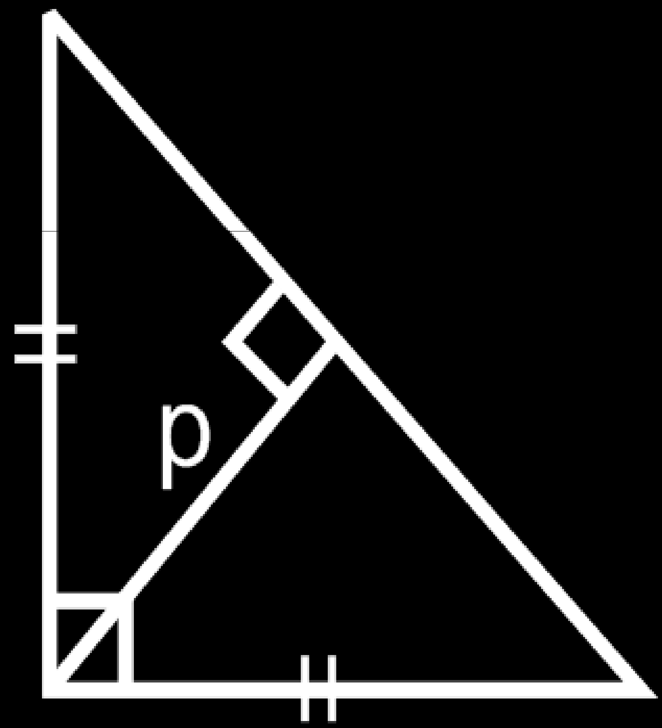
Area of right isosceles Δ in term of ⊥ from vertex

Area of right isosceles Δ in term of ⊥ from vertex



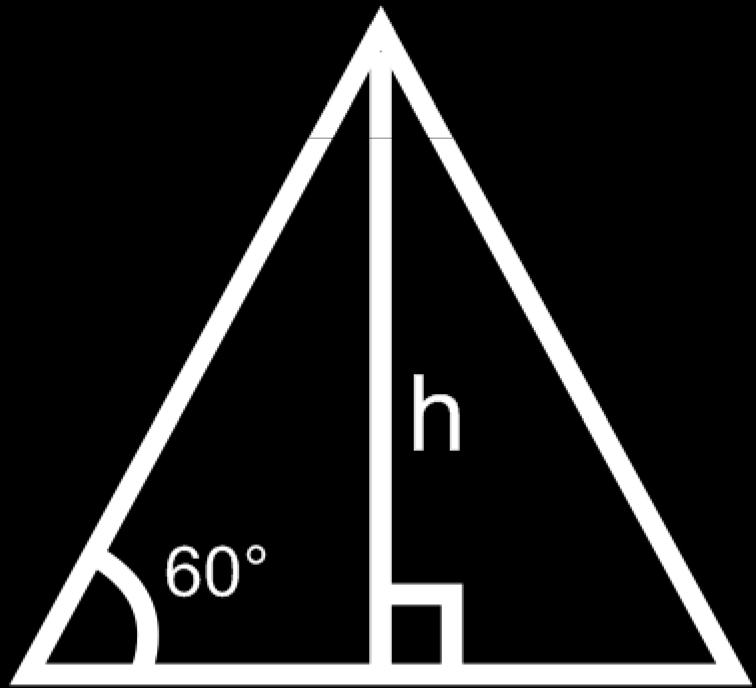
Area of right isosceles Δ in term of ⊥ from vertex

$\Delta = p^2$



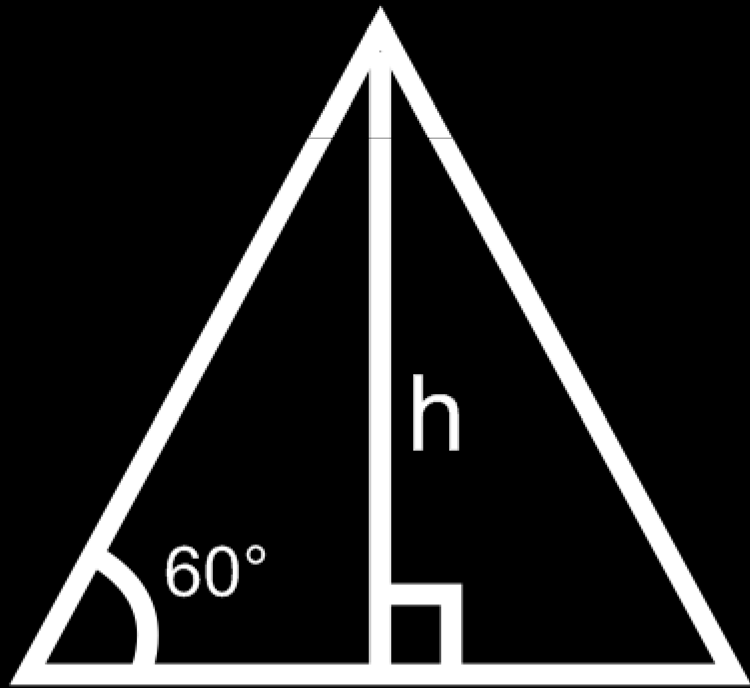
Area of equilateral Δ in terms of median / angle bisector / ⊥ bisector / altitude

Area of equilateral Δ in terms of median / angle bisector / ⊥ bisector / altitude



Area of equilateral Δ in terms of median / angle bisector / ⊥ bisector / altitude

$$\Delta = \frac{h^2}{\sqrt{3}}$$



Example

Q.1 Find area of equilateral Δ whose one vertex is (7, 0) & a side lies along line $y = x$

Q.2 Two mutually $\perp$ lines are drawn passing through points (a, b) and enclosed in an isosceles Δ together with the line $x \cos \alpha + y \sin \alpha = p$, Find the area of Δ

Q.3 The three lines $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form the three sides of a square, Find the equation of the fourth side

Q.4 Find area of quadrilateral formed by the lines

$3x - 4y + 10 = 0$, $3x - 4y + 20 = 0$,

$4x + 3y + 10 = 0$, $4x + 3y + 20 = 0$

Q.5 Find area of quadrilateral formed by the lines

$3x - 4y + 1 = 0,$ $\quad$ $3x - 4y + 2 = 0,$

$x - 2y + 3 = 0,$ $\quad$ $x - 2y + 7 = 0$

Parametric Form of Line / Distance Form

Parametric Form of Line / Distance Form

$$\frac{y - y_1}{\sin\theta} = \frac{x - x_1}{\cos\theta} = r$$

Parametric Form of Line / Distance Form

$$\frac{y-y_1}{\sin\theta}=\frac{x-x_1}{\cos\theta}=r$$

or

$$x = x_1 + r\cos\theta\ ,\ \ y = y_1 + r\sin\theta$$

Example

Q.1 In what direction a line through point (1, 2) must be drawn so that its intersection point P with the line x + y = 4 may be at a distance of $\frac{\sqrt{6}}{3}$ from A

Q.2 If $A(3,2)$, $B(7,4)$, Find coordinate of C such that ΔABC is equilateral.

Q.3 A line passing through A (-5, -4) meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B,C,D

If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$, find the equation of line

[IIT-JEE 1993]

Q.4 Two side of a rhombus lying in 1st quadrant are given by $y = \frac{3x}{4}$ & $y = \frac{4x}{3}$. If the length of longer diagonal OC = 12, Find the equation of other two sides

Assignments - 6

Find the length of the perpendicular drawn from

Q.1 The point (4, 5) upon the straight line $3x + 4y = 10$

Q.2 The origin upon the straight line $\frac{x}{3} - \frac{y}{4} = 1$.

Q.3 The point (-3, -4) upon the straight line

$12(x + 6) = 5(y - 2)$

Q.4 The point (b, a) upon the straight line $\frac{x}{a} - \frac{y}{b} = 1$.

Q.5 Find the length of the perpendicular from the origin upon the straight line joining the two points whose coordinates are $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$

Q.6 Shew that the product of the perpendiculars drawn from the two poin $\left(\pm\sqrt{a^2-b^2},\ 0\right)$ upon the straight line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2

Q.7 If p and p′ be the perpendicular from the origin upon the straight lines whose equations are

$x \sec\theta + y \operatorname{cosec}\theta = a$ and

$x \cos\theta - y \sin\theta = a \cos 2\theta$

Prove that : $4p^2 + p'^2 = a^2$

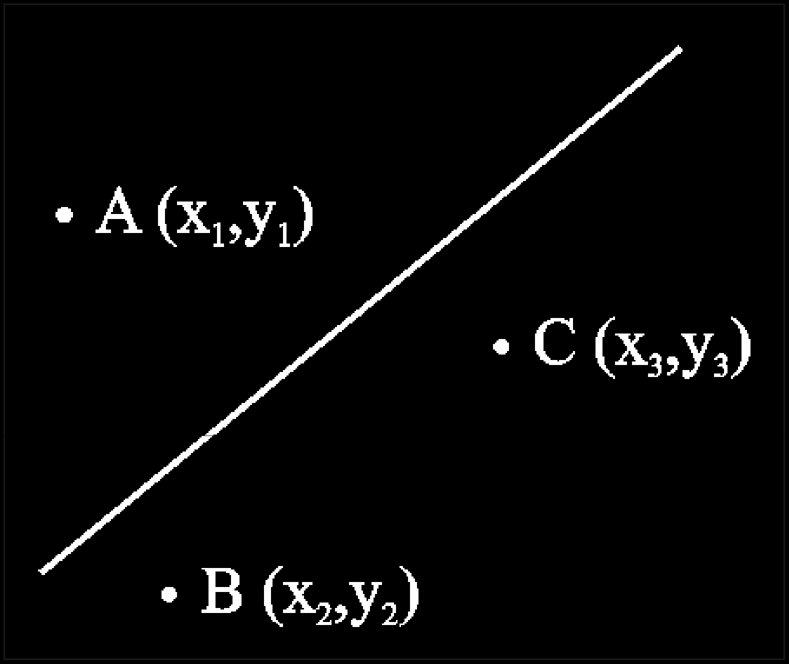
Q.8 Find the distance between the two parallel straight line $y = mx + c$ and $y = mx + d$

Q.9 What are the point on the axis of x whose perpendicular distance from the straight line $\frac{x}{a}+\frac{y}{b}=1$ is a ?

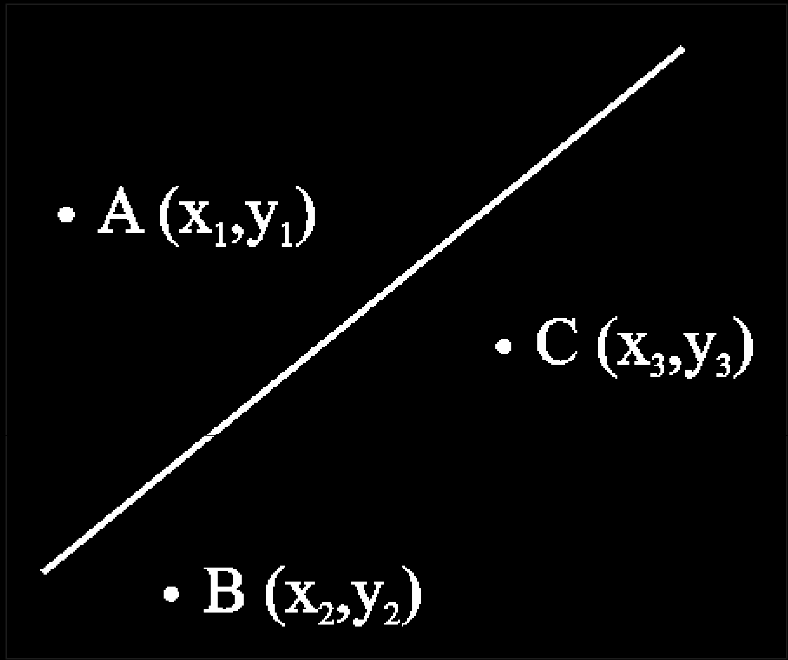
Q.10 Find the perpendicular distance from the origin of the perpendicular from the point (1, 2) upon the straight line $x-\sqrt{3}y+4=0$

Position of point w.r.t. a Line

Position of point w.r.t. a Line

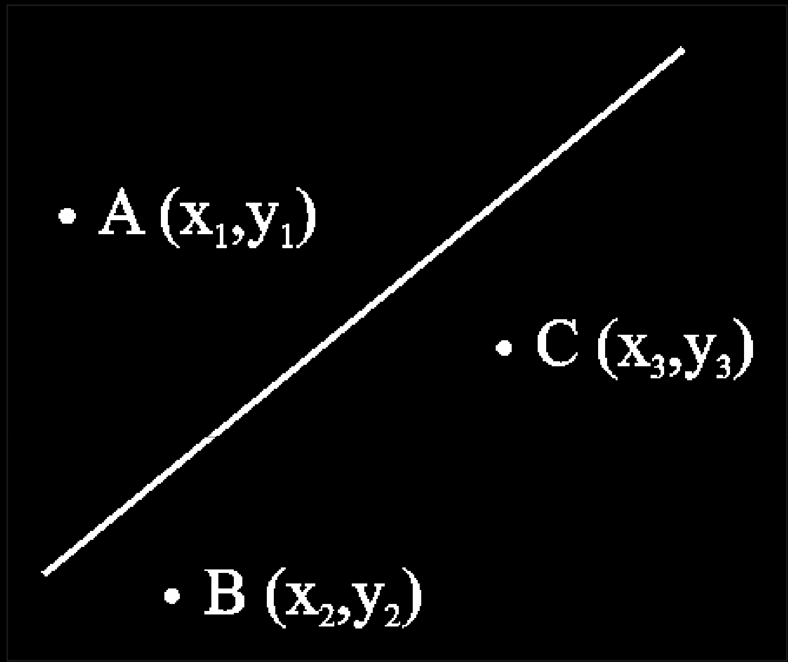


Position of point w.r.t. a Line



$$(ax_2+by_2+c)\ (ax_3+by_3+c) > 0$$

Position of point w.r.t. a Line



$$(ax_2+by_2+c)\,(ax_3+by_3+c) > 0$$

$$(ax_1+by_1+c)\,(ax_2+by_2+c) < 0$$

Example

Q.1 Find range of a for which (a, a^2) and origin lie on same side of line $4x + 4y - 3 = 0$

Q.2 If point (a, a^2) lies between lines

$x + y - 2 = 0$ & $4x + 4y - 3 = 0$,

Find the range of a.

Q.3 Determine values of α for which point (α, α^2) lies inside the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$ & $5x - 6y - 1 = 0$ [IIT-JEE 1992]

Condition of Concurrency

Condition of Concurrency

$$\left.\begin{array}{l} l_1 \equiv a_1x + b_1y + c_1 = 0 \\ l_2 \equiv a_2x + b_2y + c_2 = 0 \\ l_3 \equiv a_3x + b_3y + c_3 = 0 \end{array}\right\} \textit{are concurrent}$$

Condition of Concurrency

$$\left.\begin{aligned} l_1 &\equiv a_1x + b_1y + c_1 = 0 \\ l_2 &\equiv a_2x + b_2y + c_2 = 0 \\ l_3 &\equiv a_3x + b_3y + c_3 = 0 \end{aligned}\right\} \textit{are concurrent}$$

$$\textit{If} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Example

Q.1 Find k if lines $x-y=3$, $x + y = 7$, $kx + 3y = 4$ are concurrent

Q.2 Prove that in any Δ altitudes are concurrent

[IIT-JEE 1998]

Q.3 Let $\lambda, \alpha \in R$ the lines

$$\lambda x + \sin\alpha\, y + \cos\alpha = 0$$
$$x + \cos\alpha\, y + \sin\alpha = 0$$
$$-x + \sin\alpha\, y - \cos\alpha = 0$$

If these lines are concurrent find the range of λ

If $\lambda = 1$ find general solution for α

Q.4 If $bc \neq ad$ and the lines

$$(\sin 3\theta)x+ay+b=0$$
$$(\cos 2\theta)x+cy+d=0$$
$$2x+(a+2c)y+(b+2d)=0$$

are concurrent then find the most general values of θ

Family of lines

Family of lines

(i) Family of concurrent lines

Family of lines

(ii) Family of parallel lines

Example

Q.1 Find the point through which the line $x - 1 + \lambda y = 0$ always passes through $\forall \lambda \in R$

Q.2 Find the point through which the line $x-2+\lambda(y-1) = 0$ always passes through $\forall \lambda \in R$

Q.3 Find the point through which the line $2x - 3\lambda = y+7$ always passes through $\forall \lambda \in R$

Type - 1

Equation of line always passing through point of intersection of $l_1 = 0$ & $l_2 = 0$

is $\quad l_1 + \lambda l_2 = 0 \quad \forall \lambda \in R$

Example

Q.1 Find equation of line passing through intersection of $3x - 4y + 6 = 0$ & $x + y + 2 = 0$ and

(i) Parallel to line $y = 0$

Example

Q.1 Find equation of line passing through intersection of $3x - 4y + 6 = 0$ & $x + y + 2 = 0$ and

(ii) Parallel to line $x = 7$

Example

Q.1 Find equation of line passing through intersection of $3x - 4y + 6 = 0$ & $x + y + 2 = 0$ and

(iii) At a distance of 5 units from origin

Example

Q.1 Find equation of line passing through intersection of $3x - 4y + 6 = 0$ & $x + y + 2 = 0$ and

(iv) Situated at maximum distance from point (2,3)

Type – 2
(Converse of Type - 1)

Type – 2
(Converse of Type - 1)

$l_1 + \lambda l_2 = 0$ always passes through intersection of $l_1 = 0$ & $l_2 = 0$

Example

Q.1 If a, b, c are in A.P. Find the point through which $ax+by+c = 0$ always passes through.

Q.2 If $a+3b = 5c$, find the fixed point through which line $ax+by+c=0$ passes

Q.3 If $a^2 + 9b^2 = 6ab + 4c^2$ then $ax + by + c = 0$ passes through one or the other of which two fixed points ?

Q.4 If algebraic sum of the perpendiculars from $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ on a variable line $ax + by + c = 0$ vanishes then the variable line always passes through.

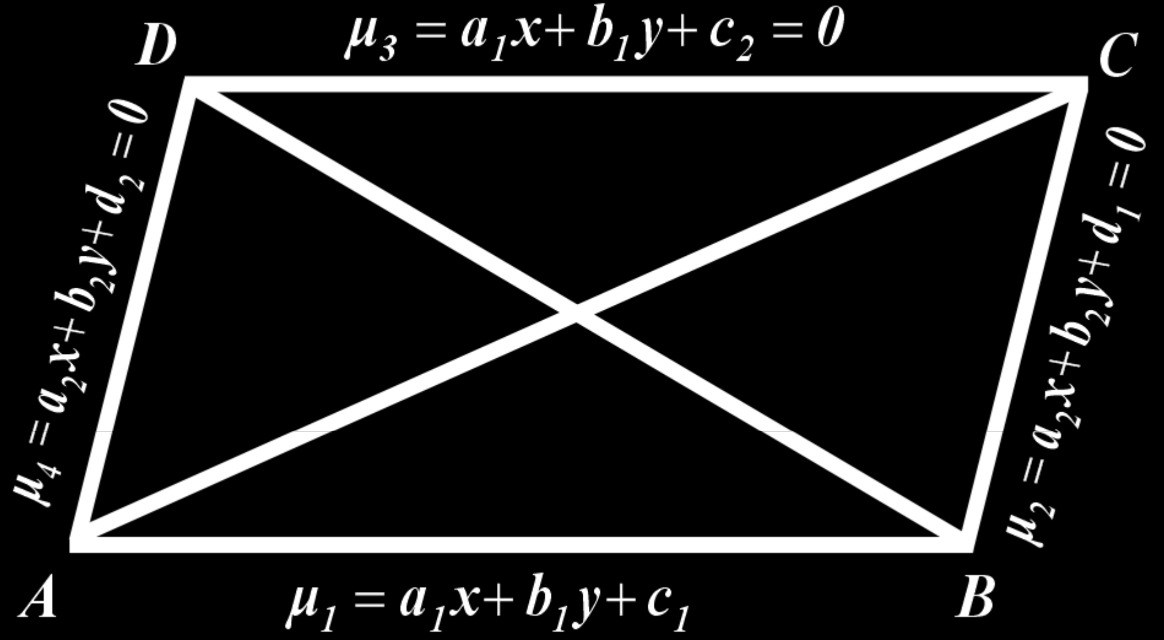
(A) G of ΔABC (B) O of ΔABC

(C) I of ΔABC (D) H of ΔABC

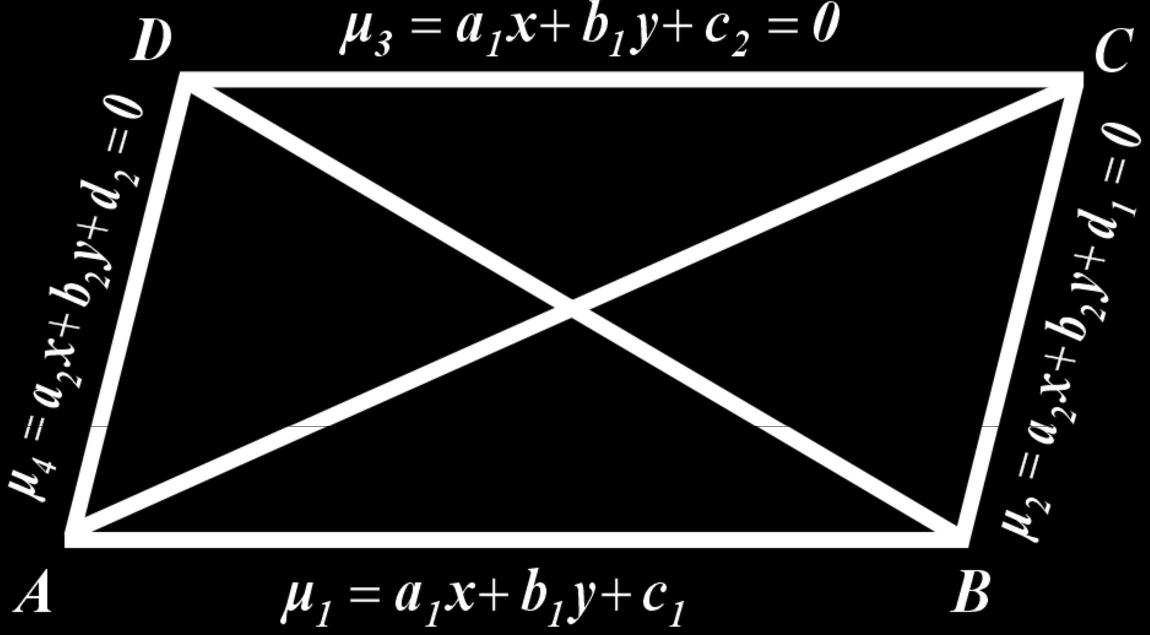
Q.5 The family of lines $x(a+2b)+y(a+3b) = a+b$ always passes through a fixed point. Find the point.

Q.6 The equations to the sides of a triangle are $x + 2y = 0$, $4x + 3y = 5$ and $3x + y = 0$. Find the coordinates of the orthocentre of the triangle without finding vertices of triangle.

Type - 3


D
$\mu_3 = a_1x + b_1y + c_2 = 0$
C
$\mu_4 = a_2x + b_2y + d_2 = 0$
$\mu_2 = a_2x + b_2y + d_1 = 0$
A
$\mu_1 = a_1x + b_1y + c_1$
B

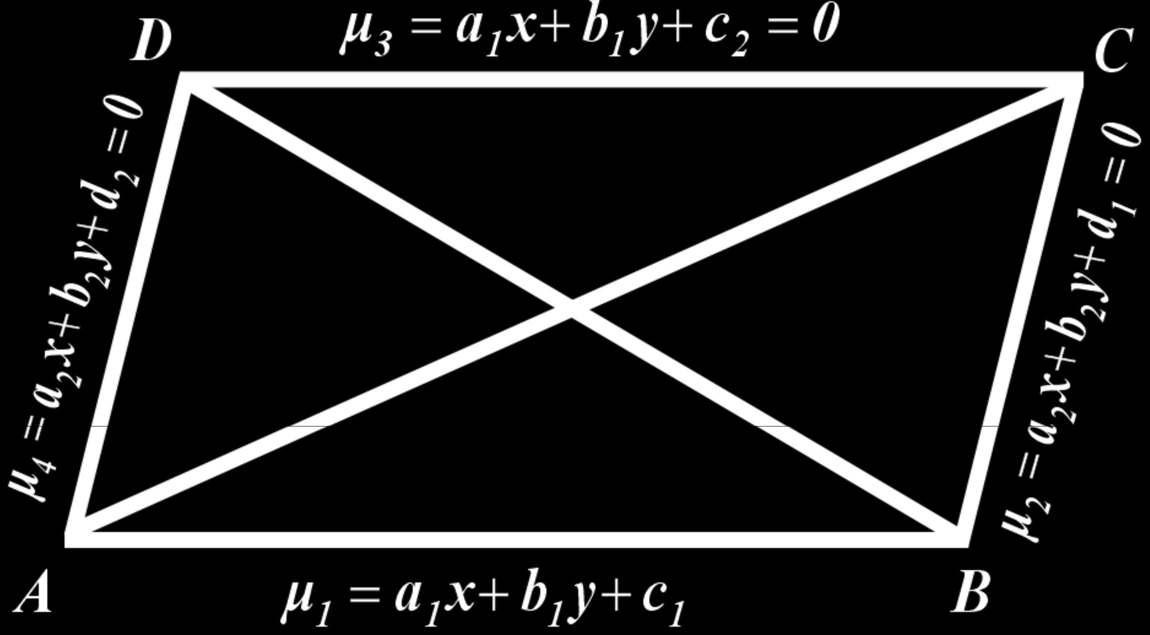
Type - 3



Equation of diagonals of ||[gm]

AC = $\mu_1\mu_2 - \mu_3\mu_4 = 0$

Type - 3



Equation of diagonals of ||[gm]

$AC = \mu_1\mu_2 - \mu_3\mu_4 = 0$

$BD = \mu_1\mu_4 - \mu_2\mu_3 = 0$

Example

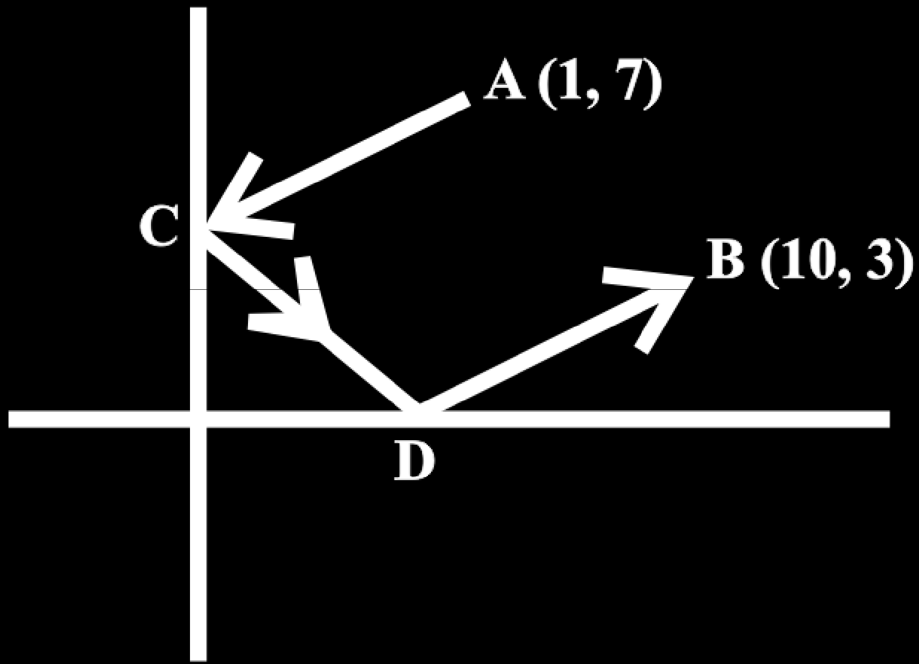
Q. Find the equations of the diagonals of the parallelogram formed by the lines

$2x - y + 7 = 0$, $2x - y - 5 = 0$, $3x + 2y - 5 = 0$ & $3x + 2y + 4 = 0$

Optics Problems

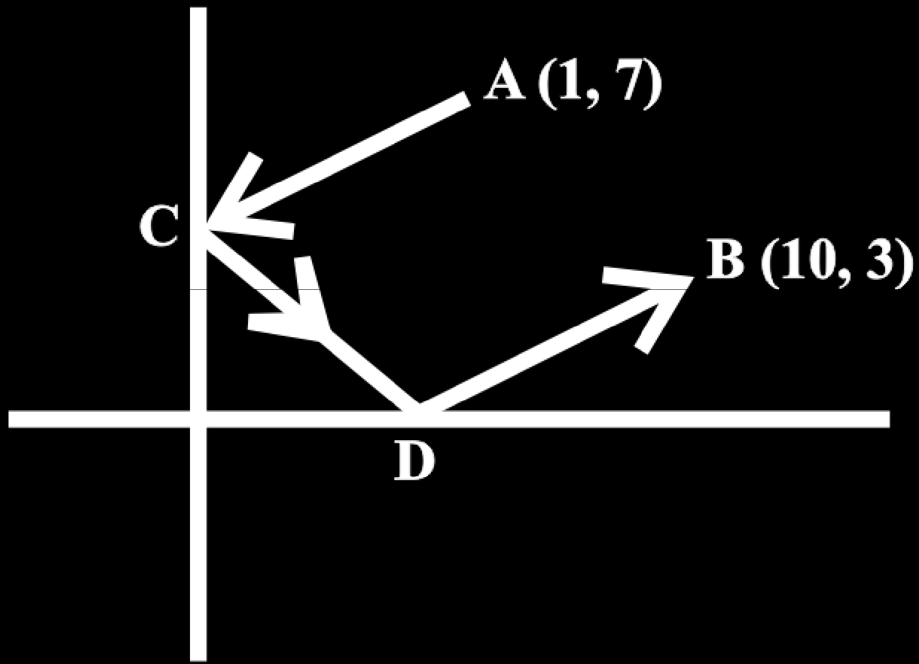
Example

Q.1 (i) Find reflection of point A (1,7) about y axes



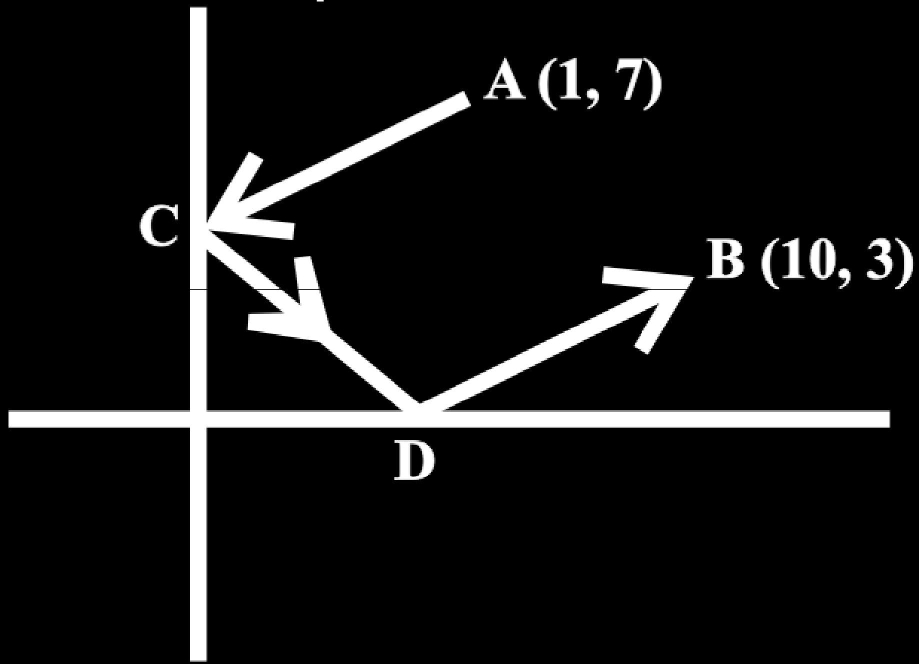
Example

Q.1 (ii) Find reflection of point (10,3) about x axes



Example

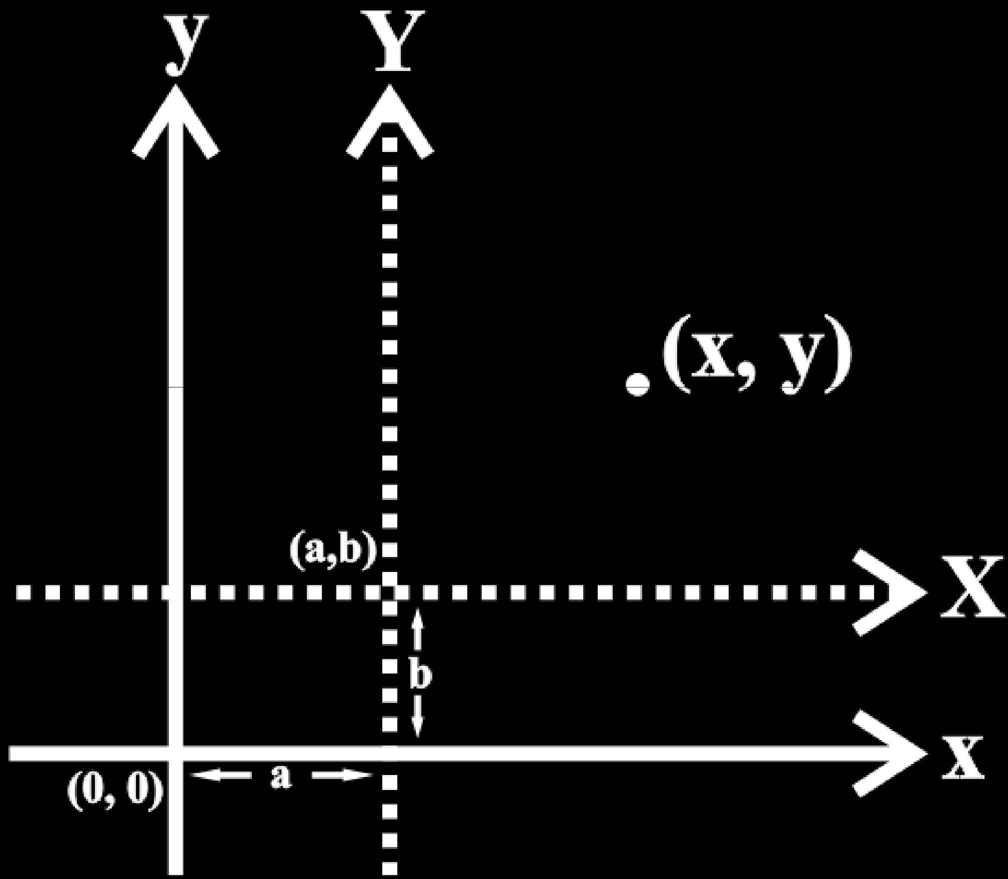
Q.1 (iii) If A (1, 7) B (10, 3). Find coordinate of point C & D



Q.2 If A (1, 2) & B (3, 5), point P lies on x axis find P such that AP + PB is minimum

Shifting of Origin

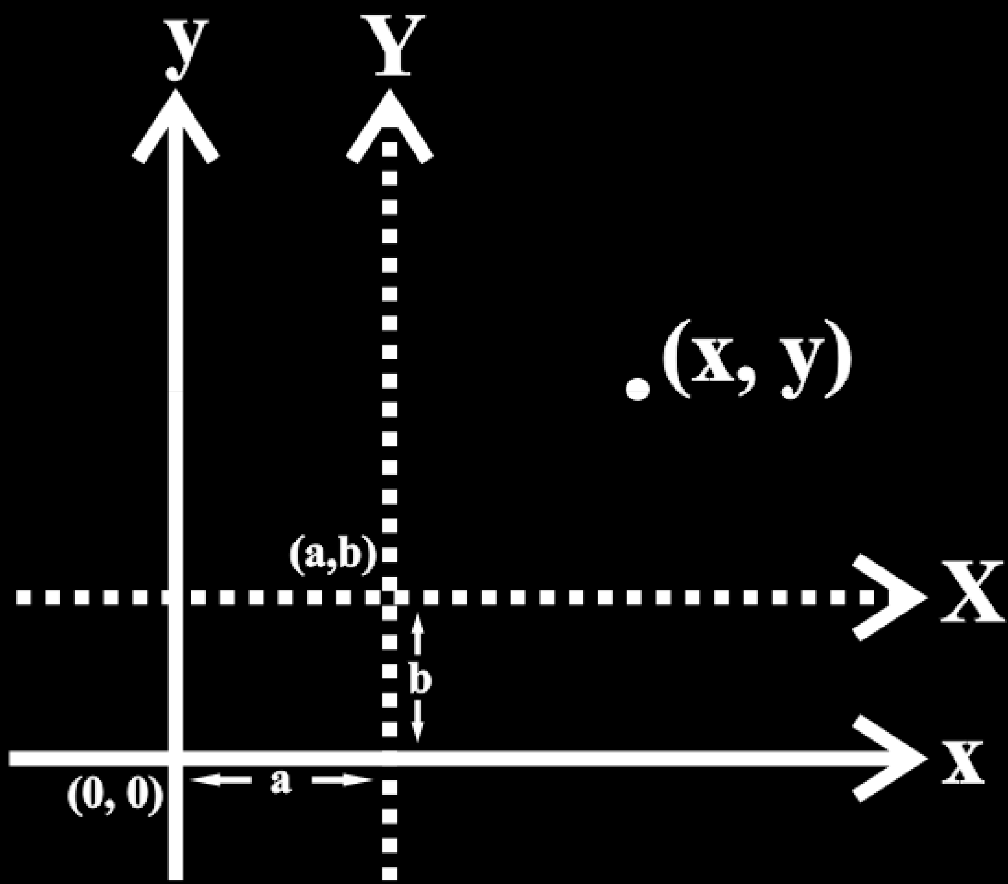
Shifting of Origin



Shifting of Origin

$X = x - a$

$Y = y - b$



Example

Q.1 Find the new coordinate of point (3, -4) if origin is shifted to (1,2)

Q.2 Find transformed equation of the straight line $2x-3y+5 = 0$ if origin is shifted to $(3, -1)$

Q.3 Find the point to which the origin should be shifted so that the equation $x^2 + y^2 - 5x + 2y - 5 = 0$ has no one degree terms

Q.4 Find the point to which the origin should be shifted so that the equation $y^2-6y-4x+13 = 0$ is transformed to $y^2 = AX$

Q.5 Find area of triangle formed with vertices (2,0), (0,0), (1,4) if origin is shifted to (2010, 2012)

Note

Slope of line remains same after changing the origin

Example

Q.1 If the axes are shifted to (1,1) then what do the following equation becomes

(i) $x^2 + xy - 3y^2 - y + 2 = 0$

Example

Q.1 If the axes are shifted to (1,1) then what do the following equation becomes

(ii) $xy - x - y + 1 = 0$

Example

Q.1 If the axes are shifted to (1,1) then what do the following equation becomes

(iii) $x^2 - y^2 - 2x + 2y = 0$

Rotation of Co-ordinate System

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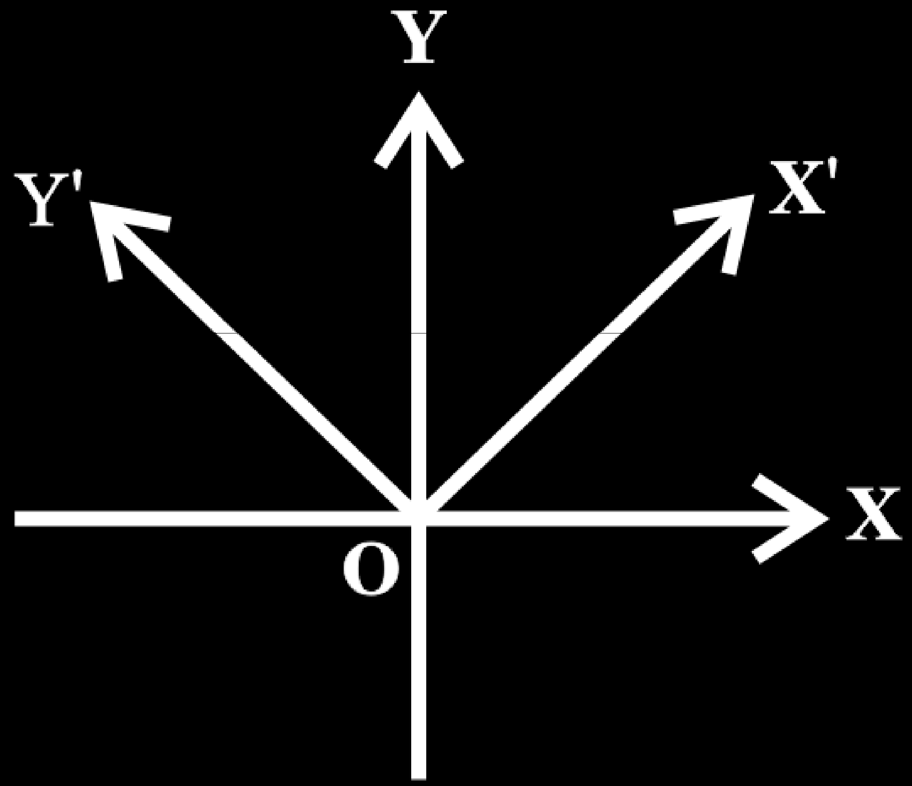

Rotation of Co-ordinate System

	x	y
x'	cosθ	sinθ
y'	-sinθ	cosθ

Example

Q. If the axes are rotated through an angle of 30° in the anticlockwise direction about the origin. The co-ordinates of point are $\left(4, -2\sqrt{3}\right)$ in the in new system. Find its old coordinates.

Angle Bisector

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Locus of point such that its distance from two intersecting lines is equal

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Locus of point such that its distance from two intersecting lines is equal

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Example

Q. Find equation of angle bisector for lines $3x + 4y + 1 = 0$, $12x + 5y + 3 = 0$

To discriminate between the acute & obtuse angle bisector (Method – 2)

To discriminate between the acute & obtuse angle bisector (Method – 3)

Example

Q. Find the bisectors between the line

$4x + 3y - 7 = 0$ and $24x+7y-31=0$.

Identify acute/obtuse and origin containing/not containing

To discriminate between the bisector of angle containing origin and that of the angle not containing origin.

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(i) Rewrite lines with same sign of absolute term.

To discriminate between the bisector of angle containing origin and that of the angle not containing origin.

(ii) Now positive sign will give you origin containing angle bisector

Example

Q.1 The vertices of a ΔABC are

$A(-1, 11)$, $B(-9, -8)$ and $C(15, -2)$

find the equation of the bisector of the angle at vertex A.

Q.2 Find bisectors between the lines

$x + \sqrt{3}\,y = 6 + 2\sqrt{3}$ and

$x - \sqrt{3}\,y = 6 - 2\sqrt{3}$

Note

If $m_1 + m_2 = 0 \Rightarrow$ lines equally inclined with the axes.

Pair of Straight Line

Pair of Line Through Origin

Pair of Line Through Origin

$ax^2 + 2h\,xy + by^2 = 0$ (2° equation)

(i) $h^2 > ab \Rightarrow$ lines are real & distinct

Pair of Line Through Origin

$ax^2 + 2h\,xy + by^2 = 0$ (2° equation)

(ii) $h^2 = ab \Rightarrow$ lines are coincidental

Pair of Line Through Origin

$ax^2 + 2h\,xy + by^2 = 0$ (2° equation)

(iii) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection

Note

A homogeneous equation of degree n represent n straight lines passing through origin.

Note

If $y = m_1x$ & $y=m_2x$ be two equation represented by $ax^2 + 2h\,xy + by^2 = 0$ then

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1\,m_2 = \frac{a}{b}$$

Angle between two lines

$$ax^2 + 2hxy + by^2 = 0$$

Angle between two lines
$ax^2 + 2hxy + by^2 = 0$

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

Lines being perpendicular

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coefficient of x^2 + coefficient of y^2 = 0

i.e. $a + b = 0$

Lines are || / Coincident

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$$h^2 = ab$$

Lines are equally inclined to X axis or coordinate axes are bisectors

Lines are equally inclined to X axis or coordinate axes are bisectors

Coefficient of xy = 0

Examples

Q.1 Find angle between lines given by $x^2 + 4xy + 4y^2 = 0$

Q.2 Find angle between lines given by $x^2 + 4xy + y^2 = 0$

Q.3 Find angle between lines given by $y^2 - 3x^2 = 0$

Q.4 Find angle between lines given by $xy = 0$

Q.5 Find angle between lines given by

$$3x^2 + 10xy - 3y^2 = 0$$

Assignments - 7

Find what straight lines are represented by the following equations and determine the angles between them.

Q.1 $x^2 - 7xy + 12y^2 = 0$

Q.2 $4x^2 - 24xy + 11y^2 = 0$

Q.3 $33x^2 - 71xy - 14y^2 = 0$

Q.4 $x^3 - 6x^2 + 11x - 6 = 0$

Q.5 $y^2 - 16 = 0$

Q.6 $y^3 - xy^2 2 - 14x^2 y + 24x^3 = 0$

Q.7 $x^2 + 2xy\sec\theta + y^2 = 0$

Q.8 $x^2 + 2xy\cot\theta + y^2 = 0$

Q.9 Find the equations of the straight lines bisecting the angles between the pairs of straight lines given in example 2, 3, 7 and 8.

General Equation of 2°

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$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Condition that 2° equation represents pair of lines

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$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Condition that 2° equation represents pair of lines

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

or
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Example

Q.1 Find whether $2x^2 - xy - y^2 - x + 4y - 3 = 0$ can be factorized in two linears. If yes find the factors. Also find the angle between the lines.

Q.2 $12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = 0$ factorize this in two linears.

Q.3 Find condition for which
$$ax^2 + bx^2y + cxy^2 + dy^3 = 0$$
represent three lines two of which are at right angle.

Q.4 Prove that $3x^2 - 8xy - 3y^2 = 0$ and $x + 2y = 3$ enclose a right isosceles Δ. Also find area of Δ.

Q.5 Prove that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 1$ enclose an equilateral triangle. Find also its area.

Q.6 Find centroid of Δ the equation of whose sides are $12x^2 - 20xy + 7y^2 = 0$ & $2x - 3y + 4 = 0$

Q.7 Find distance between parallel lines

(i) $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

Q.7 Find distance between parallel lines

(ii) $x^2 + 2\sqrt{3}\,xy + 3y^2 - 3x - 3\sqrt{3}\,y - 4 = 0$

Q.8 The equation $ax^2 + 6xy - 5y^2 - 4x + 6y + c = 0$ represents two perpendicular straight lines find ‘a’ and ‘c’.

Equation of angles bisectors of $ax^2 + 2hxy + by^2 = 0$

Equation of angles bisectors of $ax^2 + 2hxy + by^2 = 0$

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Example

Q.1 Find equation of angle bisector of straight lines $xy = 0$

Q.2 Find equation of angle bisector of straight lines $x^2 - y^2 = 0$

Product of $\perp$ dropped from (x_1, y_1) to line pair given by $ax^2 + 2hxy + by^2 = 0$

Product of ⊥ dropped from (x_1, y_1) to line pair given by $ax^2 + 2hxy + by^2 = 0$

$$\frac{\left|ax_1^2 + 2hx_1y_1 + by_1^2\right|}{\sqrt{(a-b)^2 + 4h^2}}$$

Homogenization

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$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$lx + my + n = 0$$

Homogenization

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$lx + my + n = 0$$

$$ax^2 + 2hxy + by^2 + 2yx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-n}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0$$

Example

Q.1 Find the equation of the line pair joining origin and the point of intersection of the line $2x - y = 3$ and the curve $x^2 - y^2 - xy + 3x - 6y + 18 = 0$. Also find the angle between these two lines.

Q.2 Find the value of ‘m’ if the lines joining the origin to the points common to

$x^2 + y^2 + x - 2y - m = 0$ & $x + y = 1$

are at right angles.

Q.3 Show that all chords of the curve

$$3x^2 - y^2 - 2x + 4y = 0$$

subtending right angles at the origin pass through a fixed point. Find also the coordinates of the fixed point. [IIT-JEE 1991]

Q.4 A line L passing through the point (2, 1) intersects the curve $4x^2 + y^2 - x + 4y - 2 = 0$ at the points A, B. If the lines joining origin and the points A, B are such that the coordinate axis are the bisectors between them then find the equation of line L.

Q.5 A straight line is drawn from the point (1,0) to intersect the curve $x^2 + y^2 + 6x - 10y + 1 = 0$ such that the intercept made by it on the curve subtend a right angle at the origin. Find the equation of the line L.

Assignments - 8

Prove that the following equations represent two straight lines; find also their point of intersection and the angle between them.

Q.1 $6y^2 - xy - x^2 + 30y + 36 = 0$

Q.2 $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$

Q.3 $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$

Q.4 $y^2 + xy - 2x^2 - 5x - y - 2 = 0$

Q.5 Prove that the equation,

$$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$$

represent two parallel lines.

Find the value of k so that the following equations may represent pairs of straight lines :

Q.6 $6x^2 + 11xy - 10y^2 + x + 31y + k = 0$

Q.7 $12x^2 - 10xy + 2y^22 + 11x - 5y + k = 0$

Q.8 $12x^2 + kxy + 2y^2 + 11x - 5y + 2 = 0$

Q.9 $6x^2 + xy + ky^2 - 11x + 43y - 35 = 0$

Q.9 $kxy - 8x + 9y - 12 = 0$

Q.10 $x^2 + \frac{10}{3}xy + y^2 - 5x - 7y + k = 0$

Q.11 $12x^2 + xy - 6y^2 - 29x + 8y + k = 0$

Q.12 $2x^2 + xy - y^2 + kx + 6y - 9 = 0$

Q.13 $x^2 + kxy + y^2 - 5x - 7y + 6 = 0$

Q.14 Prove that the equations to the straight lines passing through the origin which make an angle α with the straight lines $y + x = 0$ are given by the equation, $x^2 + 2xy \sec 2\alpha + y^2 = 0$

Q.15 The equations to a pair of opposite sides of a parallelogram are :

$x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$

find the equations to its diagonals.