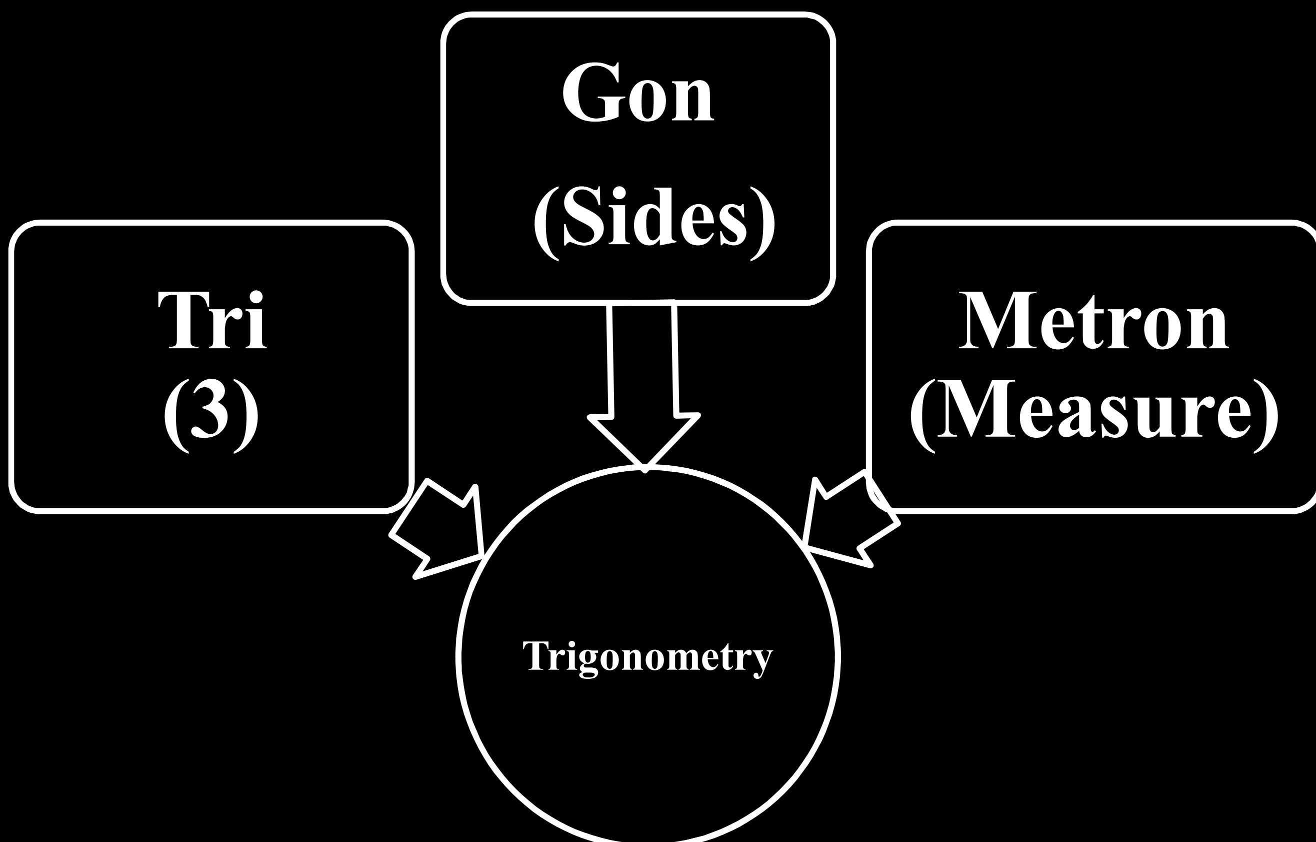
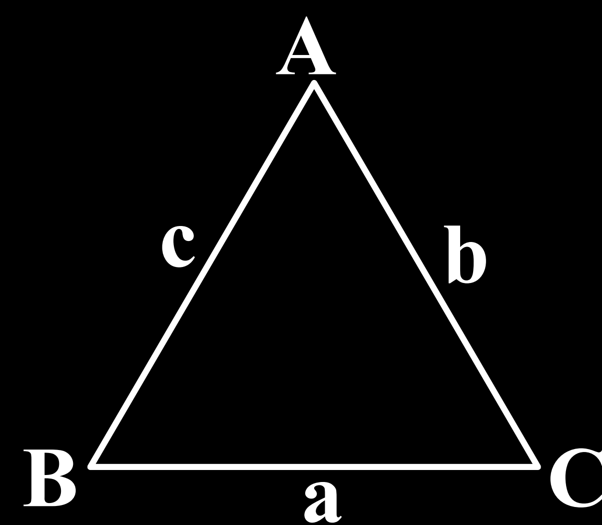
Trigonometry Ph-I

[Trigonometric Ratios and Identities]



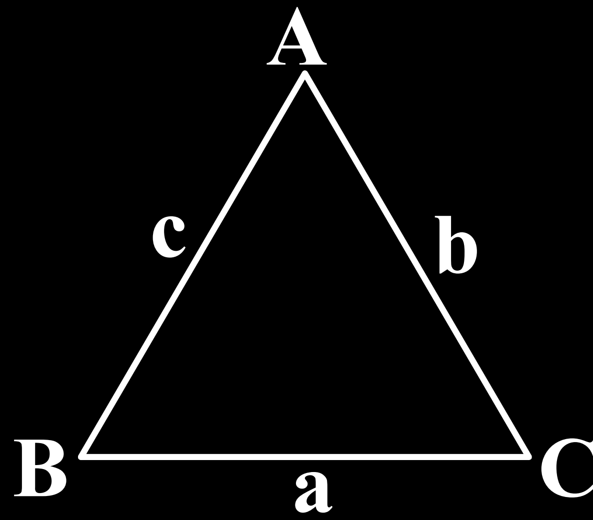
[Properties of Triangle]

[Properties of Triangle]



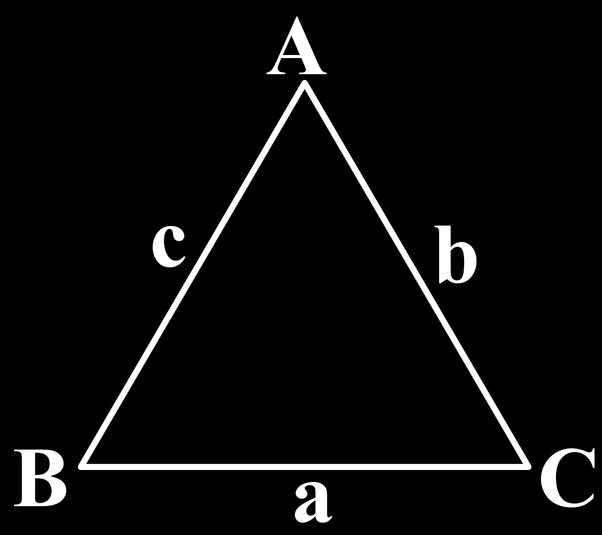
[Properties of Triangle]

- $\angle A + \angle B + \angle C = 180^0$



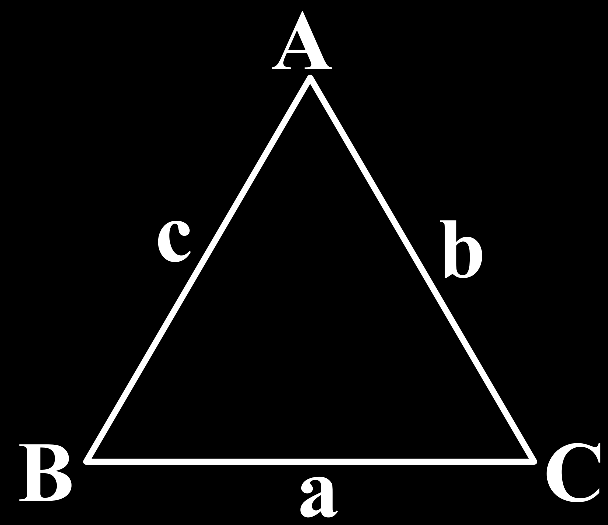
[Properties of Triangle]

- $\angle A + \angle B + \angle C = 180^0$
- $\angle B > \angle C \Rightarrow \boldsymbol{b} > \boldsymbol{c}$



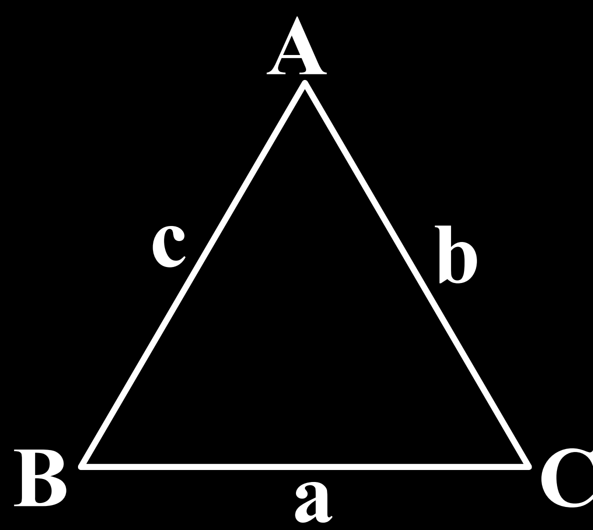
[Properties of Triangle]

- $\angle A + \angle B + \angle C = 180^0$
- $\angle B > \angle C \Rightarrow \boldsymbol{b} > \boldsymbol{c}$
- $a + b > c$

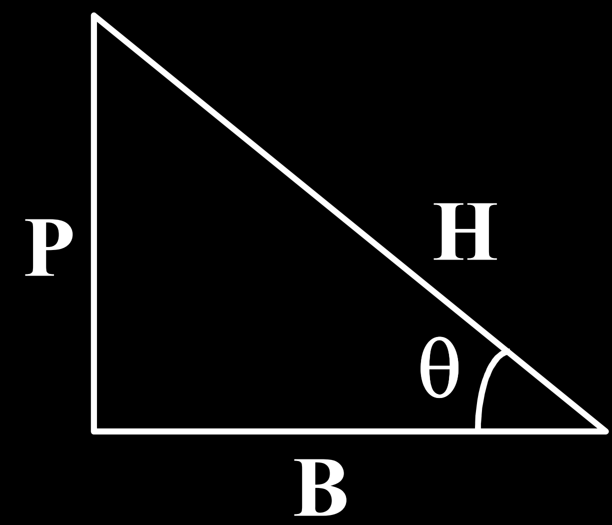


[Properties of Triangle]

- $\angle A + \angle B + \angle C = 180^0$
- $\angle B > \angle C \Rightarrow \boldsymbol{b} > \boldsymbol{c}$
- $a + b > c$
- $|a - c| < b$

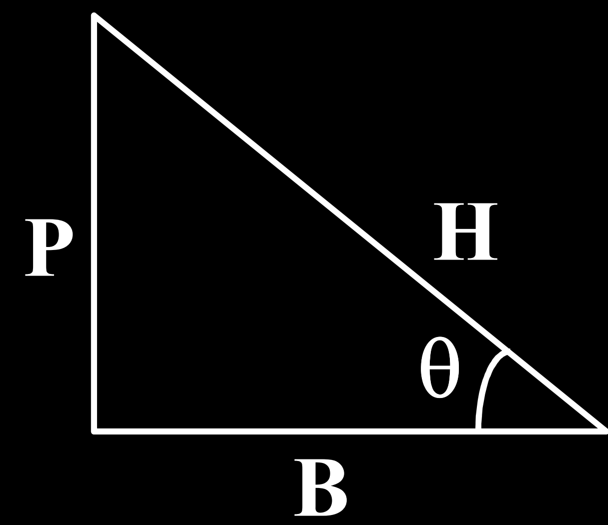


[Fundamental Ratios]



[Fundamental Ratios]

sin θ	cos θ	tan θ
P	B	P
H	H	B
cosec θ	sec θ	cot θ



[3 Important Identities]

- $\sin^2\theta + \cos^2\theta = 1$

[3 Important Identities]

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$

[3 Important Identities]

- $\sin^2\theta + \cos^2\theta = 1$

- $\sec^2\theta - \tan^2\theta = 1$

- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

[Note]

- **Reciprocal of**

  $(\sec\theta - \tan\theta)$ is $\sec\theta + \tan\theta$

[Note]

- Reciprocal of

  $(\sec\theta - \tan\theta)$ is $\sec\theta + \tan\theta$

- Reciprocal of

  $(\text{cosec}\,\theta - \cot\theta)$ is $\text{cosec}\,\theta + \cot\theta$

[Example]

- Find the value : for $0^o < A < 90^o$

  $(\sec^2 A - 1)\cot^2 A$

[Example]

- **Find the value : for $0^o < A < 90^o$**

  $(\sec^2 A - 1)\cot^2 A$

- **Prove that : for $0^o < A < 90^o$**

  $(\sec\theta + \mathrm{cosec}\,\theta)(\sin\theta + \cos\theta) = \sec\theta\,\mathrm{cosec}\,\theta + 2$

[Example]

- Find the value : for $0^o < A < 90^o$

  $(\sec^2 A - 1)\cot^2 A$

- Prove that : for $0^o < A < 90^o$

  $(\sec\theta + \operatorname{cosec}\theta)(\sin\theta + \cos\theta) = \sec\theta \operatorname{cosec}\theta + 2$

- If $\tan\theta + \sec\theta = 1.5$: for $0^o < A < 90^o$

  Find $\sin\theta$, $\tan\theta$ and $\sec\theta$

[Example]

- Prove that :

$$\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\sec\alpha}{1+\operatorname{cosec}\alpha}\right)=\tan\alpha$$

[Example]

- Prove that :

$$\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\sec\alpha}{1+\operatorname{cosec}\alpha}\right)=\tan\alpha$$

- $\dfrac{\sin x+\cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$

[More Example]

- If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ then (MCQ)

(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

[JEE – 2009]

[Measurement of Angle and Sign Convention]

2 units of angle measurement are

Degree

Radians

[Relation Between Degree & Radian]

$$1^c = \left(\frac{180^\circ}{\pi}\right) \approx 57^\circ$$

Arc length

$$l = \theta\, r$$

where θ *in radian*

Sum of all interior angles of n sided polygon

$$(n - 2)\,\pi$$

Example

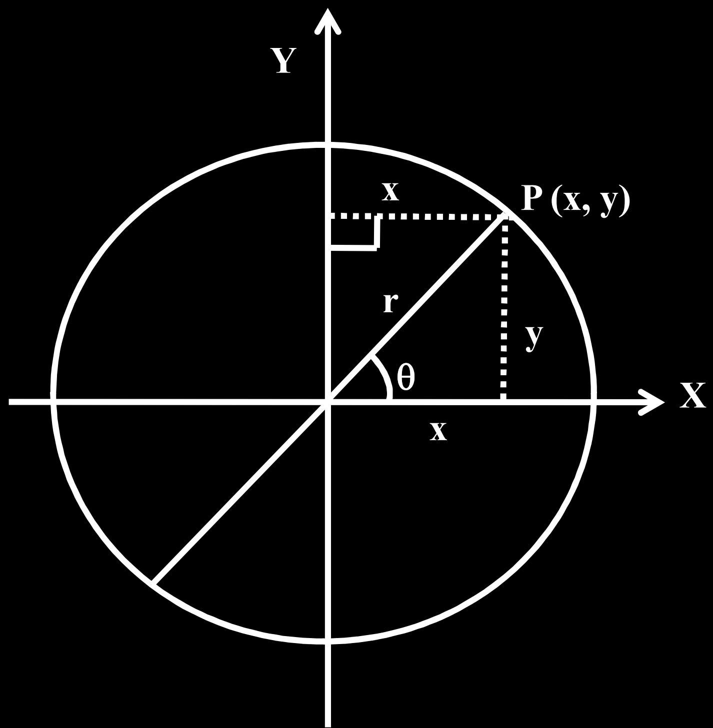
- ***A garden is in shape of a square of side length 40 meter. Now if a man runs around the garden in such a way that his distance from the side of square is 1 meter. How much distance will he travel after 1 round.***

Example

- *A garden is in shape of a square of side length 40 meter. Now if a man runs around the garden in such a way that his distance from the side of square is 1 meter. How much distance will he travel after 1 round.*

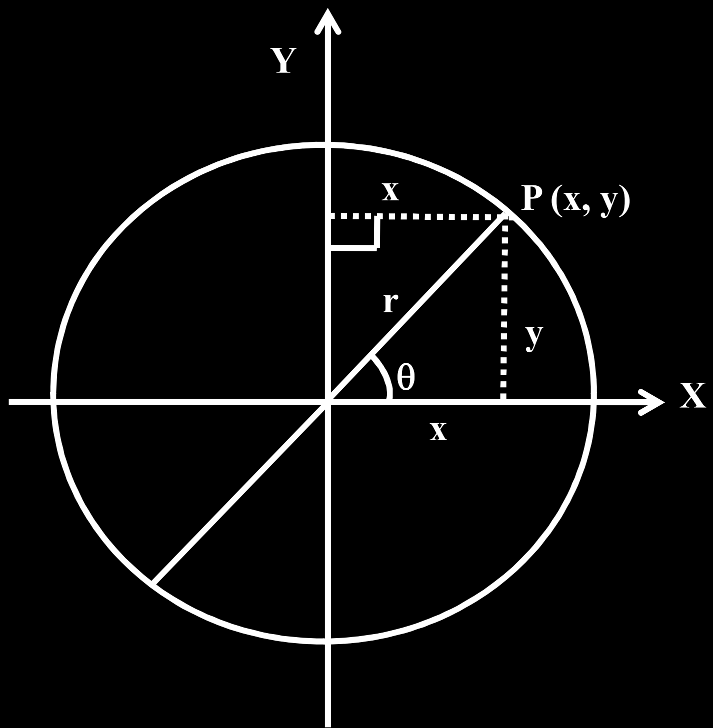
- *An equilateral triangle of sides 60 meter is in shape of a garden. Now if a man runs in such a way that his distance from the side of triangle is always 1 meter. How much distance he has covered after 1 round.*

[Real Definition of 2 Basic Function's]



[Real Definition of 2 Basic Function's]

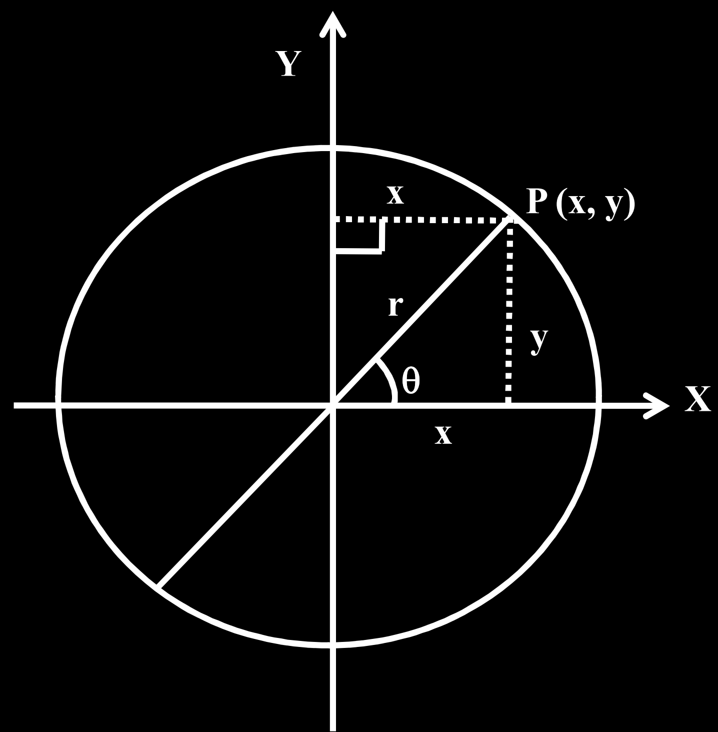
- $\sin\theta = \dfrac{(y\ co-ordinate)}{(radius)} = \dfrac{y}{r}$



[Real Definition of 2 Basic Function's]

- $$\sin\theta = \frac{(y\,co-ordinate)}{(radius)} = \frac{y}{r}$$

- $$\cos\theta = \frac{(x\,co-ordinate)}{radius} = \frac{x}{r}$$

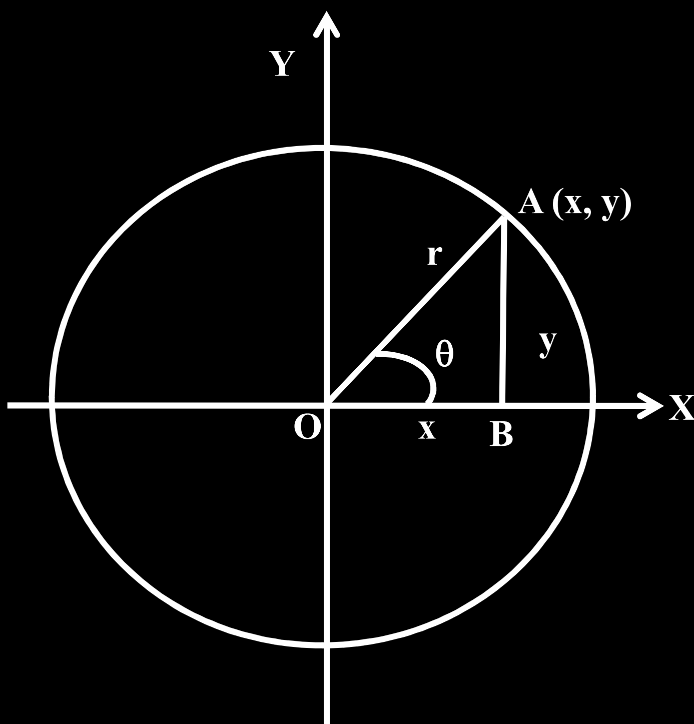


[Reduction Formulas]

- (90° - θ) Reduction

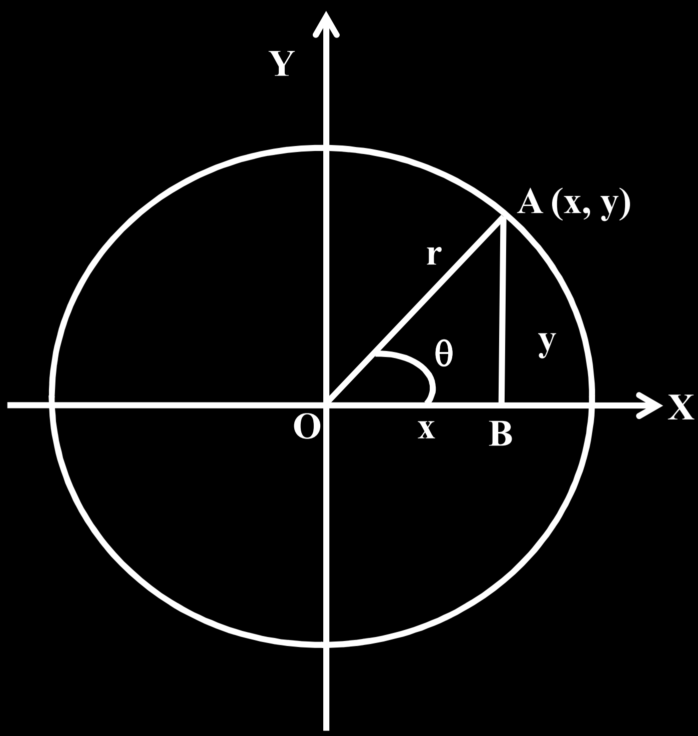
[Reduction Formulas]

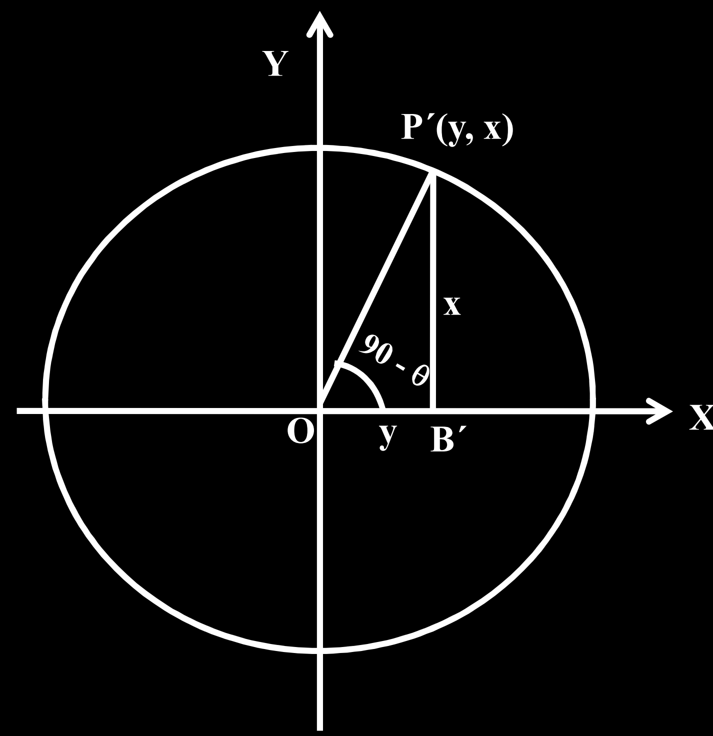
- (90° - θ) Reduction



[Reduction Formulas]

- $(90° - \theta)$ Reduction





[Reduction Formulas]

- **(90° - θ) Reduction**

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

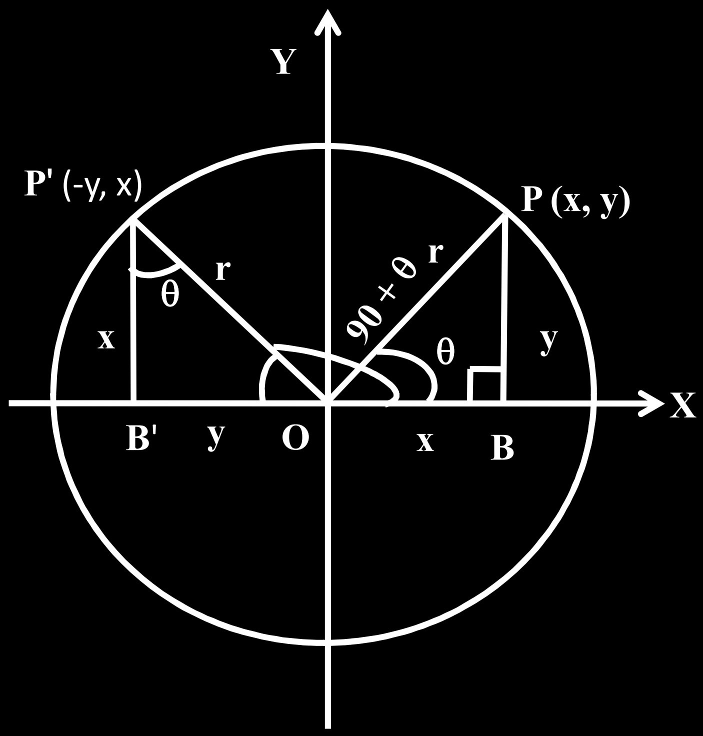
$\tan(90° - \theta) = \cot\theta$

[Reduction Formulas]

- $(90° + \theta)$ Reduction

[Reduction Formulas]

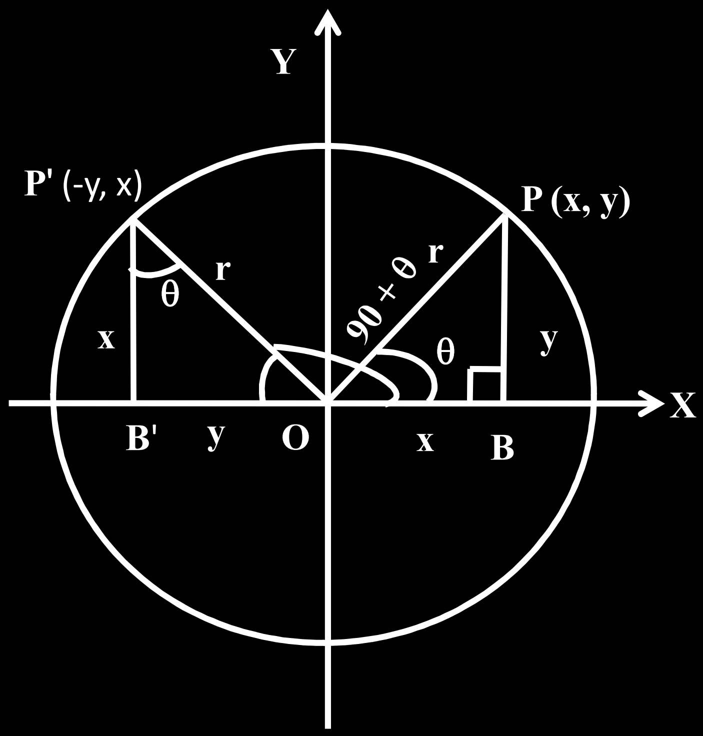
- **(90° + θ) Reduction**



[Reduction Formulas]

- **(90° + θ) Reduction**

  $\sin(90° + \theta) = \cos\theta$

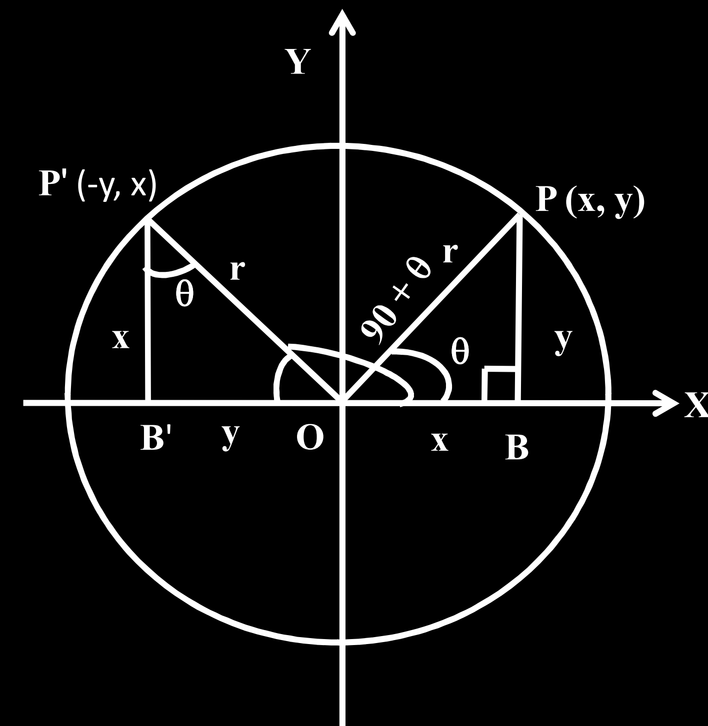


[Reduction Formulas]

- **(90° + θ) Reduction**

$\sin(90° + \theta) = \cos\theta$

$\cos(90° + \theta) = -\sin\theta$



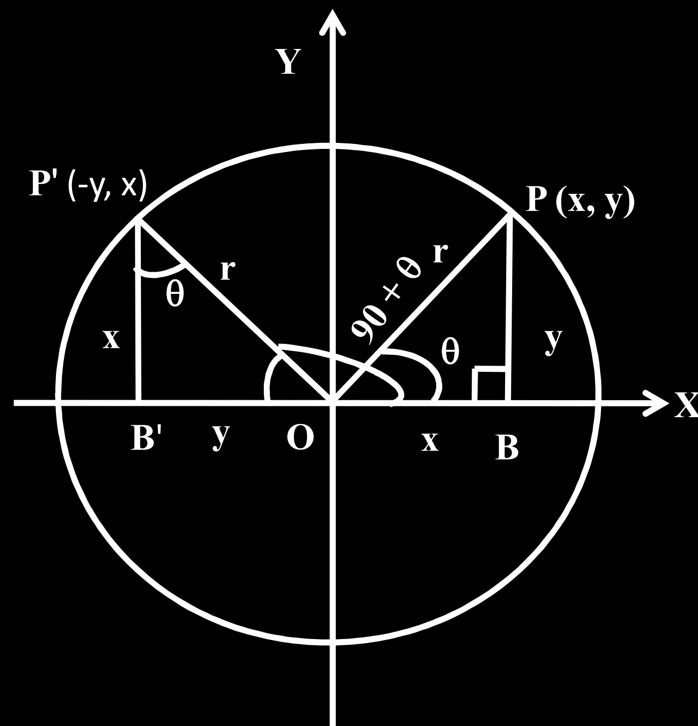
[Reduction Formulas]

- **(90° + θ) Reduction**

$\sin(90° + \theta) = \cos\theta$

$\cos(90° + \theta) = -\sin\theta$

$\tan(90° + \theta) = -\cot\theta$

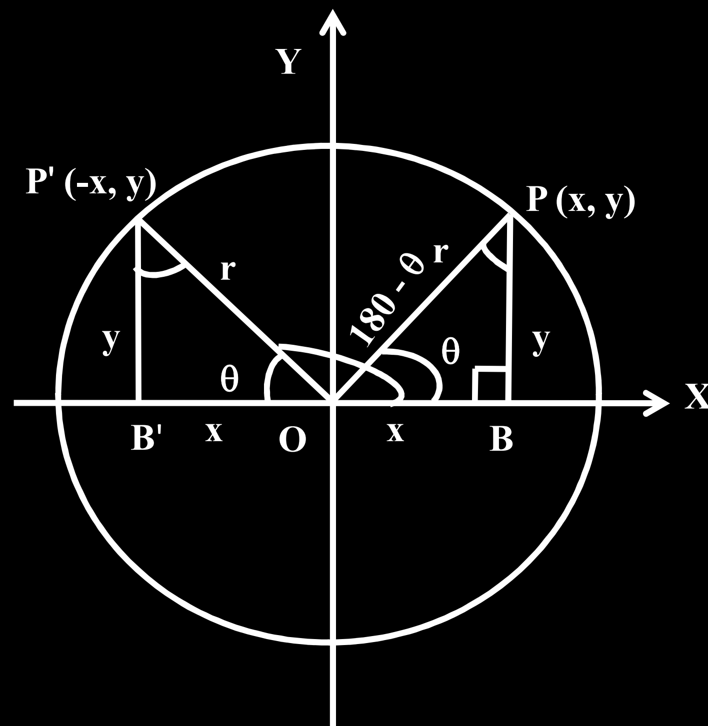


[Reduction Formulas]

- (180° - θ) Reduction

[Reduction Formulas]

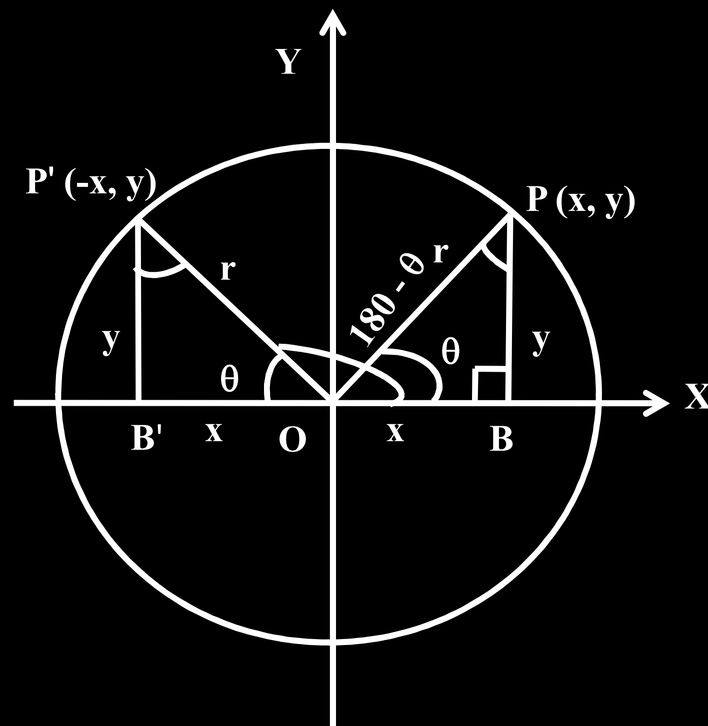
- (180° - θ) Reduction



[Reduction Formulas]

- **(180° - θ) Reduction**

  $\sin(180° - \theta) = \sin\theta$

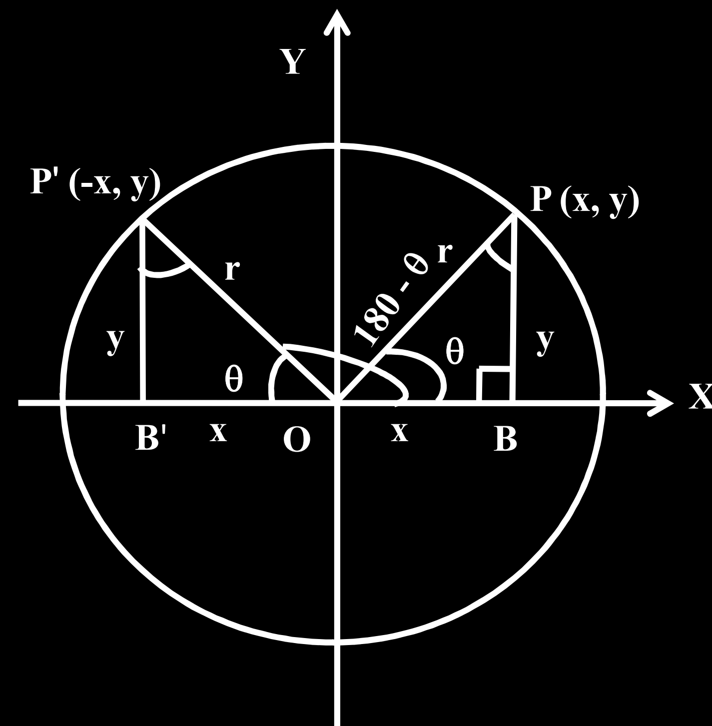


[Reduction Formulas]

- **(180° - θ) Reduction**

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$



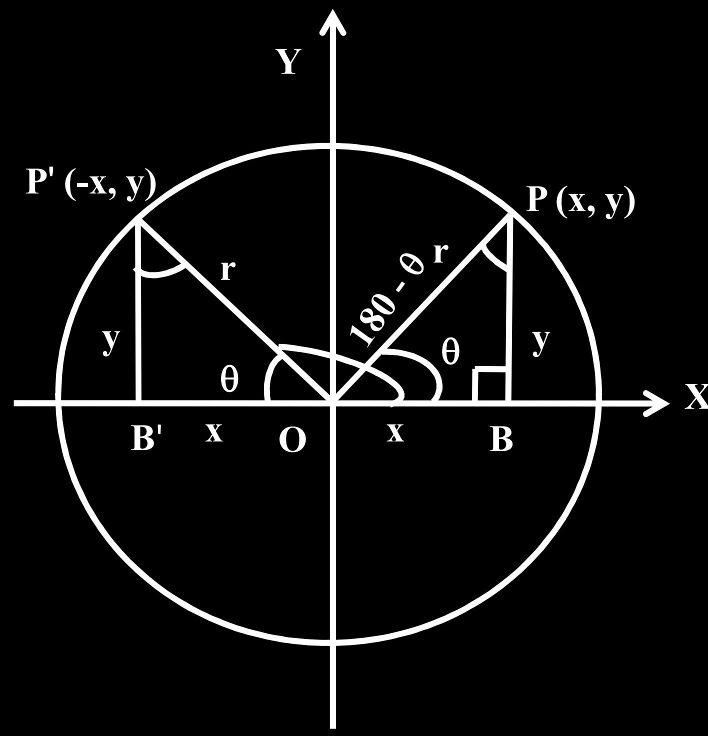
[Reduction Formulas]

- **(180° - θ) Reduction**

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

$\tan(180° - \theta) = -\tan\theta$



[Example's]

- $\sin 120^{o}$

[Example's]

- $\sin 120^o$
- $\tan 135^o$

[Example's]

- $\sin 120^o$
- $\tan 135^o$
- $\cos 150^o$

[Example's]

- $\sin 120^o$
- $\tan 135^o$
- $\cos 150^o$
- $\cos 10^o + \cos 20^o + \ldots\ldots\ldots\ldots + \cos 170^o$

[Example's]

- $\sin 120^o$
- $\tan 135^o$
- $\cos 150^o$

- $\cos 10^o + \cos 20^o + \ldots\ldots\ldots\ldots + \cos 170^o$

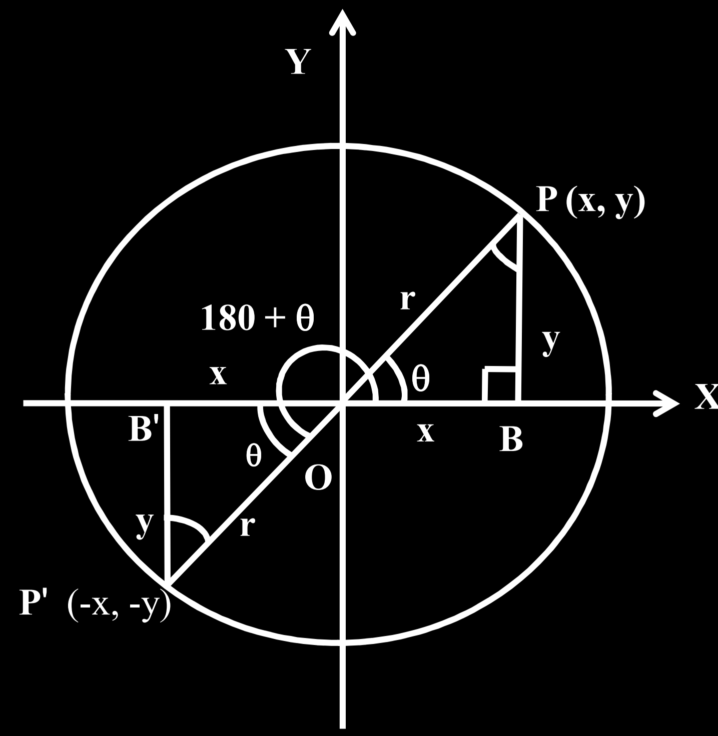
- $\tan\frac{\pi}{11} + \tan\frac{2\pi}{11} + \tan\frac{4\pi}{11} + \tan\frac{7\pi}{11} + \tan\frac{9\pi}{11} + \tan\frac{10\pi}{11}$

[Reduction Formulas]

- $(180° + \theta)$ Reduction

[Reduction Formulas]

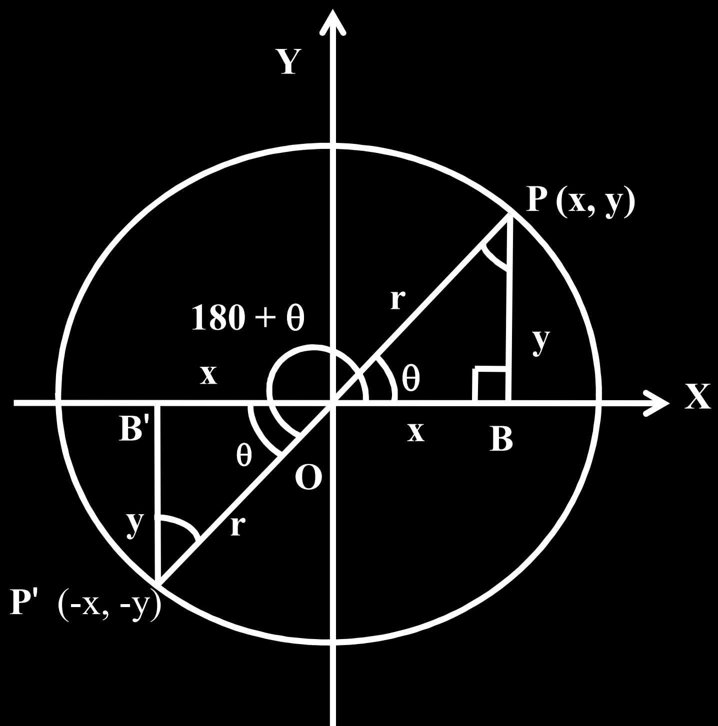
- (180° + θ) Reduction



[Reduction Formulas]

- **(180° + θ) Reduction**

$\sin(180° + \theta) = -\sin\theta$

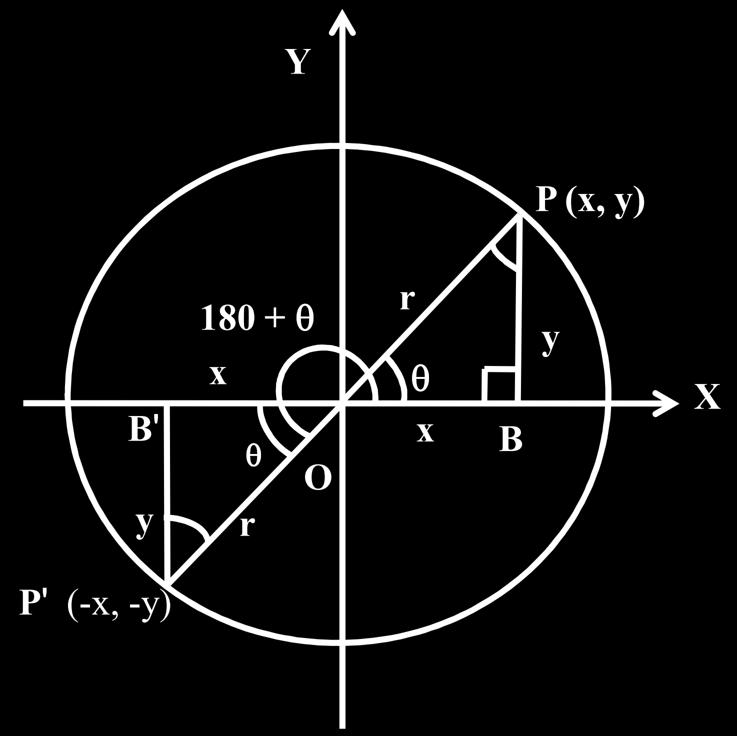


[Reduction Formulas]

- **(180° + θ) Reduction**

$\sin(180° + \theta) = -\sin\theta$

$\cos(180° + \theta) = -\cos\theta$



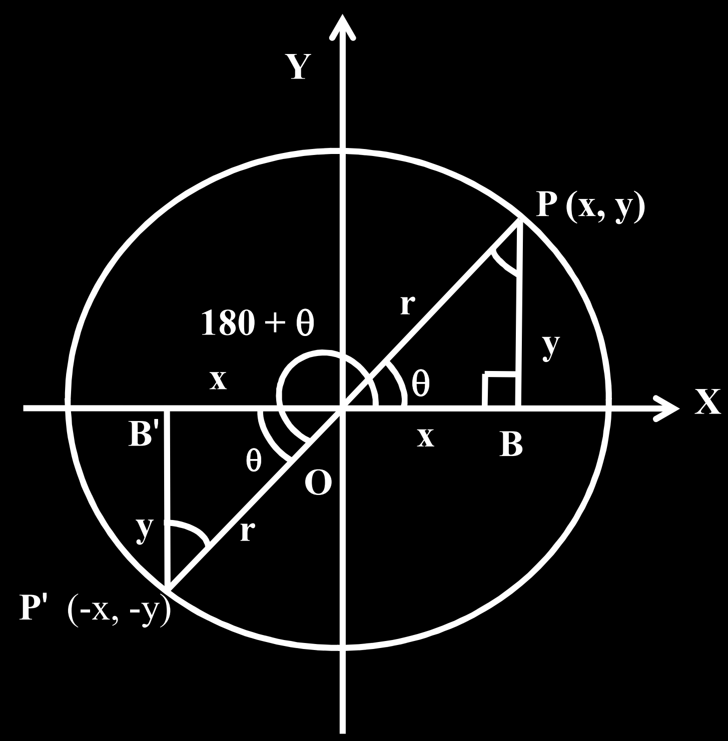
[Reduction Formulas]

- **$(180° + \theta)$ Reduction**

$\sin(180° + \theta) = -\sin\theta$

$\cos(180° + \theta) = -\cos\theta$

$\tan(180° + \theta) = \tan\theta$

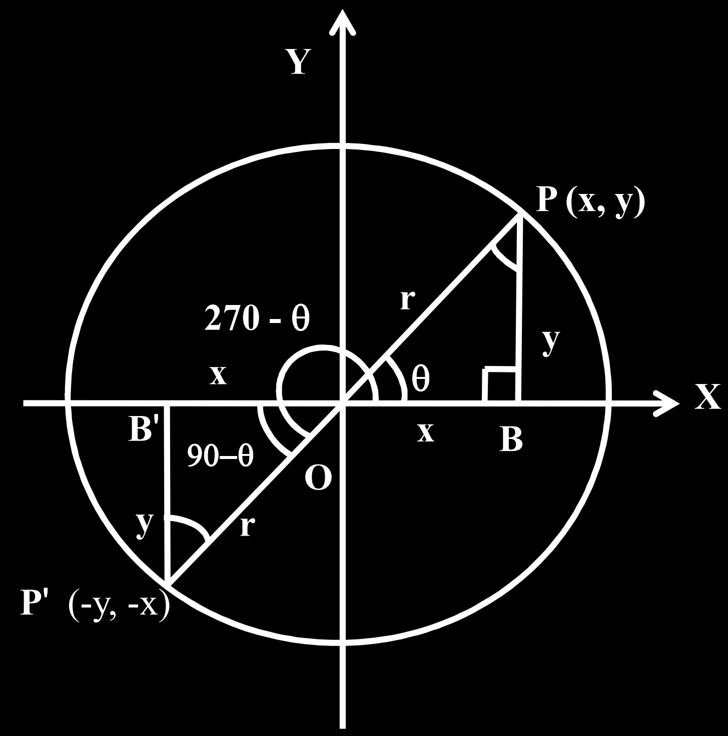


[Reduction Formulas]

- $(270° - \theta)$ Reduction

[Reduction Formulas]

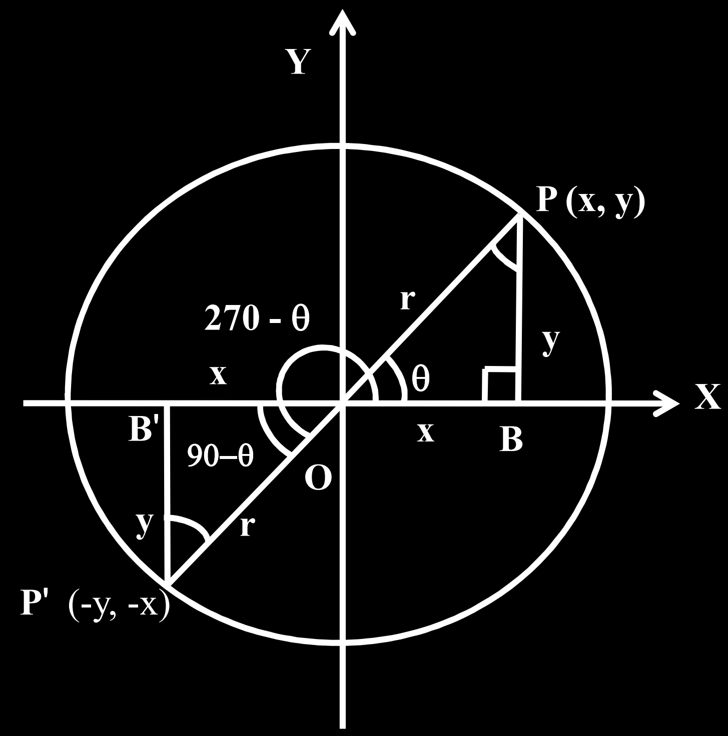
- (270° – θ) Reduction



[Reduction Formulas]

- **(270° – θ) Reduction**

  $\sin(270° - \theta) = -\cos\theta$

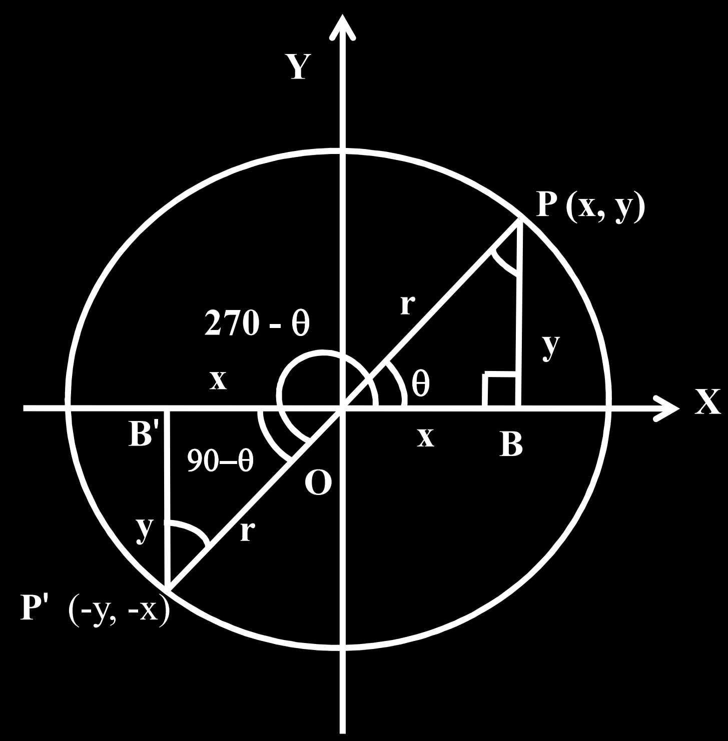


[Reduction Formulas]

- **(270° – θ) Reduction**

$\sin(270° - \theta) = -\cos\theta$

$\cos(270° - \theta) = -\sin\theta$



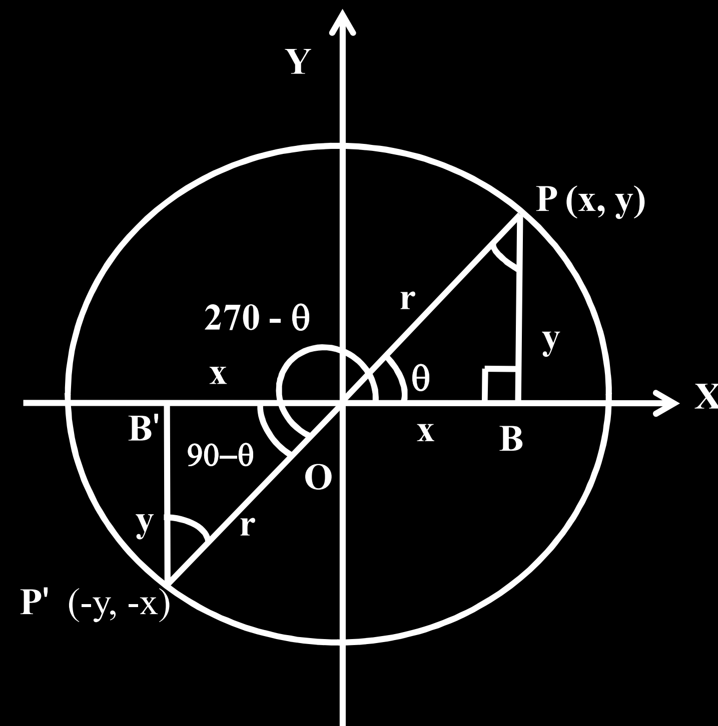
[Reduction Formulas]

- **(270° – θ) Reduction**

$\sin(270° - \theta) = -\cos\theta$

$\cos(270° - \theta) = -\sin\theta$

$\tan(270° - \theta) = \cot\theta$



[Examples]

- $\sin 210^{o}$

[Examples]

- $\sin 210^{o}$
- $\operatorname{cosec} 4\pi/3$

[Examples]

- $\sin 210^o$
- $\cos 240^o$
- $\operatorname{cosec} 4\pi/3$

[Examples]

- $\sin 210^o$
- $\operatorname{cosec} 4\pi/3$
- $\cos 240^o$
- $\cot 5\pi/4$

[Examples]

- $\sin 210^o$
- $\cos 240^o$
- $\tan 225^o$
- $\mathrm{cosec}\ 4\pi/3$
- $\cot 5\pi/4$

[Examples]

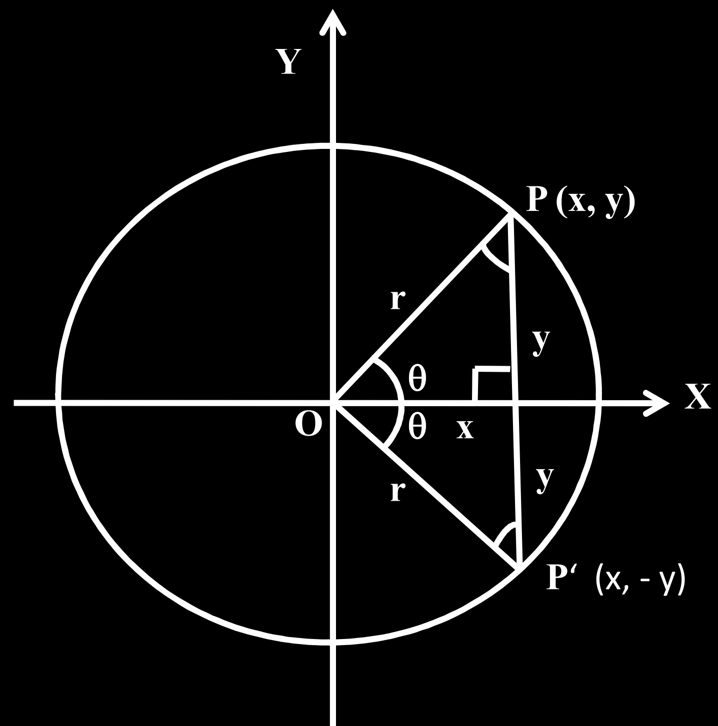
- $\sin 210^{o}$
- $\cos 240^{o}$
- $\tan 225^{o}$
- $\operatorname{cosec} 4\pi/3$
- $\cot 5\pi/4$
- $\sec 7\pi/6$

[Reduction Formulas]

- $(360° - \theta)$ Reduction

[Reduction Formulas]

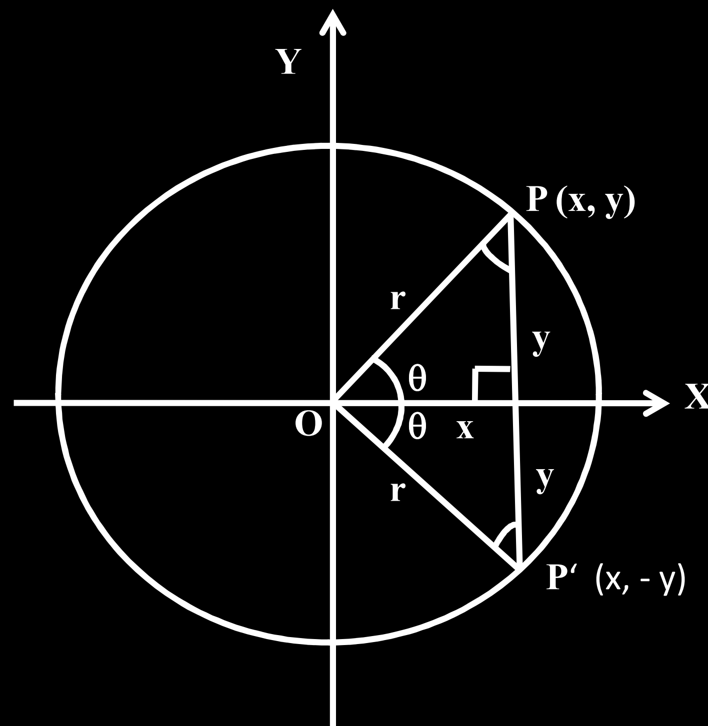
- (360° – θ) Reduction



[Reduction Formulas]

- **(360° – θ) Reduction**

$\sin(360° - \theta) = -\sin\theta$

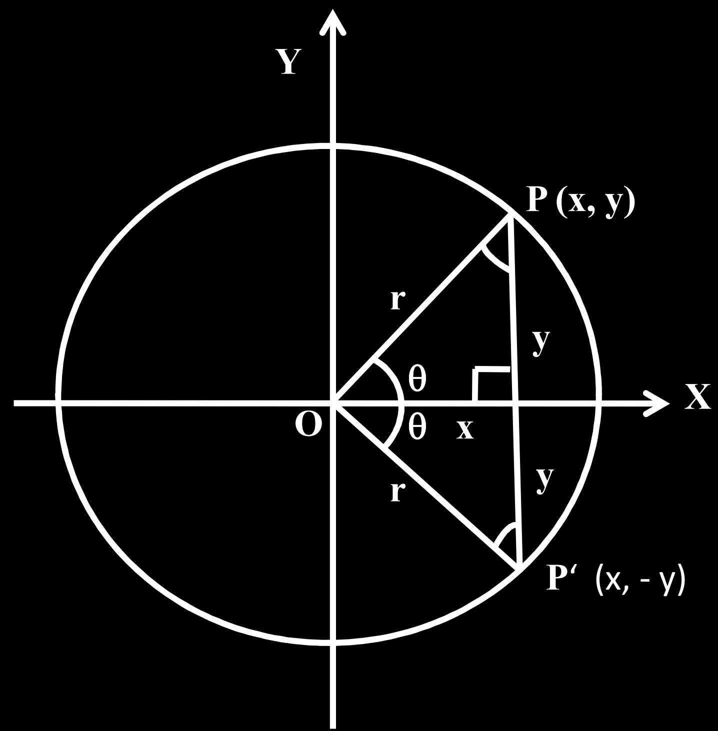


[Reduction Formulas]

- **(360° – θ) Reduction**

$\sin(360° - \theta) = -\sin\theta$

$\cos(360° - \theta) = \cos\theta$



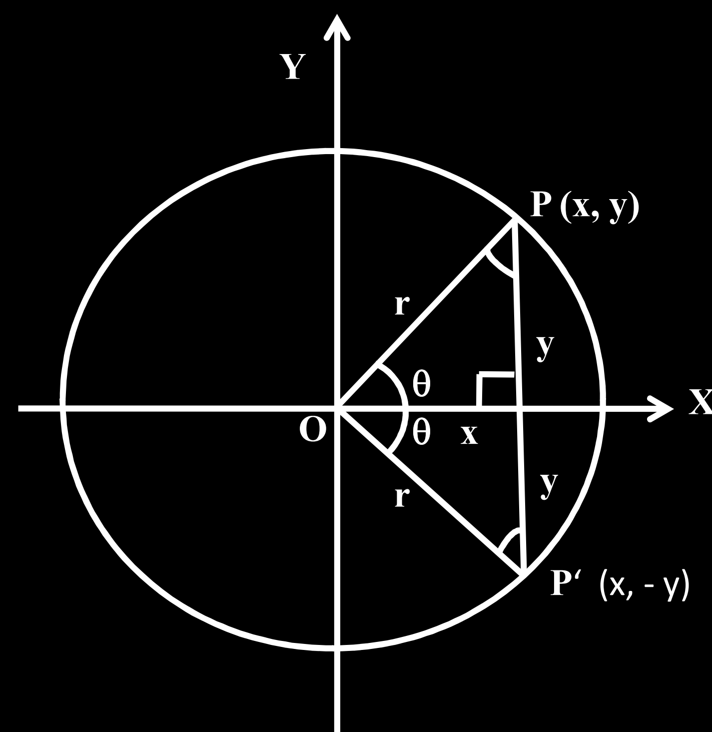
[Reduction Formulas]

- **(360° – θ) Reduction**

$\sin(360° - \theta) = -\sin\theta$

$\cos(360° - \theta) = \cos\theta$

$\tan(360° - \theta) = -\tan\theta$



[Example's]

- $\cos 315^{o}$

[Example's]

- $\cos 315^{o}$
- $\tan 5\pi/3$

[Example's]

- $\cos 315^o$
- $\tan 330^o$
- $\tan 5\pi/3$

[Example's]

- $\cos 315^{o}$
- $\tan 5\pi/3$
- $\tan 330^{o}$
- $\sin 7\pi/4$

[Example's]

- $\cos 315^o$
- $\tan 330^o$
- $\cot 5\pi/3$
- $\tan 5\pi/3$
- $\sin 7\pi/4$

[Example's]

- $\cos 315^{o}$
- $\tan 5\pi/3$
- $\tan 330^{o}$
- $\sin 7\pi/4$
- $\cot 5\pi/3$
- $\text{cosec}\ 11\pi/6$

[Reduction Formulas]

- **(360° + θ) Reduction**

[Reduction Formulas]

- **(360° + θ) Reduction**

  $\sin(360° + \theta) = \sin\theta$

[Reduction Formulas]

- **($360° + \theta$) Reduction**

$$\sin(360° + \theta) = \sin\theta$$

$$\cos(360° + \theta) = \cos\theta$$

[Reduction Formulas]

- **($360° + \theta$) Reduction**

$\sin(360° + \theta) = \sin\theta$

$\cos(360° + \theta) = \cos\theta$

$\tan(360° + \theta) = \tan\theta$

[Reduction Formulas]

- **($-\theta$) Reduction**

[Reduction Formulas]

- **($-\theta$) Reduction**

$\sin(-\theta) = -\sin\theta$

[Reduction Formulas]

- **$(-\theta)$ Reduction**

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

[Reduction Formulas]

- **(– θ) Reduction**

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

[To remember the signs we use]

sin +ve students	All +ve All
tan +ve Take	cos +ve Coffee

[Example]

True / False :

- $\sin 1^c > 0$
- $\sin 2^c > 0$
- $\sin 3^c > 0$
- $\sin 4^c > 0$
- $\sin 5^c > 0$
- $\sin 6^c > 0$
- $\sin 7^c > 0$

[Example]

True / False :

- $\cos 1^c > 0$
- $\cos 2^c > 0$
- $\cos 3^c > 0$
- $\cos 4^c > 0$
- $\cos 5^c > 0$
- $\cos 6^c > 0$
- $\cos 7^c > 0$

[Example]

True / False :

- $\tan 1^c > 0$
- $\tan 2^c > 0$
- $\tan 3^c > 0$
- $\tan 4^c > 0$
- $\tan 5^c > 0$
- $\tan 6^c > 0$
- $\tan 7^c > 0$

[Example]

If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$

(b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$

(d) None of these

[IIT-JEE 1979]

[Trigonometry Ratios Table]

[Trigonometry Ratios Table]

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degree	0	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

[Trigonometry Ratios Table]

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degree	0	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1

[Trigonometry Ratios Table]

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degree	0	30º	45º	60º	90º	120º	135º	150º	180º
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

[Trigonometry Ratios Table]

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degree	0	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND

[Highlight Of The Table]

- $\sin\theta, \cos\theta \in [-1, 1]$

[Highlight Of The Table]

- **sin θ, cos θ $\in$ [-1, 1]**

- **tan θ, cot θ $\in$ (- ∞, ∞)**

[Highlight Of The Table]

- $\sin\theta, \cos\theta \in [-1, 1]$
- $\tan\theta, \cot\theta \in (-\infty, \infty)$
- Sec θ, cosec $\theta \in (-\infty, -1] \cup [1, \infty)$

[Highlight Of The Table]

- $\sin\theta = 0 \Rightarrow \theta = n\pi \quad ; \quad n \in I$

[Highlight Of The Table]

- $\sin\theta = 0 \Rightarrow \theta = n\pi \quad ; \quad n \in I$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\dfrac{\pi}{2} \quad ; \quad n \in I$

[Highlight Of The Table]

- $\sin\theta = 0 \Rightarrow \theta = n\pi$; $n \in I$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\dfrac{\pi}{2}$; $n \in I$
- $\sin\theta = 1 \Rightarrow \theta = \left(2n\pi + \dfrac{\pi}{2}\right)$; $n \in I$

[Highlight Of The Table]

- $\sin\theta = 0 \Rightarrow \theta = n\pi$; $n \in I$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\dfrac{\pi}{2}$; $n \in I$
- $\sin\theta = 1 \Rightarrow \theta = \left(2n\pi + \dfrac{\pi}{2}\right)$; $n \in I$
- $\cos\theta = 1 \Rightarrow \theta = 2n\pi$; $n \in I$

[Highlight Of The Table]

- $\sin\theta = 0 \Rightarrow \theta = n\pi$; $n \in I$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\dfrac{\pi}{2}$; $n \in I$
- $\sin\theta = 1 \Rightarrow \theta = \left(2n\pi + \dfrac{\pi}{2}\right)$; $n \in I$
- $\cos\theta = 1 \Rightarrow \theta = 2n\pi$; $n \in I$
- $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$; $n \in I$

[Example]

True / False :

- **$\cos 1 > \sin 1$**

[Example]

True / False :

- **$\cos 1 > \sin 1$**
- **$\sin 314° < 0$**

[Example]

- **Find distance between orthocenter and circumventer in a triangle with sides 17, 15, 8.**

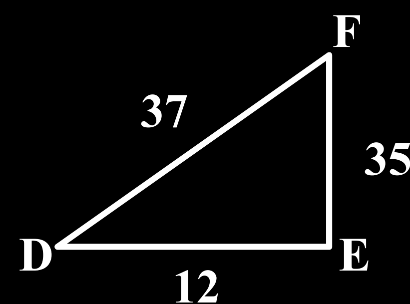
[Example]

- **Find distance between orthocenter and circumventer in a triangle with sides 17, 15, 8.**

- **Where is the orthocenter of △ABC with sides 12, 35, 37.**

(A)D (B) E

(C) F (D) none



[Example]

- Let $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$,

  $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$, then

  (A) $t_1 > t_2 > t_3 > t_4$ (B) $t_4 > t_3 > t_1 > t_2$

  (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

[JEE 2006]

[Example]

- If θ and φ are acute angles satisfying $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$ then $\theta + \phi \in$

(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$

[JEE – 2004 (Screening)]

[Values of Trigonometry ratios of 75°, 15°]

- $\sin 15° = \cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

[Values of Trigonometry ratios of 75°, 15°]

- $\sin 15° = \cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$
- $\sin 75° = \cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

[Values of Trigonometry ratios of 75°, 15°]

- $\sin 15° = \cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$
- $\sin 75° = \cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
- $\tan 15° = \cot 75° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

[Values of Trigonometry ratios of 75°, 15°]

- $\sin 15° = \cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$
- $\sin 75° = \cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
- $\tan 15° = \cot 75° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$
- $\tan 75° = \cot 15° = 2+\sqrt{3}$

[Compound Angles]

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

[Compound Angles]

- $\sin\left(A \pm B\right) = \sin A \cos B \pm \cos A \sin B$

- $\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$

[Compound Angles]

- $$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

- $$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

- $$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

[Example]

- $\cos(-300°)\cos 60° + \sin(-300°)\sin 60°$

[Example]

- $\cos(-300°)\cos 60° + \sin(-300°)\sin 60°$

- $\sin 99°\cos 21° + \cos 99°\sin 21°$

[Example]

- $\cos(-300°)\cos 60° + \sin(-300°)\sin 60°$

- $\sin 99° \cos 21° + \cos 99° \sin 21°$

- $\sin a = \frac{5}{13}$ $\cos b = \frac{-3}{5}$ Find $\sin(a-b)$

[Example]

- $\cos(-300°)\cos 60° + \sin(-300°)\sin 60°$
- $\sin 99° \cos 21° + \cos 99° \sin 21°$
- $\sin a = \dfrac{5}{13}$ $\cos b = \dfrac{-3}{5}$ Find $\sin(a-b)$
- If $\cos(\alpha + \beta) = 4/5$, $\sin(\alpha - \beta) = 5/13$ & α, β lie between 0 & $\pi/4$. Find $\tan 2\alpha$

[IIT-JEE 1979]

[Example]

- *If* $A + B = 45°$

  Prove that : $(1 + \tan A)(1 + \tan B) = 2$

[Example]

- *If* $A + B = 45°$

  Prove that : $(1 + \tan A)(1 + \tan B) = 2$

- *If* $x + y = \pi/4$ & $\tan(x + 2y) = 3$

  Then find the value $\tan x \tan y$.

[Example]

- **Given that**

  $(1 + \tan 1°)(1+\tan 2°) \ldots (1 + \tan 45°) = 2^n$

  find *n*

[Example]

- **Given that**

  $(1 + \tan 1°)(1+\tan 2°) \ldots (1 + \tan 45°) = 2^n$

  find *n*

- **$x - y = \frac{\pi}{4}$ and $\cot x + \cot y = 2$**

  Find smallest positive x and y

[REE 2000, 3]

[Example]

- *If* $\alpha + \beta = \pi/2$ *and* $\beta + \gamma = \alpha$ *then* $\tan\alpha$ *equal*

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$

(C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

[JEE 2001 (Screening)]

[Compound Angles]

- $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$

[Compound Angles]

- $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
- $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

[Compound Angles]

- $\sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B$

- $\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B$

- $2\sin A \cos B = \sin (A + B) + \sin (A - B)$

[Compound Angles]

- $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$
- $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$
- $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A\cos B = \cos(A + B) + \cos(A - B)$

[Compound Angles]

- $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$
- $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$
- $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

[Example]

- ***True / False***
  $\cos^2\theta + \cos^2(\theta+\alpha) - 2\cos\theta\cos\alpha\cos(\theta+\alpha)$
  is independent of θ

[Example]

- ***True / False***
  $\cos^2\theta + \cos^2(\theta + \alpha) - 2\cos\theta\cos\alpha\cos(\theta + \alpha)$
  is independent of θ

- ***Find 'x' in first quadrant for which***
  $\cos(x + 30°)\cos(x - 30°) = \sin 30°$

  If solutions are $\alpha, \beta, \gamma, \delta$ ***s.t.*** $0 < \alpha < \beta < \gamma < \delta$
  Find $\alpha, \beta, \gamma, \delta$
  where $\alpha, \beta, \gamma, \delta$ ***are smallest angles***

C & D Formula

- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

C & D Formula

- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\sin C - \sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$

C & D Formula

- $\sin C + \sin D = 2\sin \frac{C+D}{2} \cos \frac{C-D}{2}$
- $\sin C - \sin D = 2 \sin \frac{C-D}{2} \cos \frac{C+D}{2}$
- $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

C & D Formula

- $$\sin C + \sin D = 2\sin \frac{C+D}{2} \cos \frac{C-D}{2}$$
- $$\sin C - \sin D = 2\sin \frac{C-D}{2} \cos \frac{C+D}{2}$$
- $$\cos C + \cos D = 2\cos \frac{C+D}{2} \cos \frac{C-D}{2}$$
- $$\cos C - \cos D = 2\sin \frac{D-C}{2} \sin \frac{C+D}{2}$$

EXAMPLES

- *Prove that :*

$$\frac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta} = \tan\theta$$

EXAMPLES

- *Prove that :*

$$\frac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta} = \tan\theta$$

- *If* $\theta = 7.5°$

Find the value $\dfrac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta - 2\sin 3\theta\sin 4\theta}$

EXAMPLES

- $$Prove\ that: \frac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$$

EXAMPLES

- *Prove that :* $\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$

Find the value of expression,

- $\cos^2 73° + \cos^2 47° + \cos 73° \cos 47°$

EXAMPLES

- *Prove that :* $$\frac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$$

Find the value of expression,

- $\cos^2 73° + \cos^2 47° + \cos 73° \cos 47°$

- $\cos 55° + \cos 65° + \cos 175°$

EXAMPLES

1) *If* $\alpha = \dfrac{\pi}{19}$

Find the value $\dfrac{\sin 23\alpha - \sin 3\alpha}{\sin 16\alpha + \sin 4\alpha}$

EXAMPLES

1) *If* $\alpha = \dfrac{\pi}{19}$

Find the value $\dfrac{\sin 23\alpha - \sin 3\alpha}{\sin 16\alpha + \sin 4\alpha}$

2) $\dfrac{\sin 1° + \sin 3° + \sin 5° + \sin 7°}{\cos 1° + \cos 3° + \cos 5° + \cos 7°} = \tan\theta$

Find θ, *where* $\theta \in III^{rd}$ *quadrant*

Trigonometric Ratio of Multiple Angle

- $$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

Trigonometric Ratio of Multiple Angle

- $$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$
- $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

Trigonometric Ratio of Multiple Angle

- $$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$
- $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$
- $$1 + \cos 2\theta = 2\cos^2\theta$$

Trigonometric Ratio of Multiple Angle

- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- $1 + \cos 2\theta = 2\cos^2\theta$
- $1 - \cos 2\theta = 2\sin^2\theta$

Trigonometric Ratio of Multiple Angle

- $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$

Trigonometric Ratio of Multiple Angle

- $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$
- $\dfrac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

Trigonometric Ratio of Multiple Angle

- $$\frac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$$
- $$\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$$
- $$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Trigonometric Ratio of Multiple Angle

- $$\frac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$$
- $$\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$$
- $$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
- $$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$$

Trigonometric Ratio of Multiple Angle

- $$\frac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$$
- $$\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$$
- $$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
- $$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$$
- $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Trigonometric Ratio of Multiple Angle

- $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$
- $\dfrac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$
- $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
- $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\cot\theta - \tan\theta = 2\cot 2\theta$

Tangent and Secant value of $\frac{\pi}{8}$ & $\frac{3\pi}{8}$

Tangent and Secant value of $\frac{\pi}{8}$ & $\frac{3\pi}{8}$

- $\tan\frac{\pi}{8} = \sqrt{2} - 1 = \cot\frac{3\pi}{8}$

Tangent and Secant value of $\frac{\pi}{8}$ & $\frac{3\pi}{8}$

- $$\tan\frac{\pi}{8} = \sqrt{2} - 1 = \cot\frac{3\pi}{8}$$

- $$\tan\frac{3\pi}{8} = \sqrt{2} + 1 = \cot\frac{\pi}{8}$$

Example

(i) $\cos A = \frac{1}{2}$ Find $\cos 2A$

Example

(i) $\cos A = \frac{1}{2}$ Find $\cos 2A$

(ii) $\sin A = \frac{3}{5}$ Find $\sin 2A$

Example

(i) $\cos A = \dfrac{1}{2}$ Find $\cos 2A$

(ii) $\sin A = \dfrac{3}{5}$ Find $\sin 2A$

(iii) $\tan A = \dfrac{1}{3}$ Find $\tan 2A$

Example

(i) $\cos A = \frac{1}{2}$ Find $\cos 2A$

(ii) $\sin A = \frac{3}{5}$ Find $\sin 2A$

(iii) $\tan A = \frac{1}{3}$ Find $\tan 2A$

(iv) $\cos A = \frac{4}{5}$ Find $\tan A/2$

Example

If $\theta \in (0, \pi/2)$ Prove that :

- $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

Example

If $\theta \in (0, \pi/2)$ Prove that :

- $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

- $\dfrac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta$

Example

If $\theta \in (0, \pi/2)$ Prove that :

- $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$
- $\dfrac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta$
- $\cos 18° - \sin 18° = \sqrt{2}\ \sin 27°$

Example

Prove that :

- $$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

Example

Prove that :

- $$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

- $(1 + \sin 2A + \cos 2A)^2 = 4\cos^2 A\,(1 + \sin 2A)$

Example

Prove that :

- $$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$
- $(1 + \sin 2A + \cos 2A)^2 = 4\cos^2 A\,(1 + \sin 2A)$
- If $\tan^4 A - \tan^2 A + 3 = 0$

  Then find all the angles $\in [0, 2\pi]$

Example

Prove that :

- $\tan 70° = 2\tan 50° + \tan 20°$

Example

Prove that :

- $\tan 70° = 2\tan 50° + \tan 20°$

- $\sin x = 2^n \sin\left(\frac{x}{2^n}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\ldots\ldots\ldots\cos\left(\frac{x}{2^n}\right)$

Example

Prove that :

- $\tan 70° = 2\tan 50° + \tan 20°$

- $\sin x = 2^n \sin\left(\frac{x}{2^n}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\ldots\ldots\ldots\cos\left(\frac{x}{2^n}\right)$

- $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

[IIT-JEE 1988]

Example

- $$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

is equal to

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$

(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

(IIT-JEE 1984)

Example

- The expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left(\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right)$$

is equal to

(a) 0 (b) 1

(c) 3 (d) sin 4α + cos 6α

(IIT-JEE 1986)

Example

- Which of the following numbers is rational ?

(a) $\sin 15°$ (b) $\cos 15°$

(c) $\sin 15° \cos 15°$ (d) $\sin 15° \cos 75°$

(IIT-JEE 1998)

Example

(MCQ)

- For a positive integer n let $f_n(\theta) =$ $\tan\theta/2\,(1 + \sec\theta)(1 + \sec 2\theta)(1 + \sec 2^2\theta)\ldots\ldots\ldots(1 + \sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

(IIT-JEE 1999)

Sine, Cosine and Tangent of 3A

- $\sin 3A = 3\sin A - 4\sin^3 A$

Sine, Cosine and Tangent of 3A

- $\sin 3A = 3\sin A - 4\sin^3 A$

- $\cos 3A = 4\cos^3 A - 3\cos A$

Sine, Cosine and Tangent of 3A

- $\sin 3A = 3\sin A - 4\sin^3 A$

- $\cos 3A = 4\cos^3 A - 3\cos A$

- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Remember

- $\dfrac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$

Remember

- $$\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$$
- $$\frac{\cos 3A}{\cos A} = 4\cos^2 A - 3$$

Remember

- $$\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$$
- $$\frac{\cos 3A}{\cos A} = 4\cos^2 A - 3$$
- $$\frac{\tan 3A}{\tan A} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

Remember

- $$\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$$

- $$\frac{\cos 3A}{\cos A} = 4\cos^2 A - 3$$

- $$\frac{\tan 3A}{\tan A} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

- $\tan(x + y)\tan x \tan y = \tan(x + y) - \tan x - \tan y$

cos5A & sin5A

- $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

cos5A & sin5A

- $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$
- $\sin 5A = 16\sin^5 A - 20\sin^3 A + 5\sin A$

Examples

Find the value :

- **$8\sin^3 40° - 6\sin 40°$**

Examples

Find the value :

- $8\sin^3 40° - 6\sin 40°$
- $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A}$

Examples

Find the value :

- $8\sin^3 40° - 6\sin 40°$
- $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A}$
- $\dfrac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \dfrac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha}$

Examples

Find the value :

- $8\sin^3 40° - 6\sin 40°$
- $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A}$
- $\dfrac{\cos^3\alpha - \cos 3\alpha}{\cos\alpha} + \dfrac{\sin^3\alpha + \sin 3\alpha}{\sin\alpha}$
- Prove that :

$(4\cos^2 9° - 3)\,(4\cos^2 27° - 3) = \tan 9°$

Examples

- Let $f(\theta) = \sin\theta\,(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is

  (a) ≥ 0 only when $\theta \geq 0$ (b) ≤ 0 for all real θ

  (c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \leq 0$

  [IIT-JEE 2000]

Examples

Fill in the blank

- Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos nx$ is an identity in x, where $C_0, C_1, \ldots\ldots, C_n$ are constant and $C_n \neq 0$. Then the value of n is ……….

[IIT-JEE 1981]

Remember

- $\sin\theta \sin(60 - \theta) \sin(60 + \theta) = \dfrac{\sin 3\theta}{4}$

Remember

- $\sin\theta \sin(60-\theta)\sin(60+\theta) = \dfrac{\sin 3\theta}{4}$

- $\cos\theta \cos(60-\theta)\cos(60+\theta) = \dfrac{\cos 3\theta}{4}$

Remember

- $\sin\theta \sin(60-\theta)\sin(60+\theta) = \dfrac{\sin 3\theta}{4}$

- $\cos\theta \cos(60-\theta)\cos(60+\theta) = \dfrac{\cos 3\theta}{4}$

- $\tan\theta \tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

Examples

Find the value of following :

- $\cos 5° \cos 55° \cos 65°$

Examples

Find the value of following :

- $\cos 5° \cos 55° \cos 65°$
- $\sin 20° \sin 40° \sin 80°$

Examples

Find the value of following :

- $\cos 5° \cos 55° \cos 65°$
- $\sin 20° \sin 40° \sin 80°$
- $\tan 6° \tan 42° \tan 66° \tan 78°$

Examples

Fill in the blank :

- If $k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$,

  then the numerical value of k is …….

[IIT-JEE 1993, 3]

Continued Product

$$\prod_{r=1}^{n} \cos(r\theta) = \cos\theta \cos 2\theta \ldots\ldots\ldots\ldots \cos n\theta$$

Note that

If continued product of cosine series is given such that each angle is double of previous angle, not necessarily the last one then multiply and divide the series by sine of smallest angle.

Purpose

To simplify product of 2 or more terms

Whose product in cosine is given

Rules

(i) Only for cosine

Rules

(i) Only for cosine

(ii) If "sines" are given create into cosine

Idea

- Multiply and divide by sine of smallest angle

Idea

- Multiply and divide by sine of smallest angle

- $\sin\theta \cos\theta = \dfrac{\sin 2\theta}{2}$

Examples

Find the value of following :

- $\cos 36° \cos 72°$

Examples

Find the value of following :

- $\cos 36° \cos 72°$

- $\sin 6° \sin 42° \sin 66° \sin 78°$

Examples

Find the value of following :

- $\cos 36° \cos 72°$

- $\sin 6° \sin 42° \sin 66° \sin 78°$

- $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7}$

Examples

- *Find the value :*

  $\sin 20° \sin 40° \sin 60° \sin 80°$

Examples

- *Find the value :*

  $sin20° \, sin40° \, sin60° \, sin80°$

- $sin\frac{\pi}{14} \, sin\frac{3\pi}{14} \, sin\frac{5\pi}{14} \, sin\frac{7\pi}{14} \, sin\frac{9\pi}{14} \, sin\frac{11\pi}{14} \, sin\frac{13\pi}{14}$

[IIT-JEE 1991]

Examples

- *Find the value :*

  sin20° sin40° sin60° sin80°

- $$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

  [IIT-JEE 1991]

- $$16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right)=1$$

  [IIT-JEE 1983]

$\sin 18° \;\&\; \cos 36°$

- $$\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$$

sin18° & cos36°

- $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4} = \cos 72^\circ$

- $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4} = \sin 54^\circ$

Examples

Find the value :

- **$(\sin 132° \sin 12°)$**

Examples

Find the value :

- **(sin 132° sin 12°)**
- **$\cos^2 48° - \sin^2 12°$**

Examples

Find the value :

- $(\sin 132° \sin 12°)$
- $\cos^2 48° - \sin^2 12°$
- $4 \cos 18° - 3 \sec 18° - 2 \tan 18°$

Examples

- **Find the value :**

$$\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5}$$

Examples

- Find the value :

$$\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5}$$

- Without using tables, prove that :

(sin 12°) (sin 48°) (sin 54°) = 1/8

[IIT-JEE 1982]

Examples

- *Find the value of* $\sin^4\theta + \sin^4 3\theta + \sin^4 5\theta + \sin^4 7\theta$ *where* $\theta = \pi/16$

Examples

- *Find the value of* $\sin^4\theta + \sin^4 3\theta + \sin^4 5\theta + \sin^4 7\theta$ *where* $\theta = \pi/16$

- *Find the value of expression* $\operatorname{cosec} 10° - \sqrt{3} \sec 10°$

Examples

- *The value of the expression $\sqrt{3}$ cosec 20° - sec20° is equal to*

  (a) 2 *(b) 2sin 20° / sin 40°*

  (c) 4 *(d) 4sin 20° / sin 40°*

[IIT-JEE 1988, 2M]

Examples

- Given $\sin\theta + \sin\varphi = a$

  $\cos\theta + \cos\varphi = b$

  Find the value of

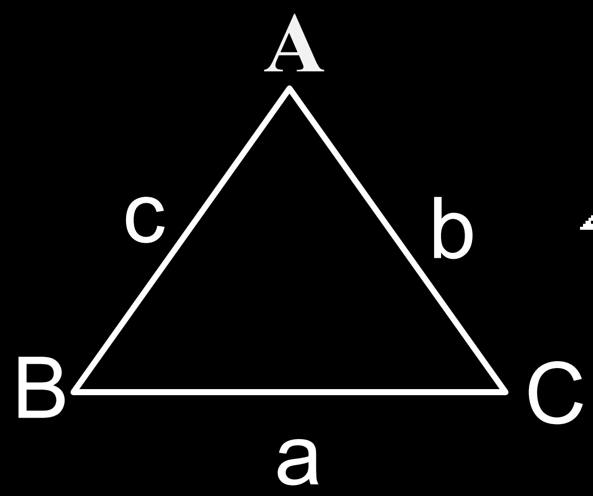
  (i) $\tan\left(\frac{\theta - \varphi}{2}\right)$

  (ii) $\tan(\theta + \varphi)$

  (iii) $\cos(\theta + \varphi)$

  (iv) $\cos\left(\frac{\theta - \varphi}{2}\right)$

Trigonometric Identities in $\triangle$



$$\angle A + \angle B + \angle C = \pi$$

Method of Identities in sine and cosine

(i) Combined any two by C & D formula

Method of Identities in sine and cosine

(i) Combined any two by C & D formula

(ii) In C & D one angle would be sum and other in difference

Method of Identities in sine and cosine

(i) Combined any two by C & D formula

(ii) In C & D one angle would be sum and other in difference

(iii) Convert third into some form of (sum angle) using formulas

Examples

- **If $A + B + C = \pi$ then prove that**

  $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

Examples

- If $A + B + C = \pi$ then prove that

  $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

- If $A + B + C = \pi$ then Find the value :

$$\frac{\sin 50^\circ + \sin 100^\circ + \sin 210^\circ}{\sin 25^\circ + \sin 50^\circ + \sin 105^\circ}$$

Examples

- **If α, β and γ are the angles of a triangle then show that :**

$$\frac{\cos\alpha}{\sin\beta\sin\gamma}+\frac{\cos\beta}{\sin\alpha\sin\gamma}+\frac{\cos\gamma}{\sin\alpha\sin\beta}=2$$

Examples

- If α, β and γ are the angles of a triangle then show that :

$$\frac{\cos\alpha}{\sin\beta\sin\gamma}+\frac{\cos\beta}{\sin\alpha\sin\gamma}+\frac{\cos\gamma}{\sin\alpha\sin\beta}=2$$

- If $A + B + C = \pi$ then prove that

$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$$

[IIT-JEE 1980]

Examples

If $A + B + C = \pi$ then prove that

- **$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$**

Examples

If $A + B + C = \pi$ then prove that

- $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$
- $\cos A + \cos B – \cos C = -1 + 4 \cos A/2 \cos B/2 \sin C/2$

Remember

In a right angle

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

Identity

$$\tan\left(\alpha+\beta+\gamma\right)=\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\left(\tan\alpha\tan\beta+\tan\beta\tan\gamma+\tan\gamma\tan\alpha\right)}$$

Examples

If $A + B + C = \pi$ then prove that

- **$\Sigma \tan A = \Pi \tan A$** **[IIT-JEE 1979]**

Examples

If $A + B + C = \pi$ then prove that

- **$\Sigma \tan A = \Pi \tan A$** **[IIT-JEE 1979]**
- **$\Sigma \cot A \cot B = 1$**

Examples

If $A + B + C = \pi$ then prove that

- **$\Sigma \tan A = \Pi \tan A$ [IIT-JEE 1979]**
- **$\Sigma \cot A \cot B = 1$**
- **$\Sigma \tan A/2 \tan B/2 = 1$**

Examples

If $A + B + C = \pi$ then prove that

- $\Sigma \tan A = \Pi \tan A$ [IIT-JEE 1979]
- $\Sigma \cot A \cot B = 1$
- $\Sigma \tan A/2 \tan B/2 = 1$
- $\Sigma \cot A/2 = \Pi \cot A/2$ [IIT-JEE 2000]

More Examples

True / False

- **There exists a ΔABC, Tangents of whose Interior angles are 1,2,3**

More Examples

True / False

- **There exists a ΔABC, Tangents of whose Interior angles are 1,2,3**

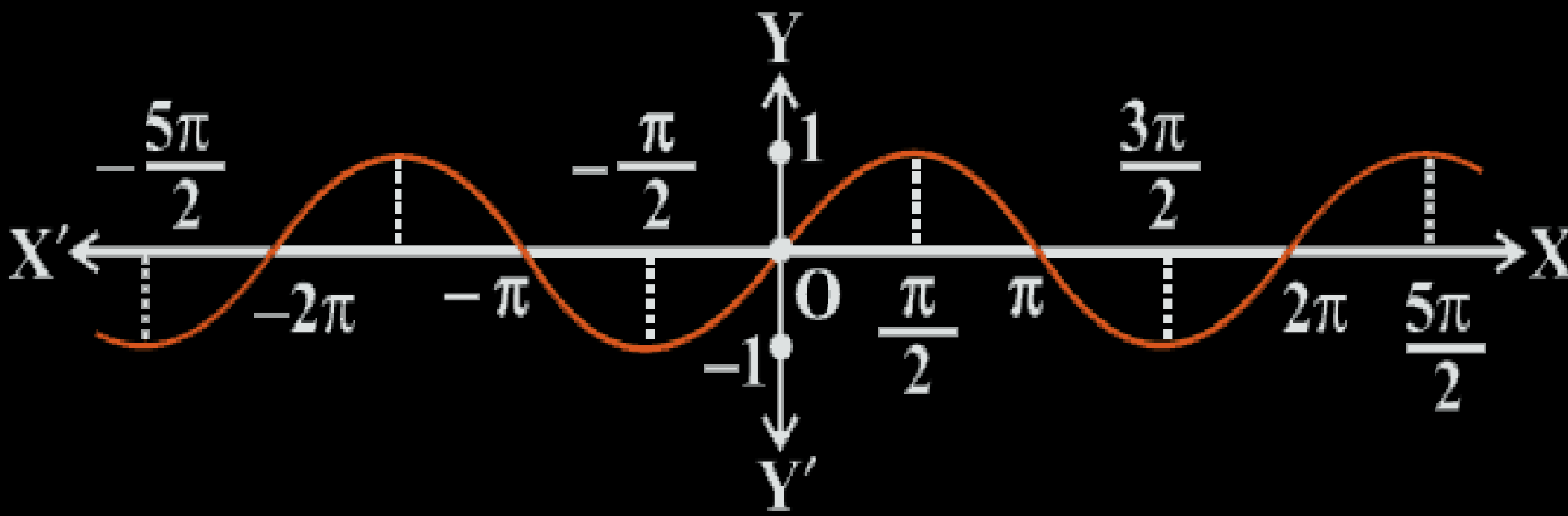
- **If $x + y + z = xyz$ $\quad x, y, z \in R$**

  Prove that :

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}\times\frac{2y}{1-y^2}\times\frac{2z}{1-z^2}$$

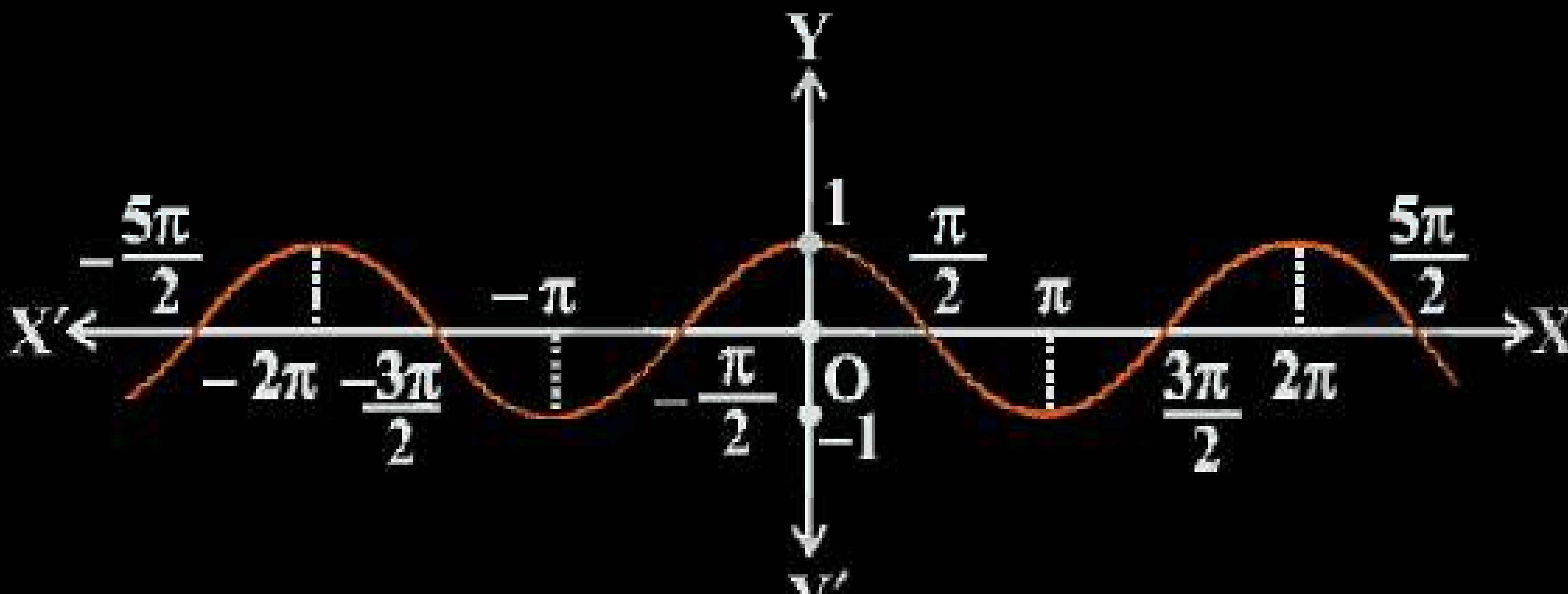
Graph of Trigonometric Function

$y = f(x) = \sin x$



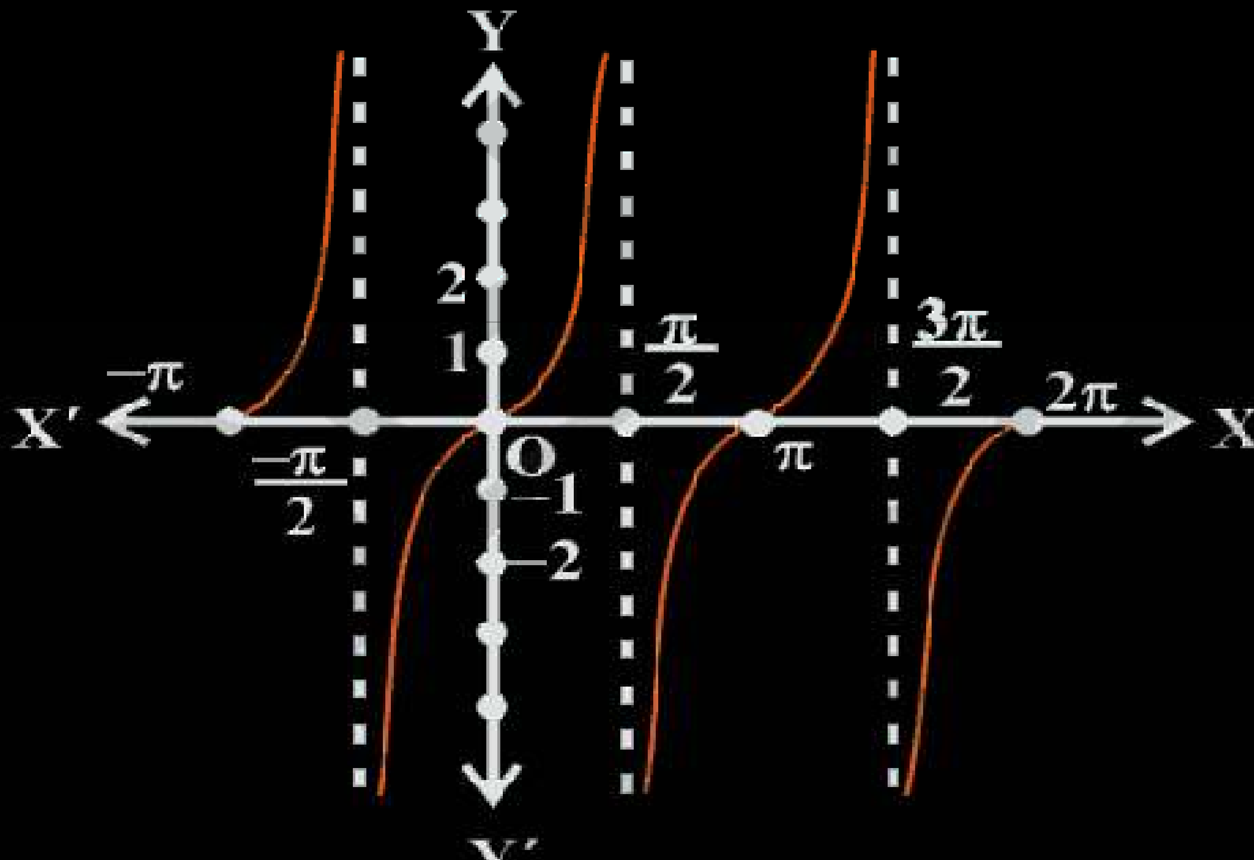
$y = \sin x$

$$y = f(x) = \cos x$$



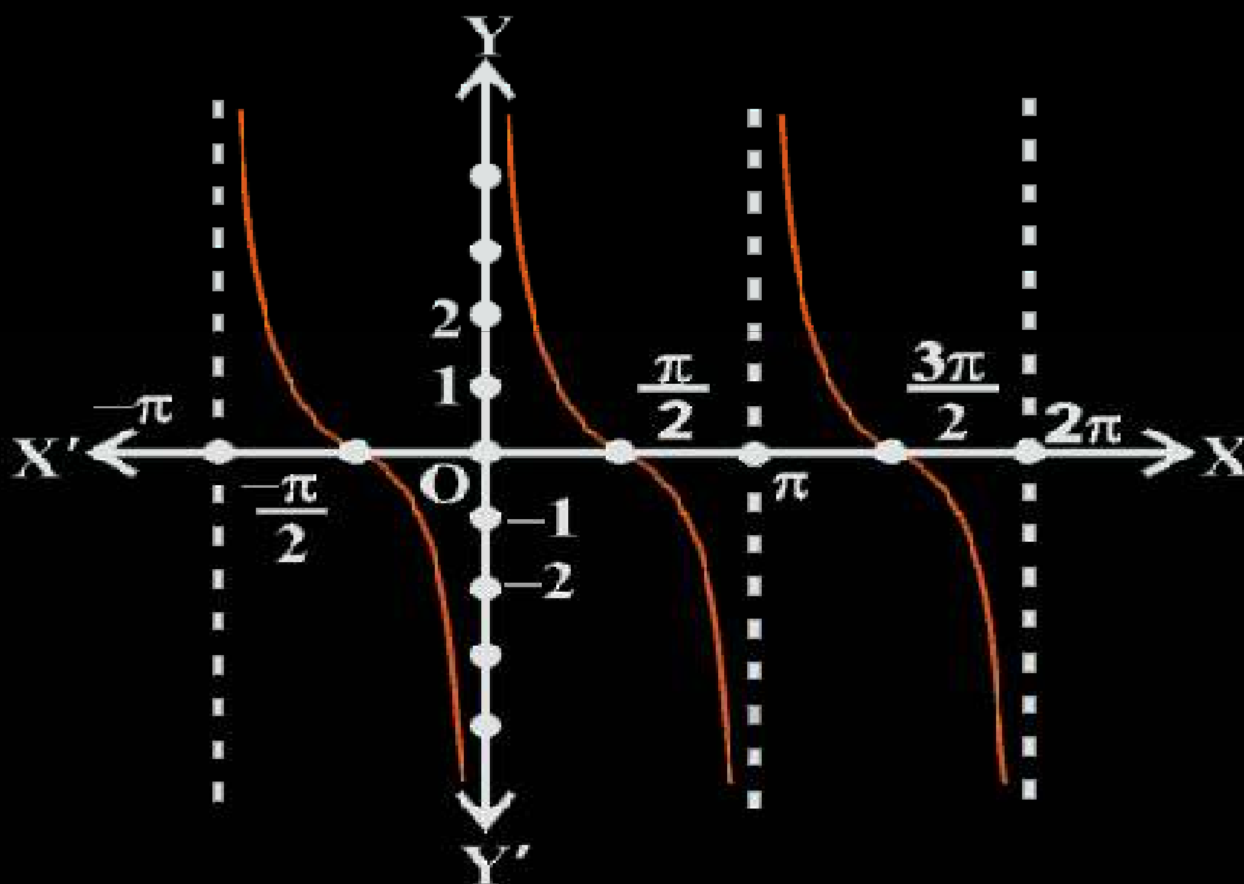
$y = \cos x$

$y = f(x) = \tan x$



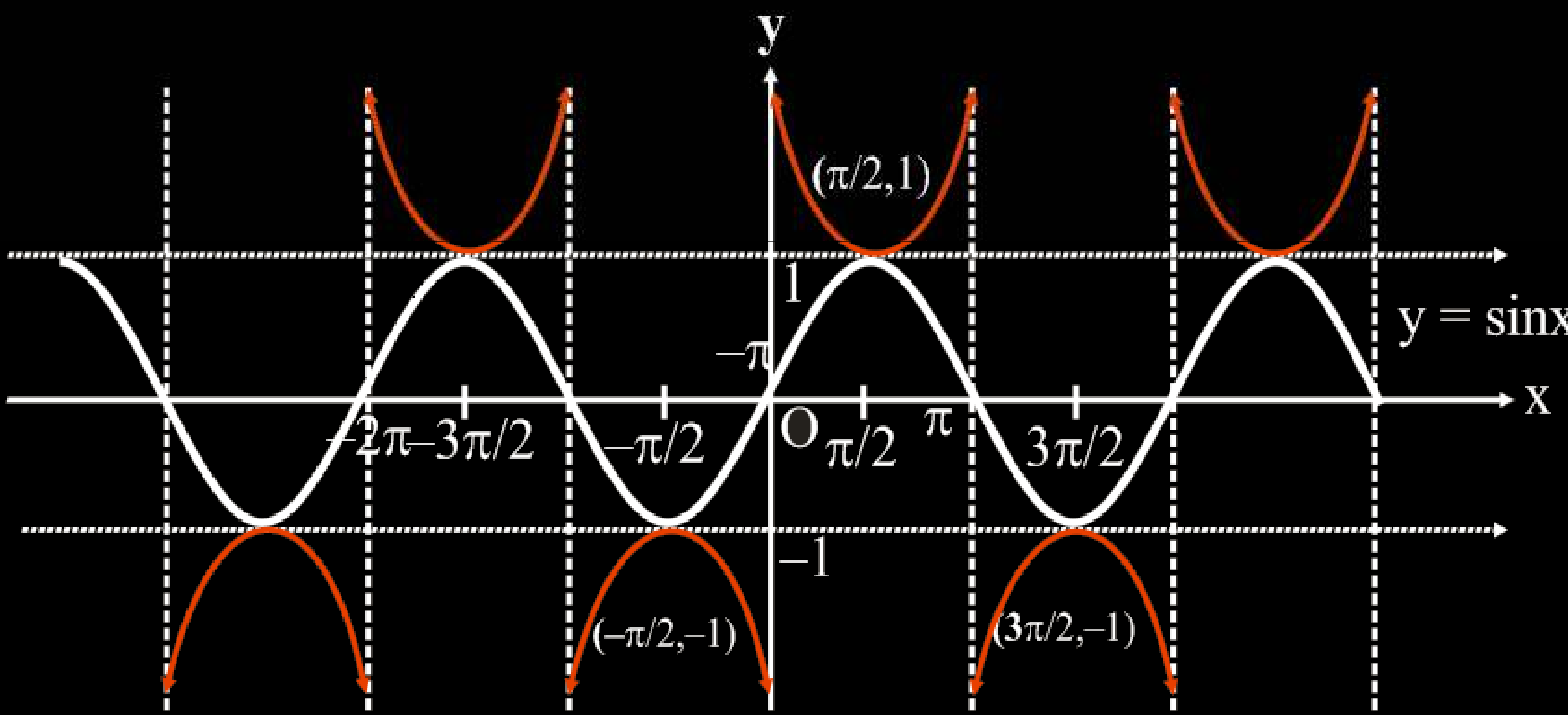
$y = \tan x$

$$y = f(x) = \cot x$$

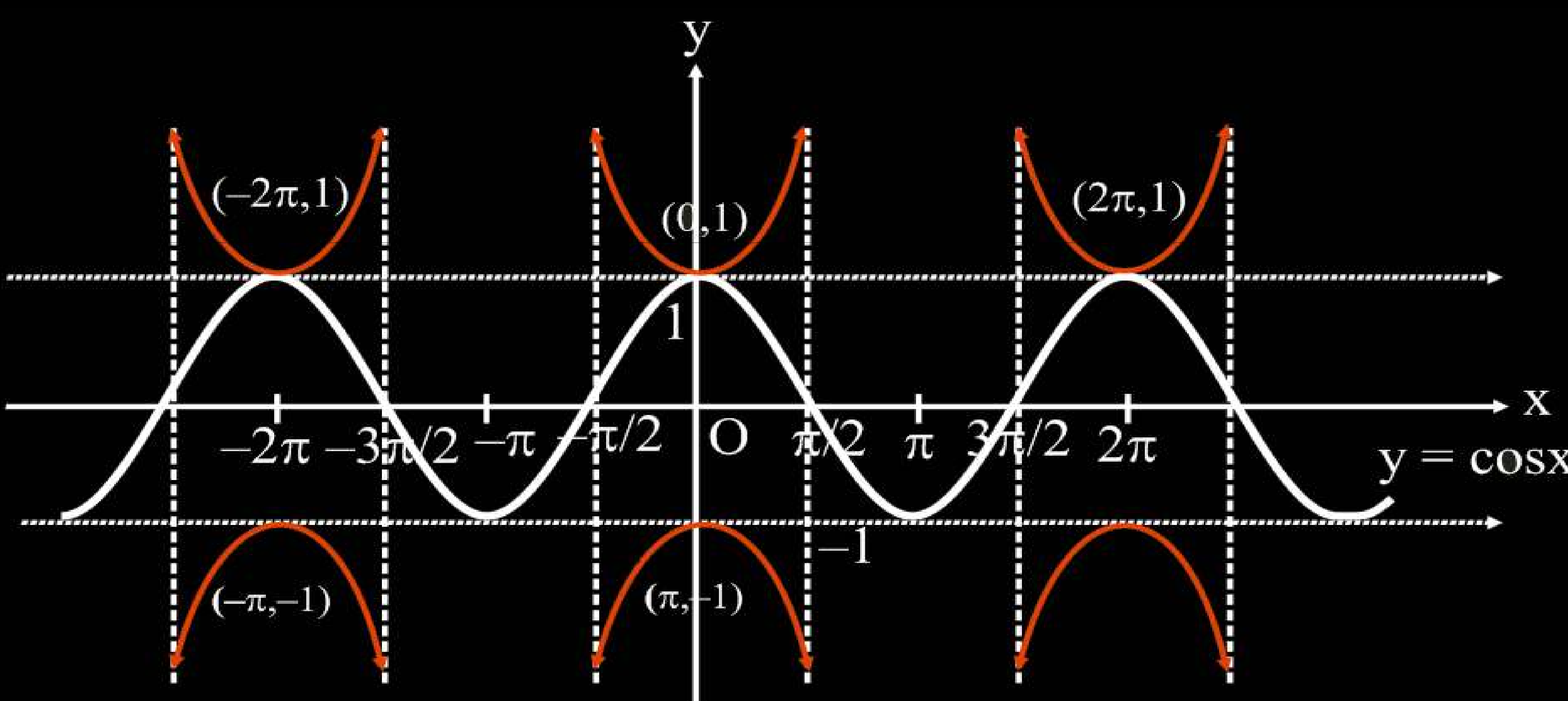


$y = \cot x$

y = f(x) = cosec x



$y = f(x) = \sec x$



Maximising and Minimising using property of boundedness of Trigonometric Function

- **$\sin x, \cos x \in [-1, 1]$**

- **$\sin x, \cos x \in [-1, 1]$**

- **$\sec x, \operatorname{cosec} x \in (-\infty, -1] \cup [1, \infty)$**

- **$\sin x, \cos x \in [-1, 1]$**

- **$\sec x, \operatorname{cosec} x \in (-\infty, -1] \cup [1, \infty)$**

- **$\tan x, \cot x \in (-\infty, \infty)$**

Examples

Find range of y

- $y = \sin(2x)$

Examples

Find range of y

- $y = \sin(2x)$
- $y = \sin(x^2)$

Examples

Find range of y

- $y = \sin(2x)$
- $y = \sin(x^2)$
- $y = \sin\sqrt{x}$

Examples

Find range of y

- $y = \sin(2x)$
- $y = \sin(x^2)$
- $y = \sin\sqrt{x}$
- $y = \cos^2 x$

Examples

Find range of y

- $y = \sin(2x)$
- $y = \sin(x^2)$
- $y = \sin\sqrt{x}$
- $y = \cos^2 x$
- $y = \cos^2 x - \sin^2 x$

Examples

Find range of y

- $y = \sin(2x)$
- $y = \sin(x^2)$
- $y = \sin\sqrt{x}$
- $y = \cos^2 x$
- $y = \cos^2 x - \sin^2 x$
- $y = \cos^4 \frac{x}{2} - \sin^4 \frac{x}{2}$

Examples

Find range of y

- $y = cos^4 x - sin^4 x$

Examples

Find range of y

- $y = \cos^4 x - \sin^4 x$
- $y = (\sin x + 2)^2 + 1$

Examples

Find range of y

- $y = \cos^4 x - \sin^4 x$
- $y = (\sin x + 2)^2 + 1$
- $y = 4\tan x \cos x$

Examples

Find range of y

- $y = \cos^4 x - \sin^4 x$
- $y = (\sin x + 2)^2 + 1$
- $y = 4\tan x \cos x$
- $y = \cos(\sin x)$

Examples

Find range of y

- $y = \cos^4 x - \sin^4 x$
- $y = (\sin x + 2)^2 + 1$
- $y = 4\tan x \cos x$
- $y = \cos(\sin x)$
- $y = \cos(2\sin x)$

Examples

Find range of y

- $y = \cos^4 x - \sin^4 x$
- $y = (\sin x + 2)^2 + 1$
- $y = 4\tan x \cos x$
- $y = \cos(\sin x)$
- $y = \cos(2\sin x)$
- $y = \cos(3 \sin x)$

Examples

Find range of y

- $y = \cos(4\sin x)$

Examples

Find range of y

- $y = \cos(4\sin x)$
- $y = \sin^2\left(\dfrac{15\pi}{8} - 4x\right) - \sin^2\left(\dfrac{17\pi}{8} - 4x\right)$

Examples

Find range of y

- $y = \cos(4\sin x)$

- $y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$

- $y = \tan^2 x$

Range

$$y = a\cos\theta + b\sin\theta$$

Range

$$y = a \cos\theta + b \sin \theta$$

$$y \in \left[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}\right]$$

Examples

Find Range of y

- $y = 3\cos x - 4\sin x$

Examples

Find Range of y

- $y = 3 \cos x - 4 \sin x$

- $y = 17 + 5 \sin x + 12 \cos x$

Examples

Find Range of y

- $y = 3\cos x - 4\sin x$

- $y = 17 + 5\sin x + 12\cos x$

- $y = \log_2\left(\dfrac{3\sin x - 4\cos x + 15}{10}\right)$

Examples

- $y = 27^{\cos 2x} \times 81^{\sin 2x}$ *[REE 2000, 3]*

Examples

- $y = 27^{\cos 2x} \times 81^{\sin 2x}$ *[REE 2000, 3]*

- *Minimum vertical distance between the graphs of* $y = 2 + \sin x$, $y = \cos x$

Range

$$f(x) = a \cos(\alpha + x) + b \cos(\beta + x)$$

$$f(x) \in \left[-\sqrt{a^2 + b^2 + 2ab\cos(\alpha - \beta)}, \sqrt{a^2 + b^2 + 2ab\cos(\alpha - \beta)}\right]$$

Example

Find range of y :

- $y = sin\left(x+\dfrac{\pi}{3}\right)+3\,cos\left(x-\dfrac{\pi}{3}\right)$

Example

Find range of y :

- $y = sin\left(x+\frac{\pi}{3}\right)+3\,cos\left(x-\frac{\pi}{3}\right)$

- **Prove that $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$ lies between -4 and 10.**

[IIT – JEE 1979]

Making Perfect Square

Argument of sine & cosine are different or a quadratic in sine cosine is given then we make perfect square sine/cosine & interperates

Examples

Find minimum & maximum value of y

- $y = \cos^2 x - 4 \cos x + 13$

Examples

Find minimum & maximum value of y

- $y = \cos^2 x - 4 \cos x + 13$
- $y = 4 \cos^2\theta - 4 \cos \theta + 9$

Examples

Find minimum & maximum value of y

- $y = \cos^2 x - 4\cos x + 13$
- $y = 4\cos^2\theta - 4\cos\theta + 9$
- $y = \cos 2x + 3\sin x$

Making use of Reciprocal Relationship between tan & cot, sin & cosec, sec & cos

- $$y = x + \frac{1}{x} \Rightarrow y \in (-\infty, -2] \bigcup [2, \infty)$$

Examples

Find Range

- $y = tan\theta + cot\theta$

Examples

Find Range

- $y = tan\theta + cot\theta$
- $y = \dfrac{\sin 3x}{\sin x}$

Examples

Find Range

- $y = \tan\theta + \cot\theta$
- $y = \dfrac{\sin 3x}{\sin x}$
- $y = a^2 \sec^2\theta + b^2 \operatorname{cosec}^2\theta$

Important

- $(\sin\theta + \cos\theta)^2 \quad = \quad (1 + \sin 2\theta)$

Important

- $(sin\theta + cos\theta)^2 \quad = \quad (1 + sin\ 2\theta)$

- $(sin\theta - cos\theta)^2 \quad = \quad (1 - sin\ 2\theta)$

Eliminant

Solve :

- $$\sin\theta + \cos\theta = a$$
  $$\sin^3\theta + \cos^3\theta = b$$

Eliminant

Solve :

- $$\sin\theta + \cos\theta = a$$
  $$\sin^3\theta + \cos^3\theta = b$$

- $$\sin\theta + \cos\theta = a$$
  $$\sin^4\theta + \cos^4\theta = b$$

Parameter

- $x^2 + y^2 = 1$
  Find Minimum & Maximum value of (5x – 12y)

Parameter

- $x^2 + y^2 = 1$
  Find Minimum & Maximum value of (5x – 12y)

- $x^2 + y^2 = 9$
  Find Maximum & Minimum of (4x – 3y)

Parameter

- $x^2 + y^2 = 4$ & $a^2 + b^2 = 16$
  Find Range of $(ax + by)$

Parameter

- $x^2 + y^2 = 4$ & $a^2 + b^2 = 16$
  Find Range of $(ax + by)$

- $x^2 + y^2 - 2x - 2y + 1 = 0$
  Find Maximum & Minimum value of $(15x - 8y)$

Sum of Sine / Cosine Series

$$\sin\alpha + \sin(\alpha+\beta) \ldots\ldots \sin(\alpha+(n-1)\beta)$$

$$= \frac{\sin n\frac{\beta}{2}}{\sin\frac{\beta}{2}} \sin \frac{\alpha+\alpha+(n-1)\beta}{2}$$

Sum of Sine / Cosine Series

$$\cos\alpha + \cos(\alpha+\beta) \ldots\ldots \cos(\alpha+(n-1)\beta)$$

$$= \frac{\sin n\frac{\beta}{2}}{\sin\frac{\beta}{2}} \cos \frac{\alpha+\alpha+(n-1)\beta}{2}$$

Examples

Find Values

- $$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\ldots\ldots+\cos\frac{9\pi}{11}$$

Examples

Find Values

- $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\ldots\ldots+\cos\frac{9\pi}{11}$
- $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$

Examples

Find Values

- $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\ldots\ldots+\cos\frac{9\pi}{11}$
- $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$
- *If* $\alpha = 2\pi/17$

  Find the value $\sum_{r=1}^{8}\cos(r\alpha)$

Note

If two cosine or two sine are given in denominator then multiply and divide by sine of difference of angle in denominator.

Examples

- $$\sum_{n=0}^{88} \frac{1}{\cos n \cos(n+1)} = \cot 1^\circ \ \operatorname{cosec} 1^\circ$$

Examples

(MCQ)

- For $0 < \theta < \frac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2} \text{ is (are)}$$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

[JEE 2009]

Important Inequalities in Triangle

Prove that

- $\cos A + \cos B + \cos C \leq 3/2$

where A,B,C are angle's of triangle

Important Inequalities in Triangle

Prove that

- $\cos A + \cos B + \cos C \leq 3/2$

  where A,B,C are angle's of triangle

- $\tan A + \tan B + \tan C \geq 3\sqrt{3}$

  where A,B,C are acute angle's

Examples

- *Prove that*

  The triangle is equilateral iff

  $\cot A + \cot B + \cot C = \sqrt{3}$

Examples

- *Prove that*

  The triangle is equilateral iff

  $cot\ A + cot\ B + cot\ C = \sqrt{3}$

- *If* $A + B + C = \pi$ *then prove that*

  $tan^2\ A/2 + tan^2\ B/2 + tan^2\ C/2 \geq 1$

ASSIGNMENT

(SL LONEY)

Examples

Prove the following :

- $\left(\sin\alpha + \operatorname{cosec}\alpha\right)^2 + \left(\cos\alpha + \sec\alpha\right)^2 = \tan^2\alpha + \cot^2\alpha + 7$

Examples

Prove the following :

- $\left(\sin\alpha + \operatorname{cosec}\alpha\right)^2 + \left(\cos\alpha + \sec\alpha\right)^2 = \tan^2\alpha + \cot^2\alpha + 7$

- If $\tan\theta + \cot\theta = 2$ find $\sin\theta$

Examples

Prove the following :

- $$\left(\sin\alpha + \operatorname{cosec}\alpha\right)^2 + \left(\cos\alpha + \sec\alpha\right)^2 = \tan^2\alpha + \cot^2\alpha + 7$$
- If $\tan\theta + \cot\theta = 2$ find $\sin\theta$
- If $\tan\theta = \dfrac{2x(x+1)}{2x+1}$, find $\sin\theta$ and $\cos\theta$

Examples

Prove the following :

- $$\left(\sin\alpha + \operatorname{cosec}\alpha\right)^2 + \left(\cos\alpha + \sec\alpha\right)^2 = \tan^2\alpha + \cot^2\alpha + 7$$
- If $\tan\theta + \cot\theta = 2$ find $\sin\theta$
- If $\tan\theta = \dfrac{2x(x+1)}{2x+1}$, find $\sin\theta$ and $\cos\theta$
- $$\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = 4\frac{1}{3}$$

Examples

- $\cos\alpha \cos\left(\gamma-\alpha\right)-\sin\alpha \sin\left(\gamma-\alpha\right)=\cos\gamma$

Examples

- $\cos\alpha \cos(\gamma-\alpha) - \sin\alpha \sin(\gamma-\alpha) = \cos\gamma$
- $\sin(n+1)A\sin(n-1)A + \cos(n+1)A\cos(n-1)A = \cos 2A$

Examples

- $\cos\alpha\cos(\gamma-\alpha)-\sin\alpha\sin(\gamma-\alpha)=\cos\gamma$
- $\sin(n+1)A\sin(n-1)A+\cos(n+1)A\cos(n-1)A=\cos 2A$
- $\sin(n+1)A\sin(n+2)A+\cos(n+1)A\cos(n+2)A=\cos A$

Examples

- $\cos\alpha\cos(\gamma-\alpha)-\sin\alpha\sin(\gamma-\alpha)=\cos\gamma$
- $\sin(n+1)A\sin(n-1)A+\cos(n+1)A\cos(n-1)A=\cos 2A$
- $\sin(n+1)A\sin(n+2)A+\cos(n+1)A\cos(n+2)A=\cos A$
- $\dfrac{(\cos\theta-\cos 3\theta)(\sin 8\theta+\sin 2\theta)}{(\sin 5\theta-\sin\theta)(\cos 4\theta-\cos 6\theta)}=1$

Examples

- $\cos\alpha \cos(\gamma - \alpha) - \sin\alpha \sin(\gamma - \alpha) = \cos\gamma$
- $\sin(n+1)A \sin(n-1)A + \cos(n+1)A \cos(n-1)A = \cos 2A$
- $\sin(n+1)A \sin(n+2)A + \cos(n+1)A \cos(n+2)A = \cos A$
- $\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$
- $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \tan\dfrac{A+B}{2} \cot\dfrac{A-B}{2}$

Examples

- $$\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$$

Examples

- $$\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$$

- $$\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$$

Examples

- $$\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$$

- $$\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$$

- $$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

Examples

- $$\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$$

- $$\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$$

- $$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

- $$\sin(\beta-\gamma)\cos(\alpha-\delta) + \sin(\gamma-\alpha)\cos(\beta-\delta) + \sin(\alpha-\beta)\cos(\gamma-\delta) = 0$$

Examples

- $$1+\tan A\tan\frac{A}{2}=\tan A\cot\frac{A}{2}-1=\sec A$$

Examples

- $1+\tan A \tan \frac{A}{2} = \tan A \cot \frac{A}{2} - 1 = \sec A$

- $\tan A + \cot A = 2 \operatorname{cosec} 2A$

Examples

- $$1+\tan A\tan\frac{A}{2}=\tan A\cot\frac{A}{2}-1=\sec A$$

- tanA + cotA = 2 cosec 2 A

- $$\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$$

Examples

- $1+\tan A\tan\frac{A}{2}=\tan A\cot\frac{A}{2}-1=\sec A$
- $\tan A+\cot A=2\operatorname{cosec}2A$
- $\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$
- $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$

Examples

- $$\frac{2\cos 2^{n}\theta+1}{2\cos\theta+1}=(2\cos\theta-1)\left(2\cos 2\theta-1\right)\left(2\cos 2^{2}\theta-1\right)$$
  $$\ldots\left(2\cos 2^{n-1}\theta-1\right)$$

Examples

- $$\frac{2\cos 2^{n}\theta+1}{2\cos\theta+1}=(2\cos\theta-1)\left(2\cos 2\theta-1\right)\left(2\cos 2^{2}\theta-1\right)\ldots\left(2\cos 2^{n-1}\theta-1\right)$$

- $$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\frac{\theta}{2}$$

Examples

- If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

  prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Examples

- If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

  prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

- If $\tan A = \dfrac{n}{n+1}$ and $\tan B = \dfrac{1}{2n+1}$, find $\tan(A+B)$

Examples

- If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

  prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

- If $\tan A = \dfrac{n}{n+1}$ and $\tan B = \dfrac{1}{2n+1}$, find $\tan(A+B)$

- If $\tan\theta = \dfrac{b}{a}$, find the value of $a\cos 2\theta + b\sin 2\theta$

Examples

- Two parallel chords of a circle, which are on the same side of the centre, subtend angles of 72° and 144° respectively at the centre. Prove that the perpendicular distance between the chords is half the radius of the circle.

Examples

- Two parallel chords of a circle, which are on the same side of the centre, subtend angles of 72° and 144° respectively at the centre. Prove that the perpendicular distance between the chords is half the radius of the circle.

- In any circle prove that the chord which subtends 108° at the centre is equal to the sum of the two chords which subtend angles of 36° and 60°.

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$
- $\cos A = -\frac{1}{2}$

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$
- $\cos A = -\frac{1}{2}$
- $\sec A = -\frac{2}{\sqrt{3}}$

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$
- $\cos A = -\frac{1}{2}$
- $\sec A = -\frac{2}{\sqrt{3}}$
- $\tan A = -1$

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$
- $\cos A = -\frac{1}{2}$
- $\sec A = -\frac{2}{\sqrt{3}}$
- $\tan A = -1$
- $\cot A = -\sqrt{3}$

Examples

What values between 0° and 360° may A have when

- $\sin A = \frac{1}{\sqrt{2}}$
- $\cos A = -\frac{1}{2}$
- $\sec A = -\frac{2}{\sqrt{3}}$
- $\tan A = -1$
- $\cot A = -\sqrt{3}$
- $\operatorname{cosec} A = -2$

Examples

- $$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Examples

- $$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

- $$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \ \sin B \ \cos C$$

Examples

- $$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

- $$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A\ \sin B\ \cos C$$

- $$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\ \cos\frac{B}{2}\ \sin\frac{C}{2}$$

Solve Sheet

To Attain Advance Level