

KEY CONCEPTS (COMPOUND ANGLES)

1. BASIC TRIGONOMETRIC IDENTITIES :

- (a) $\sin^2\theta + \cos^2\theta = 1$; $-1 \leq \sin \theta \leq 1$; $-1 \leq \cos \theta \leq 1 \quad \forall \theta \in \mathbb{R}$
 (b) $\sec^2\theta - \tan^2\theta = 1$; $|\sec \theta| \geq 1 \quad \forall \theta \in \mathbb{R}$
 (c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$; $|\operatorname{cosec} \theta| \geq 1 \quad \forall \theta \in \mathbb{R}$

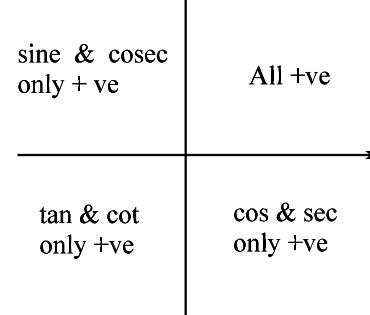
2. IMPORTANT T' RATIOS:

- (a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in \mathbb{I}$
 (b) $\sin \frac{(2n+1)\pi}{2} = (-1)^n$ & $\cos \frac{(2n+1)\pi}{2} = 0$ where $n \in \mathbb{I}$
 (c) $\sin 15^\circ$ or $\sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos \frac{5\pi}{12}$;
 $\cos 15^\circ$ or $\cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin \frac{5\pi}{12}$;
 $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$; $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$
 (d) $\sin \frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$; $\cos \frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$; $\tan \frac{\pi}{8} = \sqrt{2}-1$; $\tan \frac{3\pi}{8} = \sqrt{2}+1$
 (e) $\sin \frac{\pi}{10}$ or $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ & $\cos 36^\circ$ or $\cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

3. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

If θ is any angle, then $-\theta$, $90^\circ \pm \theta$, $180^\circ \pm \theta$, $270^\circ \pm \theta$, $360^\circ \pm \theta$ etc. are called **ALLIED ANGLES**.

- (a) $\sin(-\theta) = -\sin \theta$; $\cos(-\theta) = \cos \theta$
 (b) $\sin(90^\circ - \theta) = \cos \theta$; $\cos(90^\circ - \theta) = \sin \theta$
 (c) $\sin(90^\circ + \theta) = \cos \theta$; $\cos(90^\circ + \theta) = -\sin \theta$
 (d) $\sin(180^\circ - \theta) = \sin \theta$; $\cos(180^\circ - \theta) = -\cos \theta$
 (e) $\sin(180^\circ + \theta) = -\sin \theta$; $\cos(180^\circ + \theta) = -\cos \theta$
 (f) $\sin(270^\circ - \theta) = -\cos \theta$; $\cos(270^\circ - \theta) = -\sin \theta$
 (g) $\sin(270^\circ + \theta) = -\cos \theta$; $\cos(270^\circ + \theta) = \sin \theta$



4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES :

- (a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
 (b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
 (c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \cdot \sin(A-B)$
 (d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$
 (e) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ (f) $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

5. FACTORISATION OF THE SUM OR DIFFERENCE OF TWO SINES OR COSINES :

- (a) $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$ (b) $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$
 (c) $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$ (d) $\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$

6. TRANSFORMATION OF PRODUCTS INTO SUM OR DIFFERENCE OF SINES & COSINES :

- (a) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$ (b) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$
 (c) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ (d) $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$