MC SIR

CLASS : XII (ABCD) ***SPECIAL DPP ON VECTOR*** ***DPP. NO.- 1***

Q.1 A (1, –1, –3), B (2, 1, –2) & C (–5, 2, –6) are the position vectors of the vertices of a triangle ABC. The length of the bisector of its internal angle at A is :

(A) $\sqrt{10}/4$ (B*) $3\sqrt{10}/4$ (C) $\sqrt{10}$ (D) none

Q.2 Let $\vec{r}=\vec{a}+\lambda\vec{l}$ and $\vec{r}=\vec{b}+\mu\vec{m}$ be two lines in space where $\vec{a}=5\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=-\hat{i}+7\hat{j}+8\hat{k}$, $\vec{l}=-4\hat{i}+\hat{j}-\hat{k}$ and $\vec{m}=2\hat{i}-5\hat{j}-7\hat{k}$ then the p.v. of a point which lies on both of these lines, is

(A*) $\hat{i}+2\hat{j}+\hat{k}$ (B) $2\hat{i}+\hat{j}+\hat{k}$ (C) $\hat{i}+\hat{j}+2\hat{k}$ (D) non existent as the lines are skew

[Hint: $\lambda=\mu=1$ (point of intersection of two lines)]

Q.3 P, Q have position vectors $\vec{a}$ & $\vec{b}$ relative to the origin 'O' & X, Y divide $\overrightarrow{PQ}$ internally and externally respectively in the ratio 2 : 1. Vector $\overrightarrow{XY}$ =

(A) $\frac{3}{2}\left(\vec{b}-\vec{a}\right)$ (B) $\frac{4}{3}\left(\vec{a}-\vec{b}\right)$ (C) $\frac{5}{6}\left(\vec{b}-\vec{a}\right)$ (D*) $\frac{4}{3}\left(\vec{b}-\vec{a}\right)$

Q.4 Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p}=K\vec{g}$, then K =

(A*) 3 (B) 2 (C) 1/3 (D) 2/3

Q.5 A vector $\vec{a}$ has components 2p & 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components p + 1 & 1 then ,

(A) p = 0 (B*) p = 1 or p = – 1/3

(C) p = – 1 or p = 1/3 (D) p = 1 or p = – 1

[Hint : Equate the magnitude i.e. $4p^2 + 1 = (p + 1)^2 + 1 = p^2 + 2p + 2$

$\Rightarrow \quad 3p^2 - 2p - 1 = 0 \Rightarrow p = 1$ or $- 1/3$]

Q.6 The number of vectors of unit length perpendicular to vectors $\vec{a}$ = (1, 1, 0) & $\vec{b}$ (0, 1, 1) is:

(A) 1 (B*) 2 (C) 3 (D) ∞

[Hint: Two collinear vector always denotes a plane]

Q.7 Four points A(+1, –1, 1) ; B(1, 3, 1) ; C(4, 3, 1) and D(4, – 1, 1) taken in order are the vertices of

(A) a parallelogram which is neither a rectangle nor a rhombus

(B) rhombus

(C) an isosceles trapezium

(D*) a cyclic quadrilateral.

[Hint: It is a rectangle]

Q.8 Let α, β & γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$; $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$

(A) are collinear (B*) form an equilateral triangle

(C) form a scalene triangle (D) form a right angled triangle

Q.9 If the vectors $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$ & $\vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ constitute the sides of a ΔABC, then the length of the median bisecting the vector $\vec{c}$ is

(A) $\sqrt{2}$ (B) $\sqrt{14}$ (C) $\sqrt{74}$ (D*) $\sqrt{6}$

[Hint : $\vec{m}=\vec{b}+\frac{\vec{c}}{2}=\hat{i}+2\hat{j}+\hat{k}$, hence $|\vec{m}|=16$]

Q.10 P be a point interior to the acute triangle ABC. If $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is a null vector then w.r.t. the triangle ABC, the point P is, its

(A*) centroid (B) orthocentre (C) incentre (D) circumcentre

[Hint: $\vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}=0 \Rightarrow \vec{p}=\frac{\vec{a}+\vec{b}+\vec{c}}{3} \Rightarrow$ (A)] **[12th, 24-08-2008]**

Q.11 A vector of magnitude 10 along the normal to the curve $3x^2+8xy+2y^2-3=0$ at its point P(1, 0) can be

(A*) $6\hat{i}+8\hat{j}$ (B) $-8\hat{i}+3\hat{j}$ (C) $6\hat{i}-8\hat{j}$ (D) $8\hat{i}+6\hat{j}$

[Sol. differentiate the curve **[13th, 14-09-2008]**

$6x + 8(xy_1 + y) + 4yy_1 = 0$

m_T at (1, 0) is $6 + 8(y_1(0)) = 0$

$y_1(0) = -\frac{3}{4}$

$m_N = \frac{4}{3}$

unit vector $= \pm\frac{(3\hat{i}+4\hat{j})}{5}$

again normal vector of magnitude $10 = \pm(6\hat{i}+8\hat{j})$ Ans.]

Q.12 Consider the points A, B and C with position vectors $\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $7\hat{i}-\hat{k}$ respectively.

Statement-1: The vector sum, $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

because

Statement-2: A, B and C form the vertices of a triangle.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C*) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[Hint: Note that although $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ but A, B and C do not form the vertices of a triangle, infact they are collinear as, $\overrightarrow{AB}=(3,-1,-2)$; $\overrightarrow{BC}=(6,-2,-4)$ and $\overrightarrow{CA}=(-9,3,6)$

$\left|\overrightarrow{AB}\right|=\sqrt{14}$; $\left|\overrightarrow{BC}\right|=2\sqrt{14}$; $\left|\overrightarrow{CA}\right|=3\sqrt{14}$] **[13th, 16-12-2007]**

Q.1 If the three points with position vectors (1, a, b) ; (a, 2, b) and (a, b, 3) are collinear in space, then the value of a + b is
(A) 3 (B*) 4 (C) 5 (D) none

[Hint: A (1, a, b) ; B (a, 2, b) ; C (a, b, 3)

$\overrightarrow{AB} = (a-1)\,\hat{i} + (2-a)\,\hat{j} + 0\,\hat{k}$; $\overrightarrow{BC} = \hat{i} + (b-2)\hat{j} + (3-b)\hat{k}$

$\overrightarrow{AB} = \lambda\,\overrightarrow{BC} = \lambda\left(0\,\hat{i} + (b-2)\hat{j} + (3-b)\hat{k}\right)$ where $\lambda \neq 0$

hence $a - 1 = 0 \Rightarrow a = 1$ ....(1)
$2 - a = \lambda(b-2)$ ....(2)
and $3 - b = 0 \Rightarrow b = 3$ ....(3)
with $a = 1$ and $b = 3$, $\lambda = 1$
hence $a + b = 4 \Rightarrow$ (B)] **[12th (25-9-2005)]**

Q.2 Consider the following 3 lines in space

$L_1 : \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k} + \lambda(2\hat{i} + 4\hat{j} - \hat{k})$

$L_2 : \vec{r} = \hat{i} + \hat{j} - 3\hat{k} + \mu(4\hat{i} + 2\hat{j} + 4\hat{k})$

$L_3 : \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + t(2\hat{i} + \hat{j} + 2\hat{k})$

Then which one of the following pair(s) are in the same plane.
(A) only L_1L_2 (B) only L_2L_3 (C) only L_3L_1 (D*) L_1L_2 and L_2L_3

[Hint: L_1L_2 intersecting ; L_2L_3 parallel ; L_3L_1 skew] **[13th (25-9-2005)]**

Q.3 The acute angle between the medians drawn from the acute angles of an isosceles right angled triangle is:
(A) $\cos^{-1}(2/3)$ (B) $\cos^{-1}(3/4)$ (C*) $\cos^{-1}(4/5)$ (D) none

Q.4 If $\vec{e}_1$ & $\vec{e}_2$ are two unit vectors and θ is the angle between them, then $\cos(\theta/2)$ is

(A*) $\frac{1}{2}|\vec{e}_1 + \vec{e}_2|$ (B) $\frac{1}{2}|\vec{e}_1 - \vec{e}_2|$ (C) $\frac{\vec{e}_1 \cdot \vec{e}_2}{2}$ (D) $\frac{|\vec{e}_1 \times \vec{e}_2|}{2|\vec{e}_1||\vec{e}_2|}$

[Sol. $(\hat{e}_1 + \hat{e}_2)^2 = 2 + 2\cos\theta = 4\cos^2\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|\vec{e}_1 + \vec{e}_2|$ Ans.]

Q.5 The vectors $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ & $2\hat{i} + \hat{j} - 4\hat{k}$ form the sides of a triangle. Then triangle is
(A) an acute angled triangle (B) an obtuse angled triangle
(C) an equilateral triangle (D*) a right angled triangle

Q.6 If the vectors $3\vec{p} + \vec{q}$; $5\vec{p} - 3\vec{q}$ and $2\vec{p} + \vec{q}$; $4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors then $\sin(\vec{p}\,\hat{}\,\vec{q})$ is
(A) $\sqrt{55}/4$ (B*) $\sqrt{55}/8$ (C) $3/16$ (D) $\sqrt{247}/16$

[Sol. $(3\vec{p} + \vec{q}) \cdot (5\vec{p} - 3\vec{q}) = 0$ or $15\vec{p}^2 - 3\vec{q}^2 = 4\vec{p} \cdot \vec{q}$ ....(1)

$(2\vec{p} + \vec{q}) \cdot (4\vec{p} - 2\vec{q}) = 0$ or $8\vec{p}^2 = 2\vec{q}^2 \Rightarrow \vec{q}^2 = 4\vec{p}^2$ ....(2)

now $\cos\theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}$; substituting $\vec{q}^2 = 4\vec{p}^2$ in (1) $3\vec{p}^2 = 4\vec{p} \cdot \vec{q}$

$$\cos\theta = \frac{3}{4}\cdot\frac{\vec{p}^2}{|\vec{p}|2|\vec{p}|} = \frac{3}{8} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{55}}{8} \quad \Rightarrow \quad \text{(B)} \quad]$$

Q.7 Consider the points A, B and C with position vectors $\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $7\hat{i}-\hat{k}$ respectively.

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$\left|\overrightarrow{AB}\right|=\sqrt{14}$; $\left|\overrightarrow{BC}\right|=2\sqrt{14}$; $\left|\overrightarrow{CA}\right|=3\sqrt{14}$] **[13th, 16-12-2007]**

Q.8 The set of values of c for which the angle between the vectors $cx\hat{i}-6\hat{j}+3\hat{k}$ & $x\hat{i}-2\hat{j}+2cx\hat{k}$ is acute for every $x \in R$ is

(A) (0, 4/3) (B) [0, 4/3] (C) (11/9, 4/3) (D*) [0, 4/3)

[Hint: $cx^2+12+6cx>0$; $c=0$ is obviously]

Q.9 Let $\vec{u}=\hat{i}+\hat{j}, \vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to

(A) 1 (B) 2 (C*) 3 (D) 0

[Hint: $\hat{n}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, where $a_1^2+a_2^2+a_3^2=1$ **[12th (16-1-2005)]**

$\vec{u}\cdot\hat{n}=0 \Rightarrow a_1+a_2=0$

also $\vec{v}\cdot\hat{n}=0 \Rightarrow a_1-a_2=0$

hence, $a_1=a_2=0$

$\therefore$ $a_3=1$ or -1

$\therefore$ $\hat{n}=\hat{k}$ or $-\hat{k}$

$|\vec{w}\cdot\hat{n}|=3$]

Q.10 If the vector $6\hat{i}-3\hat{j}-6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ then the vectors are :

(A*) $-\left(\hat{i}+\hat{j}+\hat{k}\right)$ & $7\hat{i}-2\hat{j}-5\hat{k}$ (B) $-2\left(\hat{i}+\hat{j}+\hat{k}\right)$ & $8\hat{i}-\hat{j}-4\hat{k}$

(C) $+2\left(\hat{i}+\hat{j}+\hat{k}\right)$ & $4\hat{i}-5\hat{j}-8\hat{k}$ (D) none

[Hint : A vector $\vec{a}$ which is decomposed into parallel and perpendicular to the vector $\vec{b}$ is $\left(\frac{\vec{a}\,.\,\vec{b}}{\vec{b}^2}\right)\vec{b}$ &

$\vec{a}-\left(\frac{\vec{a}\,.\,\vec{b}}{\vec{b}^2}\right)\vec{b}$ or $\frac{\vec{b}\text{ x }(\vec{a}\text{ x }\vec{b})}{\vec{b}^2}$]

MC SIR

CLASS : XII (ABCD) ***SPECIAL DPP ON VECTOR*** ***DPP. NO.- 3***

Q.1 If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then the angle between $\vec{a}$ & $\vec{b}$ is :

(A) $\pi/6$ (B) $2\pi/3$ (C) $5\pi/3$ (D*) $\pi/3$

[Hint: $\vec{a}+\vec{b}=-\vec{c}$ Square $|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{c}|^2$

$$\therefore \quad \cos\theta=\frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2|\vec{a}||\vec{b}|}$$]

Q.2 The lengths of the diagonals of a parallelogram constructed on the vectors $\vec{p}=2\vec{a}+\vec{b}$ & $\vec{q}=\vec{a}-2\vec{b}$, where $\vec{a}$ & $\vec{b}$ are unit vectors forming an angle of 60° are :

(A) 3 & 4 (B*) $\sqrt{7}$ & $\sqrt{13}$ (C) $\sqrt{5}$ & $\sqrt{11}$ (D) none

[Hint: $|\vec{d}_1|=|\vec{p}+\vec{q}|$; $|\vec{d}_2|=|\vec{p}-\vec{q}|$]

Q.3 Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ to $\vec{c}+\vec{a}$ & $\vec{c}$ to $\vec{a}+\vec{b}$. Then $|\vec{a}+\vec{b}+\vec{c}|$ is :

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D*) $5\sqrt{2}$

[Hint: $\left.\begin{array}{l}\vec{a}\cdot(\vec{b}+\vec{c})=0\\ \vec{b}\cdot(\vec{c}+\vec{a})=0\\ \vec{c}\cdot(\vec{a}+\vec{b})=0\end{array}\right\} \Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$

$\therefore\ \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ now square $|\vec{a}+\vec{b}+\vec{c}|$ to get the result]

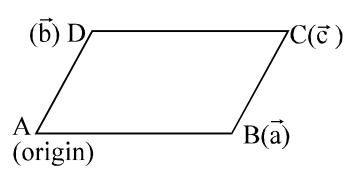
Q.4 Given a parallelogram ABCD. If $|\overrightarrow{AB}|=a$, $|\overrightarrow{AD}|=b$ & $|\overrightarrow{AC}|=c$, then $\overrightarrow{DB}\cdot\overrightarrow{AB}$ has the value

(A*) $\frac{3a^2+b^2-c^2}{2}$ (B) $\frac{a^2+3b^2-c^2}{2}$ (C) $\frac{a^2-b^2+3c^2}{2}$ (D) none

[Hint: To find $(\vec{a}-\vec{b})\cdot\vec{a}$ i.e. $|\vec{a}|^2-\vec{a}\cdot\vec{b}$ —(1)

now $\vec{a}+\vec{b}=\vec{c} \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{c}|^2$ —(2)

substitute the value of $\vec{a}\cdot\vec{b}$ from (2) in (1)]



Q.5 The set of values of x for which the angle between the vectors $\vec{a}=x\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=2x\hat{i}+x\hat{j}-\hat{k}$ acute and the angle between the vector $\vec{b}$ and the axis of ordinates is obtuse, is

(A) $1<x<2$ (B) $x>2$ (C) $x<1$ (D*) $x<0$

Q.6 If a vector $\vec{a}$ of magnitude 50 is collinear with vector $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with positive z-axis then :

(A) $\vec{a} = 4\vec{b}$ (B*) $\vec{a} = -4\vec{b}$ (C) $\vec{b} = 4\vec{a}$ (D) none

[Hint : Let $\vec{a} = \lambda\left(6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}\right)$ $|\vec{a}| = 50 \Rightarrow \lambda = \pm 4$ $\vec{a}\,.\,\hat{k} > 0 \Rightarrow \lambda = -4$]

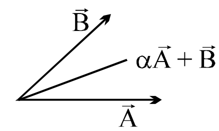
Q.7 A, B, C & D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ respectively such that $(\vec{a} - \vec{d})\cdot(\vec{b} - \vec{c}) = (\vec{b} - \vec{d})\cdot(\vec{c} - \vec{a}) = 0$. Then for the triangle ABC, D is its

(A) incentre (B) circumcentre (C*) orthocentre (D) centroid

Q.8 Let $\vec{A}$ & $\vec{B}$ be two non parallel unit vectors in a plane. If $(\alpha\vec{A} + \vec{B})$ bisects the internal angle between $\vec{A}$ & $\vec{B}$, then α is equal to

(A) 1/2 (B*) 1 (C) 2 (D) – 1

[Hint : $\vec{A}\,.\,(\alpha\vec{A} + \vec{B}) = \vec{B}\,.\,(\alpha\vec{A} + \vec{B}) \Rightarrow \alpha + \vec{A}\,.\,\vec{B} = \alpha\vec{A}\,.\,\vec{B} + 1$



$(\vec{A}\,.\,\vec{B})(1 - \alpha) = (1 - \alpha)$

$(1 - \alpha)\ [\vec{A}\cdot\vec{B} - 1] = 0$ if $\vec{A}\cdot\vec{B} = 1 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$ which is not possible $\Rightarrow \alpha = 1$]

Q.9 Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is, $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector

(A) (– 9, 5, 2) (B*) (9, 5, – 2) (C) (9, – 5, – 2) (D) none

Q.10 Let $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector. If pairwise angles between $\hat{a}, \hat{b}, \hat{c}$ are θ_1, θ_2 and θ_3 rexpectively then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ equals

(A) 3 (B) – 3 (C) 1 (D*) – 1

[Hint : consider $(\hat{a} + \hat{b} + \hat{c})^2 = 3 + 2(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)$]

Q.11 A tangent is drawn to the curve $y = \frac{8}{x^2}$ at a point $A(x_1, y_1)$, where $x_1 = 2$. The tangent cuts the x-axis at point B. Then the scalar product of the vectors $\vec{AB}$ & $\vec{OB}$ is

(A*) 3 (B) – 3 (C) 6 (D) – 6

Q.12 L_1 and L_2 are two lines whose vector equations are

$$L_1 : \vec{r} = \lambda\left[(\cos\theta + \sqrt{3})\hat{i} + (\sqrt{2}\sin\theta)\hat{j} + (\cos\theta - \sqrt{3})\hat{k}\right]$$

$$L_2 : \vec{r} = \mu(a\hat{i} + b\hat{j} + c\hat{k}),$$

where λ and μ are scalars and α is the acute angle between L_1 and L_2.
If the angle 'α' is independent of θ then the value of 'α' is

(A*) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

[Sol. Both the lines pass through origin **[12th, 28-09-2008]**

Line L_1 is parallel to the vector

$$\vec{V}_1 = \left(\cos\theta + \sqrt{3}\right)\hat{i} + \left(\sqrt{2}\sin\theta\right)\hat{j} + \left(\cos\theta - \sqrt{3}\right)\hat{k}$$

and L_2 is parallel to the vector]

$$\vec{V}_2 = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\therefore \quad \cos\alpha = \frac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1||\vec{V}_2|} = \frac{a(\cos\theta + \sqrt{3}) + (b\sqrt{2})\sin\theta + c(\cos\theta - \sqrt{3})}{\sqrt{a^2+b^2+c^2}\sqrt{(\cos\theta+\sqrt{3})^2 + 2\sin^2\theta + (\cos\theta - \sqrt{3})^2}}$$

$$= \frac{(a+c)\cos\theta + b\sqrt{2}\sin\theta + (a-c)\sqrt{3}}{\sqrt{a^2+b^2+c^2}\ \sqrt{2+6}}$$

in order that $\cos\alpha$ is independent of θ

$a + c = 0$ and $b = 0$

$$\therefore \quad \cos\alpha = \frac{2a\sqrt{3}}{a\sqrt{2}\cdot 2\sqrt{2}} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{6} \text{ Ans.]}$$

MC SIR

Q.1 Cosine of an angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ if $|\vec{a}|=2, |\vec{b}|=1$ and $\vec{a}\hat{}\vec{b}=60°$ is

(A*) $\sqrt{3/7}$ (B) $9/\sqrt{21}$ (C) $3/\sqrt{7}$ (D) none

Q.2 An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\overrightarrow{OA}=\vec{a}$ & $\overrightarrow{OB}=\vec{b}$, then the vector $\overrightarrow{OC}$ in terms of $\vec{a}$ & $\vec{b}$, is

(A) $\sqrt{3}\,\vec{a}-2\vec{b}$ (B*) $-\sqrt{3}\,\vec{a}+2\vec{b}$ (C) $2\vec{a}-\sqrt{3}\,\vec{b}$ (D) $-2\vec{a}+\sqrt{3}\,\vec{b}$

[12th 18-12-2005]

Q.3 For two particular vectors $\vec{A}$ and $\vec{B}$ it is known that $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$. What must be true about the two vectors?

(A) At least one of the two vectors must be the zero vector.

(B) $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ is true for any two vectors.

(C*) One of the two vectors is a scalar multiple of the other vector.

(D) The two vectors must be perpendicular to each other.

[Sol. $\vec{A}\times\vec{B}=-\vec{A}\times\vec{B}$ **[12th, 21-10-2007]**

$\vec{A}\times\vec{B}=0$

either $\vec{A}=0$ or $\vec{B}=0$

or $\vec{A}$ and $\vec{B}$ are collinear]

Q.4 'P' is a point inside the triangle ABC, such that $BC\left(\overrightarrow{PA}\right)+CA\left(\overrightarrow{PB}\right)+AB\left(\overrightarrow{PC}\right)=0$, then for the triangle ABC the point P is its :

(A*) incentre (B) circumcentre (C) centroid (D) orthocentre

Q.5 The vector equations of two lines L_1 and L_2 are respectivly

$\vec{r}=17\hat{i}-9\hat{j}+9\hat{k}+\lambda(3\hat{i}+\hat{j}+5\hat{k})$ and $\vec{r}=15\hat{i}-8\hat{j}-\hat{k}+\mu(4\hat{i}+3\hat{j})$

I L_1 and L_2 are skew lines

II (11, –11, –1) is the point of intersection of L_1 and L_2

III (–11, 11, 1) is the point of intersection of L_1 and L_2

IV $\cos^{-1}\left(3/\sqrt{35}\right)$ is the acute angle between L_1 and L_2

then, which of the following is true?

(A*) II and IV (B) I and IV (C) IV only (D) III and IV

Q.6 Given three vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ each two of which are non collinear. Further if $\left(\vec{a}+\vec{b}\right)$ is collinear with $\vec{c}$, $\left(\vec{b}+\vec{c}\right)$ is collinear with $\vec{a}$ & $|\vec{a}|=|\vec{b}|=|\vec{c}|=\sqrt{2}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$:

(A) is 3 (B*) is –3 (C) is 0 (D) cannot be evaluated

[Hint: $\vec{a}+\vec{b}=\lambda\vec{c}$ ....(1)

and $\vec{b}+\vec{c}=\mu\vec{a}$ ....(2)

$\therefore \quad (\lambda\vec{c}-\vec{a}) + \vec{c} = \mu\vec{a} \qquad$ [putting $\vec{b} = \lambda\vec{c}-\vec{a}$]

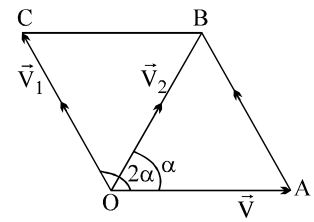
$(\lambda+1)\ \vec{c} = (\mu+1)\ \vec{a} \quad \Rightarrow \quad \lambda = \mu = -1$

$\therefore \quad \vec{a}+\vec{b}+\vec{c}=0$ now proceed]

Q.7 For some non zero vector $\vec{V}$, if the sum of $\vec{V}$ and the vector obtained from $\vec{V}$ by rotating it by an angle 2α equals to the vector obtained from $\vec{V}$ by rotating it by α then the value of α, is

(A*) $2n\pi \pm \frac{\pi}{3}$ (B) $n\pi \pm \frac{\pi}{3}$ (C) $2n\pi \pm \frac{2\pi}{3}$ (D) $n\pi \pm \frac{2\pi}{3}$

where n is an integer.



[Sol. Given $\vec{V} + \vec{V}_1 = \vec{V}_2$; Also $\vec{V} \wedge \vec{V}_1 = 2\alpha$

$(\vec{V}_1)^2 = (\vec{V}_2 - \vec{V})^2$ and $\vec{V} \wedge \vec{V}_2 = \alpha$

also, $|\vec{V}| = |\vec{V}_1| = |\vec{V}_2| = \lambda$ say

hence, $\lambda^2 = 2\lambda^2 - 2\lambda^2 \cos\alpha \ \Rightarrow\ \cos\alpha = 1/2 \Rightarrow\ \alpha = 2n\pi \pm (\pi/3)$ Ans.] **[12th & 13th 03-03-2007]**

Q.8 Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1,\ |\vec{v}| = 2,\ |\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) 2 (B) $\sqrt{7}$ (C*) $\sqrt{14}$ (D) 14

[Sol. $\vec{v}\cdot\hat{u} = \vec{w}\cdot\hat{u}$

$\vec{v} \perp \vec{w} \ \Rightarrow \quad \vec{v}\cdot\vec{w} = 0$

now, $|\vec{u}-\vec{v}+\vec{w}|^2 = \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u}\cdot\vec{v} - 2\vec{w}\cdot\vec{v} + 2\vec{u}\cdot\vec{w} = 1 + 4 + 9$

so $|\vec{u}-\vec{v}+\vec{w}| = \sqrt{14}$ Ans.] **[29-01-2005, 12th & 13th]**

Q.9 If $\vec{a}$ and $\vec{b}$ are non zero, non collinear, and the linear combination

$(2x-y)\vec{a} + 4\vec{b} = 5\vec{a} + (x-2y)\vec{b}$ holds for real x and y then x + y has the value equal to

(A) – 3 (B*) 1 (C) 17 (D) 3

[Sol. $(2x-y-5)\vec{a} = (x-2y-4)\vec{b}$ **[12th 17-9-2006]**

$\therefore \quad 2x - y = 5 \quad ....(1); \qquad x - 2y = 4 \quad ....(2)$

from (1) and (2)

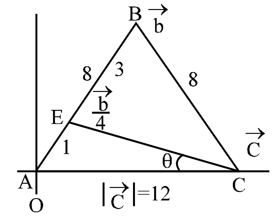
$2(2y + 4) - y = 5 \quad \Rightarrow \quad 3y = -3 \quad \Rightarrow \quad y = -1$

and $x = 2$; hence $x + y = 1$ Ans.]

Q.10 In the isosceles triangle ABC $|\vec{AB}| = |\vec{BC}| = 8$, a point E divides AB internally in the ratio 1 : 3, then the cosine of the angle between $\vec{CE}$ & $\vec{CA}$ is (where $|\vec{CA}| = 12$)

(A) $-\frac{3\sqrt{7}}{8}$ (B) $\frac{3\sqrt{8}}{17}$ (C*) $\frac{3\sqrt{7}}{8}$ (D) $\frac{-3\sqrt{8}}{17}$

[Sol. Given $|\vec{b}| = |\vec{b}-\vec{c}| = 8$ and $|\vec{c}| = 12$; $\cos\theta = \dfrac{\vec{c}\cdot\left(\vec{c}-\dfrac{\vec{b}}{4}\right)}{|\vec{c}|\left|\vec{c}-\dfrac{\vec{b}}{4}\right|} = \dfrac{\vec{c}^{\,2}-\dfrac{\vec{c}\cdot\vec{b}}{4}}{12\left|\vec{c}-\dfrac{\vec{b}}{4}\right|}$



now proceed.]

Q.11 If $\vec{p} = 3\vec{a} - 5\vec{b}$; $\vec{q} = 2\vec{a} + \vec{b}$; $\vec{r} = \vec{a} + 4\vec{b}$; $\vec{s} = -\vec{a} + \vec{b}$ are four vectors such that $\sin\left(\vec{p} \wedge \vec{q}\right) = 1$ and $\sin\left(\vec{r} \wedge \vec{s}\right) = 1$ then $\cos\left(\vec{a} \wedge \vec{b}\right)$ is :

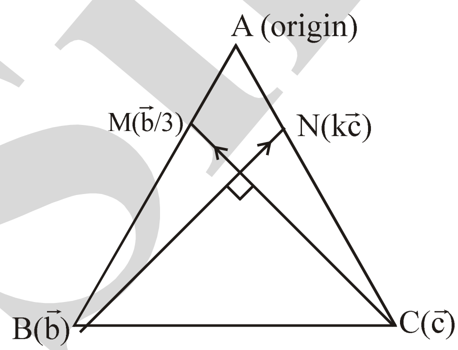
(A) $-\dfrac{19}{5\sqrt{43}}$ (B) 0 (C) 1 (D*) $\dfrac{19}{5\sqrt{43}}$

[Hint : $\vec{p}\cdot\vec{q} = 0$ & $\vec{r}\cdot\vec{s} = 0$. Form two simultaneous equations and get a relation between $|\vec{a}|$ & $|\vec{b}|$ as $25|\vec{a}|^2 = 43|\vec{b}|^2$. Now compute $\cos\left(\vec{a} \wedge \vec{b}\right) = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$]

Q.12 Given an equilateral triangle ABC with side length equal to 'a'. Let M and N be two points respectively on the side AB and AC much that $\overrightarrow{AN} = K\overrightarrow{AC}$ and $\overrightarrow{AM} = \dfrac{\overrightarrow{AB}}{3}$. If $\overrightarrow{BN}$ and $\overrightarrow{CM}$ are orthogonal then the value of K is equal to

(A*) $\dfrac{1}{5}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$

[Sol. $\overrightarrow{BN}\cdot\overrightarrow{CM} = 0$ **[13th, 01-02-2009, P-2]**



$$\left(k\vec{c}-\vec{b}\right)\cdot\left(\frac{\vec{b}}{3}-\vec{c}\right) = 0$$

$$\frac{k}{3}\cdot\frac{a^2}{2} - ka^2 - \frac{a^2}{3} + \frac{a^2}{2} = 0$$

$$\frac{k}{6} - k + \frac{1}{6} = 0$$

$$\frac{5k}{6} = \frac{1}{6} \quad\Rightarrow\quad k = \frac{1}{5}$$ **Ans.**]

[QUIZ-79 XII]

MC SIR

CLASS : XII (ABCD) **SPECIAL DPP ON VECTOR** **DPP. NO.- 5**

Q.1 If $\vec{e}_1$ & $\vec{e}_2$ are two unit vectors and θ is the angle between them , then $\sin(\theta/2)$ is :

(A) $\frac{1}{2}|\vec{e}_1+\vec{e}_2|$ (B*) $\frac{1}{2}|\vec{e}_1-\vec{e}_2|$ (C) $\frac{\vec{e}_1 . \vec{e}_2}{2}$ (D) $\frac{|\vec{e}_1 \text{ x } \vec{e}_2|}{2|\vec{e}_1||\vec{e}_2|}$

[Hint: consider $|\hat{e}_1-\hat{e}_2|^2 = 2 - 2\cos\theta = 2.\frac{\sin^2\theta}{2}$ $\therefore$ $\frac{1}{2}$ $|\hat{e}_1-\hat{e}_2| = \frac{\sin\theta}{2}$]

Q.2 If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r} \text{ x } \vec{p} = \vec{q} \text{ x } \vec{p}$ & $\vec{r} . \vec{s} = 0$, then $\vec{r}$ =

(A) $\vec{p} . \vec{s}$ (B) $\vec{q} + \left(\frac{\vec{q} . \vec{p}}{\vec{p} . \vec{s}}\right)\vec{p}$ (C*) $\vec{q} - \left(\frac{\vec{q} . \vec{s}}{\vec{p} . \vec{s}}\right)\vec{p}$ (D) $\vec{q} + \mu\vec{p}$ for all scalars μ

[Hint: $(\vec{r}-\vec{q})\times\vec{p} = 0 \Rightarrow \vec{r}-\vec{q} = \lambda\,\vec{p}$ $\quad \vec{r} = \lambda\,\vec{p} + \vec{q}$ — (1) now $\vec{r} . \vec{s} = 0$

$\Rightarrow (\lambda\,\vec{p}+\vec{q}) . \vec{s} = 0$ $\quad \lambda = -\frac{\vec{q} . \vec{s}}{\vec{p} . \vec{s}}$ — (2) Put λ from (2) in (1) to get the result]

Q.3 If $\vec{u} = \vec{a}-\vec{b}$; $\vec{v} = \vec{a}+\vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$ then $|\vec{u}\times\vec{v}|$ is equal to

(A) $\sqrt{2(16-(\vec{a}.\vec{b})^2)}$ (B*) $2\sqrt{(16-(\vec{a}.\vec{b})^2)}$ (C) $2\sqrt{(4-(\vec{a}.\vec{b})^2)}$ (D) $\sqrt{2(4-(\vec{a}.\vec{b})^2)}$

[Hint: $|\vec{u}\times\vec{v}| = 2|\vec{a}\times\vec{b}|$; $\therefore$ $|\vec{u}\times\vec{v}|^2 = 4|\vec{a}\times\vec{b}|^2 = 4[\vec{a}^2 b^2 - (\vec{a}\cdot\vec{b})^2] = 4[16-(\vec{a}\cdot\vec{b})^2] \Rightarrow$ result]

Q.4 If $\vec{u}$ and $\vec{v}$ are two vectors such that $|\vec{u}| = 3$; $|\vec{v}| = 2$ and $|\vec{u}\times\vec{v}| = 6$ then the correct statement is

(A) $\vec{u}\wedge\vec{v} \in (0, 90°)$ (B) $\vec{u}\wedge\vec{v} \in (90°, 180°)$ (C*) $\vec{u}\wedge\vec{v} = 90°$ (D) $(\vec{u}\times\vec{v})\times\vec{u} = 6\vec{v}$

[Sol. $|\vec{u}\times\vec{v}|^2 = \vec{u}^2\vec{v}^2 - (\vec{u}\cdot\vec{v})^2$ **[12th, 28-09-2008]**

$36 = (9)(4) - (\vec{u}\cdot\vec{v})^2$

$\Rightarrow$ $\vec{u}\cdot\vec{v} = 0$ $\Rightarrow$ $\vec{u}$ and $\vec{v}$ are orthogonal

also $(\vec{u}\times\vec{v})\times\vec{u} = (\vec{u}\cdot\vec{u})\vec{v} - (\vec{v}\cdot\vec{u})\vec{u} = 9\,\vec{v}$ $\Rightarrow$ **(D)** is incorrect]

Q.5 If $\vec{A} = (1, 1, 1)$, $\vec{C} = (0, 1, -1)$ are given vectors, then a vector $\vec{B}$ satisfying the equation $\vec{A} \text{ x } \vec{B} = \vec{C}$ and $\vec{A} . \vec{B} = 3$ is :

(A) (5, 2, 2) (B*) $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$ (C) $\left(\frac{2}{3}, \frac{5}{3}, \frac{2}{3}\right)$ (D) $\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}\right)$

[Hint: Let $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$; $b_1 + b_2 + b_3 = 3$ and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0\hat{i} + \hat{j} + \hat{k}$;

This gives $b_3 = b_2$; $b_2 - b_1 = -1$; $b_1 - b_3 = 1 \Rightarrow$ (B)

Alternatively: take cross with $\vec{A}$;

$\vec{A}\times(\vec{A}\times\vec{B}) = \vec{A}\times\vec{C}$ or $(\vec{A}\cdot\vec{B}) - (\vec{A}\cdot\vec{A})\vec{B} = \vec{A}\times\vec{C}$

$3\vec{A} - 3\vec{B} = \vec{A}\times\vec{C}$ $\Rightarrow$ $\vec{B} = \frac{3\vec{A} + \vec{C}\times\vec{A}}{\vec{A}}$]

Q.6 Given a parallelogram OACB. The lengths of the vectors $\vec{OA}$, $\vec{OB}$ & $\vec{AB}$ are a, b & c respectively. The scalar product of the vectors $\vec{OC}$ & $\vec{OB}$ is :

(A) $\frac{a^2 - 3b^2 + c^2}{2}$ (B) $\frac{3a^2 + b^2 - c^2}{2}$ (C) $\frac{3a^2 - b^2 + c^2}{2}$ (D*) $\frac{a^2 + 3b^2 - c^2}{2}$

Q.7 Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ then $\left\{\left(\vec{a} + 3\vec{b}\right) \times \left(3\vec{a} - \vec{b}\right)\right\}^2 =$

(A) 225 (B) 250 (C) 275 (D*) 300

Q.8 In a quadrilateral ABCD, $\vec{AC}$ is the bisector of the $\left(\vec{AB} \wedge \vec{AD}\right)$ which is $\frac{2\pi}{3}$,

$15\left|\vec{AC}\right| = 3\left|\vec{AB}\right| = 5\left|\vec{AD}\right|$ then $\cos\left(\vec{BA} \wedge \vec{CD}\right)$ is :

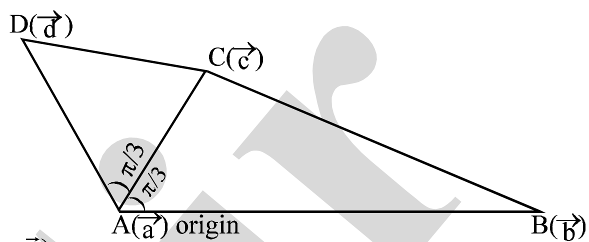
(A) $-\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $-\frac{\sqrt{21}}{7\sqrt{3}}$ (C*) $\frac{2}{\sqrt{7}}$ (D) $\frac{2\sqrt{7}}{14}$

[Hint: Given $15|\vec{AC}| = 3|\vec{AB}| = 5|\vec{AD}|$

Let $|\vec{AC}| = \lambda > 0$

$\therefore |\vec{AB}| = 5\lambda$

$|\vec{AD}| = 3\lambda$



Now $\cos\left(\vec{BA} \wedge \vec{CD}\right) = \frac{\vec{BA}.\vec{CD}}{\left|\vec{BA}\right|\left|\vec{CD}\right|} = \frac{\vec{b}.(\vec{d} - \vec{c})}{|\vec{b}||\vec{d} - \vec{c}|}$ .....(1)

Now numerator of (1) = $\vec{b}.\vec{c} - \vec{b}.\vec{d}$

$= |\vec{b}||\vec{c}|\cos\frac{\pi}{3} - |\vec{b}||\vec{d}|\cos\frac{2\pi}{3}$

$= (5\lambda)(\lambda)\frac{1}{2} + 5\lambda(3\lambda)\frac{1}{2} = \frac{5\lambda^2 + 15\lambda^2}{2} = 10\lambda^2$

Denominator of (1) = $|\vec{b}|\,|\vec{d} - \vec{c}|$

Now $|\vec{d} - \vec{c}|^2 = \vec{d}^2 + \vec{c}^2 - 2\vec{c}.\vec{d} = 9\lambda^2 + \lambda^2 - 2(\lambda)(3\lambda)1/2 = 10\lambda^2 - 3\lambda^2 = 7\lambda^2$

$\therefore \left|\vec{d} - \vec{c}\right| = \sqrt{7}\,\lambda$

Denominator of (1) = $(5\lambda)(\sqrt{7}\,\lambda) = 5\sqrt{7}\,\lambda^2$

$\therefore \cos\left(\vec{BA} \wedge \vec{CD}\right) = \frac{10\lambda^2}{5\sqrt{7}\,\lambda^2} = \frac{2}{\sqrt{7}} \Rightarrow$ (C)]

Q.9 If the two adjacent sides of two rectangles are represented by the vectors $\vec{p} = 5\vec{a} - 3\vec{b}\,;\,\vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}\,;\,\vec{s} = -\vec{a} + \vec{b}$ respectively, then the angle between the vectors $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$

(A) is $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ (B*) is $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

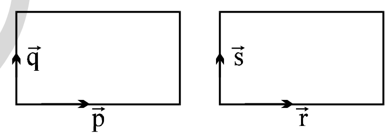
(C) is $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ (D) cannot be evaluated

[Sol. $\vec{p}.\vec{q} = 0$

$(5\vec{a} - 3\vec{b}).(-\vec{a} - 2\vec{b}) = 0$

$-5\vec{a}^2 - 10\vec{a}\,\vec{b} + 3\vec{a}\,\vec{b} + 6\vec{b}^2 = 0$

$6\vec{b}^2 - 5\vec{a}^2 - 7\vec{a}.\vec{b} = 0$ **....(1)**



Also $\vec{r}.\vec{s} = 0$

$(-4\vec{a} - \vec{b})(-\vec{a} + \vec{b}) = 0$ or $4\vec{a}^2 - 4\vec{a}.\vec{b} + \vec{a}.\vec{b} - \vec{b}^2 = 0$

or $4\vec{a}^2 - \vec{b}^2 - 3\vec{a}.\vec{b} = 0$ **....(2)**

now $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s}) = \frac{1}{3}(5\vec{a} - 3\vec{b} - 4\vec{a} - \vec{b} - \vec{a} + \vec{b}) = -\vec{b}$

$\vec{y} = \frac{1}{5}(\vec{r} + \vec{s}) = \frac{1}{5}(-5\vec{a}) = -\vec{a}$

Angle between $\vec{x}$ and $\vec{y}$ i.e. $\cos\theta = \frac{\vec{x}.\vec{y}}{|\vec{x}||\vec{y}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ **....(3)**

From (1) and (2) $|\vec{a}| = \sqrt{\frac{25}{19}}\sqrt{\vec{a}.\vec{b}}$ and $|\vec{b}| = \sqrt{\frac{43}{19}}\sqrt{\vec{a}.\vec{b}}$

$\therefore$ $|\vec{a}||\vec{b}| = \frac{\sqrt{25\times 43}}{19}.\vec{a}.\vec{b}$; $\therefore$ $\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{19}{5\sqrt{43}}$ $\Rightarrow$ $\theta = \cos^{-1}\frac{19}{5\sqrt{43}}$]

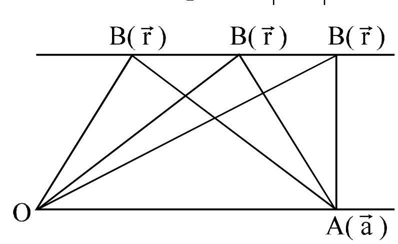
Q.10 If the vector product of a constant vector $\vec{OA}$ with a variable vector $\vec{OB}$ in a fixed plane OAB be a constant vector, then locus of B is :

(A) a straight line perpendicular to $\vec{OA}$ (B) a circle with centre O radius equal to $|\vec{OA}|$

(C*) a straight line parallel to $\vec{OA}$ (D) none of these

[Hint : $|\vec{a} \times \vec{r}| = |\vec{c}|$

Triangles on the same base and between the same parallel will have the same area]

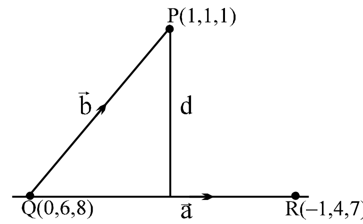


Q.11 If the distance from the point P(1, 1, 1) to the line passing through the points Q(0, 6, 8) and R(–1, 4, 7) is expressed in the form $\sqrt{p/q}$ where p and q are coprime, then the value of $\frac{(p+q)(p+q-1)}{2}$ equals

(A*) 4950 (B) 5050 (C) 5150 (D) none

[Sol. $\vec{a} = -\hat{i} - 2\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - 5\hat{j} - 7\hat{k}$



$$\left|\vec{d}\right| = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|}$$

$|\vec{a}| = \sqrt{6}$

$|\vec{a}\times\vec{b}|^2 = \vec{a}^2\,\vec{b}^2 - (\vec{a}\cdot\vec{b})^2 = (6)(75) - (-1 + 10 + 7)^2 = 450 - 256 = 194$

$|\vec{a}\times\vec{b}| = \sqrt{194}$

$$\therefore\ d = \sqrt{\frac{194}{6}} = \sqrt{\frac{97}{3}}$$

$$\therefore\ p + q = 100 \qquad \Rightarrow \qquad \frac{(p+q)(p+q-1)}{2} = \frac{100\times 99}{2} = 4950 \ ]$$

MC SIR

CLASS : XII (ABCD) **SPECIAL DPP ON VECTOR** **DPP. NO.- 6**

Q.1 For non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $|\vec{a} \times \vec{b}.\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds if and only if ;

(A) $\vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0$ (B) $\vec{c}.\vec{a} = 0,\ \vec{a}.\vec{b} = 0$

(C) $\vec{a}.\vec{c} = 0,\ \vec{b}.\vec{c} = 0$ (D*) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

[Hint: $\sin\theta\cos\phi = 1 \Rightarrow \theta = \frac{\pi}{2}$ and $\phi = 0 \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular]

Q.2 The vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are

(A) not coplanar (B*) coplanar but cannot form a triangle

(C) coplanar but can form a triangle (D) coplanar & can form a right angled triangle

[Hint: Note that $\vec{a} + \vec{b} = \vec{c}$]

Q.3 Given the vectors

$$\vec{u} = 2\hat{i} - \hat{j} - \hat{k}$$
$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$$
$$\vec{w} = \hat{i} - \hat{k}$$

If the volume of the parallelopiped having $-c\vec{u}$, $\vec{v}$ and $c\vec{w}$ as concurrent edges, is 8 then 'c' can be equal to

(A*) ± 2 (B) 4 (C) 8 (D) can not be determined

[Sol. $V = -c^2[\vec{u}\,\vec{v}\,\vec{w}] = -c^2\begin{vmatrix} 2 & -1 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & -1 \end{vmatrix} = -c^2[2(1-0) - 1(1) + (-2-1)] = -c^2[2-1-3] = 8$

$\therefore\ 2c^2 = 8 \Rightarrow c = 2$ or -2 Ans.] **[12th & 13th 11-02-2007]**

Q.4 Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j}$; $(\vec{a} \wedge \vec{b}) = \pi/2, \vec{a}\cdot\vec{c} = 4$ then

(A) $[\bar{a}\ \bar{b}\ \bar{c}]^2 = |\bar{a}|$ (B) $[\bar{a}\ \bar{b}\ \bar{c}] = |\bar{a}|$ (C) $[\bar{a}\ \bar{b}\ \bar{c}] = 0$ (D*) $[\bar{a}\ \bar{b}\ \bar{c}] = |\bar{a}|^2$

[Hint: $\left.\begin{array}{l} x - y + 2 = 0 \quad \text{and} \\ x + 2y = 4 \end{array}\right] \Rightarrow x = 0 ; y = 2 \Rightarrow \vec{a} = 2\hat{j} + 2\hat{k}$ & $|\vec{a}|^2 = 8$

$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |\bar{a}|^2 \Rightarrow$ (D)]

Q.5 The set of values of m for which the vectors $\hat{i} + \hat{j} + m\hat{k}$, $\hat{i} + \hat{j} + (m+1)\hat{k}$ & $(\hat{i} - \hat{j} + m\hat{k})$ are non-coplanar :

(A*) R (B) $R - \{1\}$ (C) $R - \{-2\}$ (D) ϕ

[Hint : The value of $\begin{vmatrix} 1 & 1 & m \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix} = 1 \therefore$ non coplanar $\forall\ m \in R$]

Q.6 Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ & $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is :

(A) the A.M. of a & b (B*) the G. M. of a & b

(C) the H. M. of a & b (D) equal to zero.

[Sol. Vectors in the same plane, hence

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$-1\,(ab - c^2) - 1(ac - ac) = 0$

$c^2 = ab$]

Q.7 Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$; $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$. If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{6}$ then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$

(A) 0 (B) 1

(C*) $\frac{1}{4}\,(a_1^2 + a_2^2 + a_3^2)\,(b_1^2 + b_2^2 + b_3^2)$ (D) $\frac{3}{4}\,(a_1^2 + a_2^2 + a_3^2)\,(b_1^2 + b_2^2 + b_3^2)\,(c_1^2 + c_2^2 + c_3^2)$

[Hint: $(\vec{a} \times \vec{b} \cdot \vec{c})^2 = |\vec{a}|^2\,|\vec{b}|^2\,|\vec{c}|^2 \sin^2\theta \cos^2\phi = (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)\frac{1}{4} \Rightarrow$ (C)]

Q.8 For three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ which of the following expressions is not equal to any of the remaining three?

(A) $\vec{u} . (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) . \vec{u}$ (C*) $\vec{v} . (\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v}) . \vec{w}$

[JEE '98, 2]

Q.9 The vector $\vec{c}$ is perpendicular to the vectors $\vec{a} = (2, -3, 1)$, $\vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{c} . (\hat{i} + 2\hat{j} - 7\hat{k}) = 10$. Then the vector $\vec{c}$ =

(A*) (7, 5, 1) (B) (– 7, – 5, –1) (C) (1, 1, –1) (D) none

[Hint: Let $\vec{c} = \lambda(\vec{a} \times \vec{b})$. Hence $\lambda\,(\vec{a} \times \vec{b}).(\hat{i} + 2\hat{j} - 7\hat{k}) = 10$

$$\lambda\begin{vmatrix} 2 & -3 & 1 \\ 1 & -2 & 3 \\ 1 & 2 & -7 \end{vmatrix} = 10 \Rightarrow \quad \lambda = -1 \quad \Rightarrow \quad \vec{c} = -(\vec{a} \times \vec{b})$$

$\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & -2 & 3 \end{vmatrix} = (-9+2)\hat{i} - (5)\hat{j} + (-4+3)\hat{k} \quad \Rightarrow \quad (-7, -5, -1) \text{ Ans.]}$$

Q.10 Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ & $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors, $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c}$ are coplanar then $\frac{\alpha}{\beta}$ is :

(A) 1 (B) 2 (C) 3 (D*) – 3

[Sol. $\vec{V}_1 = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{V}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$

$\vec{V}_3 = \vec{c} = \alpha\vec{a} + \beta\vec{b} = \alpha(\hat{i}+\hat{j}) + \beta(\hat{j}+\hat{k}) = \alpha\hat{i} + (\alpha+\beta)\hat{j} + \beta\hat{k} = \vec{c}$

since $\vec{V}_1, \vec{V}_2, \vec{V}_3$ are coplanar

now $\begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ \alpha & \alpha+\beta & \beta \end{vmatrix} = 0$, using $C_2 \to C_2 - (C_1 + C_3)$, we get

$\begin{vmatrix} 1 & -4 & 1 \\ 3 & 0 & -1 \\ \alpha & 0 & \beta \end{vmatrix} = 0$, hence $4(3\beta + \alpha) = 0 \Rightarrow 3\beta + \alpha = 0 \Rightarrow \dfrac{\alpha}{\beta} = -3$ Ans.]

Q.11 A rigid body rotates about an axis through the origin with an angular velocity $10\sqrt{3}$ radians/sec. If $\vec{\omega}$ points in the direction of $\hat{i} + \hat{j} + \hat{k}$ then the equation to the locus of the points having tangential speed 20 m/sec. is :

(A) $x^2 + y^2 + z^2 - xy - yz - zx - 1 = 0$

(B) $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx - 1 = 0$

(C*) $x^2 + y^2 + z^2 - xy - yz - zx - 2 = 0$

(D) $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx - 2 = 0$

[Hint: $\vec{n} = \dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$; $\vec{\omega} = |\vec{\omega}|\,\vec{n} = 10\left(\hat{i}+\hat{j}+\hat{k}\right)$

Now $\vec{v} = \vec{\omega} \times \vec{r} = 10\left(\hat{i}+\hat{j}+\hat{k}\right) \times \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$ where $\vec{r}$ is the position vector of the point whose locus is to be determined.

Hence $\vec{v} = 10\left((z-y)\hat{i} - (z-x)\hat{j} + (y-x)\hat{k}\right)$

$\therefore |\vec{v}| = 10\sqrt{(x-y)^2 + (y-z)^2 + (z-x)^2}$ hence $2(x^2 + y^2 + z^2 - xy - yz - zx) = 4$

$\Rightarrow x^2 + y^2 + z^2 - zy - yz - zx - 2 = 0$ which is the equation of a cylinder]

Q.12 A rigid body rotates with constant angular velocity ω about the line whose vector equation is, $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. The speed of the particle at the instant it passes through the point with p.v. $2\hat{i} + 3\hat{j} + 5\hat{k}$ is :

(A*) $\omega\sqrt{2}$ (B) 2ω (C) $\omega/\sqrt{2}$ (D) none

[Hint: $\vec{n} = \dfrac{\hat{i}+2\hat{j}+2\hat{k}}{3}$; $\vec{\omega} = \dfrac{\omega}{3}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

$$\vec{v} = \vec{\omega} \times \vec{r} = \frac{\omega}{3}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 5 \end{vmatrix} = \frac{\omega}{3}\left(4\hat{i} - \hat{j} - \hat{k}\right)$$

$\therefore |\vec{v}| = \dfrac{\omega}{3}\sqrt{18} = \omega\sqrt{2}$]

Q.13 Given 3 vectors

$$\vec{V}_1 = a\hat{i} + b\hat{j} + c\hat{k}; \qquad \vec{V}_2 = b\hat{i} + c\hat{j} + a\hat{k}; \qquad \vec{V}_3 = c\hat{i} + a\hat{j} + b\hat{k}$$

In which one of the following conditions $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$ are linearly independent?

(A) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
(D*) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

[Sol. only in (D) $[\vec{V}_1\ \vec{V}_2\ \vec{V}_3] \neq 0$]

Q.14 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then the vector $\vec{c}$ such that $\vec{a} \cdot \vec{c} = 2$ & $\vec{a} \times \vec{c} = \vec{b}$ is

(A) $\frac{1}{3}\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$ (B*) $\frac{1}{3}\left(-\hat{i} + 2\hat{j} + 5\hat{k}\right)$ (C) $\frac{1}{3}\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$ (D) $\frac{1}{3}\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$

[Hint: $\vec{a} \times \vec{b} = \vec{a} \times \left(\vec{a} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{c}\right)\vec{a} - \left(\vec{a} \cdot \vec{a}\right)\vec{c} = 2\vec{a} - 3\vec{c}$

But $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{k}$

Hence $3\vec{c} = 2\vec{a} - \left(3\hat{i} - 3\hat{k}\right) = \left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(3\hat{i} - 3\hat{k}\right) = -\hat{i} + 2\hat{j} + 5\hat{k}$

$\Rightarrow \quad \vec{c} = \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) \Rightarrow$ (B)]

One or more than one is/are correct:

Q.15 If $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ & $\vec{b} \times \vec{c} = \vec{a}$ then which of the following always hold(s) good?

(A*) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs (B) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left|\vec{b}\right|$

(C*) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left|\vec{c}\right|^2$ (D) $\left|\vec{b}\right| = \left|\vec{c}\right|$

[Sol. Clearly $\vec{a} \cdot \vec{c} = 0$ and $\vec{b} \cdot \vec{c} = 0$ Also $\vec{a} \cdot \vec{b} = 0 \Rightarrow$ A

$\vec{a} \times \vec{b} = \vec{c}$

dot with $\vec{b} \Rightarrow \vec{b} \cdot \vec{c} = 0$; |||ly $\vec{b} \times \vec{c} = \vec{a}$

dot with $\vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$; dot with $\vec{c} \Rightarrow \vec{a} \cdot \vec{c} = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

Again $\left.\begin{matrix} |\vec{a}|\,|\vec{b}| = |\vec{c}| \\ |\vec{b}|\,|\vec{c}| = |\vec{a}| \end{matrix}\right] \Rightarrow \frac{|\vec{a}|}{|\vec{c}|} = \frac{|\vec{c}|}{|\vec{a}|} \Rightarrow |\vec{a}| = |\vec{c}|$ & $|\vec{b}| = 1$

$\Rightarrow \vec{a} \times \vec{b} \cdot \vec{c} = |\vec{a}|\,|\vec{b}|\,|\vec{c}| = |\vec{a}|^2 = |\vec{c}|^2$

(children will assume $\vec{a} = \hat{i}$; $\vec{b} = \hat{j}$ and $\vec{c} = \hat{k}$ but in this case all the four will be correct which will be wrong)] **[12th, 09-11-2008]**

Q.1 The altitude of a parallelopiped whose three coterminous edges are the vectors, $\vec{A} = \hat{i} + \hat{j} + \hat{k}$; $\vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ & $\vec{C} = \hat{i} + \hat{j} + 3\hat{k}$ with $\vec{A}$ and $\vec{B}$ as the sides of the base of the parallelopiped, is

(A) $2/\sqrt{19}$ (B) $4/\sqrt{19}$ (C*) $2\sqrt{38}/19$ (D) none

[Hint: $h = \dfrac{2\left[\vec{A}\,\vec{B}\,\vec{C}\right]}{\left|\vec{A}\times\vec{B}\right|}$ & $\left|\vec{A}\times\vec{B}\right| = \sqrt{a^2b^2 - (\vec{a}.\vec{b})^2}$]

Q.2 Consider Δ ABC with A ≡ $(\vec{a})$; B ≡ $(\vec{b})$ & C ≡ $(\vec{c})$. If $\vec{b}.(\vec{a}+\vec{c}) = \vec{b}.\vec{b} + \vec{a}.\vec{c}$; $\left|\vec{b}-\vec{a}\right| = 3$; $\left|\vec{c}-\vec{b}\right| = 4$ then the angle between the medians $\vec{AM}$ & $\vec{BD}$ is

(A*) $\pi - \cos^{-1}\left(\dfrac{1}{5\sqrt{13}}\right)$ (B) $\pi - \cos^{-1}\left(\dfrac{1}{13\sqrt{5}}\right)$

(C) $\cos^{-1}\left(\dfrac{1}{5\sqrt{13}}\right)$ (D) $\cos^{-1}\left(\dfrac{1}{13\sqrt{5}}\right)$

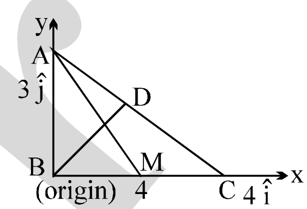
[Hint: $\vec{b}.\vec{a} + \vec{b}.\vec{c} = \vec{b}.\vec{b} + \vec{a}.\vec{c}$

or $\vec{b}.(\vec{a}-\vec{b}) - \vec{c}.(\vec{a}-\vec{b}) = 0$

or $(\vec{b}-\vec{c}).(\vec{a}-\vec{b}) = 0$

⇒ BC & AB are perpendicular

Now find angle between AM & BM]



Q.3 If A (– 4, 0, 3) ; B (14, 2, –5) then which one of the following points lie on the bisector of the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ ('O' is the origin of reference)

(A) (2, 1, –1) (B) (2, 11, 5) (C) (10, 2, –2) (D*) (1, 1, 2)

[Hint: $\overrightarrow{OA} = -4\hat{i} + 3\hat{k}$; $\overrightarrow{OB} = 14\hat{i} + 2\hat{j} - 5\hat{k}$

$\hat{a} = \dfrac{-4\hat{i}+3\hat{k}}{5}$; $\hat{b} = \dfrac{14\hat{i}+2\hat{j}-5\hat{k}}{15}$

$\vec{r} = \dfrac{\lambda}{15}\left[-12\hat{i} + 9\hat{j} + 14\hat{i} + 2\hat{j} - 5\hat{k}\right]$

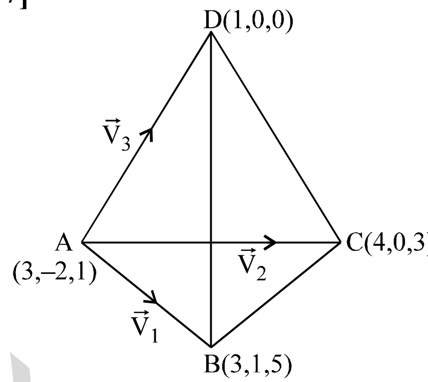
$\vec{r} = \dfrac{\lambda}{15}\left[2\hat{i} + 2\hat{j} + 4\hat{k}\right]$

$\vec{r} = \dfrac{2\lambda}{15}\left[\hat{i} + \hat{j} + 2\hat{k}\right]$ ⇒ answer is (D) with $\lambda = \dfrac{15}{2}$]

Q.4 Position vectors of the four angular points of a tetrahedron ABCD are A(3, –2, 1); B(3, 1, 5); C(4, 0, 3) and D(1, 0, 0). Acute angle between the plane faces ADC and ABC is

(A*) $\tan^{-1}(5/2)$ (B) $\cos^{-1}(2/5)$ (C) $\text{cosec}^{-1}(5/2)$ (D) $\cot^{-1}(3/2)$

[Sol. $\vec{V}_1 = 0\hat{i} + 3\hat{j} + 4\hat{k}$ **[12th & 13th 11-3-2007]**



$\vec{V}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{V}_3 = -2\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{n}_1 = \vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ 1 & 2 & 2 \end{vmatrix}$$

$\vec{n}_1 = (-2)\hat{i} - (0-4)\hat{j} + (0-3)\hat{k}$

$\vec{n}_1 = -2\hat{i} + 4\hat{j} - 3\hat{k}$

$$\vec{n}_2 = \vec{V}_2 \times \vec{V}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ -2 & 2 & -1 \end{vmatrix}$$

$\vec{n}_2 = (-2-4)\hat{i} - (-1+4)\hat{j} + (2+4)\hat{k}$

$\vec{n}_2 = -6\hat{i} - 3\hat{j} + 6\hat{k} = -3(2\hat{i} + \hat{j} - 2\hat{k})$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{-4+4+6}{\sqrt{29}\cdot 3} = \frac{2}{\sqrt{29}}$$

hence $\tan\theta = \dfrac{5}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{5}{2}\right)$]

Q.5 The volume of the tetrahedron formed by the coterminus edges $\vec{a}, \vec{b}, \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ is

(A) 6 (B) 18 (C*) 36 (D) 9

[Sol. $\dfrac{1}{6}[\bar{a}\,\bar{b}\,\bar{c}] = 3 \Rightarrow [\bar{a}\,\bar{b}\,\bar{c}] = 18$

volume of the required parallelepiped = $[\bar{a}+\bar{b} \;\; \bar{b}+\bar{c} \;\; \bar{c}+\bar{a}] = \{(\bar{a}+\bar{b})\times(\bar{b}+\bar{c})\}\cdot(\bar{c}+\bar{a})$

$= 2[\bar{a}\,\bar{b}\,\bar{c}] = 36$]

Q.6 Given unit vectors $\vec{m}$, $\vec{n}$ & $\vec{p}$ such that angle between $\vec{m}$ & $\vec{n}$ = angle between $\vec{p}$ and $(\vec{m}\times\vec{n}) = \pi/6$ then $[\vec{n}\;\vec{p}\;\vec{m}] =$

(A*) $\sqrt{3}/4$ (B) 3/4 (C) 1/4 (D) none

[Hint: $[\vec{n}\,\vec{p}\,\vec{m}] = \sin\theta\cos\phi = \sin\dfrac{\pi}{6}.\cos\dfrac{\pi}{6} = \dfrac{1}{2}.\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}$]

Q.7 $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$, then the acute angle between $\vec{a}$ & $\vec{c}$ is :

(A*) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $5\pi/12$

[Hint : $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$; $|\vec{a}| = |\vec{b}| = 1$; $|\vec{c}| = 2$

$\vec{a} \times \vec{d} = -\vec{b} \Rightarrow (\vec{a} \times \vec{d})^2 = \vec{b}^2 = 1$

or $|\vec{a}|^2 |\vec{d}|^2 - (\vec{a} . \vec{d})^2 = 1$

or $(\vec{a} \times \vec{c})^2 - 0 = 1 \Rightarrow |\vec{a}|^2 |\vec{c}|^2 - (\vec{a} . \vec{c})^2 = 1$

$\Rightarrow 4 - 2\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{3}{4}$; $\theta = \pi/6$]

[Alternative: $(\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})\vec{c} = -\vec{b}$

$(\lambda\vec{a} - \vec{c})^2 = 1$ or $\lambda^2 \vec{a}^2 + \vec{c}^2 - 2\lambda\vec{a}.\vec{c} = 1$ (where $\lambda = \vec{a} \cdot \vec{c}$)

$\Rightarrow \lambda^2 + 4 - 2\lambda^2 = 1$ or $\lambda^2 = 3$

$\vec{a}^2 \ \vec{c}^2 \cos^2\theta = 3$

$\cos^2\theta = 3/4$

$\theta = \pi/6$]

Q.8 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors & $|\vec{c}| = \sqrt{3}$, then

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = -1, \beta = \pm 1$ (D*) $\alpha = \pm 1, \beta = 1$

[JEE '98 , 2]

[Hint: $\alpha^2 + \beta^2 + 1 = 3 \Rightarrow \alpha^2 + \beta^2 = 2$; and $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$]

Q.9 A vector of magnitude $5\sqrt{5}$ coplanar with vectors $\hat{i} + 2\hat{j}$ & $\hat{j} + 2\hat{k}$ and the perpendicular vector $2\hat{i} + \hat{j} + 2\hat{k}$ is

(A) $\pm 5 (5\hat{i} + 6\hat{j} - 8\hat{k})$ (B) $\pm \sqrt{5} (5\hat{i} + 6\hat{j} - 8\hat{k})$

(C) $\pm 5\sqrt{5} (5\hat{i} + 6\hat{j} - 8\hat{k})$ (D*) $\pm (5\hat{i} + 6\hat{j} - 8\hat{k})$

[Hint : Unit vector coplanar with $\vec{a}$ & $\vec{b}$ and perpendicular to vector $\vec{c}$ is $\pm \frac{(\vec{a} \times \vec{b}) \times \vec{c}}{|(\vec{a} \times \vec{b}) \times \vec{c}|}$]

Paragraph for questions nos. 10 to 12

Consider three vectors $\vec{p} = \hat{i} + \hat{j} + \hat{k}$, $\vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$ and let $\vec{s}$ be a unit vector, then

Q.10 vectors $\vec{p}$, $\vec{q}$ and $\vec{r}$ are

(A) linearly dependent

(B) can form the sides of a possible triangle

(C*) such that the vectors $(\vec{q} - \vec{r})$ is orthogonal to $\vec{p}$

(D) such that each one of these can be expressed as a linear combination of the other two

[Hint: $\underbrace{(\hat{i}+3\hat{j}-4\hat{k})}_{(\vec{q}-\vec{r})\cdot\vec{p}} \cdot (\hat{i}+\hat{j}+\hat{k}) = 1 + 3 - 4 = 0 \quad \Rightarrow \quad$ (C) **[12th, 23-9-2007]**

since $[\vec{p}\,\vec{q}\,\vec{r}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = (12 + 1) - 1(6 + 1) + 1(2 - 4) = 13 - 7 - 2 = 4 \Rightarrow$ A, B, D are wrong]

Q.11$_{\text{vectors}}$ if $(\vec{p}\times\vec{q}) \times \vec{r} = u\vec{p} + v\vec{q} + w\vec{r}$, then $(u + v + w)$ equals to

(A) 8 (B*) 2 (C) – 2 (D) 4

[Hint: $(\vec{p}\times\vec{q}) \times \vec{r} = (\vec{p}\cdot\vec{r})\vec{q} - (\vec{q}\cdot\vec{r})\vec{p}$ **[12th, 23-9-2007]**

$\Rightarrow \quad u = -(\vec{q}\cdot\vec{r}) = -(2 + 4 - 3) = -3$

$v = \vec{p}\cdot\vec{r} = 1 + 1 + 3 = 5 \quad \& \quad w = 0.$

Hence $u = -3,\ v = 5,\ w = 0 \Rightarrow \quad u + v + w = 2$ Ans.]

Q.12$_{\text{vectors}}$ the magnitude of the vector $(\vec{p}\cdot\vec{s})(\vec{q}\times\vec{r}) + (\vec{q}\cdot\vec{s})(\vec{r}\times\vec{p}) + (\vec{r}\cdot\vec{s})(\vec{p}\times\vec{q})$ is

(A*) 4 (B) 8 (C) 18 (D) 2

[Sol. $\because$ $\vec{p}$, $\vec{q}$ and $\vec{r}$ are non-coplanar therefore $\vec{q}\times\vec{r}$, $\vec{r}\times\vec{p}$ and $\vec{p}\times\vec{q}$ are also non-coplanar

Hence, $\vec{s} = l(\vec{q}\times\vec{r}) + w(\vec{r}\times\vec{p}) + n(\vec{p}\times\vec{q})$ **[12th, 23-9-2007]**

$\therefore \quad l = \dfrac{\vec{s}\cdot\vec{p}}{[\vec{p}\,\vec{q}\,\vec{r}]}, \quad w = \dfrac{\vec{s}\cdot\vec{q}}{[\vec{p}\,\vec{q}\,\vec{r}]} \quad \& \quad n = \dfrac{\vec{s}\cdot\vec{r}}{[\vec{p}\,\vec{q}\,\vec{r}]}$

Hence, $\vec{s}[\vec{p}\,\vec{q}\,\vec{r}] = (\vec{s}\cdot\vec{p})(\vec{q}\times\vec{r}) + (\vec{s}\cdot\vec{q})(\vec{r}\times\vec{p}) + (\vec{s}\cdot\vec{r})(\vec{p}\times\vec{q})$

$\therefore \quad \left|(\vec{s}\cdot\vec{p})(\vec{q}\times\vec{r}) + (\vec{s}\cdot\vec{q})(\vec{r}\times\vec{p}) + (\vec{s}\cdot\vec{r})(\vec{p}\times\vec{q})\right| = |\vec{s}[\vec{p}\,\vec{q}\,\vec{r}]| = [\vec{p}\,\vec{q}\,\vec{r}]$ (as $|\vec{s}| = 1$)

$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = (12 + 1) - 1(6 + 1) + 1(2 - 4) = 13 - 7 - 2 = 4$ **Ans.**]

One or more than one is/are correct:

Q.13$_{25}$ Given the following information about the non zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$

(i) $(\vec{A}\times\vec{B})\times\vec{A} = \vec{0}$ (ii) $\vec{B}\cdot\vec{B} = 4$

(iii) $\vec{A}\cdot\vec{B} = -6$ (iv) $\vec{B}\cdot\vec{C} = 6$

Which one of the following holds good?

(A*) $\vec{A}\times\vec{B} = \vec{0}$ (B*) $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$ (C) $\vec{A}\cdot\vec{A} = 8$ (D*) $\vec{A}\cdot\vec{C} = -9$

[Sol. Given $|\vec{A}||\vec{B}|\cos\theta = -6$; $\quad |\vec{B}| = 2$ (given) **[12th 15-10-2006]**

$\vec{B}\cdot\vec{C} = |\vec{B}||\vec{C}|\cos\phi = 6$

and $(\vec{A}\times\vec{B})\times\vec{A} = 0$

$(\vec{A}\cdot\vec{A})\vec{B} - (\vec{B}\cdot\vec{A})\vec{A} = 0$

$(\vec{A}\cdot\vec{A})\vec{B} = -6\vec{A}$ .....(1)

$\therefore$ $\vec{A}$ and $\vec{B}$ are collinear and θ between $\vec{A}$ and $\vec{B}$ is π $\Rightarrow$ $\vec{A}\times\vec{B} = 0$ $\Rightarrow$ A is correct

$\Rightarrow \quad \vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C} = 0 \quad \Rightarrow$ B is correct also $\vec{A}\cdot\vec{B} = -6$ and $|\vec{B}| = 2$

$\therefore$ $|\vec{A}||\vec{B}|\cos\pi = -6$

$|\vec{A}|\cdot(2) = 6$ $\Rightarrow$ $|\vec{A}| = 3$ $\Rightarrow$ $\vec{A}\cdot\vec{A} = 9$ $\Rightarrow$ C is not correct

again $\vec{A}\cdot\vec{C} = ?$

dot with $\vec{C}$ the equation (1)

$9(\vec{B}\cdot\vec{C}) = -6\,\vec{A}\cdot\vec{C}$

$9)(6) = -6(\vec{A}\cdot\vec{C})$ $\Rightarrow$ $\vec{A}\cdot\vec{C} = -9$ $\Rightarrow$ (D) is correct]

Q.14$_{vec}$ Let $\vec{a}, \vec{b}, \vec{c}$ are non zero vectors and $\vec{V}_1 = \vec{a}\times(\vec{b}\times\vec{c})$ and $\vec{V}_2 = (\vec{a}\times\vec{b})\times\vec{c}$. If $\vec{V}_1 = \vec{V}_2$ then which of the following hold(s) good?

(A) $\vec{a}$ and $\vec{b}$ are orthogonal

(B*) $\vec{a}$ and $\vec{c}$ are collinear

(C) $\vec{b}$ and $\vec{c}$ are orthogonal

(D*) $\vec{b} = \lambda(\vec{a}\times\vec{c})$ when λ is a scalar.

[Sol. $\vec{V}_1 = \vec{V}_2$ **[12th, 28-09-2008]**

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$\therefore$ $(\vec{a}\cdot\vec{b})\vec{c} = (\vec{b}\cdot\vec{c})\vec{a}$

$\Rightarrow$ either $\vec{c}$ and $\vec{a}$ are collinear or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$ $\Rightarrow$ $\vec{b} = \lambda(\vec{a}\times\vec{c})$ $\Rightarrow$ **B, D**]

Q.15$_{vec}$ If $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are four non zero vectors in the same plane no two of which are collinear then which of the following hold(s) good?

(A) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = 0$

(B*) $(\vec{A}\times\vec{C})\cdot(\vec{B}\times\vec{D}) \neq 0$

(C*) $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D}) = \vec{0}$

(D) $(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{D}) \neq \vec{0}$

[Hint: Obviously (B) and (C)] **[12th, 28-09-2008]**

DPP-1

Q.1	B	Q.2	A	Q.3	D	Q.4	A	Q.5	B
Q.6	B	Q.7	D	Q.8	B	Q.9	D	Q.10	A
Q.11	A	Q.12	C						

DPP-2

Q.1	B	Q.2	D	Q.3	C	Q.4	A	Q.5	D
Q.6	B	Q.7	C	Q.8	D	Q.9	C	Q.10	A

DPP-3

Q.1	D	Q.2	B	Q.3	D	Q.4	A	Q.5	D
Q.6	B	Q.7	C	Q.8	B	Q.9	B	Q.10	D
Q.11	A	Q.12	A						

DPP-4

Q.1	A	Q.2	B	Q.3	C	Q.4	A	Q.5	A
Q.6	B	Q.7	A	Q.8	C	Q.9	B	Q.10	C
Q.11	D	Q.12	A						

DPP-5

Q.1	B	Q.2	C	Q.3	B	Q.4	C	Q.5	B
Q.6	D	Q.7	D	Q.8	C	Q.9	B	Q.10	C
Q.11	A								

DPP-6

Q.1	D	Q.2	B	Q.3	A	Q.4	D	Q.5	A
Q.6	B	Q.7	C	Q.8	C	Q.9	A	Q.10	D
Q.11	C	Q.12	A	Q.13	D	Q.14	B	Q.15	A, C

DPP-7

Q.1	C	Q.2	A	Q.3	D	Q.4	A	Q.5	C
Q.6	A	Q.7	A	Q.8	D	Q.9	D	Q.10	C
Q.11	B	Q.12	A	Q.13	A, B, D	Q.14	B, D	Q.15	B, C