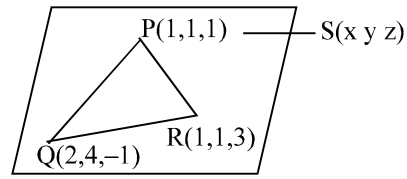
Q.1$_{new}$ Consider three vectors $\vec{p} = \hat{i} + \hat{j} + \hat{k}$, $\vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$. If $\vec{p}$, $\vec{q}$ and $\vec{r}$ denotes the position vector of three non-collinear points then the equation of the plane containing these points is

(A) $2x - 3y + 1 = 0$ (B) $x - 3y + 2z = 0$
(C) $3x - y + z - 3 = 0$ (D*) $3x - y - 2 = 0$



[Sol. Equation of plane $\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 3 & -2 \\ 0 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad 2(3x - 3 - y + 1) = 0 \quad \Rightarrow \quad 3x - y = 2$]

Q.2 The intercept made by the plane $\vec{r}.\vec{n} = q$ on the x-axis is

(A*) $\dfrac{q}{\hat{i}.\vec{n}}$ (B) $\dfrac{\hat{i}.\vec{n}}{q}$ (C) $(\hat{i}.\vec{n})q$ (D) $\dfrac{q}{|\vec{n}|}$

[Hint: x intercept is say $x_1 \Rightarrow$ plane passes through $x_1\hat{i}$

$\therefore \quad x_1\hat{i}.\vec{n} = q \Rightarrow x_1 = \dfrac{q}{\hat{i}.\vec{n}}$]

Q.3 If the distance between the planes

$8x + 12y - 14z = 2$

and $4x + 6y - 7z = 2$

can be expressed in the form $\dfrac{1}{\sqrt{N}}$ where N is natural then the value of $\dfrac{N(N+1)}{2}$ is

(A) 4950 (B) 5050 (C) 5150 (D*) 5151

[Sol. $P_1 = 4x + 6y - 7z - 1 = 0$; $P_2 = 4x + 6y - 7z - 2 = 0$ **[12th, 23-9-2007]**

$d = \dfrac{1}{\sqrt{16+36+49}} = \dfrac{1}{\sqrt{101}}$

Hence, $\dfrac{101 \times 102}{2} = 5151 \quad \Rightarrow \quad$ (D)]

Q.4 A plane passes through the point P(4, 0, 0) and Q(0, 0, 4) and is parallel to the y-axis. The distance of the plane from the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D*) $2\sqrt{2}$

[Sol. x and z intercept of the plane is 4 and it is parallel to y-axis, hence equation of the plane is $x + z = 4$.
Its distance from (0, 0, 0) is $2\sqrt{2}$ **Ans.**] **[12th & 13th 11-3-2007]**

Q.5 If from the point P (f, g, h) perpendiculars PL, PM be drawn to yz and zx planes then the equation to the plane OLM is

(A*) $\dfrac{x}{f} + \dfrac{y}{g} - \dfrac{z}{h} = 0$ (B) $\dfrac{x}{f} + \dfrac{y}{g} + \dfrac{z}{h} = 0$ (C) $\dfrac{x}{f} - \dfrac{y}{g} + \dfrac{z}{h} = 0$ (D) $-\dfrac{x}{f} + \dfrac{y}{g} + \dfrac{z}{h} = 0$

[Hint: coordinate of L (0, g, h) and M (f, 0, h). Now find the equation of OLM] **[12th (26-12-2004)]**

Q.6 If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(k)$ with x-axis, then k is equal to

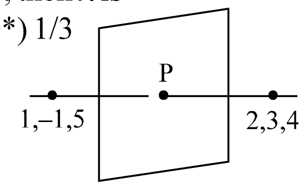
(A) $\sqrt{3}/2$ (B*) 2/7 (C) $\sqrt{2}/3$ (D) 1

[Hint: $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{V} = \hat{i}$; now $\sin\theta = \dfrac{\vec{V}.\vec{n}}{|\vec{n}|} = \dfrac{2}{7} \Rightarrow K = \dfrac{2}{7}$]

Q.7 The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio $\lambda : 1$, then λ is

(A) – 3 (B) – 1/3 (C) 3 (D*) 1/3

[Hint: y coordinate of P is zero $\Rightarrow 0 = \dfrac{3\lambda + (-1)}{\lambda + 1} \Rightarrow \lambda = \dfrac{1}{3}$



]

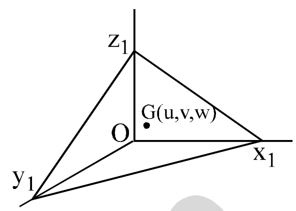
Q.8 A variable plane forms a tetrahedron of constant volume $64\,K^3$ with the coordinate planes and the origin, then locus of the centroid of the tetrahedron is

(A) $x^3 + y^3 + z^3 = 6K^3$ (B*) $xyz = 6k^3$

(C) $x^2 + y^2 + z^2 = 4K^2$ (D) $x^{-2} + y^{-2} + z^{-2} = 4K^{-2}$ **[13th (24-03-2005)]**

[Sol. $\dfrac{x_1}{4} = u, \quad \dfrac{y_1}{4} = v, \quad \dfrac{z_1}{4} = w$

$x_1 = 4u, \quad y_1 = 4v, \quad z_1 = 4w$



$$V = \frac{1}{6}\begin{vmatrix} 4u & 0 & 0 \\ 0 & 4v & 0 \\ 0 & 0 & 4w \end{vmatrix} = \left(\frac{64}{6}\right) uvw$$

$\therefore \quad 64 \cdot \left(\dfrac{uvw}{6}\right) = 64\,k^3 \quad \Rightarrow \quad xyz = 6k^3$]

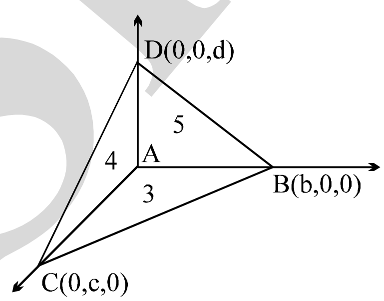
Q.9 Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and 5 sq. units respectively. Then the area of the triangle BCD, is

(A*) $5\sqrt{2}$ (B) 5 (C) $5/\sqrt{2}$ (D) 5/2

[Sol. Area of Δ BCD $= \dfrac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BD}\right| = \dfrac{1}{2}\left|(b\hat{i} - c\hat{j}) \times (b\hat{i} - d\hat{k})\right|$

$= \dfrac{1}{2}\left|bd\,\hat{j} + bc\,\hat{k} + dc\,\hat{i}\right|$

$= \dfrac{1}{2}\sqrt{b^2c^2 + c^2d^2 + d^2b^2}$ ....(1)



now $6 = bc$; $8 = cd$; $10 = bd$

$b^2c^2 + c^2d^2 + d^2b^2 = 200$

substituting the value in (1)

$A = \dfrac{1}{2}\sqrt{200} = 5\sqrt{2}$ Ans.] **[29-01-2005, 12th & 13th]**

Q.10$_{\text{vectors}}$ Equation of the line which passes through the point with p. v. (2, 1, 0) and perpendicular to the plane containing the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

(A*) $\vec{r}$ = (2, 1, 0) + t (1, –1, 1)
(B) $\vec{r}$ = (2, 1, 0) + t (–1, 1, 1)
(C) $\vec{r}$ = (2, 1, 0) + t (1, 1, –1)
(D) $\vec{r}$ = (2, 1, 0) + t (1, 1, 1)

where t is a parameter

[Sol. $\vec{r} = 2\hat{i}+\hat{j}+0\hat{k} + t\,(\hat{i}+\hat{j})\times(\hat{j}+\hat{k}) = (2, 1, 0) + t\,(\hat{k}-\hat{j}+\hat{i}) = (2, 1, 0) + t\,(1, -1, 1) \Rightarrow$ (A)]

Q.11$_{\text{vectors}}$ Which of the following planes are parallel but not identical?

$P_1 : 4x - 2y + 6z = 3$
$P_2 : 4x - 2y - 2z = 6$
$P_3 : -6x + 3y - 9z = 5$
$P_4 : 2x - y - z = 3$

(A) P_2 & P_3 (B) P_2 & P_4 (C*) P_1 & P_3 (D) P_1 & P_4

[Hint. In A, $-2/3 = -2/3 \neq 2/9$
B, $2 = 2 = 2 = 2$ identical
C, $-2/3 = -2/3 = -2/3 \neq 3/5 \Rightarrow$ (C) is correct
D, $2 = 2 \neq -6$]

Q.12 A parallelopiped is formed by planes drawn through the points (1, 2, 3) and (9, 8, 5) parallel to the coordinate planes then which of the following is not the length of an edge of this rectangular parallelopiped

(A) 2 (B*) 4 (C) 6 (D) 8

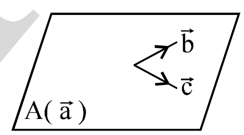
[Sol. $\left.\begin{array}{l} x=9\ ;\ x=1 \\ y=8\ ;\ y=2 \\ z=5\ ;z=3 \end{array}\right\} \Rightarrow$ edges of the cuboid are 8, 6, 2 $\Rightarrow$ (B) is correct]

Q.13 Vector equation of the plane $\vec{r} = \hat{i}-\hat{j}+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ in the scalar dot product form is

(A) $\vec{r}.(5\hat{i}-2\hat{j}+3\hat{k}) = 7$
(B) $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k}) = 7$
(C*) $\vec{r}.(5\hat{i}-2\hat{j}-3\hat{k}) = 7$
(D) $\vec{r}.(5\hat{i}+2\hat{j}+3\hat{k}) = 7$

[Hint : plane through $\vec{a}$ and || to two non collinear vector $(\vec{r}-\vec{a}).(\vec{b}\times\vec{c}) = 0$

(Take dot with $\vec{b}\times\vec{c}$ both sides]



Q.14 The vector equations of the two lines L_1 and L_2 are given by

$L_1 : \vec{r} = 2\hat{i}+9\hat{j}+13\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$; $L_2 : \vec{r} = -3\hat{i}+7\hat{j}+p\hat{k}+\mu(-\hat{i}+2\hat{j}-3\hat{k})$

then the lines L_1 and L_2 are

(A) skew lines for all $p \in R$
(B) intersecting for all $p \in R$ and the point of intersection is (–1, 3, 4)
(C*) intersecting lines for $p = -2$
(D) intersecting for all real $p \in R$

[Hint: Intersecting if $\begin{vmatrix} 5 & 2 & 13-p \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{vmatrix} = \begin{vmatrix} 5 & 2 & 13-p \\ 0 & 4 & 0 \\ -1 & 2 & -3 \end{vmatrix}$ or $-4(-15 + 13 - p) = 0$

$p = -2$

Alternatively: $(\lambda + 2) = -(\mu + 3)$ ....(1)

$2\lambda + 9 = 2\mu + 7$ ....(2)

$3\lambda + 13 = p - 3\mu$ ....(3)

from(1) $\mu = -(\lambda + 5)$

put in (2) $2\lambda + 9 = -2(\lambda + 5) + 7 \Rightarrow \lambda = -3$

now from (3) $-9 + 13 = p + 6 \Rightarrow p = -2$ Ans.]

Q.15 Consider the plane $(x, y, z) = (0, 1, 1) + \lambda(1, -1, 1) + \mu(2, -1, 0)$. The distance of this plane from the origin is

(A) 1/3 (B) $\sqrt{3}/2$ (C*) $\sqrt{3/2}$ (D) $2/\sqrt{3}$

[Sol. $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ **[13th, 20-01-2008]**

taking dot with $\vec{b} \times \vec{c}$

$[\vec{r}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$ where $\vec{a} = (0,1,1)$; $\vec{b} = (1,-1,1)$ and $\vec{c} = (2,-1,0)$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = 0 - (0 - 2) + 1(-1 + 2) = 3$$

and $$[\vec{r}\ \vec{b}\ \vec{c}] = \begin{vmatrix} x & y & z \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = x(0 + 1) - y(0 - 2) + z(-1 + 2) = x + 2y + z$$

hence equation of plane is $x + 2y + z = 3$; $\therefore$ $p = \left|\frac{-3}{\sqrt{6}}\right| = \sqrt{\frac{3}{2}}$ Ans.]

Q.1 The value of 'a' for which the lines $\frac{x-2}{1}=\frac{y-9}{2}=\frac{z-13}{3}$ and $\frac{x-a}{-1}=\frac{y-7}{2}=\frac{z+2}{-3}$ intersect, is

(A) – 5 (B) – 2 (C) 5 (D*) – 3

[Hint: $\begin{vmatrix} 2-a & 9-7 & 13-(-2) \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{vmatrix} = 0$]

Q.2 Given A (1, –1, 0) ; B(3, 1, 2) ; C(2, –2, 4) and D(–1, 1, –1) which of the following points neither lie on AB nor on CD?

(A*) (2, 2, 4) (B) (2, –2, 4) (C) (2, 0,1) (D) (0, –2, –1)

[Hint: Write the lines AB and CD in symmetrical form and verify]

Q.3 For the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$, which one of the following is incorrect?

(A) it lies in the plane $x-2y+z=0$ (B) it is same as line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(C*) it passes through (2, 3, 5) (D) it is parallel to the plane $x-2y+z-6=0$

[Sol. On (1, 2, 3) satisfies the plane $x-2y+z=0$ and also $(\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}-2\hat{j}+\hat{k})=0 \Rightarrow$(A)

Since the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ both satisfy (0, 0, 0) and (1, 2, 3) hence both are same $\Rightarrow$ (B). Given line is obviously | | to the plane $x-2y+z=6 \Rightarrow$ (D)]

Q.4 Given planes

$P_1 : cy + bz = x$

$P_2 : az + cx = y$

$P_3 : bx + ay = z$

P_1, P_2 and P_3 pass through one line, if

(A) $a^2 + b^2 + c^2 = ab + bc + ca$ (B*) $a^2 + b^2 + c^2 + 2abc = 1$

(C) $a^2 + b^2 + c^2 = 1$ (D) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + 2abc = 1$

[Hint: Infinite solution $\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow \vec{n}_1 \times \vec{n}_2 \cdot \vec{n}_3$

note that 3 such planes can meet only at one point i.e. (0, 0, 0) or they may have the same line of intersection i.e. at infinite solution.]

Q.5 The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) parallel to x-axis (B*) perpendicular to x-axis

(C) perpendicular to YOZ plane (D) parallel to y-axis

Q.6 The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(A) k = 0 or – 1 (B) k = 1 or – 1 (C*) k = 0 or – 3 (D) k = 3 or – 3

[Sol. The given lines are coplanar if

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix}$$

or if $2(1+k)-(k+2)(1-k)=0$ if $k^2+3k=0$

or if $k = 0$ or -3]

Q.7 The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy = c^2$, in xy plane if c is equal to

(A) ± 1 (B) $\pm 1/3$ (C*) $\pm\sqrt{5}$ (D) none

[Hint: put $z = 0$ in the line given $x = 5$ and $y = 1$ $\Rightarrow$ $5 \cdot 1 = c^2$] **[13th Test (24-03-2005)]**

Q.8 The line which contains all points (x, y, z) which are of the form $(x, y, z) = (2, -2, 5) + \lambda(1, -3, 2)$ intersects the plane $2x - 3y + 4z = 163$ at P and intersects the YZ plane at Q. If the distance PQ is $a\sqrt{b}$ where $a, b \in N$ and $a > 3$ then $(a + b)$ equals

(A*) 23 (B) 95 (C) 27 (D) none

[Sol. Equation of the line is **[12th & 13th 07-01-2007]**

$$\frac{x-2}{1}=\frac{y+2}{-3}=\frac{z-5}{2}=\lambda \quad ....(1)$$

hence any point on the line (1) can be taken as

$\Rightarrow$ $x = \lambda + 2$

$y = -(3\lambda + 2)$

$z = (2\lambda + 5)$

for some λ point lies on the plane

$2x - 3y + 4z = 163$ ....(2)

$2(\lambda + 2) + 3(3\lambda + 2) + 4(2\lambda + 5) = 163$

This gives $19\lambda = 133$ $\Rightarrow$ $\lambda = 7$

Hence, $P \equiv (9, -23, 19)$

Also (1) intersect YZ plane i.e. $x = 0$ $\Rightarrow$ $\lambda + 2 = 0$, hence $\lambda = -2$

$\therefore$ Q(0, 4, 1)

So, $PQ = \sqrt{9^2+27^2+18^2} = 9\sqrt{1+3^2+2^2} = 9\sqrt{14}$

$\therefore$ $a = 9$ and $b = 14$

Hence, $a + b = 23$]

Q.9 Let L_1 be the line $\vec{r}_1 = 2\hat{i}+\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{k})$ and let L_2 be the line $\vec{r}_2 = 3\hat{i}+\hat{j}+\mu(\hat{i}+\hat{j}-\hat{k})$. Let Π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane Π from the origin is

(A) 1/7 (B*) $\sqrt{2/7}$ (C) $\sqrt{6}$ (D) none

[Sol. Equation of the plane containing L_1

$A(x-2) + B(y-1) + C(z+1) = 0$

where $A + 2C = 0$ also, $A + B - C = 0$

$\Rightarrow$ $A = -2C$, $B = 3C$, $C = C$

plane is $-2(x-2) + 3(y-1) + z + 1 = 0$

$-2x + 3y + z + 4 - 3 + 1 = 0$

$2x - 3y - z - 2 = 0$

Hence $p = \left|\frac{-2}{\sqrt{14}}\right| = \sqrt{\frac{2}{7}}$ Ans.] **[29-01-2005, 12th & 13th]**

Q.10 The value of m for which straight line $3x - 2y + z + 3 = 0 = 4x - 3y + 4z + 1$ is parallel to the plane $2x - y + mz - 2 = 0$ is
(A*) –2 (B) 8 (C) – 18 (D) 11

[Hint: Vector $\left((3\hat{i} - 2\hat{j} + \hat{k}) \times (4\hat{i} - 3\hat{j} + 4\hat{k})\right)$ is perpendicular to $2\hat{i} - \hat{j} + m\hat{k} \Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ 4 & -3 & 4 \\ 2 & -1 & m \end{vmatrix} = 0$]

Q.11 A straight line is given by $\vec{r} = (1+t)\hat{i} + 3t\hat{j} + (1-t)\hat{k}$ where $t \in R$. If this line lies in the plane $x + y + cz = d$ then the value of $(c + d)$ is
(A) – 1 (B) 1 (C) 7 (D*) 9

[Sol. Equation of line is **[12th 15-10-2006]**

$\vec{r} = \hat{i} + 0\hat{j} + \hat{k} + t(\hat{i} + 3\hat{j} - \hat{k})$ .....(1)

(1) lies in $x + y + cz = d$ ....(2)

$\therefore$ $1 + 0 + c = d \Rightarrow 1 + c = d$

also $1 \cdot 1 + 1 \cdot 3 + c(-1) = 0$

$c = 4 \Rightarrow d = 5 \Rightarrow (c + d) = 9$ Ans.]

Q.12 The distance of the point (–1, –5, – 10) from the point of intersection of the line $\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ is
(A) $2\sqrt{11}$ (B) $\sqrt{126}$ (C*) 13 (D) 14

[Sol. Any point on $\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12}$ can be (2r + 2, 4r – 1, 12r + 2)

which lies on $x - y + z = 5$ **[Test-III, Paper-2, Apex 2007]**

$\therefore$ $(2r + 2) - (4r - 1) + 12r + 2 = 5$

$\Rightarrow$ $r = 0$

$\therefore$ Point on the plane $\equiv$ (2, – 1, 2)

Distance between (2, – 1, 2) and (–1, –5, – 10) = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$ = 13 Ans.]

Q.13 $P(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the position vector of a variable point. If R moves such that $(\vec{r} - \vec{p}) \times (\vec{r} - \vec{q}) = 0$ then the locus of R is

(A) a plane containing the origin 'O' and parallel to two non collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$
(B) the surface of a sphere described on PQ as its diameter.
(C*) a line passing through the points P and Q
(D) a set of lines parallel to the line PQ.

[Hint: Obviously **(C)**; $R(\vec{r})$ moves on PQ $\quad \overset{R(\vec{r})}{P(\vec{p}) \text{———————} Q(\vec{q})}$] **[12th, 09-11-2008]**

MATCH THE COLUMN:

Q.14 Consider the following four pairs of lines in **column-I** and match them with one or more entries in **column-II**.

	Column-I		**Column-II**
(A)	$L_1 : x = 1 + t,\ y = t,\ z = 2 - 5t$ $L_2 : \vec{r} = (2,1,-3) + \lambda(2, 2, -10)$	(P)	non coplanar lines
(B)	$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ $L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$	(Q)	lines lie in a unique plane
(C)	$L_1 : x = -6t,\ y = 1 + 9t,\ z = -3t$ $L_2 : x = 1 + 2s,\ y = 4 - 3s,\ z = s$	(R)	infinite planes containing both the lines
(D)	$L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ $L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$	(S)	lines are not intersecting

[Ans. (A) R, (B) Q, (C) Q, S, (D) P, S]

[Sol. (A) $L_1 : \frac{x-1}{1} = \frac{y-0}{1} = \frac{z-2}{-5}$; $\vec{V}_1 = \hat{i} + \hat{j} - 5\hat{k}$ **[12th, 21-10-2007]**

$L_2 : \frac{x-2}{2} = \frac{y-1}{2} = \frac{z+3}{-10}$; $\vec{V}_2 = 2(\hat{i} + \hat{j} - 5\hat{k})$

Hence lines are parallel and both contains the points (1, 0, 2) and (2, 1, – 3) ⇒ coincident line both L_1 and L_2 may lie in an infinite number of planes hence ⇒ **(R)**

(B) $\left.\begin{aligned}\vec{V}_1 &= 2\hat{i} + 2\hat{j} - \hat{k} \\ \vec{V}_2 &= \hat{i} - 2\hat{j} + 3\hat{k}\end{aligned}\right]$ ⇒ lines not parallel

Also both intersect at (3, 5, 1)

Hence lines are intersecting hence they lie on a unique plane ⇒ **(P)**

(C) $L_1 : \frac{x-0}{-6} = \frac{y-1}{9} = \frac{z-0}{-3} = t$

$L_2 : \frac{x-1}{2} = \frac{y-4}{-3} = \frac{z-0}{1} = s$

$\left.\begin{aligned}&L_1 \text{ is parallel to } -6\hat{i} + 9\hat{j} - 3\hat{k} \\ &L_2 \text{ is parallel to } 2\hat{i} - 3\hat{j} + \hat{k}\end{aligned}\right|$ ⇒ lines parallel but not conicident

as (0, 1, 0) does not line on L_2, not intersecting

Hence L_1, L_2 lies in a unique planes ⇒ (Q), (S)

(D) Lines are skew can be verified ⇒ (P), (S)]

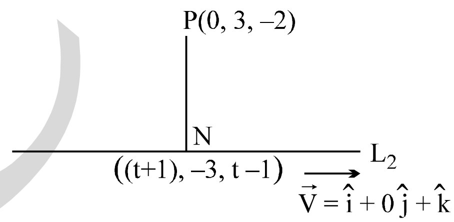
Q.15$_{65/mc}$ P(0, 3, – 2); Q(3, 7, – 1) and R(1, – 3, – 1) are 3 given points. Let L_1 be the line passing through P and Q and L_2 be the line through R and parallel to the vector $\vec{V} = \hat{i} + \hat{k}$.

	Column-I		**Column-II**
(A)	perpendicular distance of P from L_2	(P)	$7\sqrt{3}$
(B)	shortest distance between L_1 and L_2	(Q)	2
(C)	area of the triangle PQR	(R)	6
(D)	distance from (0, 0, 0) to the plane PQR	(S)	$\frac{19}{\sqrt{147}}$

[Ans. (A) R; (B) Q; (C) P ; (D) S]

[Sol. $L_1 : \frac{x}{3} = \frac{y-3}{4} = \frac{z+2}{1}$ ....(1)

(passing through P and Q)



$L_2: \frac{x-1}{1} = \frac{y-3}{0} = \frac{z+1}{1}$ ....(2)

(passing through R and parallel to $\vec{v} = \hat{i} + \hat{k}$) **[13th, 16-12-2007]**

(A) distance of P(0, 3, – 2) from L_2

$\overrightarrow{PN} = (t+1)\hat{i} - 6\hat{j} + 2(t-1)\hat{k}$

now $\overrightarrow{PN} \cdot \vec{V} = 0 \Rightarrow [(t+1)\hat{i} - 6\hat{j} + (t+1)\hat{k}] \cdot (\hat{i} + \hat{k}) = 0$

$(t + 1) + (t + 1) = 0 \Rightarrow t = - 1$

hence $\overrightarrow{PN} = -6\hat{j}$

$|\overrightarrow{PN}| = |-6\hat{j}| = 6$ **Ans.** $\Rightarrow$ **(R)**

Distance between L_1 and L_2

Equation of plane containing L_1 and parallel to L_2

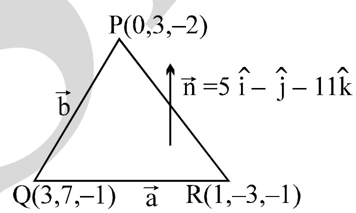
$Ax + B(y - 3) + C(z + 2) = 0$

where $3A + 4B + C = 0$

and $A + 0B + C = 0 \Rightarrow A + C = 0$

$C = \lambda, A = - \lambda, B = + \lambda/2$

$\therefore$ equation of plane



$- \lambda x + \frac{\lambda}{2}(y - 3) + \lambda(z + 2) = 0$

$2x - y + 3 - 2z - 4 = 0$

$2x - y - 2z = 1$ ....(1)

now distance of the point (1, – 3, – 1) lying on the line L_2 from the plane (1)

$d = \left|\frac{2+3+2-1}{3}\right| = 2$ Ans. $\Rightarrow$ **(Q)**

Area of ΔPQR

$\overrightarrow{QR} = \vec{a} = 2\hat{i} + 10\hat{j} + 0\hat{k}$

$\overrightarrow{QP} = \vec{b} = 3\hat{i} + 4\hat{j} + \hat{k}$

$$\vec{a}\times\vec{b} = 2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 5 & 0\\ 3 & 4 & 1\end{vmatrix} = 2[\hat{i}(5)-\hat{j}(1)+\hat{k}(4-15)] = 2[5\hat{i}-\hat{j}-11\hat{k}]$$

$$\frac{|\vec{a}\times\vec{b}|}{2} = \sqrt{25+1+121} = \sqrt{147} = \sqrt{3\cdot 49} = 7\sqrt{3} \quad \textbf{Ans.} \quad \Rightarrow \quad \textbf{(P)}$$

<u>Distance of (0, 0, 0) from PQR</u>

equation of plane PQR is

$(\vec{r}-\vec{p})\cdot\vec{n}$

$= [x\hat{i}+(y-3)\hat{j}+(z+2)\hat{k}]\cdot[5\hat{i}-\hat{j}-11\hat{k}]$

$= 5x-(y-3)-11(z+2) = 0$

$= 5x - y - 11z - 19 = 0$

distance from (0, 0, 0) of the plane

$$d = \left|\frac{19}{\sqrt{25+1+121}}\right| = \frac{19}{\sqrt{147}} \quad \textbf{Ans.} \quad \Rightarrow \quad \textbf{(S)}]$$

Q.1 If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar & $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors to $\vec{a}$, $\vec{b}$ & $\vec{c}$ respectively, then $\left(\ell\vec{a} + m\vec{b} + n\vec{c}\right)\cdot\left(\ell\vec{p} + m\vec{q} + n\vec{r}\right)$ is equal to : (where l, m, n are scalars)

(A*) $l^2 + m^2 + n^2$ (B) $l m + m n + n l$ (C) 0 (D) none of these

[Hint : $\vec{p} = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{q} = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{r} = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

Substitute the values of $\vec{p}, \vec{q}, \vec{r}$ to get the result]

Q.2 If $\vec{A}, \vec{B}$ & $\vec{C}$ are three non-coplanar vectors, then $(\vec{A} + \vec{B} + \vec{C})\cdot[(\vec{A} + \vec{B})\times(\vec{A} + \vec{C})]$ equals

(A) 0 (B) $[\vec{A}\,\vec{B}\,\vec{C}]$ (C) 2 $[\vec{A}\,\vec{B}\,\vec{C}]$ (D*) $-$ $[\vec{A}\,\vec{B}\,\vec{C}]$

[12th 17-9-2006] [JEE '95,1]

Q.3 A plane P_1 has the equation $2x - y + z = 4$ and the plane P_2 has the equation $x + ny + 2z = 11$. If the angle between P_1 and P_2 is $\dfrac{\pi}{3}$ then the value(s) of 'n' is (are)

(A) 7/2 (B) 17, –1 (C*) –17, 1 (D) – 7/2

[Hint: $\vec{n}_1 = 2\hat{i} - \hat{j} + \hat{k}$; $\vec{n}_2 = \hat{i} + n\hat{j} + 2\hat{k}$; $\cos\dfrac{\pi}{3} = \dfrac{2 - n + 2}{\sqrt{6}\sqrt{5 + n^2}} = \dfrac{1}{2}$]

Q.4 The three vectors $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$, $\hat{k} + \hat{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to these three planes form a parallelopiped of volume :

(A) 1/3 (B) 4 (C) $3\sqrt{3}/4$ (D*) $4/3\sqrt{3}$

[Hint : $\left(\hat{i} + \hat{j}\right)\times\left(\hat{j} + \hat{k}\right) = \hat{i} - \hat{j} + \hat{k}$ $\Rightarrow$ unit vector perpendicular as to the plane of $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is $\dfrac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$ similarly other two unit vectors are

$$\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} - \hat{k}\right) \text{ and } \frac{1}{\sqrt{3}}\left(-\hat{i} + \hat{j} + \hat{k}\right) \Rightarrow V = \left[\hat{n}_1\ \hat{n}_2\ \hat{n}_3\right] = \frac{1}{3\sqrt{3}}\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{4}{3\sqrt{3}}$$

Alternatively : Let $\vec{a} = \hat{i} + \hat{j}$; $\vec{b} = \hat{j} + \hat{k}$ & $\vec{c} = \hat{k} + \hat{i}$.

$$\text{Now } \left[\vec{a}\times\vec{b}\ ,\ \vec{b}\times\vec{c}\ ,\ \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}^2 = \left[1\,(1) - 1\,(0 - 1)\right]^2 = 4$$

Hence actual volume with unit vectors = $\dfrac{4}{\left|\vec{a}\times\vec{b}\right|\left|\vec{b}\times\vec{c}\right|\left|\vec{c}\times\vec{a}\right|}$

Now $\left|\vec{a}\times\vec{b}\right| = \sqrt{\vec{a}^2\ \vec{b}^2 - (\vec{a}.\vec{b})^2} = \sqrt{4 - 1} = \sqrt{3}$ etc $V_{actual} = \dfrac{4}{3\sqrt{3}}$]

Q.5 If $\vec{x}$ & $\vec{y}$ are two non collinear vectors and a, b, c represent the sides of a Δ ABC satisfying $(a-b)\vec{x} + (b-c)\vec{y} + (c-a)(\vec{x}\times\vec{y}) = 0$ then Δ ABC is

(A*) an acute angle triangle (B) an obtuse angle triangle

(C) a right angle triangle (D) a scalene triangle

[Hint : as $\vec{x}$, $\vec{y}$ and $\vec{x}\times\vec{y}$ are non coplanar vectors $\Rightarrow$ linearly independent

therefore $a-b=0=b-c=c-a \Rightarrow a=b=c \Rightarrow \Delta$ is equilateral $\Rightarrow$ A]

Q.6 Given three non – zero, non – coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{r_1} = p\vec{a} + q\vec{b} + \vec{c}$ and $\vec{r_2} = \vec{a} + p\vec{b} + q\vec{c}$ if the vectors $\vec{r_1} + 2\vec{r_2}$ and $2\vec{r_1} + \vec{r_2}$ are collinear then (p, q) is

(A) (0, 0) (B) (1, –1) (C) (–1, 1) (D*) (1, 1)

[Hint: $\vec{r_1} + 2\vec{r_2} = (p\vec{a}+q\vec{b}+\vec{c}) + 2(\vec{a}+p\vec{b}+q\vec{c})$

$\vec{r_1} + 2\vec{r_2} = (p+2)\vec{a} + (q+2p)\vec{b} + (1+2q)\vec{c}$

$2\vec{r_1} + \vec{r_2} = (2p+1)\vec{a} + (2q+p)\vec{b} + (2+q)\vec{c}$

$\frac{p+2}{2p+1} = \frac{q+2p}{2q+p} = \frac{1+2q}{2+q} = \frac{p+q+2p+2q+3}{p+q+2p+2q+3} = 1 \Rightarrow p = 1$ & $q = 1$]

Q.7 If the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar and l, m, n are distinct scalars, then

$$\left[\left(\ell\vec{a} + m\vec{b} + n\vec{c}\right)\left(\ell\vec{b} + m\vec{c} + n\vec{a}\right)\left(\ell\vec{c} + m\vec{a} + n\vec{b}\right)\right] = 0 \text{ implies :}$$

(A) $lm + mn + nl = 0$ (B*) $l + m + n = 0$

(C) $l^2 + m^2 + n^2 = 0$ (D) $l^3 + m^3 + n^3 = 0$

[Hint: $\left.\begin{array}{l}\vec{V_1} = l\vec{a} + m\vec{b} + n\vec{c}\\ \vec{V_2} = n\vec{a} + l\vec{b} + m\vec{c}\\ \vec{V_3} = m\vec{a} + n\vec{b} + l\vec{c}\end{array}\right\}$ when $\vec{a}, \vec{b}, \vec{c}$ are non – coplanar

$\therefore [\vec{V_1}\,\vec{V_2}\,\vec{V_3}] = \begin{vmatrix} l & m & n \\ n & l & m \\ m & n & l \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}] = 0;$ But $[\vec{a}\,\vec{b}\,\vec{c}] \neq 0 \Rightarrow \begin{vmatrix} l & m & n \\ n & l & m \\ m & n & l \end{vmatrix} = 0 \ \forall\ \lambda \in R$

or $(l+m+n)\,[\,(l-m)^2 + (m-n)^2 + (n-l)^2\,] = 0 \Rightarrow l + m + n = 0$]

Q.8 Let $\vec{r_1}, \vec{r_2}, \vec{r_3}........\vec{r_n}$ be the position vectors of points $P_1, P_2, P_3,.....P_n$ relative to the origin O. If the vector equation $a_1\vec{r_1} + a_2\vec{r_2} + ... + a_n\vec{r_n} = 0$ holds, then a similar equation will also hold w.r.t. to any other origin provided

(A) $a_1 + a_2 + ... + a_n = n$ (B) $a_1 + a_2 + ... + a_n = 1$

(C*) $a_1 + a_2 + ... + a_n = 0$ (D) none

[Sol. Given $a_1\vec{r_1} + a_2\vec{r_2} + ... + a_n\vec{r_n} = 0$ **[12th (18-12-2005)]**

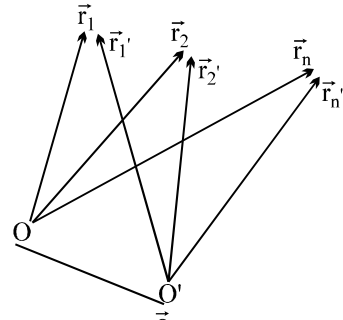
now $\vec{a} + \vec{r_1}' = \vec{r_1}$ and so on

hence $a_1(\vec{a} + \vec{r_1}') + a_2(\vec{a} + \vec{r_2}') + ... + a_n(\vec{a} + \vec{r_n}') = 0$

$a_1\vec{r_1}' + a_2\vec{r_2}' + ... + a_n\vec{r_n}' + \vec{a}(a_1 + a_2 + ... + a_n) = 0$

hence $a_1\vec{r_1}' + a_2\vec{r_2}' + ... + a_n\vec{r_n}' = 0$

if $a_1 + a_2 + ... a_n = 0$]



Q.9 The orthogonal projection A' of the point A with position vector (1, 2, 3) on the plane $3x - y + 4z = 0$ is

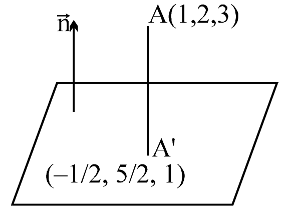
(A) (–1, 3, –1) (B*) $\left(-\frac{1}{2}, \frac{5}{2}, 1\right)$ (C) $\left(\frac{1}{2}, -\frac{5}{2}, -1\right)$ (D) (6, –7, –5)

[Sol. $\vec{n} = 3\hat{i} - \hat{j} + 4\hat{k}$ **[12th (27-11-2005)]**

line through A are parallel to $\vec{n}$ is

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + 4\hat{k})$

$= 3\lambda + 1, 2 - \lambda, 3 + 4\lambda$ ....(1)



Hence (1) must satisfy the plane $3x - y + 4z = 0$

$3(3\lambda + 1) - (2 - \lambda) + 4(3 + 4\lambda) = 0$

$26\lambda + 13 = 0 \Rightarrow \lambda = -\frac{1}{2}$

Hence A' is $\left(-\frac{1}{2}, \frac{5}{2}, 1\right)$ which is the foot of the perpendicular from A on the given plane]

Paragraph for Question Nos. 10 to 11

Consider a plane

$x + y - z = 1$ and the point A(1, 2, –3)

A line L has the equation

$x = 1 + 3r$
$y = 2 - r$
$z = 3 + 4r$

Q.10 The co-ordinate of a point B of line L, such that AB is parallel to the plane, is

(A) 10, –1, 15 (B) –5, 4, –5 (C) 4, 1, 7 (D*) –8, 5, –9

[Sol. line $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = r$ **[12th (27-11-2005)]**

Any point say $B \equiv 3r + 1, 2 - r, 3 + 4r$ (on the line L)

$\overline{AB} = 3r, -r, 4r + 6$

Hence, $\overline{AB}$ is parallel to $x + y - z = 1$

hence, $3r - r - 4r - 6 = 0$

$2r = -6 \Rightarrow r = -3$

hence B is –8, 5, –9]

Q.11 Equation of the plane containing the line L and the point A has the equation

(A) $x - 3y + 5 = 0$ (B*) $x + 3y - 7 = 0$ (C) $3x - y - 1 = 0$ (D) $3x + y - 5 = 0$

[Sol. Equation of plane containing the line L is

$A(x - 1) + B(y - 2) + C(z - 3) = 0$, where $3A - B + 4C = 0$ ....(1) **[12th (27-11-2005)]**

$\therefore$ (1) also contains the point A(1, 2, –3)

hence C = 0; 3A = B

equation of plane $x - 1 + 3(y - 2) = 0$

$x + 3y - 7 = 0$ Ans.]

Paragraph for Question Nos. 12 to 15

Consider a triangular pyramid ABCD the position vectors of whose angular points are A(3, 0, 1) ; B(–1, 4, 1); C(5, 2, 3) and D(0, –5, 4). Let G be the point of intersection of the medians of the triangle BCD.

Q.12 The length of the vector $\overrightarrow{AG}$ is

(A) $\sqrt{17}$ (B*) $\sqrt{51}/3$ (C) $\sqrt{51}/9$ (D) $\sqrt{59}/4$

Q.13 Area of the triangle ABC in sq. units is

(A) 24 (B) $8\sqrt{6}$ (C*) $4\sqrt{6}$ (D) none

Q.14 The length of the perpendicular from the vertex D on the opposite face is

(A*) $14/\sqrt{6}$ (B) $2/\sqrt{6}$ (C) $3/\sqrt{6}$ (D) none

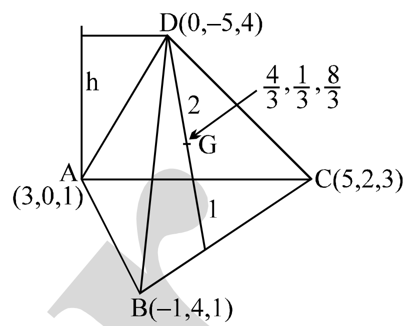
Q.15 Equation of the plane ABC is

(A) x + y + 2z = 5 (B) x – y – 2z = 1 (C) 2x + y – 2z = 4 (D*) x + y – 2z = 1

[Sol. $\left|\overrightarrow{AG}\right|^2 = \left(\frac{5}{3}\right)^2 + \frac{1}{9} + \left(\frac{5}{3}\right)^2 = \frac{51}{8} \Rightarrow \left|\overrightarrow{AG}\right| = \frac{\sqrt{51}}{3}$ **Ans.** **[12th (27-11-2005)]**

$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} + 0\hat{k}$

$\overrightarrow{AC} = 2\hat{i} + 2\hat{j} + 2\hat{k}$



$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = -8\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= -8(-\hat{i} - \hat{j} + 2\hat{k}) = 8(\hat{i} + \hat{j} - 2\hat{k}) = \vec{n}$

$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = 4\sqrt{6}$ **Ans.**

h = | projection of $\overrightarrow{AD}$ on $\vec{n}$ | ; $\overrightarrow{AD} = -3\hat{i} - 5\hat{j} + 3\hat{k}$

$= \left|\frac{\overrightarrow{AD}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{(-3\hat{i} - 5\hat{j} + 3\hat{k})(\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{6}}\right| = \left|\frac{-3-5-6}{\sqrt{6}}\right| = \frac{14}{\sqrt{6}}$ **Ans.**

Equation of the plane ABC

A(x – 3) + By + (z – 1) = 0, where A = 1, B = 1, c = – 2

$\therefore$ x – 3 + y – 2z + 2 = 0

x + y – 2z = 1 **Ans.**]

Paragraph for Question Nos. 16 to 18

The equation of line: $\frac{x-x'}{a'} = \frac{y-y'}{b'} = \frac{z-z'}{c'}$

The equation of plane : $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

Equation of plane through the intersection of the two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$:

$(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$

Q.16 The distance of the point (1, – 2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-3}{-4}$ is

(A) $\sqrt{21}/5$ (B*) $\sqrt{29}/5$ (C) $\sqrt{13}/5$ (D) $2/\sqrt{5}$

[Sol. $x - y + z = 5$ ....(1)

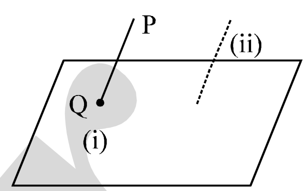
$\frac{x}{2} = \frac{y}{3} = \frac{z-3}{-4}$ ....(2)

any line passing through $P \equiv (1, -2, 3)$ and parallel to (2) is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-4}$ ....(3)

any point on (3) is given by $Q \equiv (2r+1, 3r-2, -4r+3)$

If this lies on (1) we get

$(2r+1) - (3r-2) + (-4r+3) = 5 \Rightarrow r = \frac{1}{5}$



$\therefore \quad Q = \left(\frac{7}{5}, -\frac{7}{5}, \frac{11}{5}\right)$

$(PQ)^2 = \left(1-\frac{7}{5}\right)^2 + \left(-2+\frac{7}{5}\right)^2 + \left(3-\frac{11}{5}\right)^2 = \frac{29}{25} \Rightarrow PQ = \frac{1}{5}\sqrt{29}$ Ans.]

Q.17 The equation of the plane through (0, 2, 4) and containing the line $\frac{x+3}{3} = \frac{y-1}{4} = \frac{z-2}{-2}$ is

(A) x – 2y + 4z – 12 = 0
(B) 5x + y + 9z – 38 = 0
(C*) 10x – 12y – 9z + 60 = 0
(D) 7x + 5y – 3z + 2 = 0

[Sol. Let the equation of plane passing through (0, 2, 4) is **[Test-IV, Paper-II, Apex 2007]**

$a(x-0) + b(y-2) + c(z-4) = 0$ ....(1)

(1) contain the given line then, $3a + 4b - 2c = 0$ ....(2)

If (–3, 1, 2) of a given line lies on (1) then,

$-3a - b - 2c = 0$ or $3a + b + 2c = 0$ ....(3)

solving (2) and (3) we get

$$\frac{a}{10} = \frac{b}{-12} = \frac{c}{-9}$$

substituting in (1) we get

$10x - 12y - 9z + 60 = 0$ Ans.]

Q.18 The plane x – y – z = 2 is rotated through 90° about its line of intersection with the plane x + 2y + z = 2. Then equation of this plane in new position is

(A*) 5x + 4y + z – 10 = 0
(B) 4x + 5y – 3z = 0
(C) 2x + y + 2z = 9
(D) 3x + 4y – 5z = 9

[Sol. $x - y - z = 2$ ....(1) **[Test-IV, Paper-II, Apex 2007]**

$x + 2y + z = 2$ ....(2)

Required plane passes through the common line of (1) and (2)

$\therefore$ its equation is given by

$x - y - z - 2 + k(x + 2y + z - 2) = 0$

$(1 + k)x + (-1 + 2k)y + (-1 + k)z - 2 - 2k = 0$ ....(3)

As (1) and (3) are perpendicular

$\therefore$ $1(1 + k) + (-1)(-1 + 2k) + (-1)(-1 + k) = 0 \Rightarrow k = \frac{3}{2}$

Substituting value of k in (3), we get

$5x + 4y + z - 10 = 0$ Ans.]

Paragraph for Question Nos. 19 to 21

Consider the three vectors $\vec{p}, \vec{q}$ and $\vec{r}$ such that

$\vec{p} = \hat{i} + \hat{j} + \hat{k}$; $\vec{q} = \hat{i} - \hat{j} + \hat{k}$

$\vec{p} \times \vec{r} = \vec{q} + c\vec{p}$ and $\vec{p} \cdot \vec{r} = 2$

Q.19 The value of $[\vec{p}\ \vec{q}\ \vec{r}]$ is

(A) $-\frac{5\sqrt{2}c}{|\vec{r}|}$ (B*) $-\frac{8}{3}$ (C) 0 (D) greater then zero

[Sol. Given $\vec{p} = \hat{i} + \hat{j} + \hat{k}$

$\vec{q} = \hat{i} - \hat{j} + \hat{k}$

$\vec{p} \times \vec{r} = \vec{q} + c\vec{p}$ and $\vec{p} . \vec{r} = 2$

$\therefore$ $\vec{p} \times (\vec{p} \times \vec{r}) = \vec{p} \times (\vec{q} + c\vec{p})$ and $\vec{p} . (\vec{p} \times \vec{r}) = \vec{p} . (\vec{q} + c\vec{p})$

$\therefore$ $(\vec{p} . \vec{r})\vec{p} - (\vec{p} . \vec{p})\vec{r} = \vec{p} \times \vec{q} + c\vec{0}$ $0 = \vec{p} \cdot \vec{q} + c(\vec{p} \cdot \vec{p})$

$\therefore$ $(\vec{p} \cdot \vec{p})\vec{r} = (\vec{p} \cdot \vec{r})\vec{p} - \vec{p} \times \vec{q}$ ... (i) $c = -\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}$ ... (ii)

But $\vec{p} \cdot \vec{p} = |\vec{p}| = p^2 = 3$ ... (iii)

$\vec{p} \cdot \vec{q} = 1 - 1 + 1 = 1$ ... (iv)

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{k} \text{ ... (v)}$$

using (iii), (iv), (v) in (i) and (ii), we get

$3\vec{r} = 2\vec{p} - 2i + 2k$ and $c = -\frac{1}{3}$ ... (vii)

$\therefore$ $\vec{r} = \frac{1}{3}\left[2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + 2\hat{k}\right]$

$\therefore$ $\vec{r} = \frac{2}{3}(\hat{j} + 2\hat{k})$ ... (vi)

Now $[\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & \frac{1}{2} & \frac{4}{3} \end{vmatrix} = \left(\frac{-4}{3} - \frac{2}{3}\right) - 1\left(\frac{4}{3}\right) + 1\left(\frac{2}{3}\right)$

$\therefore \quad [\vec{p}\ \vec{q}\ \vec{r}] = -2 - \frac{2}{5} = -\frac{8}{3}$ ...(viii)]

Q.20 If $\vec{x}$ is a vector such that $[\vec{p}\,\vec{q}\,\vec{r}]\,\vec{x} = (\vec{p} \times \vec{q}) \times \vec{r}$, then $\vec{x}$ is

(A) $c(\hat{i} - 2\hat{j} + \hat{k})$

(B) a unit vector

(C) indeterminate, as $[\vec{p}\,\vec{q}\,\vec{r}]$

(D*) $-\frac{1}{2}(\hat{i} - 2\hat{j} + \hat{k})$

[Sol. $[\vec{p}\ \vec{q}\ \vec{r}]\vec{x} = (\vec{p} \times \vec{q}) \times \vec{r}$

$\Rightarrow \quad \left(-\frac{8}{3}\right)\vec{x} = (\vec{p}\cdot\vec{r})\vec{q} - (\vec{q}\cdot\vec{r})\vec{p}$

or $\quad \left(-\frac{8}{3}\right)\vec{x} = 2\vec{q} - \frac{2\vec{p}}{3} \quad \left(\vec{q}\cdot\vec{r} = \frac{2}{3} \text{ Verify your self}\right)$

$\therefore \quad \vec{x} = -\frac{3}{8}\cdot\frac{2}{3}(3\vec{q} - \vec{p}) = -\frac{1}{4}(3\hat{i} - 3\hat{j} + 3\hat{k} - \hat{i} - \hat{j} - \hat{k}) = -\frac{1}{4}(2\vec{i} - 4\vec{j} + 2\vec{k})$

$\therefore \quad \vec{x} = -\frac{1}{2}(\hat{i} - 2\hat{j} + \hat{k})$]

Q.21 If $\vec{y}$ is a vector satisfying $(1 + c)\,\vec{y} = \vec{p} \times (\vec{q} \times \vec{r})$ then the vectors $\vec{x}, \vec{y}, \vec{r}$

(A) are collinear

(B) are coplanar

(C*) represent the coterminus edges of a tetrahedron whose volume is c cubic units.

(D) represent the coterminus edges of a parallelepiped whose volume is c cubic units

[Sol. As $\quad c = -\frac{1}{3}$ from (vii)

$\therefore \quad \left(1 - \frac{1}{3}\right)\vec{y} = (\vec{p}\cdot\vec{r})\vec{q} - (\vec{p}\cdot\vec{q})\vec{r}$

$\therefore \quad \frac{2}{3}\vec{y} = 2\vec{q} - (1)\vec{r} \qquad (\text{As } \vec{p}\cdot\vec{q} = 1 \text{ from .... (iv)})$

$\therefore \quad \vec{y} = \frac{3}{2}\left(2\hat{i} - 2\hat{j} + 2\hat{k} - \frac{2}{3}\hat{j} - \frac{4}{3}\hat{k}\right) = \frac{3}{2}\,\frac{(6\hat{i} - 8\hat{j} + 2\hat{k})}{3}$

$\vec{y} = 3\hat{i} - 4\hat{j} + \hat{k}$

$\therefore \quad [\vec{x}\ \vec{y}\ \vec{r}] = \begin{vmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ 3 & -4 & 1 \\ 0 & \frac{2}{3} & \frac{4}{3} \end{vmatrix} = -\frac{1}{2}\left(-\frac{16}{3} - \frac{2}{3}\right) - 1\,(4) - \frac{1}{2}(2)$

$\therefore \quad [\vec{x}\ \vec{y}\ \vec{r}] = 3 - 4 - 1 = -2$

$$\therefore \quad \left|\frac{1}{6}[\vec{x}\ \vec{y}\ \vec{r}]\right| = \left|-\frac{1}{3}\right| = |c|$$

$\Rightarrow$ $\vec{x}, \vec{y}, \vec{r}$ are the coterminus edges of a tetrahedron whose volume is $|c|$.]

[REASONING TYPE]

Q.22 Given lines $\frac{x-4}{2} = \frac{y+5}{4} = \frac{z-1}{-3}$ and $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z}{2}$

Statement-1: The lines intersect.
because
Statement-2: They are not parallel.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D*) Statement-1 is false, statement-2 is true.

[Hint: L_1 and L_2 are obviously not parallel **[12th, 28-09-2008]**
Consider the determinant

$$D = \begin{vmatrix} 2 & -4 & 1 \\ 2 & 4 & -3 \\ 1 & 3 & 2 \end{vmatrix} = 2(8+9) + 4(4+3) + 1(6-4) = 34 + 28 + 2 \Rightarrow D \neq 0 \Rightarrow \text{skew}$$

Hence S-1 is false]

Q.23 Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement-1: $\vec{a}\times\vec{b} = \left((\hat{i}\times\vec{a})\cdot\vec{b}\right)\hat{i} + \left((\hat{j}\times\vec{a})\cdot\vec{b}\right)\hat{j} + \left((\hat{k}\times\vec{a})\cdot\vec{b}\right)\hat{k}$
because
Statement-2: $\vec{c} = (\hat{i}\cdot\vec{c})\,\hat{i} + (\hat{j}\cdot\vec{c})\,\hat{j} + (\hat{k}\cdot\vec{c})\,\hat{k}$
(A*) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[Hint: think ! obvious] **[12th, 28-09-2008]**

[MULTIPLE OBJECTIVE TYPE]

Select the correct alternative(s): (More than one are correct)

Q.24 If $A(\bar{a}); B(\bar{b}); C(\bar{c})$ and $D(\bar{d})$ are four points such that

$\bar{a} = -2\hat{i}+4\hat{j}+3\hat{k}$; $\bar{b} = 2\hat{i}-8\hat{j}$; $\bar{c} = \hat{i}-3\hat{j}+5\hat{k}$; $\bar{d} = 4\hat{i}+\hat{j}-7\hat{k}$

d is the shortest distance between the lines AB and CD, then which of the following is True?
(A) d = 0, hence AB and CD intersect

(B*) $d = \frac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{BD}]}{|\overrightarrow{AB}\times\overrightarrow{CD}|}$

(C*) AB and CD are skew lines and $d = \frac{23}{13}$

(D*) $d = \frac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{AC}]}{|\overrightarrow{AB}\times\overrightarrow{CD}|}$

[Sol. $\overrightarrow{AB} = \bar{b} - \bar{a} = 4\hat{i} - 12\hat{j} - 3\hat{k}$

$\overrightarrow{CD} = \bar{d} - \bar{c} = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\overrightarrow{AC} = \bar{c} - \bar{a} = 3\hat{i} - 7\hat{j} + 2\hat{k}$

$\overrightarrow{BD} = \bar{d} - \bar{b} = 2\hat{i} + 9\hat{j} - 7\hat{k}$

By definition $d = \dfrac{(\overrightarrow{AB} \times \overrightarrow{CD}) \cdot \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$ ... (i)

$= \dfrac{(\overrightarrow{AB} \times \overrightarrow{CD}) \cdot \overrightarrow{BD}}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$ ... (ii)

$\overrightarrow{AB} \times \overrightarrow{CD} = 13(12\hat{i} + 3\hat{j} + 4\hat{k})$ $\quad \therefore \quad |\overrightarrow{AB} \times \overrightarrow{CD}| = 169$

$\therefore \quad d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (3\hat{i} - 7\hat{j} + 2\hat{k}) = \dfrac{23}{13}$ using (i)

also $\quad d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (2\hat{i} - 9\hat{j} + 7\hat{k}) = \dfrac{23}{13}$ using (ii)

Q.25 Consider four points $A(\bar{a})$; $B(\bar{b})$; $C(\bar{c})$ and $D(\bar{d})$, such that

$\overline{GA} + \overline{GB} + \overline{GC} + \overline{GD} = \bar{0}$ for a point $G(\bar{g})$, if

(A*) G is the centroid of the tetrahedron ABCD

(B*) G lies on the line joining each of A, B, C, D to the centroid of the triangle formed by the other three

(C) p.v. of G is collinear with the p.v. of the centroids of the triangle formed by any three of the four given points.

(D*) □ABCD is parallelogram with G as the point of intersection of the diagonals AC and BD.

[Sol. Given $\overline{GA} + \overline{GB} + \overline{GC} + \overline{GD} = \bar{0}$

$\Rightarrow \quad \bar{a} - \bar{g} + \bar{b} - \bar{g} + \bar{c} - \bar{g} + \bar{d} - \bar{g} = \bar{0}$

$\Rightarrow \quad \bar{g} = \dfrac{\bar{a} + \bar{b} + \bar{c} + \bar{d}}{4}$ ... (i)

If G is the centroid of the tetrahedron ABCD then (i) holds

If $\quad G_1(\bar{g}_1)$ is the centroid of Δ ABC

$G_2(\bar{g}_2)$ is the centroid of Δ BCD

$G_3(\bar{g}_3)$ is the centroid of Δ CDA

$G_4(\bar{g}_4)$ is the centroid of Δ ABD

Then, $\bar{g}_1 = \dfrac{\bar{a} + \bar{b} + \bar{c}}{3}$; $\bar{g}_2 = \dfrac{\bar{b} + \bar{c} + \bar{d}}{3}$; $\bar{g}_3 = \dfrac{\bar{c} + \bar{d} + \bar{a}}{3}$; $\bar{g}_4 = \dfrac{\bar{a} + \bar{b} + \bar{d}}{3}$

$\therefore \quad$ equation of G_1D is $\vec{r} = t_1\bar{g}_1 + (1 - t_1)\bar{d}$

equation of G_2A is $\vec{r} = t_2\bar{g}_2 + (1 - t_2)\bar{a}$

equation of G_3B is $\vec{r} = t_3\bar{g}_3 + (1-t_3)\bar{b}$

equation of G_4C is $\vec{r} = t_4\bar{g}_4 + (1-t_4)\bar{c}$

For $t_1 = t_2 = t_3 = t_4 = \frac{3}{4}$

G lies on $G_1D : G_2A, G_3B, G_4C$ think !

Also G cannot be collinear with G_1, G_2, G_3 and finally for a parallelogram ABCD

$$\bar{g} = \frac{\bar{a}+\bar{c}}{2} \text{ and } \vec{g} = \frac{\bar{b}+\bar{d}}{2} \Rightarrow 2\vec{g} = 2\vec{g} = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{2}$$

$\Rightarrow \quad \bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}$ which staisfies (i)]

Q.26 Given the equations of the line $3x - y + z + 1 = 0,\ 5x + y + 3z = 0$.
Then which of the following is correct ?

(A) symmetical form of the equations of line is $\frac{x}{2} = \frac{y-\frac{1}{8}}{-1} = \frac{z+\frac{5}{8}}{1}$

(B*) symmetrical form of the equations of line is $\frac{x+\frac{1}{8}}{1} = \frac{y-\frac{5}{8}}{1} = \frac{z}{-2}$

(C) equation of the plane through (2, 1, 4) and prependicular to the given lines is $2x - y + z - 7 = 0$

(D*) equation of the plane through (2, 1, 4) and prependicular to the given lines is $x + y - 2z + 5 = 0$

Sol. Let a, b, c be the d.r's. of the given line. Then

we have $\quad 3a - b + c = 0$

$\qquad\qquad 5a + b + 3c = 0$

Solving we get $\frac{a}{1} = \frac{b}{1} = \frac{c}{-2}$

Again, suppose the given line intersect the plane z = 0 at $(x_1, y_1, 0)$ then $3x_1 - y_1 + 1 = 0$ and $5x_1 + y_1 = 0$

Solving we get $x_1 = -\frac{1}{8}, y_1 = \frac{5}{8}$

Hence the symmetrical form of the line is $\frac{x+\frac{1}{8}}{1} = \frac{y-\frac{5}{8}}{1} = \frac{z}{-2}$

Equation of plane through (2, 1, 4) is

$A(x-2) + B(y-1) + c(z-4) = 0$

when $A = l,\ b = 1$ and $c = -2l$

$\Rightarrow \quad x - 2 + y - 1 - 2(z-4) = 0 \quad \Rightarrow \quad x + y - 2z + 5 = 0$]

Q.27 Given three vectors

$\vec{U} = 2\hat{i} + 3\hat{j} - 6\hat{k}$; $\vec{V} = 6\hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{W} = 3\hat{i} - 6\hat{j} - 2\hat{k}$

Which of the following hold good for the vectors $\vec{U}$, $\vec{V}$ and $\vec{W}$?

(A) $\vec{U}$, $\vec{V}$ and $\vec{W}$ are linearly depedent

(B*) $(\vec{U} \times \vec{V}) \times \vec{W} = \vec{0}$

(C*) $\vec{U}$, $\vec{V}$ and $\vec{W}$ form a triplet of mutually perpendicular vectors

(D*) $\vec{U} \times (\vec{V} \times \vec{W}) = \vec{0}$

[Sol. If may be observed that $[\vec{U}\,\vec{V}\,\vec{W}] = \begin{vmatrix} 2 & 3 & -6 \\ 6 & 2 & 3 \\ 3 & -6 & -2 \end{vmatrix} = 343 \neq 0$

$\Rightarrow$ $\vec{U}, \vec{V}, \vec{W}$ are non coplanar hence linearly independent

Further $\vec{U} \times \vec{V} = \vec{W}$ and $\vec{V} \times \vec{W} = \vec{U}$

$\Rightarrow$ They form a right handed triplet of mutually perpendicular vectors and of course!

$(\vec{U} \times \vec{V}) \times \vec{W} = \vec{0} = \vec{U} \times (\vec{V} \times \vec{W})$]

Q.28 Consider the family of planes $x + y + z = c$ where c is a parameter intersecting the coordinate axes at P, Q, R and α, β, γ are the angles made by each member of this family with positive x, y and z axis. Which of the following interpretations hold good for this family.

(A*) each member of this family is equally inclined with the coordinate axes.

(B*) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

(C*) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) for $c = 3$ area of the triangle PQR is $3\sqrt{3}$ sq. units.

[Sol. Vector normal to the plane

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\vec{V}_x = \hat{i}$; $\vec{V}_y = \hat{j}$; $\vec{V}_z = \hat{k}$

$\cos(90 - \alpha) = \dfrac{\vec{V}_x \cdot \vec{n}}{|\vec{n}|} \quad \Rightarrow \quad \sin\alpha = \dfrac{1}{\sqrt{3}}$

|||ly $\sin\beta = \dfrac{1}{\sqrt{3}}$ and $\sin\gamma = \dfrac{1}{\sqrt{3}}$

hence $\sum \sin^2\alpha = 1$ and $\sum \cos^2\alpha = 2$

Also plane is equally inclined with the coordinate axes.

Also $A = \dfrac{1}{2}\sqrt{9^2 + 9^2 + 9^2} = \dfrac{9\sqrt{3}}{2} \quad \Rightarrow$ (D) is not correct]

[MATCH THE COLUMN]

Q.29

	Column-I		Column-II
(A)	*Centre of the parallelopiped whose 3 coterminous edges* $\overrightarrow{OA}, \overrightarrow{OB}$ *and* $\overrightarrow{OC}$ *have position vectors* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *respectively where O is the origin, is*	(P)	$\vec{a}+\vec{b}+\vec{c}$
(B)	*OABC is a tetrahedron where O is the origin. Positions vectors of its angular points A, B and C are* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *respectively. Segments joining each vertex with the centroid of the opposite face are concurrent at a point P whose p.v.'s are*	(Q)	$\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$
(C)	*Let ABC be a triangle the position vectors of its angular points are* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *respectively. If* $\|\vec{a}-\vec{b}\|=\|\vec{b}-\vec{c}\|=\|\vec{c}-\vec{a}\|$ *then the p.v. of the orthocentre of the triangle is*	(R)	$\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$
(D)	*Let* $\vec{a}, \vec{b}, \vec{c}$ *be 3 mutually perpendicular vectors of the same magnitude. If an unknown vector* $\vec{x}$ *satisfies the equation* $\vec{a}\times\left((\vec{x}-\vec{b})\times\vec{a}\right)+\vec{b}\times\left((\vec{x}-\vec{c})\times\vec{b}\right)+\vec{c}\times\left((\vec{x}-\vec{a})\times\vec{c}\right)=0.$ *Then* $\vec{x}$ *is given by*	(S)	$\dfrac{\vec{a}+\vec{b}+\vec{c}}{2}$

[Ans. (A) S; (B) R; (C) Q; (D) S] **[13th(27-8-2006)]**

[Sol. (D) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

also $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$

consider $\vec{a}\times\left((\vec{x}-\vec{b})\times\vec{a}\right)$

$$= (\vec{a}\cdot\vec{a})(\vec{x}-\vec{b}) - \{\vec{a}\cdot(\vec{x}-\vec{b})\}\vec{a}$$

$$= (\vec{a}\cdot\vec{a})\vec{x} - (\vec{a}\cdot\vec{a})\vec{b} - \{(\vec{a}\cdot\vec{x}) - \vec{a}\cdot\vec{b}\}\vec{a}$$

$$= (\vec{a}\cdot\vec{a})\vec{x} - (\vec{a}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{x})\vec{a} + \underbrace{(\vec{a}\cdot\vec{b})}_{\text{zero}}\vec{a}$$

$$\vec{a}\times\left((\vec{x}-\vec{b})\times\vec{a}\right) = (\vec{a}\cdot\vec{a})\vec{x} - (\vec{a}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{x})\vec{a}$$

|||ly $\vec{b}\times\left((\vec{x}-\vec{c})\times\vec{b}\right) = (\vec{b}\cdot\vec{b})\vec{x} - (\vec{b}\cdot\vec{b})\vec{c} - (\vec{b}\cdot\vec{x})\vec{b}$

and $\vec{c}\times\left((\vec{x}-\vec{a})\times\vec{c}\right) = (\vec{c}\cdot\vec{c})\vec{x} - (\vec{c}\cdot\vec{c})\vec{a} - (\vec{c}\cdot\vec{x})\vec{c}$

$\therefore$ LHS $= 3\lambda^2\vec{x} - \lambda^2\left(\vec{a}+\vec{b}+\vec{c}\right) - \left((\vec{a}\cdot\vec{x})\vec{a} + (\vec{b}\cdot\vec{x})\vec{b} + (\vec{c}\cdot\vec{x})\vec{c}\right) = 0$ ....(1)

now $\vec{a}, \vec{b}, \vec{c}$ are non coplanar

$\therefore$ $\vec{x} = x_1\vec{a} + x_2\vec{b} + x_3\vec{c}$

$$\frac{\vec{a}\cdot\vec{x}}{\lambda^2} = x_1; \quad \frac{\vec{b}\cdot\vec{x}}{\lambda^2} = x_2 \quad \text{and} \quad \frac{\vec{c}\cdot\vec{x}}{\lambda^2} = x_3$$

$\therefore$ $\lambda^2\vec{x} = (\vec{a}\cdot\vec{x})\vec{a} + (\vec{b}\cdot\vec{x})\vec{b} + (\vec{c}\cdot\vec{x})\vec{c}$

substituting the value in (1) we get $\vec{x} = \dfrac{\vec{a}+\vec{b}+\vec{c}}{2}$]

Q.30 **Column-I** **Column-II**

(A) Let O be an interior point of ΔABC such that $\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$, then the ratio of the area of Δ ABC to the area of Δ AOC, is with O is the origin — (P) 0

(B) Let ABC be a triangle whose centroid is G, orthocentre is H and circumcentre is the origin 'O'. If D is any point in the plane of the triangle such that no three of O, A, B, C and D are collinear satisfying the relation $\overrightarrow{AD}+\overrightarrow{BD}+\overrightarrow{CH}+3\overrightarrow{HG}=\lambda\overrightarrow{HD}$ then the value of the scalar 'λ' is — (Q) 1, (R) 2, (S) 3

(C) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non zero vectors such that no three of them are in the same plane and no two are orthogonal then the value of the scalar

$$\frac{(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d})}{(\vec{a}\times\vec{b})\cdot(\vec{d}\times\vec{c})} \text{ is}$$

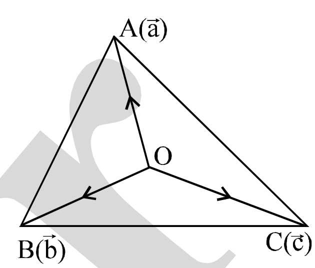
[Ans. (A) S; (B) R; (C) Q]

[Sol.**(A)** $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta AOC} = \dfrac{\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\frac{1}{2}\left|\vec{a}\times\vec{c}\right|}$ **[12th, 28-09-2008]**

now $\vec{a}+2\vec{b}+3\vec{c}=0$

cross with $\vec{b}$, $\vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=0 \Rightarrow \vec{a}\times\vec{b} = 3(\vec{b}\times\vec{c})$

cross with $\vec{a}$, $2\vec{a}\times\vec{b}+3\vec{a}\times\vec{c}=0 \Rightarrow \vec{a}\times\vec{b} = \frac{3}{2}(\vec{c}\times\vec{a})$



$\therefore$ $\vec{a}\times\vec{b} = \frac{3}{2}(\vec{c}\times\vec{a}) = 3(\vec{b}\times\vec{c})$

Let $(\vec{c}\times\vec{a})=\vec{p}$

$\vec{a}\times\vec{b}=\frac{3\vec{p}}{2}$; $\vec{b}\times\vec{c}=\frac{\vec{p}}{2}$

$\therefore$ ratio $= \dfrac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}\times\vec{a}\right|} = \dfrac{\left|\frac{3\vec{p}}{2}+\frac{\vec{p}}{2}+\vec{p}\right|}{|\vec{p}|} = \dfrac{3|\vec{p}|}{|\vec{p}|} = 3$ **Ans.** $\Rightarrow$ **(S)**

(B) LHS $= \vec{d}-\vec{a}+\vec{d}-\vec{b}+\vec{h}-\vec{c}+3(\vec{g}-\vec{h})$

$= 2\vec{d}-(\vec{a}+\vec{b}+\vec{c})+3\dfrac{(\vec{a}+\vec{b}+\vec{c})}{3}-2\vec{h}$

$= 2\vec{d}-2\vec{h}=2(\vec{d}-\vec{h})=2\overrightarrow{HD} \Rightarrow \lambda = 2$ **Ans.** $\Rightarrow$ **(R)**

(C) $(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{d})=\begin{vmatrix}\vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{d}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{d}\end{vmatrix}$

|||ly compute others which gives (1) $\Rightarrow$ **(Q)**]

[SUBJECTIVE TYPE]

Q.31$_{3d}$ If the lattice point P(x, y, z), x, y, z $\in$ I with the largest value of z such that the P lies on the planes $7x + 6y + 2z = 272$ and $x - y + z = 16$ (given x, y, z > 0), find the value of $(x + y + z)$.

[Ans. 66]

[Sol. $7x + 6y + 2z = 272$ **[12th, 28-09-2008]**

$2x - 2y + 2z = 32$

sub. $\overline{}$

$5x + 8y = 240$

$x = \dfrac{240 - 8y}{5} = 48 - \dfrac{8}{5}y$

let $y = 5k,\ k \in I$

$\therefore$ $x = 48 - 8k$

$\therefore$ $x - y + z = 16$

$(48 - 8k) - 5k + z = 16$

$z = 13k - 32 > 0 \Rightarrow k > \dfrac{32}{13} \Rightarrow k \ge 3$

now $48 - 8k > 0 \Rightarrow k < 6 \Rightarrow k \le 5$

$\therefore$ $3 \le k \le 5 \Rightarrow k = 5$

$\therefore$ $Z_{max} = 65 - 32 = 33$

$y = 25;\ x = 8$

$\therefore$ $x, y, z \equiv (8, 25, 33) \Rightarrow$ sum = 66 **Ans.**]

Q.32$_{vec}$ Given $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$.

Compute the value of $\left|\vec{A} \times \left(\vec{A} \times (\vec{A} \times \vec{B})\right) \cdot \vec{C}\right|$. [Ans. 343] **[12th, 28-09-2008]**

[Sol. $\vec{V} = \vec{A} \times \left((\vec{A} \cdot \vec{B})\vec{A} - (\vec{A} \cdot \vec{A})\vec{B}\right) \cdot \vec{C}$

$$= \left(\underbrace{\vec{A} \times (\vec{A} \cdot \vec{B})\vec{A}}_{\text{zero}} - (\vec{A} \cdot \vec{A})\vec{A} \times \vec{B}\right) \cdot \vec{C}$$

$= -|\vec{A}|^2 [\vec{A}\ \ \vec{B}\ \ \vec{C}]$

now $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$$[\vec{A}\ \ \vec{B}\ \ \vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix} = 2(1 + 4) - 1(3 - 12) + 1(-6 - 6)$$

$= 10 + 9 - 12 = 7$

$\therefore$ $\left|-|\vec{A}|^2 [\vec{A}\,\vec{B}\,\vec{C}]\right| = 49 \times 7 = 343$ **Ans.**]

DPP-8

Q.1	D	Q.2	A	Q.3	D	Q.4	D	Q.5	A
Q.6	B	Q.7	D	Q.8	B	Q.9	A	Q.10	A
Q.11	C	Q.12	B	Q.13	C	Q.14	C	Q.15	C

DPP-9

Q.1	D	Q.2	A	Q.3	C	Q.4	B	Q.5	B
Q.6	C	Q.7	C	Q.8	A	Q.9	B	Q.10	A
Q.11	D	Q.12	C	Q.13	C				

Q.14 (A) R, (B) Q, (C) Q, S, (D) P, S

Q.15 (A) R; (B) Q; (C) P ; (D) S

DPP-10

Q.1	A	Q.2	D	Q.3	C	Q.4	D	Q.5	A
Q.6	D	Q.7	B	Q.8	C	Q.9	B	Q.10	D
Q.11	B	Q.12	B	Q.13	C	Q.14	A	Q.15	D
Q.16	B	Q.17	C	Q.18	A	Q.19	B	Q.20	D
Q.21	C	Q.22	D	Q.23	A	Q.24	B, C, D	Q.25	A, B, D
Q.26	B, D	Q.27	B, C, D	Q.28	A, B, C	Q.29	(A) S; (B) R; (C) Q; (D) S		

Q.30 (A) S; (B) R; (C) Q

Q.31 66

Q.32 343