الصفحة الالكترونية لمقررات
الفيزياء لكليات التقنية

المؤسسة العامة للتدريب التقني والمهني
Technical and Vocational Training Corporation

# Chapter 2
# Coordinate Systems

Prepared by:
Abdulaziz Alanazi
Abdullah Alsahow

# Content

أولا

01

# Cartesian Coordinates

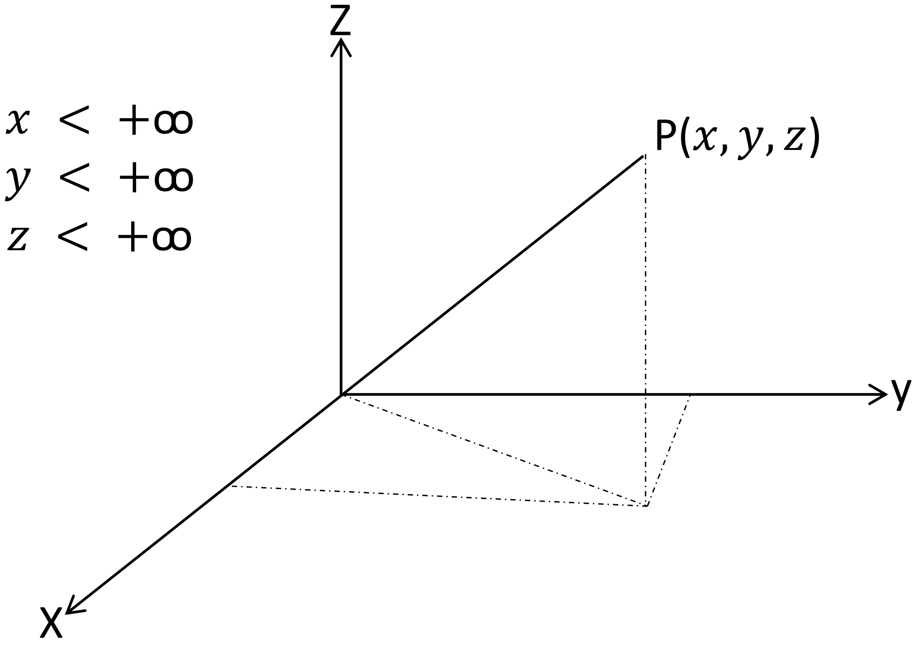
# Coordinate Systems:

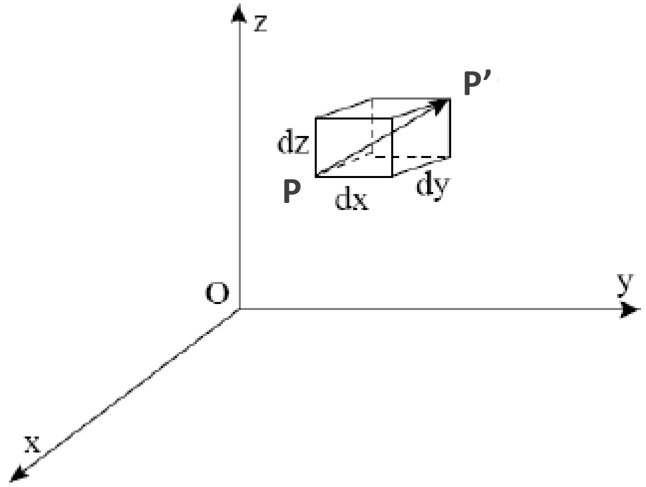
Many aspect of physics involve description of the location of an object in space.
In order to determine the location , we need coordinate systems (Cartesian , cylindrical and Spherical systems).

## 1- Cartesian Coordinates :

The point P is identified by the Cartesian coordinates $(x, y, z)$.

$$-\infty < x < +\infty$$
$$-\infty < y < +\infty$$
$$-\infty < z < +\infty$$




When an object moves from one point to another $(P \ to \ P')$, its coordinates change and form a cube:

**The elementary volume** is : $d\tau = dxdydz$

# ثانيا 02

# Cylindrical Coordinates

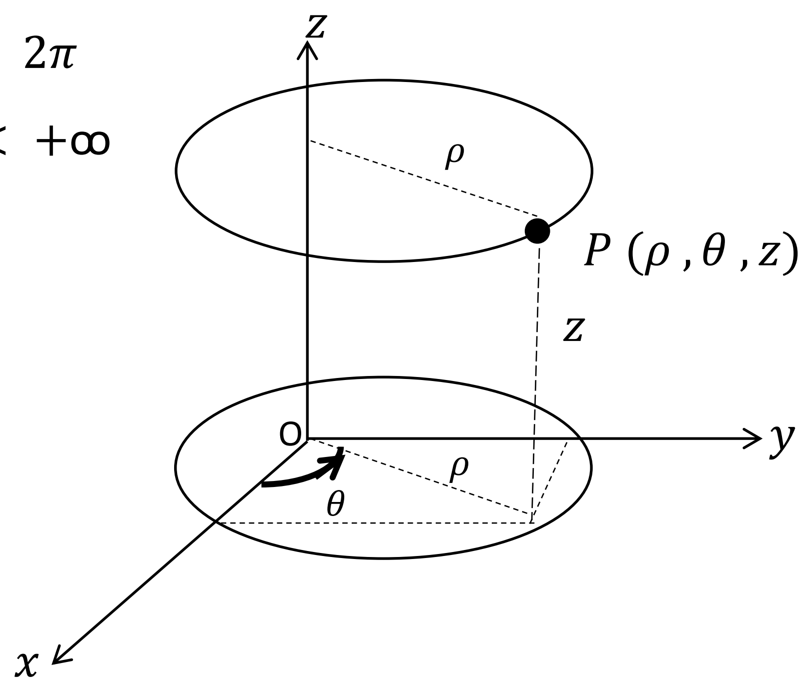
## 2- Cylindrical Coordinates :

Point P is located by cylindrical coordinate ($\rho$ , $\theta$ , $z$).
$\rho$ called radius , ϴ called azimuthal angle .

$$0 \le \rho < \infty$$

$$0 \le \Theta < 2\pi$$

$$-\infty < z < +\infty$$




Conversion formula from Cartesian coordinates to cylindrical coordinates :

$$\rho = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

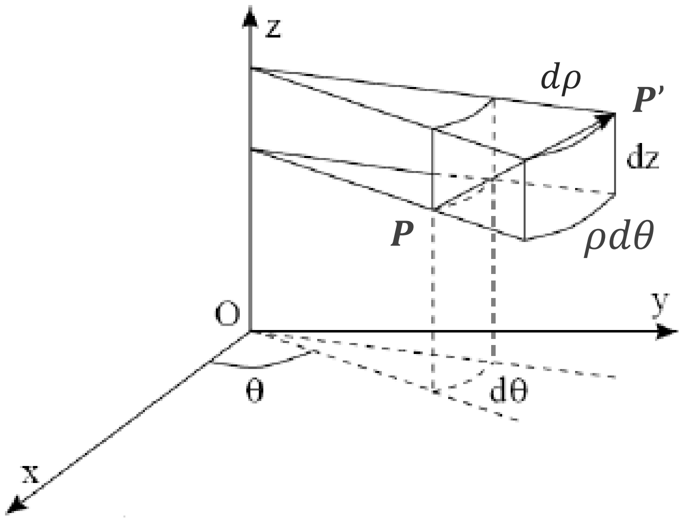
Conversion formula from cylindrical coordinates to Cartesian coordinates:

$$x = \rho \cos\theta$$

$$y = \rho \sin\theta$$

$$z = z$$

When an object moves from one point to another $(P\ to\ P')$, its coordinates change and form a cube




**The elementary volume** is : $d\tau = (d\rho)(\rho d\theta)(dz)$

Example 1: convert the point ( 1 , $\sqrt{3}$ , -2$\sqrt{3}$ ) from Cartesian coordinates to cylindrical coordinates ?

The Solution

$(x, y, z) \longrightarrow (\rho, \theta, z)$

$(1, \sqrt{3}, -2\sqrt{3}) \longrightarrow (\rho, \theta, z)$

$\rho = \sqrt{x^2+y^2} = \sqrt{(1)^2+(\sqrt{3})^2} = \sqrt{(1)+(3)} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60^{o} = \frac{\pi}{3}$

Rad=Degree x$\frac{\pi}{180^\circ}$= $60^\circ$ x$\frac{\pi}{180^\circ}$ $= \frac{6\pi}{18} = \frac{\pi}{3}$ Rad

Z= -2$\sqrt{3}$

∴ The point (1 , $\sqrt{3}$ , -2$\sqrt{3}$) = (2 , $\frac{\pi}{3}$ , -2$\sqrt{3}$)

Example 2: convert the point ( 1 , 1 , $\sqrt{2}$ ) from Cartesian coordinates to cylindrical coordinates ?

The Solution

$(x, y, z) \longrightarrow (\rho, \theta, z)$

$(1, 1, \sqrt{2}) \longrightarrow (\rho, \theta, z)$

$\rho = \sqrt{x^2+y^2} = \sqrt{(1)^2+(1)^2} = \sqrt{(1)+(1)} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = 45^{o} = \frac{\pi}{4}$

Rad=Degree x$\frac{\pi}{180^\circ}$= $45^\circ$ x$\frac{\pi}{180^\circ}$ $= \frac{\pi}{4}$ Rad

Z= $\sqrt{2}$

∴ The point (1 , 1 , $\sqrt{2}$) = ($\sqrt{2}$ , $\frac{\pi}{4}$ , $\sqrt{2}$)

Example 3: convert the point ( $\frac{-1}{2}, \frac{4\pi}{3}$ , 8 ) from cylindrical coordinates to Cartesian coordinates ?

The Solution

$(\rho, \theta, z) \longrightarrow (x, y, z)$

$\left(\frac{-1}{2}, \frac{4\pi}{3}, 8\right) \longrightarrow (x, y, z)$

$x = \rho \cos\theta = \frac{-1}{2} \times \cos(240^\circ) = \frac{-1}{2} \times \frac{-1}{2} = \frac{1}{4}$

$y = \rho \sin\theta = = \frac{-1}{2} \times \sin(240^\circ) = \frac{-1}{2} \times \frac{-\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$Z = 8$

Degree = rad x $\frac{180^\circ}{\pi} = \frac{4\pi}{3} \times \frac{180^\circ}{\pi}$
$= 4 \times 60 = 240^\circ$

$\therefore$ The point $\left(\frac{-1}{2}, \frac{4\pi}{3}, 8\right) = \left(\frac{1}{4}, \frac{\sqrt{3}}{4}, 8\right)$

Example 4: convert the point (6 , $\frac{\pi}{6}$, 2) from cylindrical coordinates to Cartesian coordinates ?

The Solution

$(\rho, \theta, z) \longrightarrow (x, y, z)$

$\left(6, \frac{\pi}{6}, 2\right) \longrightarrow (x, y, z)$

$x = \rho \cos\theta = 6 \times \cos(30^\circ) = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$y = \rho \sin\theta = 6 \times \sin(30^\circ) = 6 \times \frac{1}{2} = 3$

$Z = 2$

Degree = rad x $\frac{180^\circ}{\pi} = \frac{\pi}{6} \times \frac{180^\circ}{\pi}$
$= \frac{180^\circ}{6} = 30^\circ$

$\therefore$ The point $\left(6, \frac{\pi}{6}, 2\right) = (3\sqrt{3}, 3, 2)$

Example 5: convert the point ($\sqrt{2}$ , $\sqrt{6}$ , 1 ) from Cartesian coordinates to cylindrical coordinates ?

The Solution

$(x , y , z) \longrightarrow (\rho , \theta , z)$

$(\sqrt{2} , \sqrt{6} , 1) \longrightarrow (\rho , \theta , z)$

$\rho = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{2})^2 + (\sqrt{6})^2} = \sqrt{(2) + (6)} = \sqrt{8} = \sqrt{2X4}$

$= 2\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = 60^o = \frac{\pi}{3}$

Z= 1

$\text{Rad} = \text{Degree} \times \frac{\pi}{180°} = 60° \times \frac{\pi}{180°} = \frac{6\pi}{18} = \frac{\pi}{3}$ Rad

$\therefore$ The point ($\sqrt{2}$ , $\sqrt{6}$ , 1 ) = ( $2\sqrt{2}$ , $\frac{\pi}{3}$, 1 )

Example 6: convert the point ( 2 , $\frac{\pi}{4}$ , 4 ) from cylindrical coordinates to Cartesian coordinates ?

The Solution

$(\rho , \theta , z) \longrightarrow (x , y , z)$

$(2 , \frac{\pi}{4} , 4) \longrightarrow (x , y , z)$

$x = \rho \cos\theta = 2 X \cos(45°)\ 2 X \frac{\sqrt{2}}{2} = \sqrt{2}$

$y = \rho \sin\theta = = 2 X \sin(45°) = 2 X \frac{\sqrt{2}}{2} = \sqrt{2}$

Z= 4

$\text{Degree} = \text{rad} \times \frac{180°}{\pi} = \frac{\pi}{4} X \frac{180°}{\pi} = \frac{180°}{4} = 45°$

$\therefore$ The point ( 2 , $\frac{\pi}{4}$ , 4 ) = ($\sqrt{2}$ , $\sqrt{2}$ , 4 )

Example 6: Calculate the volume (in cm$^3$) when the point move from $(-2, 0,3)$ $to$ $(4, 5,6)$?

The Solution

$$(x_1\,, y_1\,, z_1) = (-2\,, 0\,, 3)$$

$$(x_2\,, y_2\,, z_2) = (\,4\,, 5\,, 6)$$

$$d\tau = (dx)(dy)(dz) =$$

$$= (4-(-2)) \times (5-0) \times (6-3) =$$

$$= 6 \;\times 5 \times 3 = 90\; cm^3$$

$\therefore$ The volume $=90\; cm^3$

Example 7: Calculate the volume (in cm$^3$) when the point move from $(5, \frac{\pi}{4}, 2)$ $to$ $(7, \frac{\pi}{3}, 6)$?

The Solution

$$(\rho_1\,, \theta_1\,, z_1) = (5\,, \frac{\pi}{4}\,, 2)$$

$$(\rho_2\,, \theta_2\,, z_2) = (\,7\,, \frac{\pi}{3}\,, 6)$$

$$d\tau = (d\rho)(\rho d\theta)(dz) = \rho(d\rho)(d\theta)(dz) =$$

$$= 5 \times (7-5) \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \times (6-2) =$$

$$= 5 \;\times (2) \times \left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) \times (4) =$$

$$= 5 \;\times (2) \times \left(\frac{\pi}{12}\right) \times (4) = \frac{10\pi}{3} cm^3$$

$\therefore$ The volume $= \frac{10\pi}{3} cm^3 = 10.47 cm^3$

# ثالثا 03

# Spherical Coordinates

## 3- Spherical Coordinates :

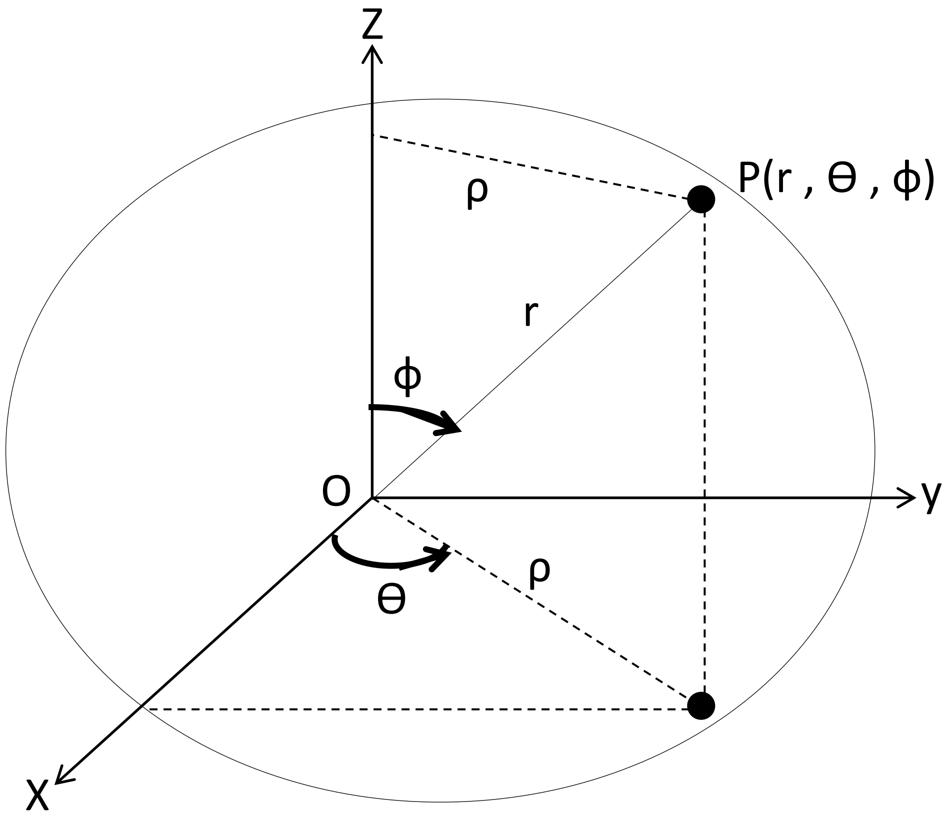
Point P is located by Spherical coordinate (r , ϴ , ϕ).

ρ called radius , ϴ called polar angle ϕ called azimuth angle .

$0 \leq r < \infty$

$0 \leq \theta < 2\pi$

$0 \leq \phi \leq \pi$




### Conversion formula from Cartesian coordinates to spherical coordinates :

$$r=\sqrt{x^2 + y^2 + z^2}$$

$$\theta= \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi=\cos^{-1}\left(\frac{z}{r}\right)$$

### Conversion formula from spherical coordinates to Cartesian coordinates:

$x=r \sin\phi \cos\theta$

$y= r \sin\phi \sin\theta$

$z= r \cos\phi$

Example 1: convert the point $(\sqrt{6}, \sqrt{6}, 2)$ from Cartesian coordinates to spherical coordinates .

The Solution

$$(x, y, z) \longrightarrow (r, \theta, \phi)$$

$$(\sqrt{6}, \sqrt{6}, 2) \longrightarrow (\rho, \theta, \phi)$$

$$r=\sqrt{x^2+y^2+z^2} = \sqrt{(\sqrt{6})^2+(\sqrt{6})^2+(2)^2}$$

$$=\sqrt{6+6+4} = \sqrt{16} = 4$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \tan^{-1}(1) = 45^o = \frac{\pi}{4}$$

$$\phi = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{2}{4}\right) = 60^o = \frac{\pi}{3}$$

$$\therefore \text{ The point } (\sqrt{6}, \sqrt{6}, 2) = \left(4, \frac{\pi}{4}, \frac{\pi}{3}\right)$$

Example 2: convert Cartesian coordinates of the point $(1, 1, \sqrt{6})$ to spherical coordinates.

The Solution

$$(x, y, z) \longrightarrow (r, \theta, \phi)$$

$$(1, 1, \sqrt{6}) \longrightarrow (\rho, \theta, \phi)$$

$$r=\sqrt{x^2+y^2+z^2} = \sqrt{(1)^2+(1)^2+(\sqrt{6})^2}$$

$$=\sqrt{1+1+6} = \sqrt{8} = \sqrt{2 X 4} = 2\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = 45^o = \frac{\pi}{4}$$

$$\phi = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{\sqrt{6}}{2\sqrt{2}}\right) = 30^o = \frac{\pi}{6}$$

$$\therefore \text{ The point } (1, 1, \sqrt{6}) = \left(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{6}\right)$$

Example 3: convert the point $(4, \frac{\pi}{6}, \frac{\pi}{4})$ from spherical coordinates to Cartesian coordinates .

The Solution

$$(r, \theta, \phi) \longrightarrow (x, y, z)$$

$$(4, \frac{\pi}{6}, \frac{\pi}{4}) \longrightarrow (x, y, z)$$

$$x = r \sin\phi \cos\theta = 4 \times \sin(45^o) \times \cos(30^o)$$

$$= 4 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = 4 \times \frac{\sqrt{6}}{4} = \sqrt{6}$$

$$y = r \sin\phi \sin\theta = 4 \times \sin(45^o) \times \sin(30^o)$$

$$= 4 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} = 4 \times \frac{\sqrt{2}}{4} = \sqrt{2}$$

$$z = r \cos\phi = 4 \times \cos(45^o) = 4 \times \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

$\therefore$ The point $(4, \frac{\pi}{6}, \frac{\pi}{4}) = (\sqrt{6}, \sqrt{2}, 2\sqrt{2})$

Example 4: convert the point $(2, \frac{\pi}{6}, \frac{\pi}{3})$ from spherical coordinates to Cartesian coordinates .

The Solution

$$(r, \theta, \phi) \longrightarrow (x, y, z)$$

$$(2, \frac{\pi}{6}, \frac{\pi}{3}) \longrightarrow (x, y, z)$$

$$x = r \sin\phi \cos\theta = 2 \times \sin(60^o) \times \cos(30^o)$$

$$= 2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = 2 \times \frac{\sqrt{3 \times 3}}{2 \times 2} = 2 \times \frac{\sqrt{9}}{4} = \frac{3}{2}$$

$$y = r \sin\phi \sin\theta = 2 \times \sin(60^o) \times \sin(30^o)$$

$$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$z = r \cos\phi = 2 \times \cos(60^o) = 2 \times \frac{1}{2} = 1$$

$\therefore$ The point $(2, \frac{\pi}{6}, \frac{\pi}{3}) = (\frac{3}{2}, \frac{\sqrt{3}}{2}, 1)$

**Example 5:** convert the point $(2\sqrt{3}, 6, 4)$ from Cartesian coordinates to spherical coordinates .

The Solution

$$(x, y, z) \longrightarrow (r, \theta, \phi)$$

$$(2\sqrt{3}, 6, 4) \longrightarrow (\rho, \theta, \phi)$$

$$r=\sqrt{x^2+y^2+z^2} = \sqrt{(2\sqrt{3})^2+(6)^2+(-4)^2}$$

$$=\sqrt{4X3+36+16}$$

$$=\sqrt{12+36+16}=\sqrt{64}=8$$

$$\Theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{6}{2\sqrt{3}}\right) = 60^o = \frac{\pi}{3}$$

$$\phi=\cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{-4}{8}\right) = 120^o = \frac{2\pi}{3}$$

$\therefore$ The point $(2\sqrt{3}, 6, -4) = (8, \frac{\pi}{3}, \frac{2\pi}{3})$

**Example 6:** convert the point $(8, \frac{\pi}{4}, \frac{\pi}{6})$ from spherical coordinates to Cartesian coordinates .

The Solution

$$(r, \theta, \phi) \longrightarrow (x, y, z)$$

$$(8, \frac{\pi}{4}, \frac{\pi}{6}) \longrightarrow (x, y, z)$$

$$x = r\sin\phi\cos\Theta = 8 X \sin(30^o) X \cos(45^o)$$

$$= 8 X \frac{1}{2} X \frac{\sqrt{2}}{2} = 8 X \frac{\sqrt{2}}{4} = 4 X \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

$$y = r\sin\phi\sin\Theta = 8 X \sin(30^o) X \sin(45^o)$$

$$= 8 X \frac{1}{2} X \frac{\sqrt{2}}{2} = 8 X \frac{\sqrt{2}}{4} = 4 X \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

$$z = r\cos\phi = 8 X \cos(30^o) = 8 X \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$\therefore$ The point $(8, \frac{\pi}{4}, \frac{\pi}{6}) = (2\sqrt{2}, 2\sqrt{2}, 4\sqrt{3})$

# Thank you