Modeling and Simulation of CPR-A as an Aerial Cable Suspended Parallel Robot

Parallel Robots Project

Ashkan Rashvand and Ehsan Damghani

Cable Suspended Parallel Robots

Simple structure | Easy assembly capability | Large workspace | Low mass and volume

Outline

01

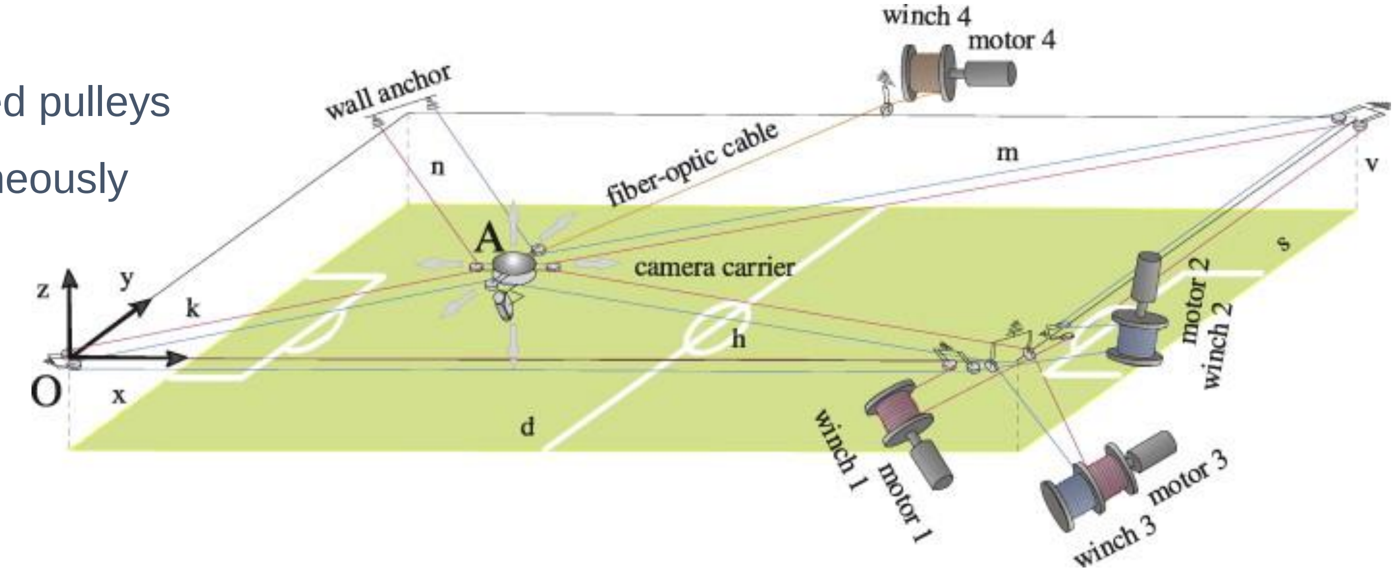
Mechanism Description

CPRA System

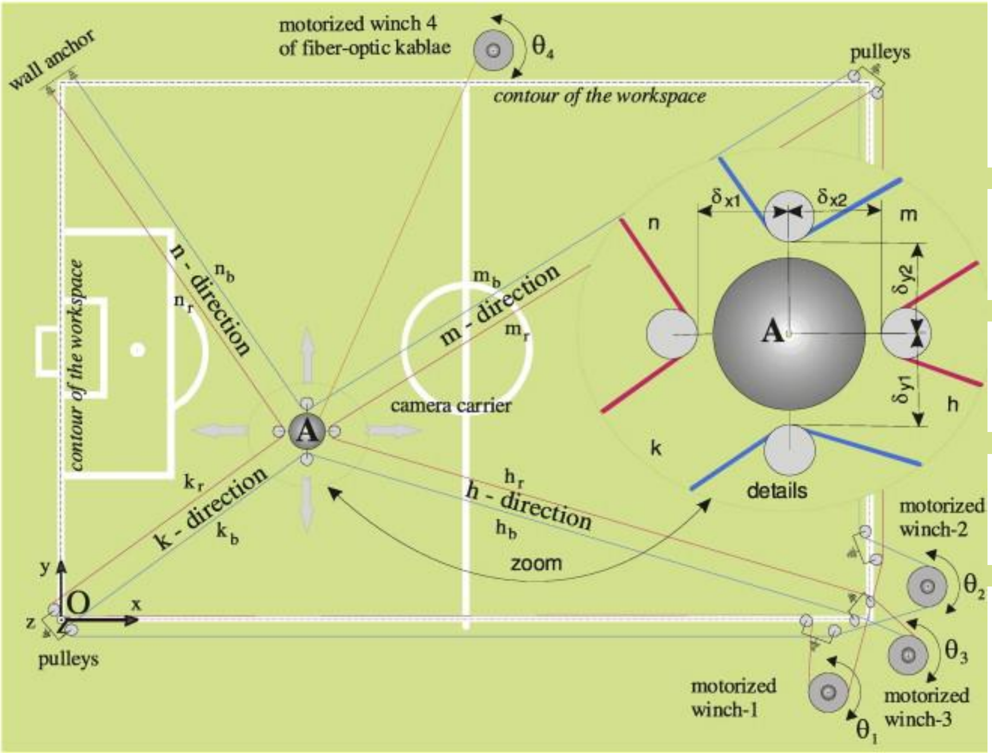
ADVANCED ROBOTICS & AUTOMATED SYSTEMS

Mechanism Description

- 3 motors and 2 cables
- 4 moving pulleys and 10 fixed pulleys
- Winds and unwinds simultaneously
- Asymmetric design
- Workspace Improvement
- Low maintenance costs



Modeling assumptions



3 degrees of freedom

Point mass moving platform

$n_r = n_b \,, m_r = m_b \,, k_r = k_b \,, h_r = h_b$

$\delta x_1 = \delta y_1 = \delta x_2 = \delta y_2 = 0$

02

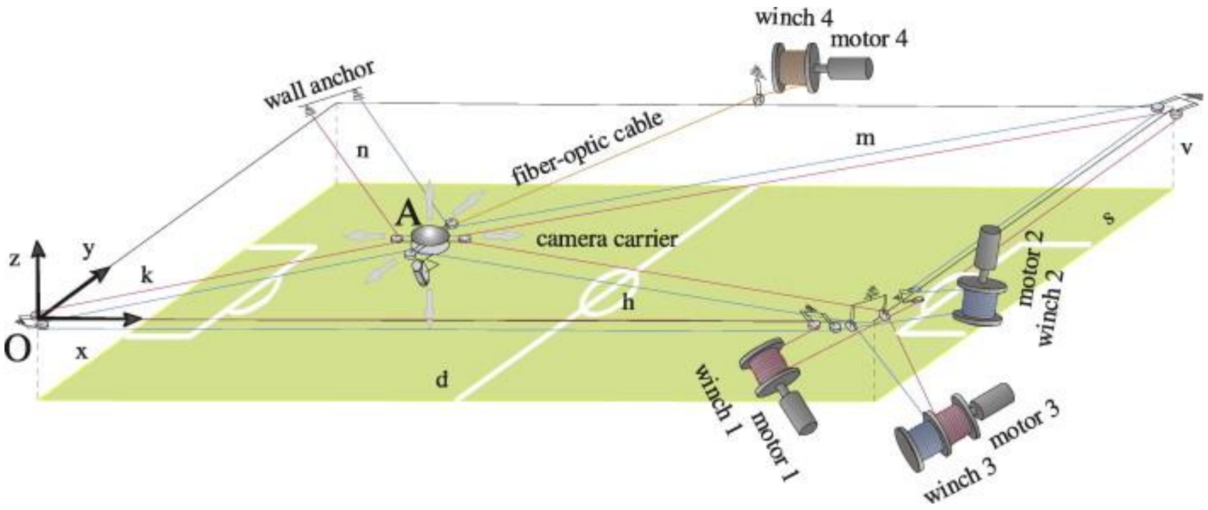
Kinematics

CPRA System

- Invers Kinematics
- Forward Kinematics
- Kinematics Simulation

ARAS HI-TECH ROBOTIC SOLUTIONS
ADVANCED ROBOTICS & AUTOMATED SYSTEMS

Inverse Kinematics



$$k = \sqrt{x^2 + y^2 + z^2}$$

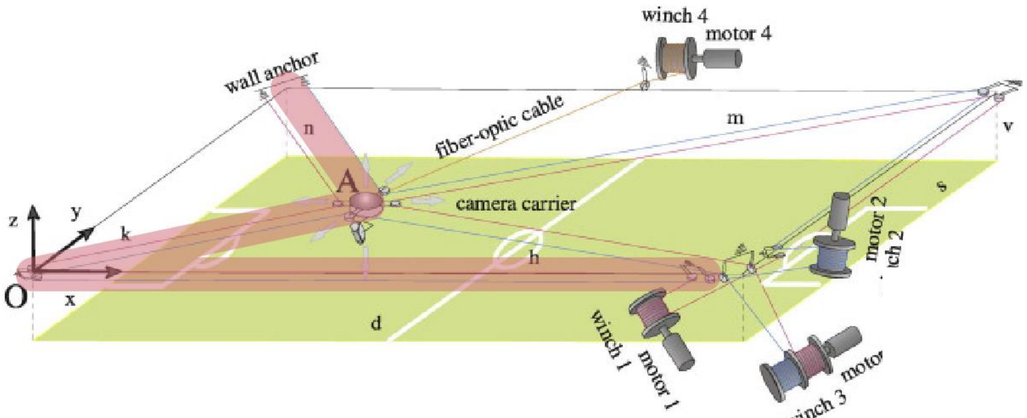
$$h = \sqrt{(d-x)^2 + y^2 + z^2}$$

$$m = \sqrt{(d-x)^2 + (s-y)^2 + z^2}$$

$$n = \sqrt{x^2 + (s-y)^2 + z^2}$$

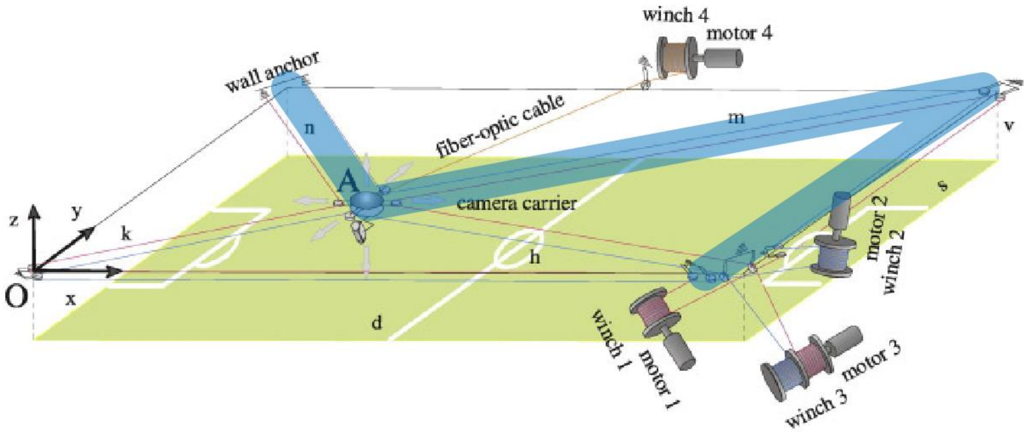
$$k_0 = h_0 = m_0 = n_0 = \sqrt{(\frac{d}{2})^2 + (\frac{s}{2})^2}$$

Inverse Kinematics

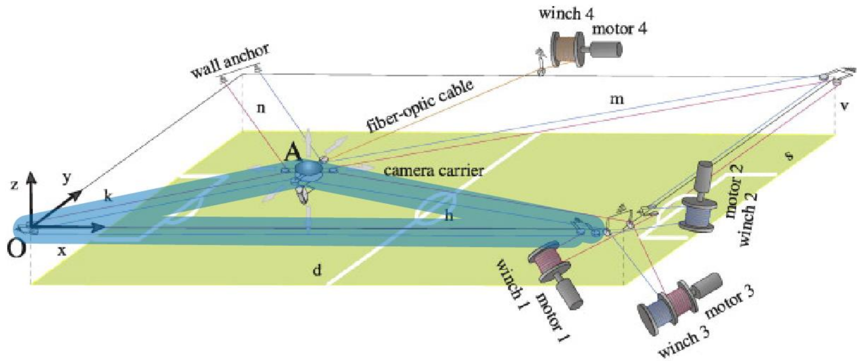


$$r_1\theta_1 = (k - k_0) + (n - n_0)$$

$$r_2\theta_2 = (m - m_0) + (n - n_0)$$

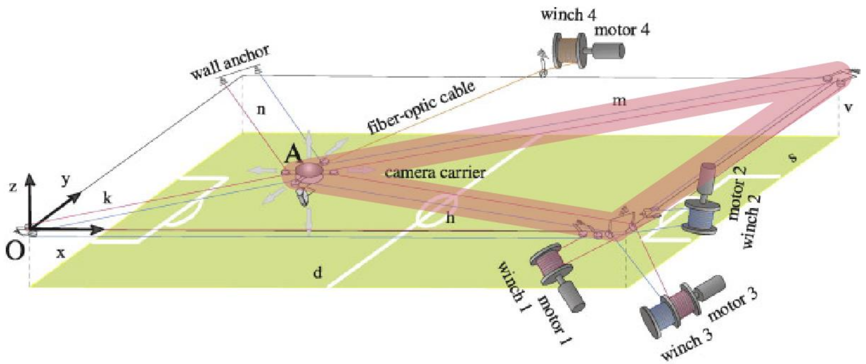


Inverse Kinematics



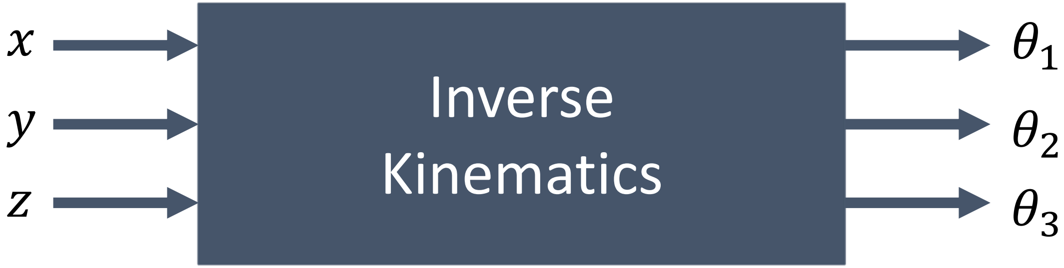
$$r_3\theta_3 = (h - h_0) + (k - k_0) + r_2\theta_2$$

$$r_3\theta_3 = (m - m_0) + (h - h_0) + r_1\theta_1$$



$$r_3\theta_3 = (k - k_0) + (h - h_0) + (m - m_0) + (n - n_0)$$

Inverse Kinematics



$$\theta_1 = \frac{1}{r_1}\left(\sqrt{x^2+y^2+z^2} + \sqrt{x^2+(s-y)^2+z^2} - k_0 - n_0\right)$$

$$\theta_2 = \frac{1}{r_2}\left(\sqrt{(d-x)^2+(s-y)^2+z^2} + \sqrt{x^2+(s-y)^2+z^2} - m_0 - n_0\right)$$

$$\begin{aligned}\theta_3 =& \frac{1}{r_3}(\sqrt{x^2+y^2+z^2} + \sqrt{(d-x)^2+y^2+z^2} + \sqrt{(d-x)^2+(s-y)^2+z^2} \\ &+ \sqrt{x^2+(s-y)^2+z^2} - k_0 - h_0 - m_0 - n_0)\end{aligned}$$

Forward Kinematics

New Variables

$$\alpha = r_3\theta_3 - r_2\theta_2 + (k_0 + h_0)$$

$$\beta = r_3\theta_3 - r_1\theta_1 + (m_0 + h_0)$$

$$\gamma = r_1\theta_1 + r_2\theta_2 - r_3\theta_3 + (n_0 - h_0)$$

$$k = \alpha - h$$

$$m = \beta - h$$

$$n = \gamma + h$$

$$k^2 + m^2 = n^2 + h^2$$

Geometric Equality

$$h = \frac{\alpha^2 + \beta^2 - \gamma^2}{2(\alpha + \beta + \gamma)}$$

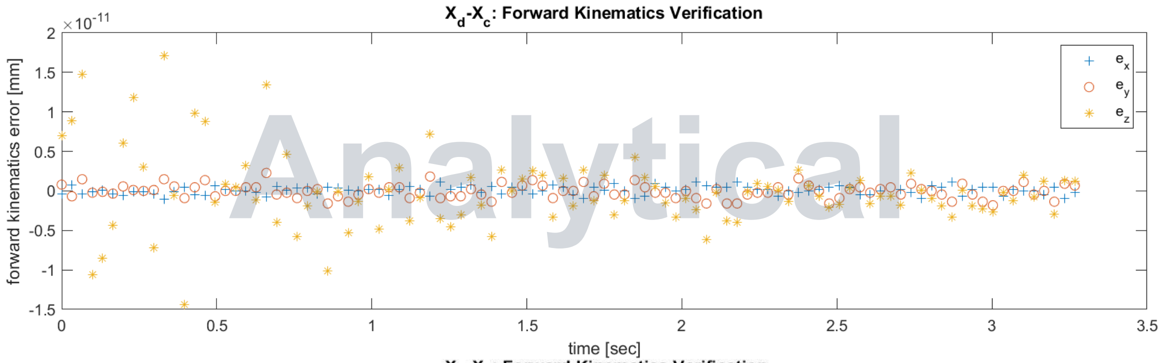
Forward Kinematics

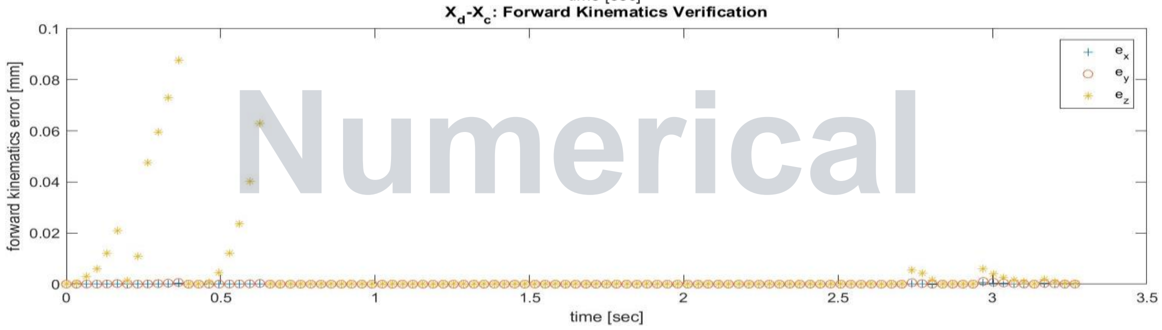
$$x = \frac{\alpha^2 - 2\alpha h + d^2}{2d}$$

$$y = \frac{s^2 + 2\beta h - \beta^2}{2s}$$

$$z = -\sqrt{((\alpha - h)^2 - (\frac{s^2 + 2\beta h - \beta^2}{2s})^2 - (\frac{\alpha^2 - 2\alpha h + d^2}{2d})^2)}$$

Forward Kinematics (Analytical vs. Numerical)





03

Jacobian Analysis

CPRA System

- Velocity Analysis
- Jacobian Simulation
- Singularity Analysis
- Stiffness Analysis
- Sensitivity Analysis

ARAS HI-TECH ROBOTIC SOLUTIONS
ADVANCED ROBOTICS & AUTOMATED SYSTEMS

Jacobian Analysis

$$\dot{\theta}_1 = \frac{1}{r_1}(\dot{k} + \dot{n})$$

$$\dot{\theta}_2 = \frac{1}{r_2}(\dot{m} + \dot{n})$$

$$\dot{\theta}_3 = \frac{1}{r_3}(\dot{k} + \dot{h} + \dot{m} + \dot{n})$$

$$\dot{\theta}_1 = \frac{1}{r_1}\left[\left(\frac{1}{k} + \frac{1}{n}\right) x\dot{x} + \left(\frac{y}{k} + \frac{y-s}{n}\right)\dot{y} + \left(\frac{1}{k} + \frac{1}{n}\right) z\dot{z}\right]$$

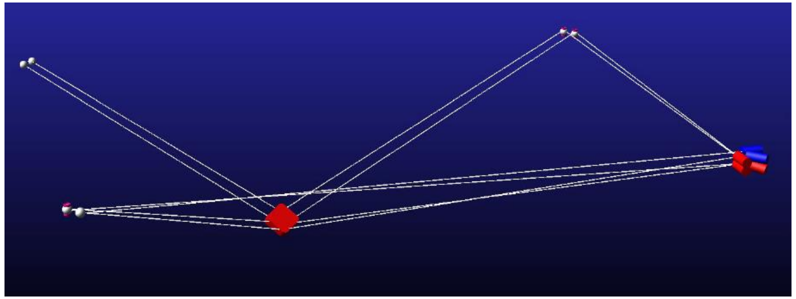
$$\dot{\theta}_2 = \frac{1}{r_2}\left[\left(\frac{x-d}{m} + \frac{x}{n}\right)\dot{x} + \left(\frac{1}{m} + \frac{1}{n}\right)(y-s)\dot{y} + \left(\frac{1}{m} + \frac{1}{n}\right) z\dot{z}\right]$$

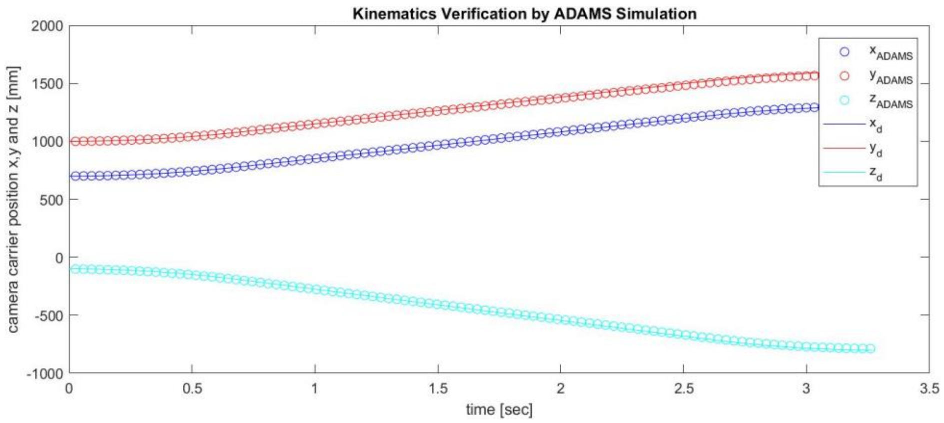
$$\dot{\theta}_3 = \frac{1}{r_3}\left[\left(\frac{x}{k} + \frac{x-d}{h} + \frac{x-d}{m} + \frac{x}{n}\right)\dot{x} + \left(\frac{y}{k} + \frac{y}{h} + \frac{y-s}{m} + \frac{y-s}{n}\right)\dot{y} + \left(\frac{1}{k} + \frac{1}{h} + \frac{1}{m} + \frac{1}{n}\right) z\dot{z}\right]$$

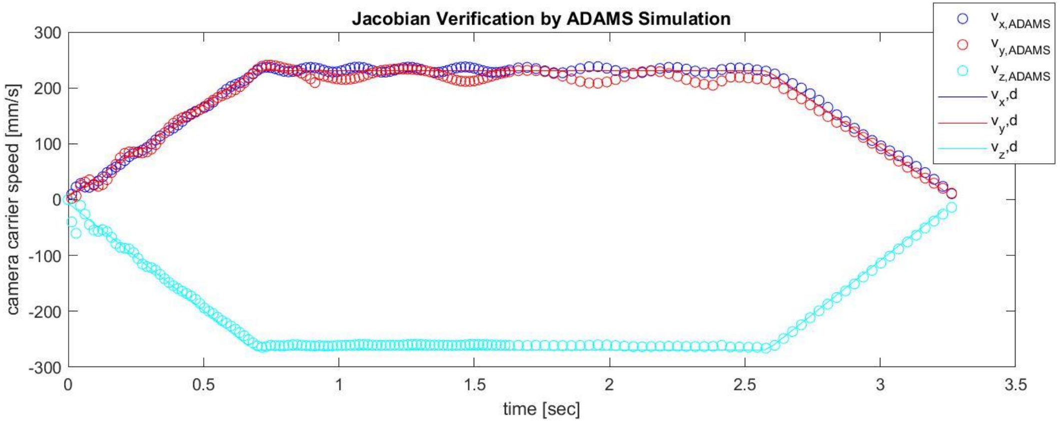
$$\dot{\theta} = J\dot{X}$$

$$J = \begin{bmatrix} \left(\frac{1}{k} + \frac{1}{n}\right) x & \left(\frac{y}{k} + \frac{y-s}{n}\right) & \left(\frac{1}{k} + \frac{1}{n}\right) z \\ \left(\frac{x-d}{m} + \frac{x}{n}\right) & \left(\frac{1}{m} + \frac{1}{n}\right)(y-s) & \left(\frac{1}{m} + \frac{1}{n}\right) z \\ \left(\frac{x}{k} + \frac{x-d}{h} + \frac{x-d}{m} + \frac{x}{n}\right) & \left(\frac{y}{k} + \frac{y}{h} + \frac{y-s}{m} + \frac{y-s}{n}\right) & \left(\frac{1}{k} + \frac{1}{h} + \frac{1}{m} + \frac{1}{n}\right) z \end{bmatrix}$$

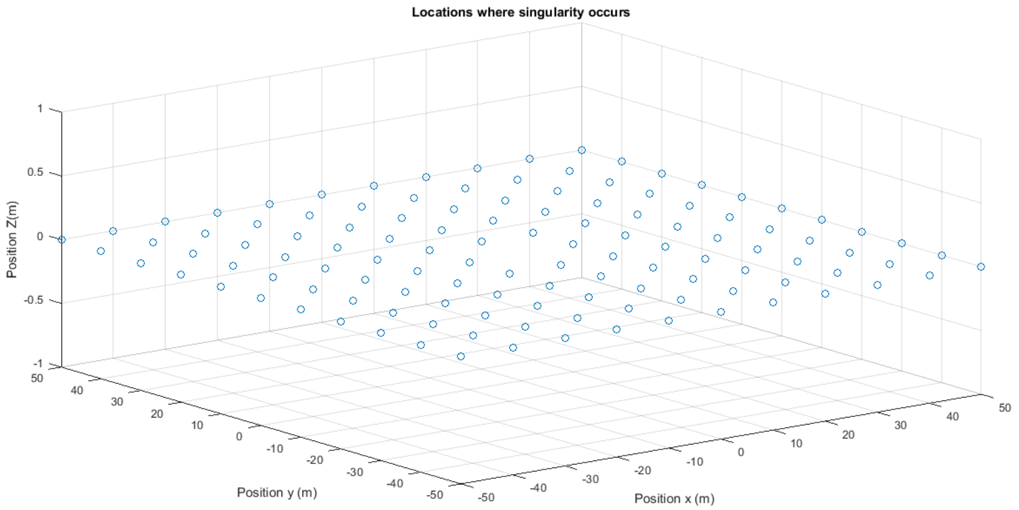
Adams Simulation

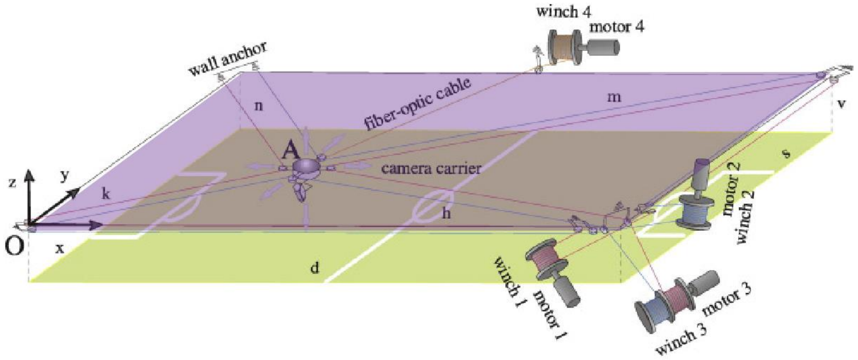




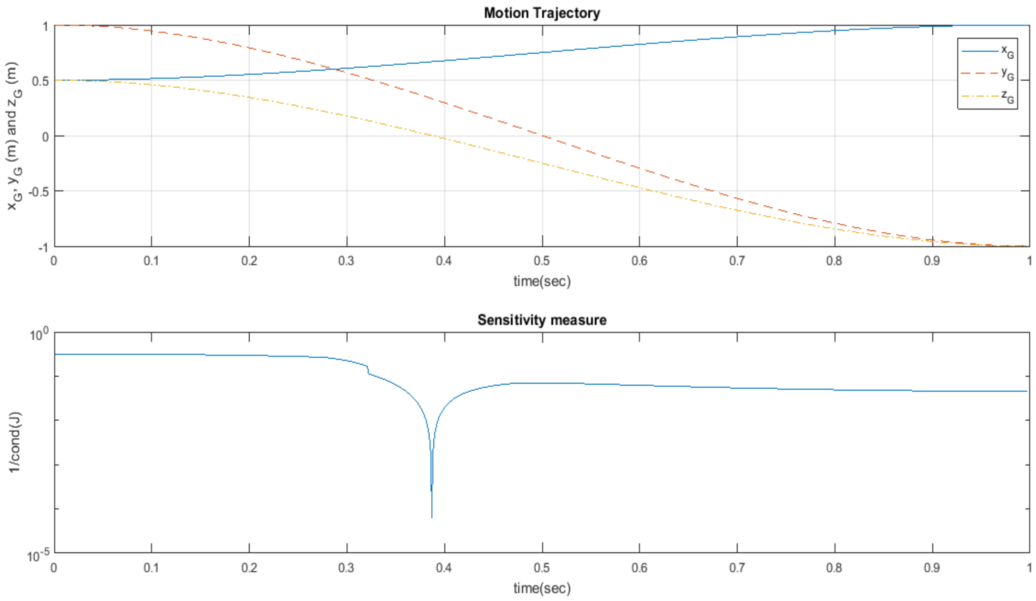


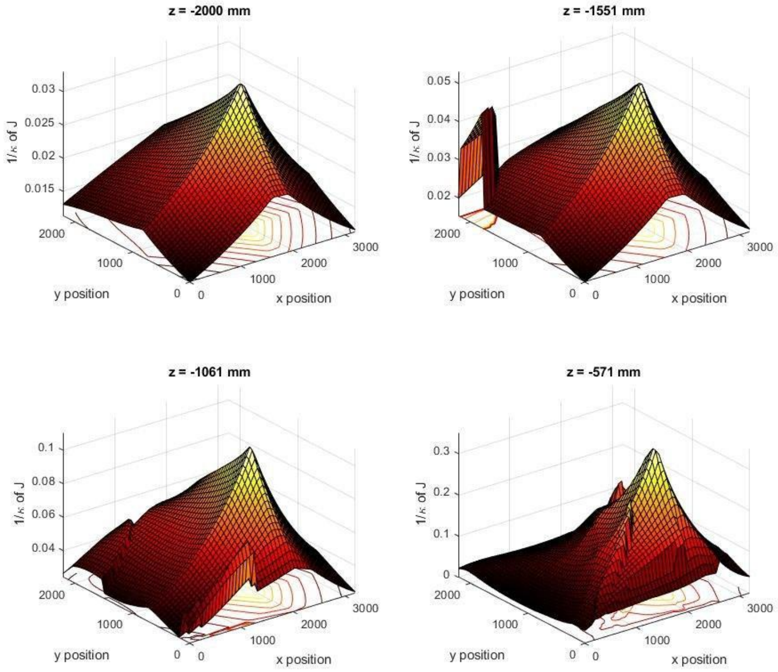
Singularity Analysis





Sensitivity and Stiffness Analysis





04

Dynamics Formulation

CPRA System

ARAS HI-TECH ROBOTIC SOLUTIONS
ADVANCED ROBOTICS & AUTOMATED SYSTEMS

Regardless of the mass of the cables

$$T_{cc} = \frac{1}{2}\dot{X}^T M \dot{X}$$

$$M = mI_{3\times 3}$$

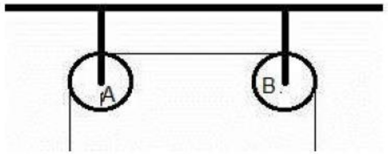
$$U_{cc} = -Mg^T X$$

$$G = \frac{\partial U_{cc}}{\partial X} = -Mg^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

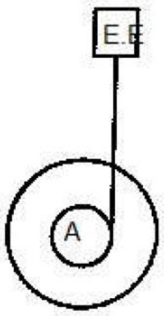
$$M\ddot{X} + G = Q$$

Dynamic Modeling with Considering The Mass of Cables

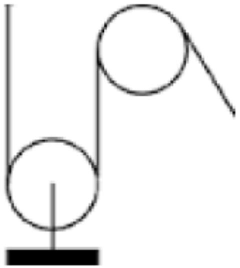
Velocity analysis of center of the mass of cables



Velocity flow

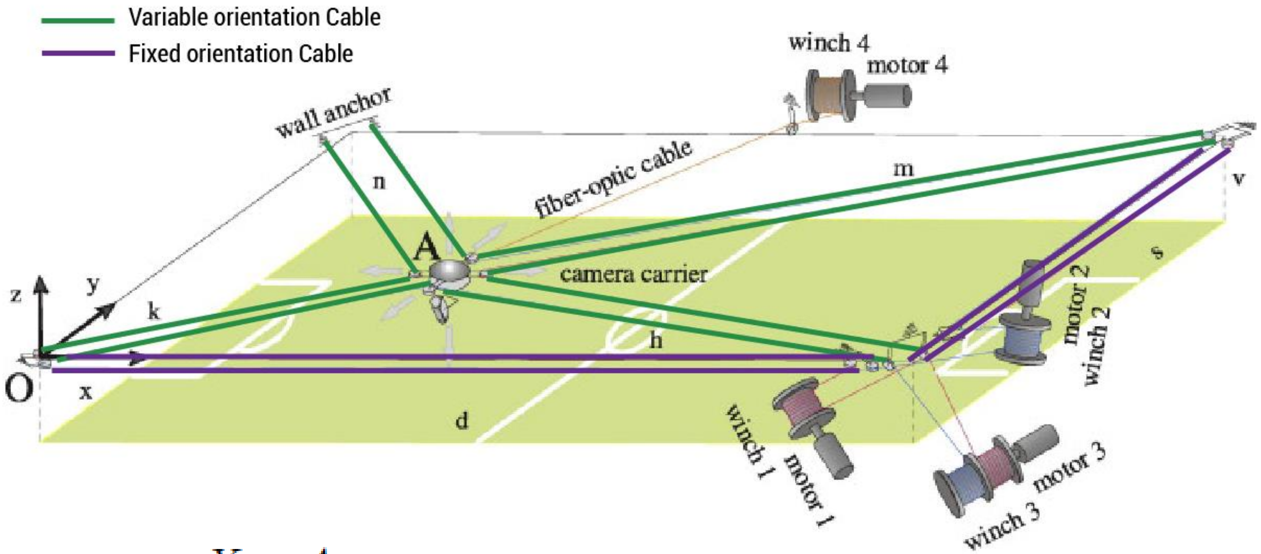


Length change



Velocity flow
+
Length change

Dynamic Modeling with Considering The Mass of Cables



$$v_{c_{h1}} = (\dot{n} + \dot{k} + \dot{m} + \frac{1}{2}\dot{h})\hat{s}_4 + \frac{1}{2}h\omega_4 \times \hat{s}_4$$

$$v_{c_{m1}} = (\dot{n} + \dot{k} + \frac{1}{2}\dot{m})\hat{s}_3 + \frac{1}{2}m\omega_3 \times \hat{s}_3$$

$$v_{c_{k1}} = (\dot{n} + \frac{1}{2}\dot{k})\hat{s}_2 + \frac{1}{2}k\omega_2 \times \hat{s}_2$$

$$v_{c_{n1}} = \frac{1}{2}\dot{n}\hat{s}_1 + \frac{1}{2}n\omega_1 \times \hat{s}_1$$

$$v_{c_{d1}} = (\dot{k} + \dot{n})\hat{s}_5$$

$$v_{c_{s1}} = (\dot{k} + \dot{n})\hat{s}_6$$

$$s_i = \frac{X - A_i}{\|X - A_i\|} \qquad s_5 = -\hat{i}, \quad s_6 = \hat{j}$$

Dynamic Formulation of the Limbs

$$K_i = \frac{1}{2} v_{ci}^T m_i v_{ci} + \frac{1}{2} \omega_i^T I_{ci} \omega_i$$

$$\frac{1}{2}\omega_i^T I_{ci}\omega_i = -\frac{1}{2L_i^2} I_{xx} \dot{x}_i^T s_{ix}^2 \dot{x}_i$$

$$\dot{L}_i = \hat{S}_i^T \dot{s}_i$$

$$\omega_i = \frac{1}{L_i} \hat{s}_{ix} \dot{x}_i$$

$$L_i \omega_i \times \hat{s}_i = \hat{s}_{ix}^2 \dot{x}_i$$

$$M_{n1} = \frac{n\rho}{4}\left[I_{3\times 3} - \frac{s_{1x}^2}{3} \right]$$

$$M_{k1} = \frac{k\rho}{4}\left[I_{3\times 3} - \frac{s_{2x}^2}{3} + 2(\hat{s}_2 \hat{s}_1^T + \hat{s}_1 \hat{s}_2^T) + 4\hat{s}_1 \hat{s}_1^T \right]$$

$$M_{m1} = \frac{m\rho}{4}\left[I_{3\times 3} - \frac{s_{3x}^2}{3} + 2(\hat{s}_3 \hat{s}_1^T + \hat{s}_1 \hat{s}_3^T + \hat{s}_3 \hat{s}_2^T + \hat{s}_2 \hat{s}_3^T) + 4(\hat{s}_1 \hat{s}_1^T + \hat{s}_2 \hat{s}_2^T) \right]$$

Dynamic Formulation of the Limbs

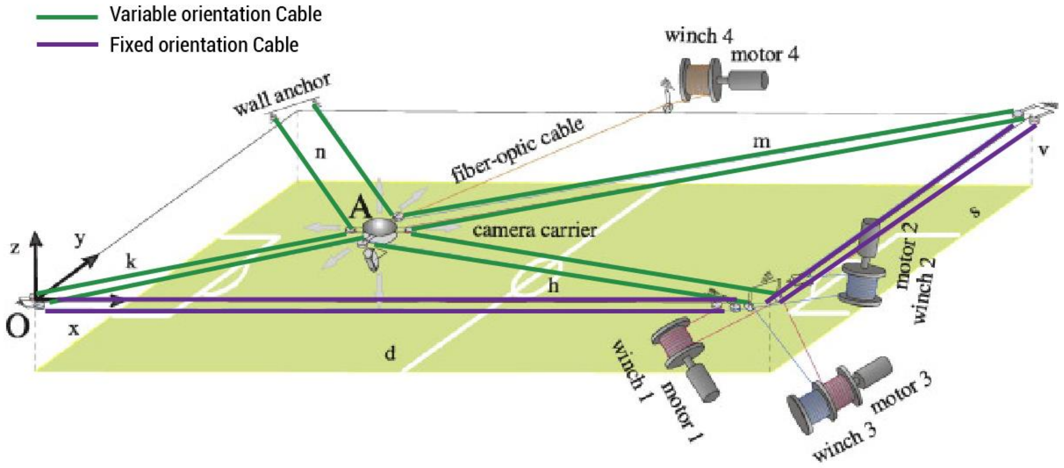
$$G_{n1} = G_{n2} = n\rho \left[\frac{1}{2} \hat{s}_{1x}^2 - \hat{s}_1 \hat{s}_1^T \right] g$$

$$G_{k1} = G_{k2} = k\rho \left[\frac{1}{2} \hat{s}_{2x}^2 - \hat{s}_2 \hat{s}_2^T \right] g$$

$$G_{m1} = G_{m2} = m\rho \left[\frac{1}{2} \hat{s}_{3x}^2 - \hat{s}_3 \hat{s}_3^T \right] g$$

$$G_{h1} = G_{h2} = n\rho \left[\frac{1}{2} \hat{s}_{4x}^2 - \hat{s}_4 \hat{s}_4^T \right] g$$

$$C_i \dot{X} = \dot{M} \dot{X} - \frac{1}{2} \frac{\partial}{\partial X} \left(\dot{X}^T M_i \dot{X} \right)$$



Dynamic Formulation of the Moving Platform

$$T_{cc} = \frac{1}{2}\dot{X}^T M \dot{X} \quad \longrightarrow \quad M = mI_{3\times 3}$$

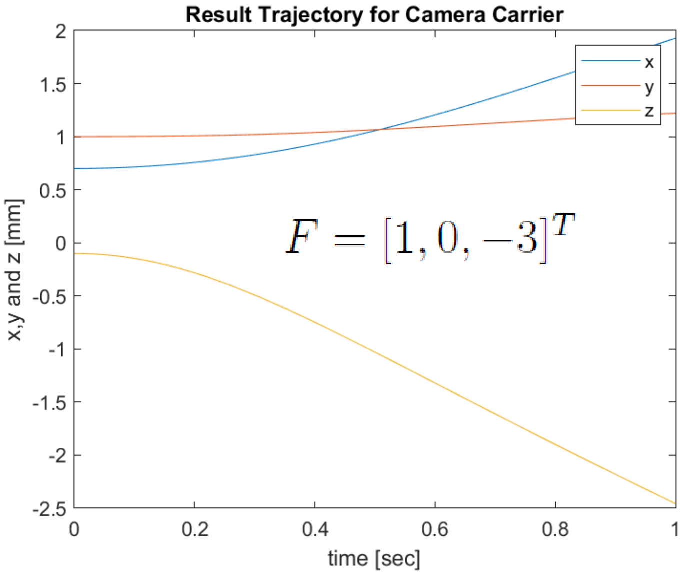
$$U_{cc} = -Mg^T X \quad \longrightarrow \quad G = \frac{\partial U_{cc}}{\partial X} = -Mg^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

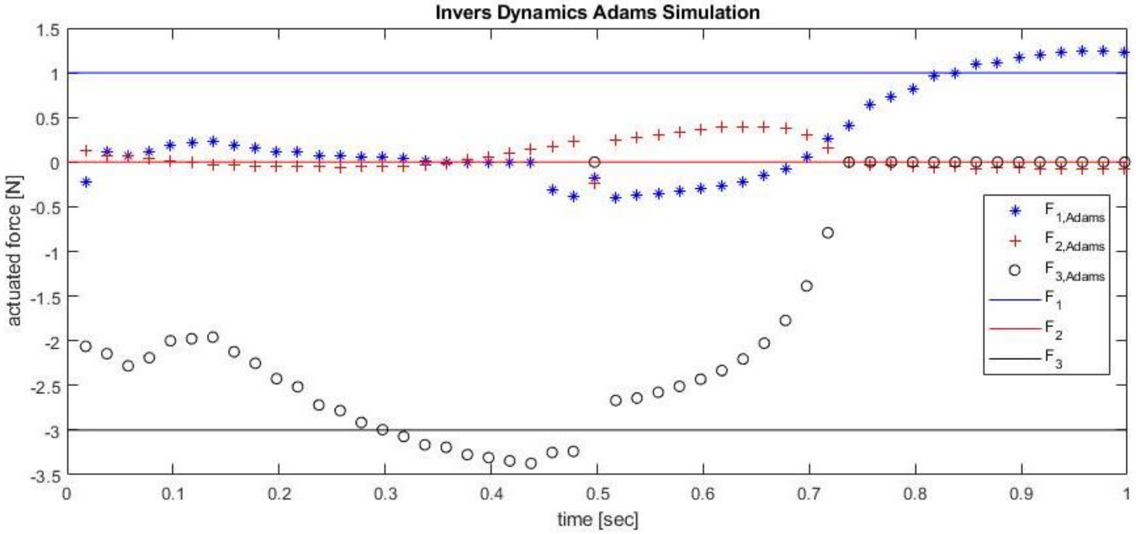
$$\dot{M}\dot{X} = 0 \quad and \quad -\frac{1}{2}\frac{\partial}{\partial X}\left(\dot{X}^T M \dot{X}\right) = 0 \quad \longrightarrow \quad C_p = 0$$

Dynamic Formulation of the Whole Manipulator

$$M(X)\ddot{X} + C(X,\dot{X})\dot{X} + G(X) = Q$$

$$Q_i = F_d + F = F_d + J^T\tau$$





05

Motion Control

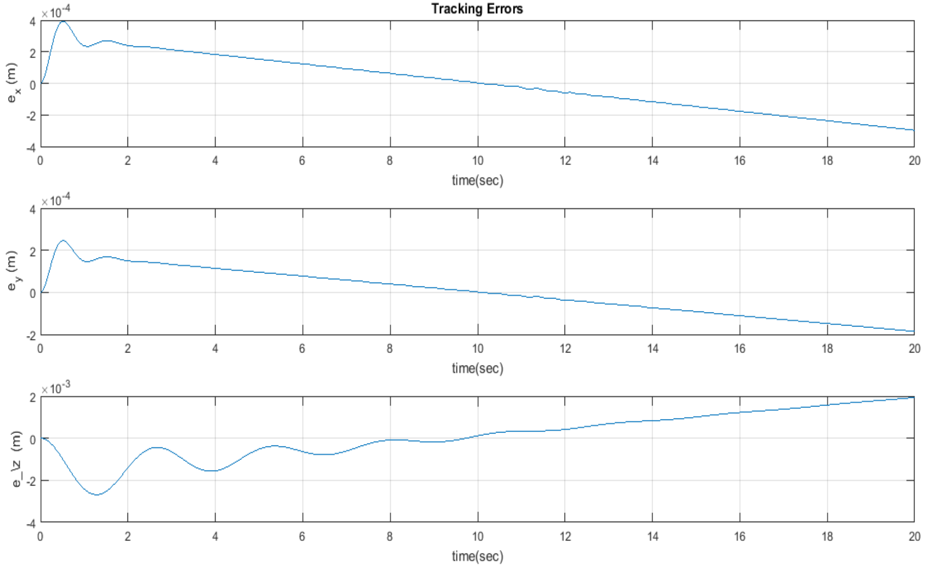
CPRA System

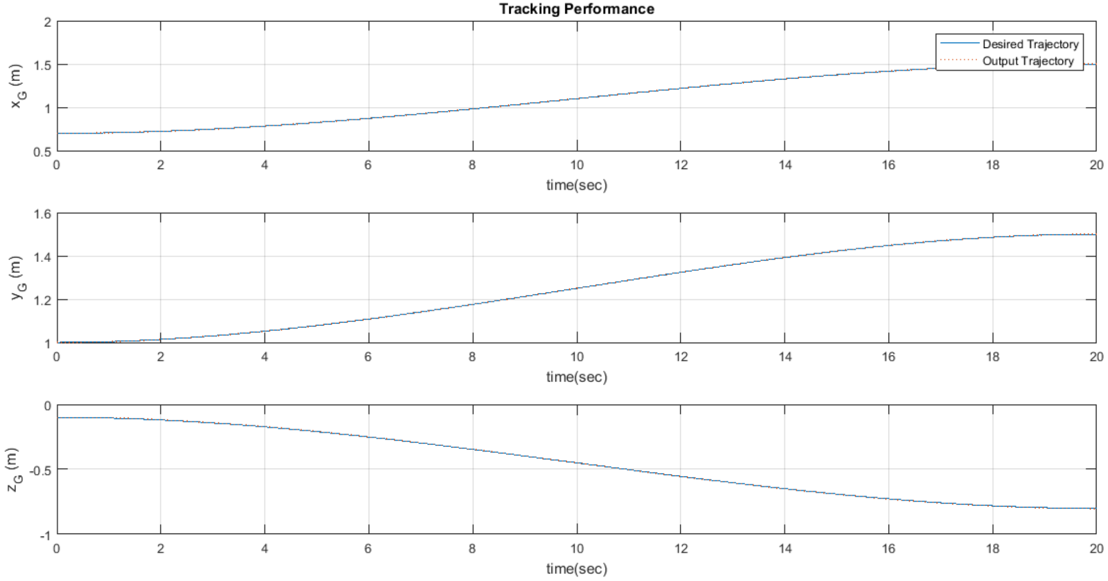
- Decentralized PD
- Feed Forward
- IDC
- Partial IDC
- Robust

ARAS HI-TECH ROBOTIC SOLUTIONS
ADVANCED ROBOTICS & AUTOMATED SYSTEMS

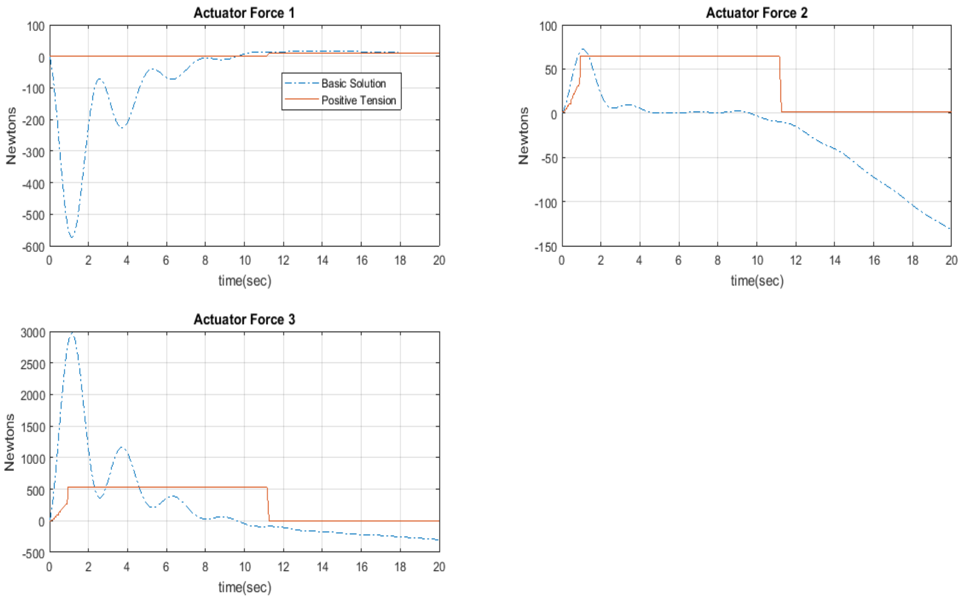
Decentralized PD Controller

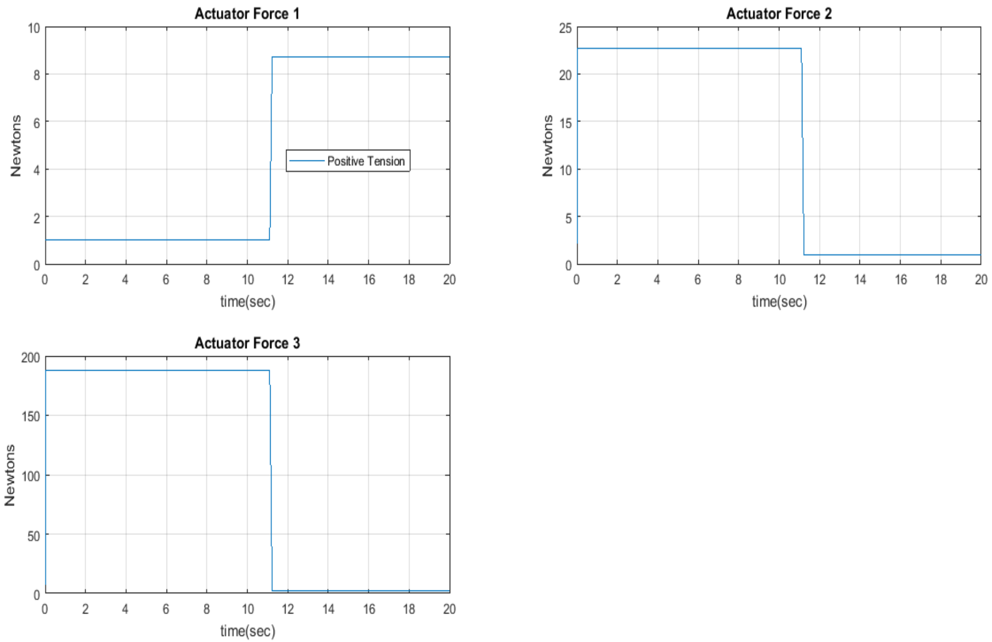
$$K_p = 10^3 \times diag[1, 1, 2000], \quad K_d = 10^2 \times diag[1, 1, 2000]$$



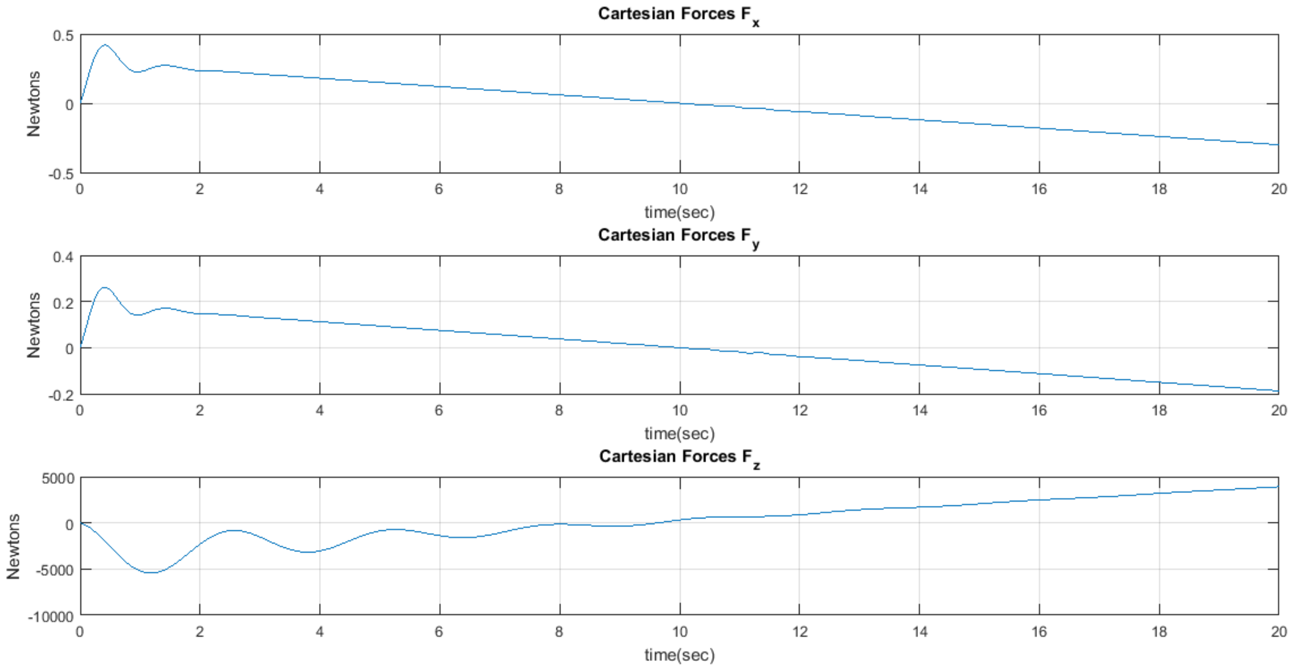


Decentralized PD Controller



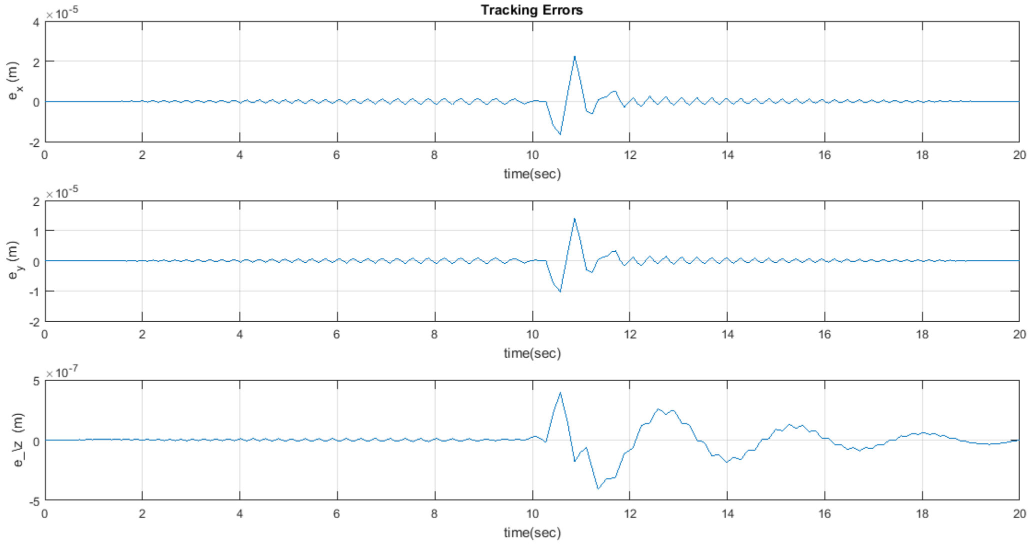


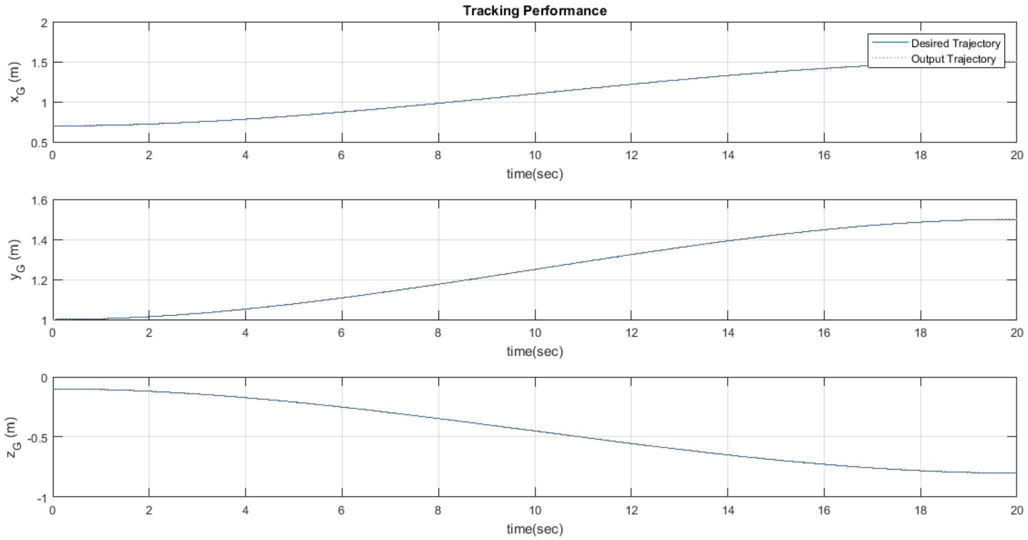
Decentralized PD Controller



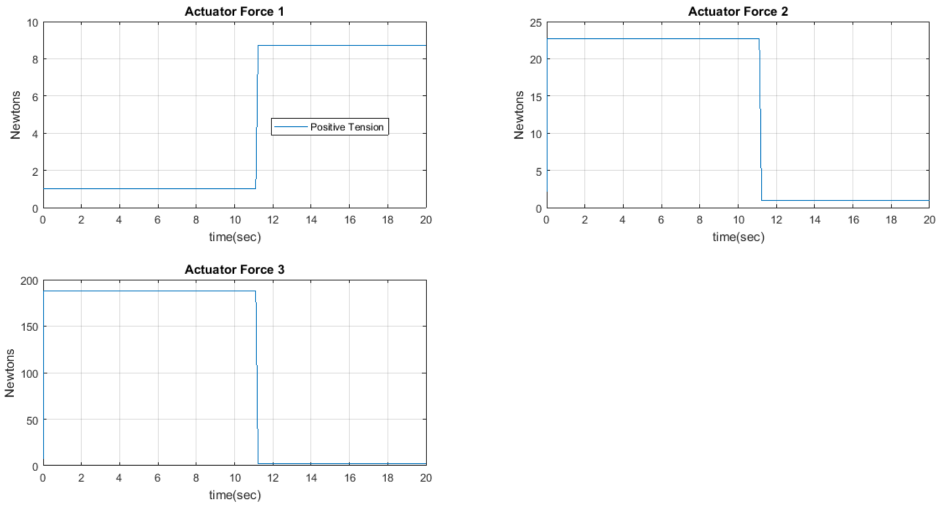
Feed Forward Control

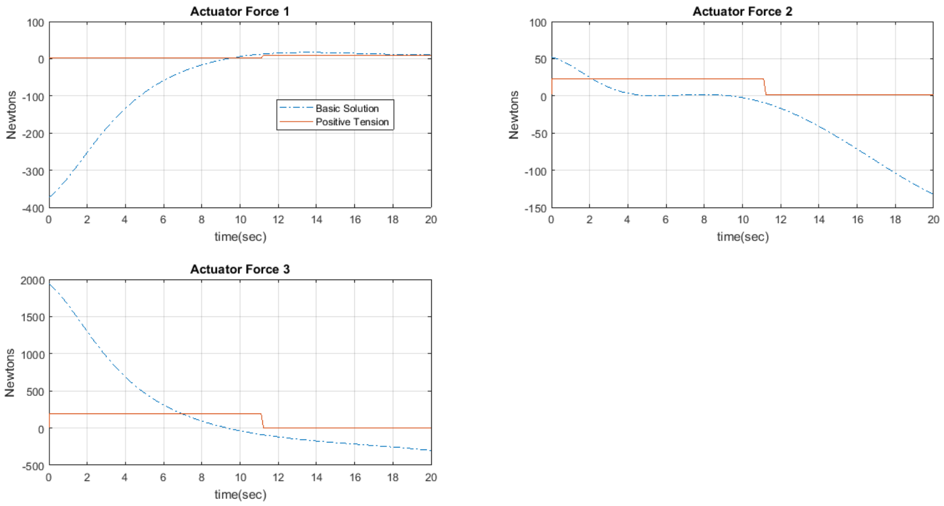
$$K_p = 10^3 \times diag[1, 1, 2000], \quad K_d = 10^2 \times diag[1, 1, 2000]$$



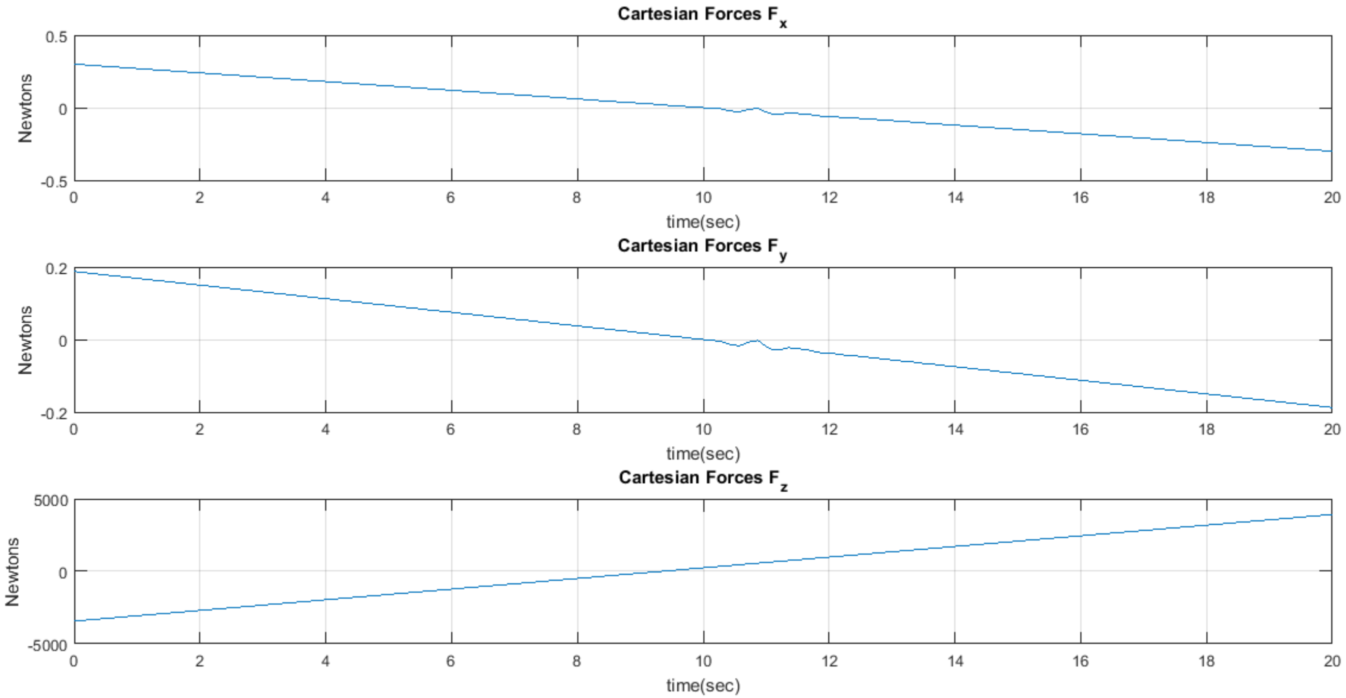


Feed Forward Control



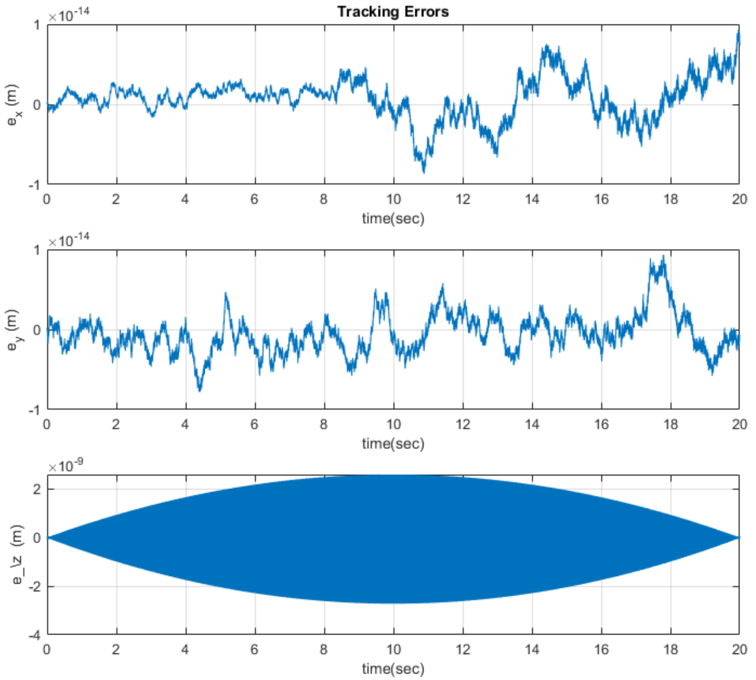


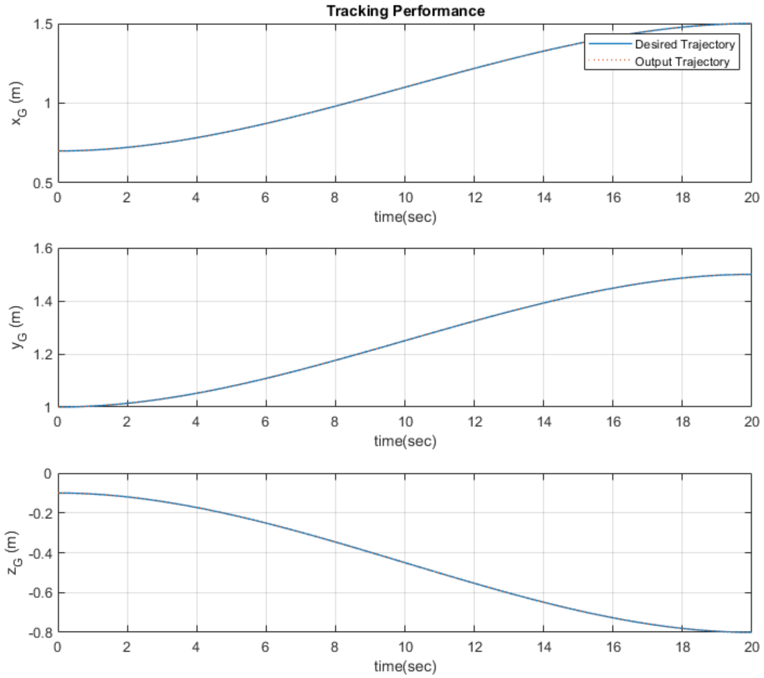
Feed Forward Control



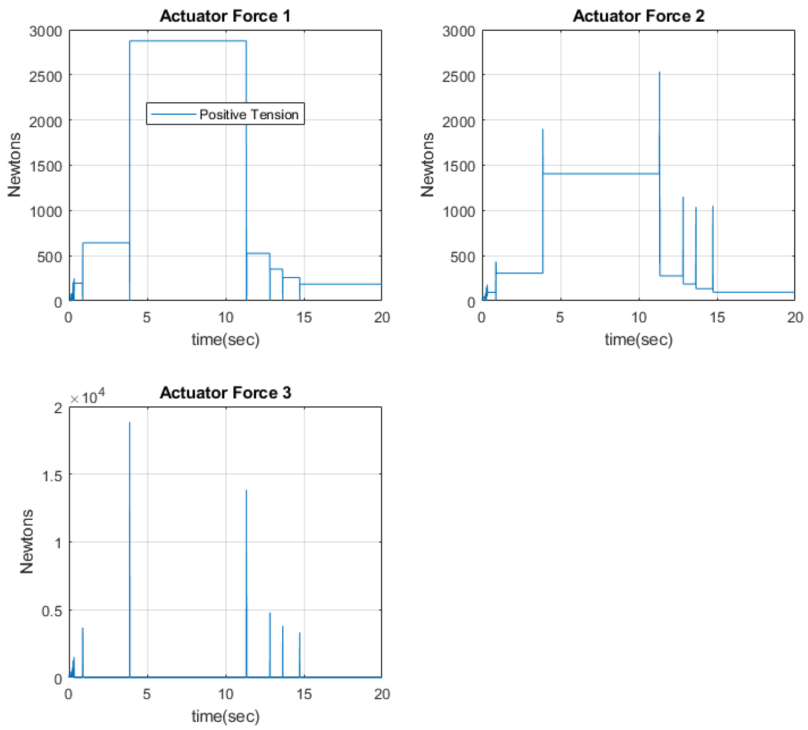
IDC Controller

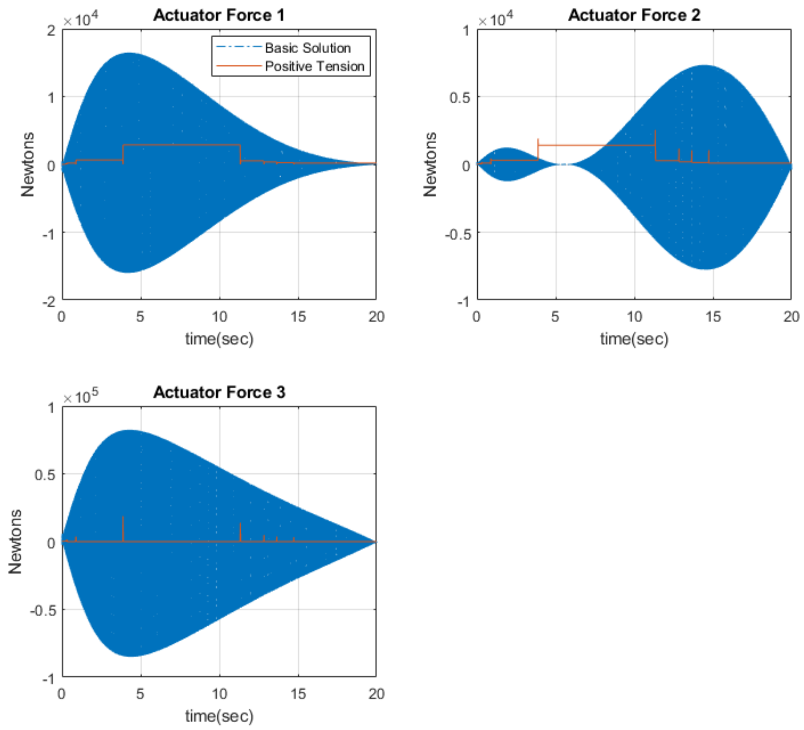
$$K_p = 10^3 \times diag[1, 1, 2000], \quad K_d = 10^2 \times diag[1, 1, 2000]$$



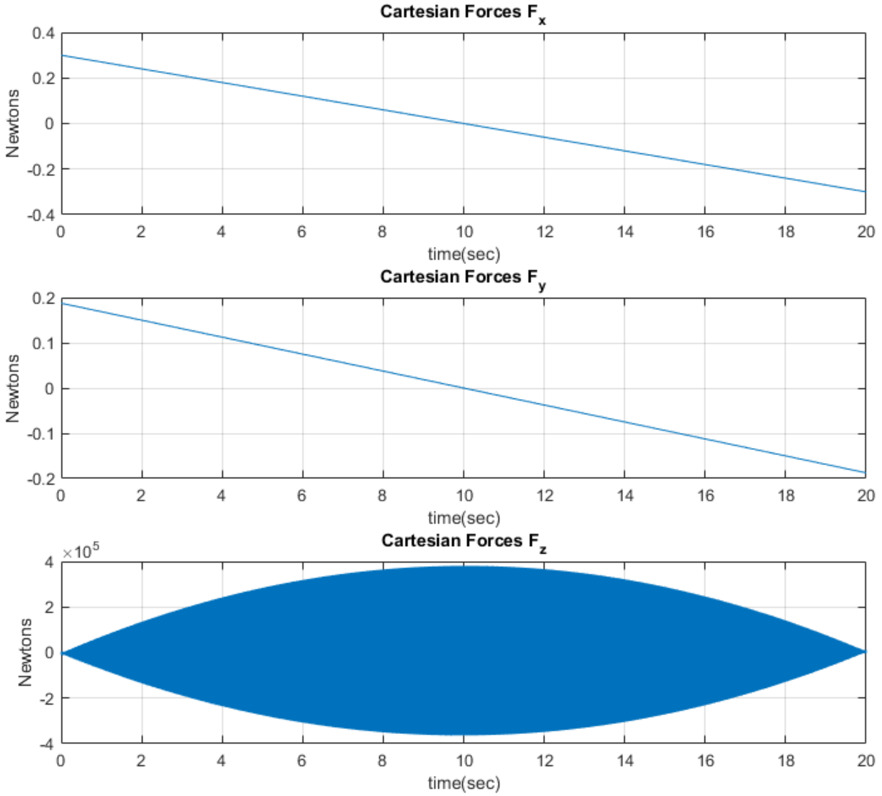


IDC Controller



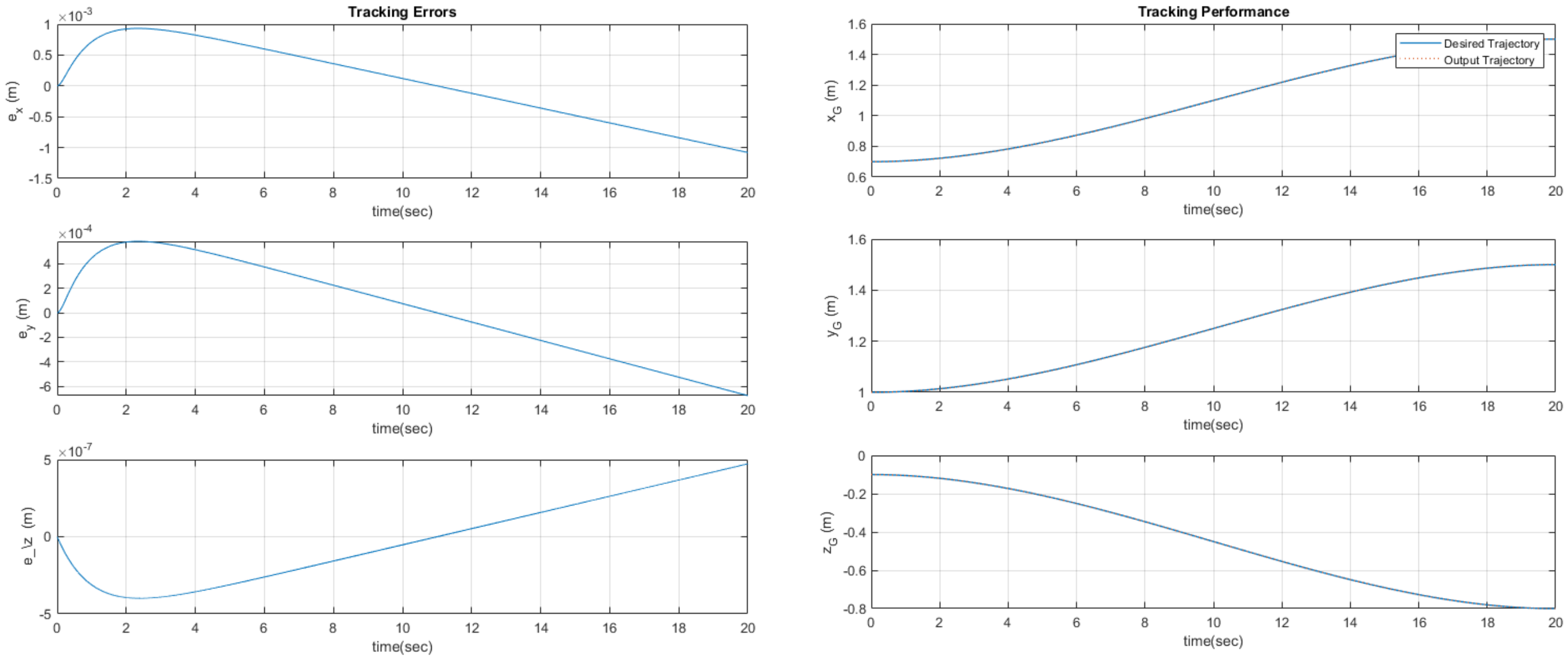


IDC Controller

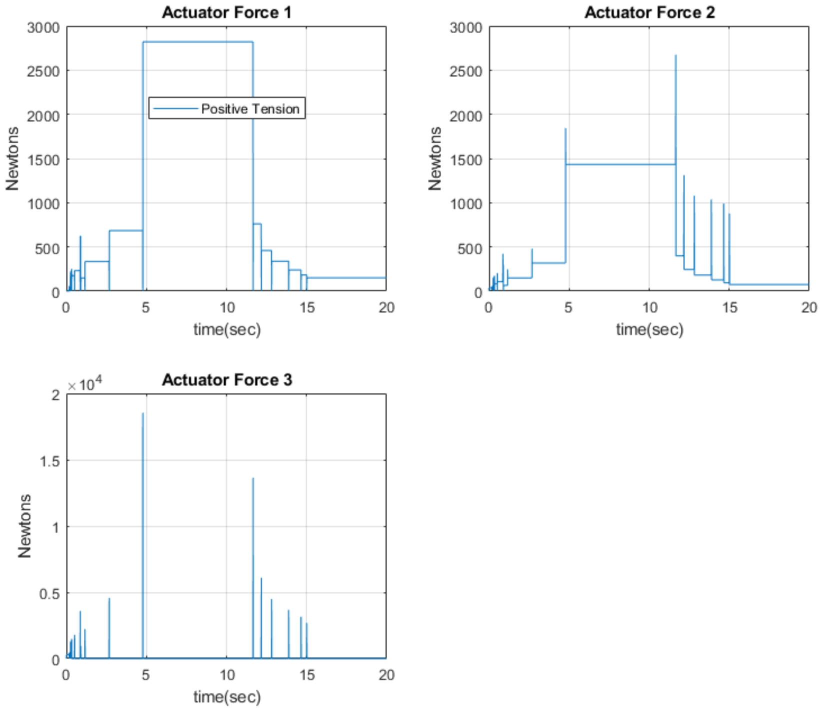


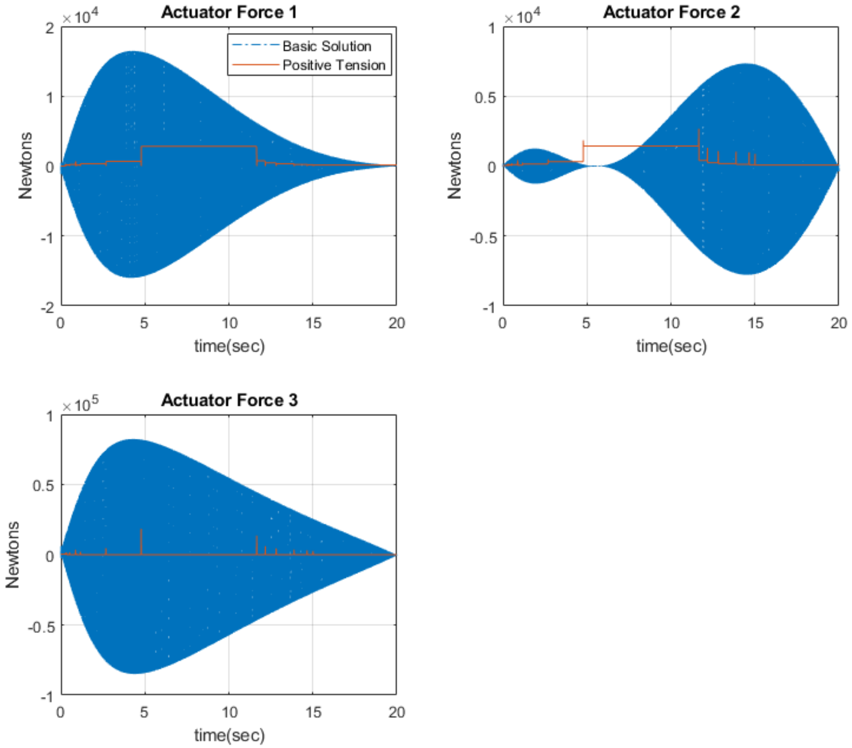
Partial IDC Controller

$$K_p = 10^3 \times diag[1, 1, 2000], \quad K_d = 10^2 \times diag[1, 1, 2000]$$

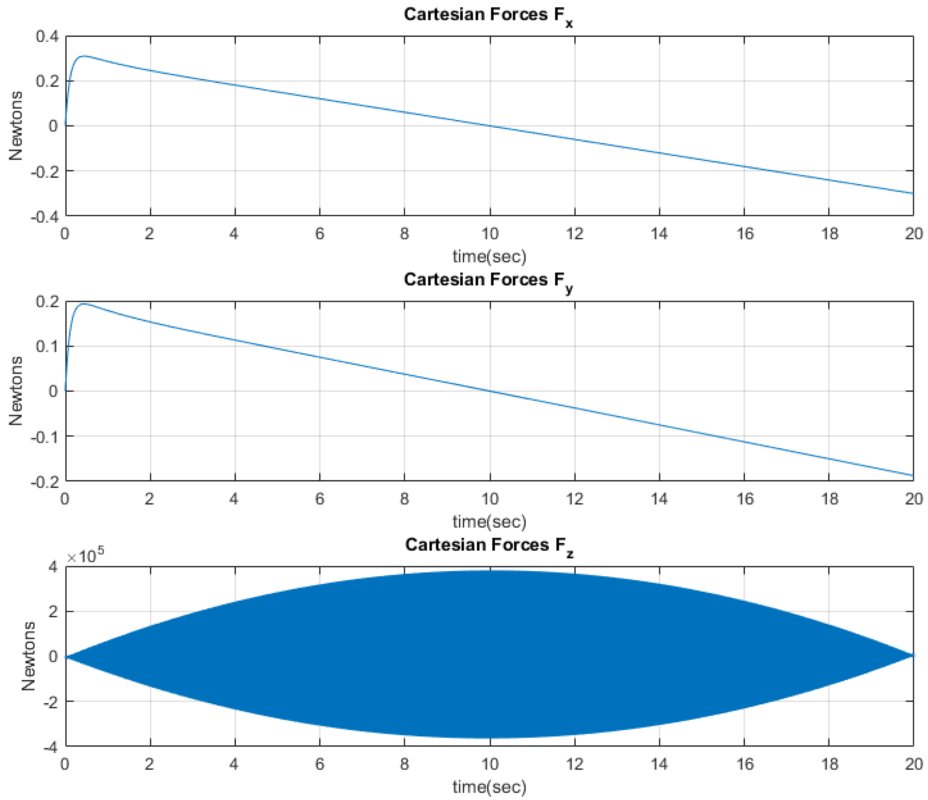


Partial IDC Controller



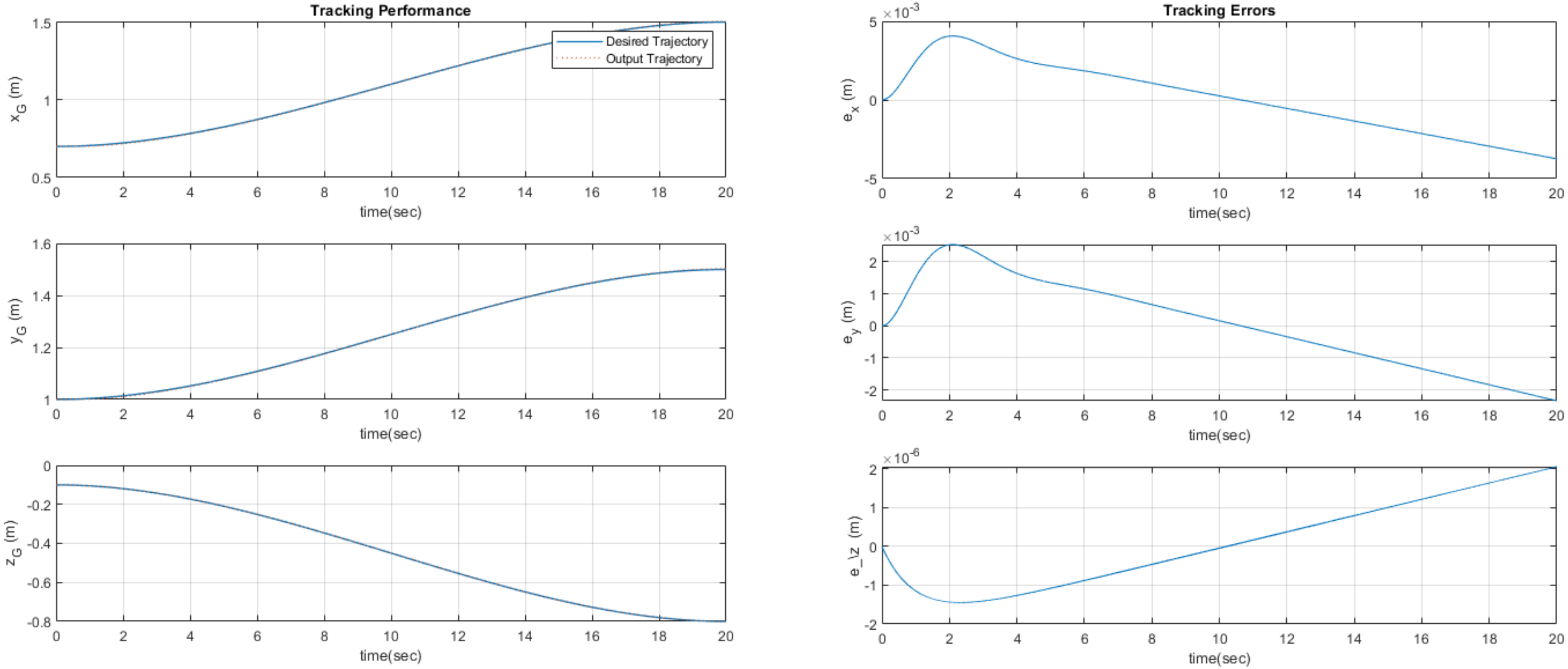


Partial IDC Controller

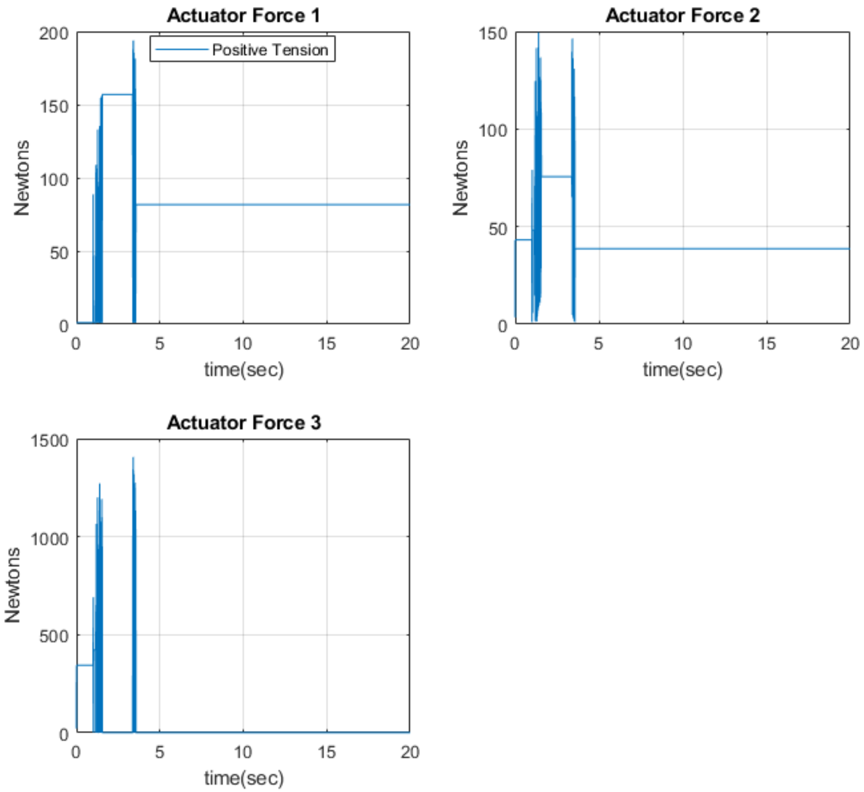


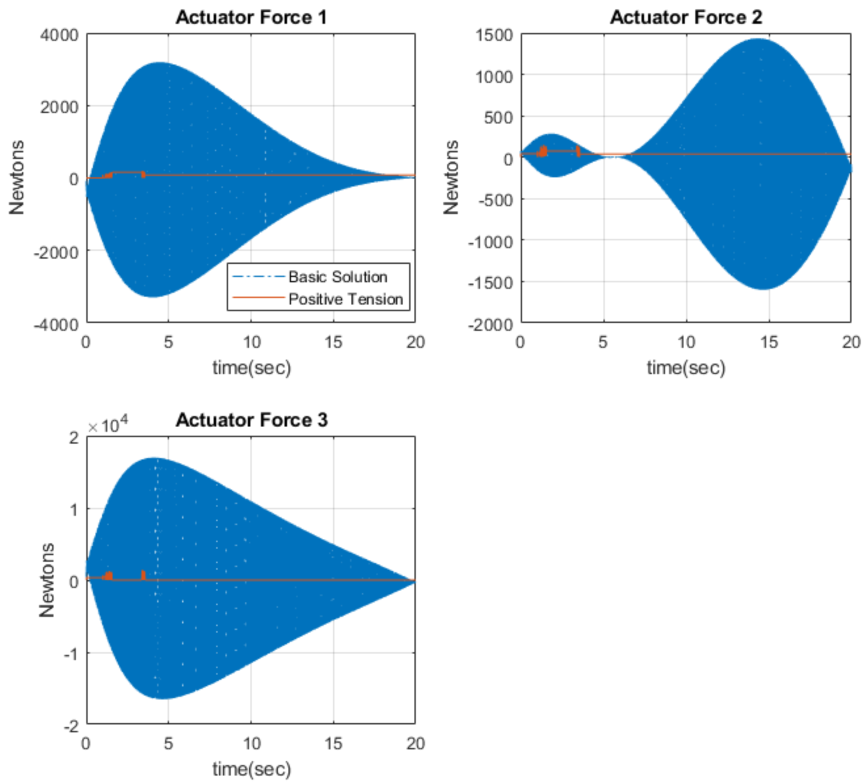
Robust Controller

$$K_p = 10^3 \times diag[1, 1, 2000], \quad K_d = 10^2 \times diag[1, 1, 2000]$$

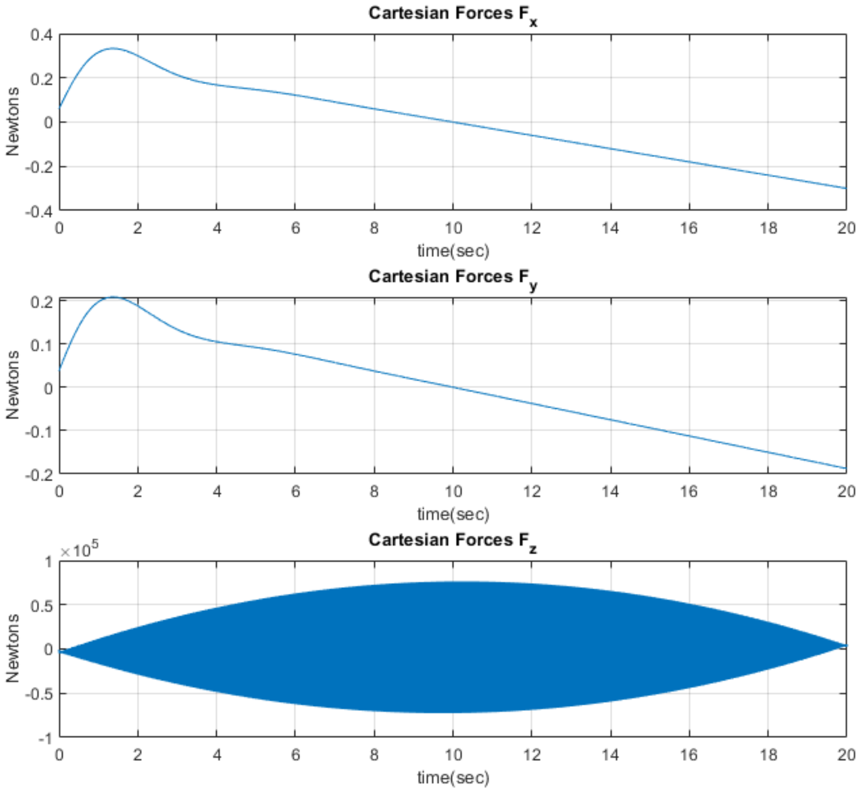


Robust Controller

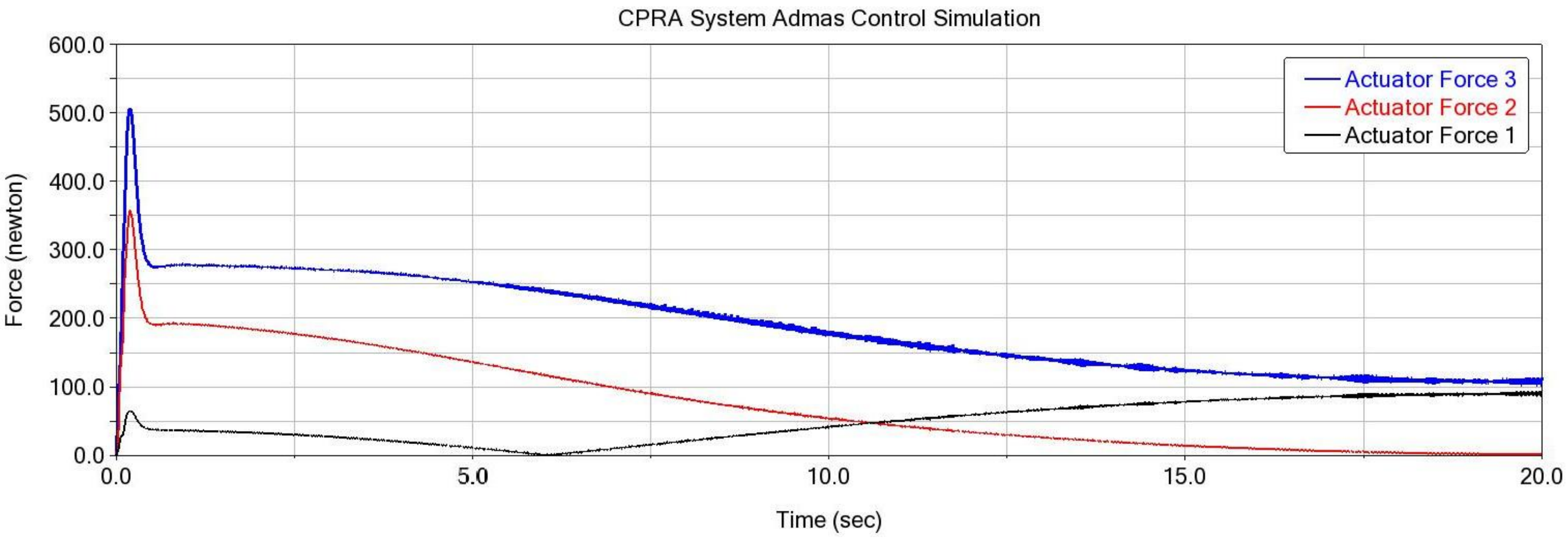




Robust Controller



Motion Control Adams Simulation



06

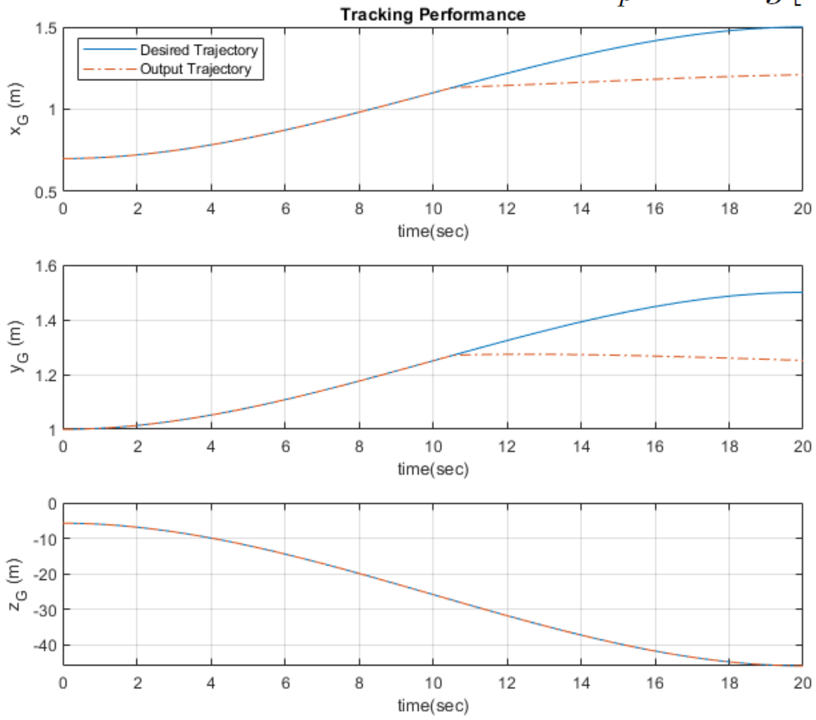
Force Control

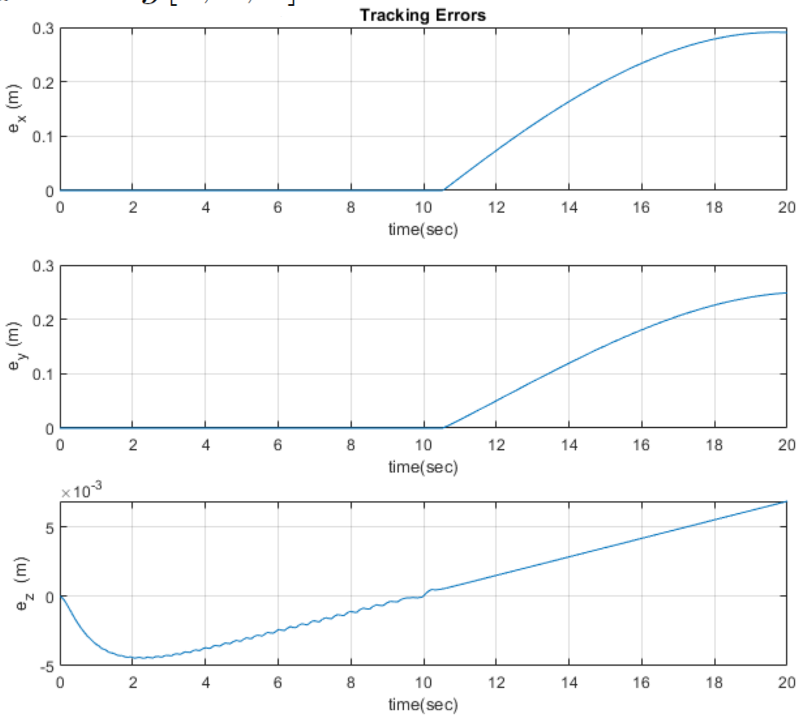
CPRA System

- Stiffness Control
- Impedance Control

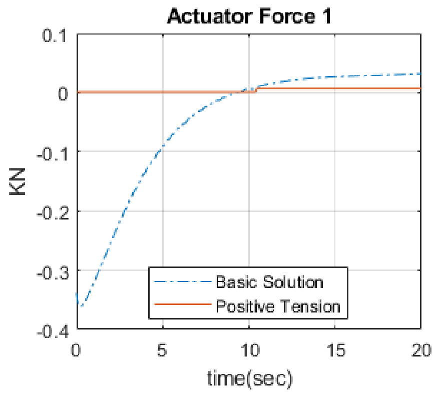
Stiffness Control

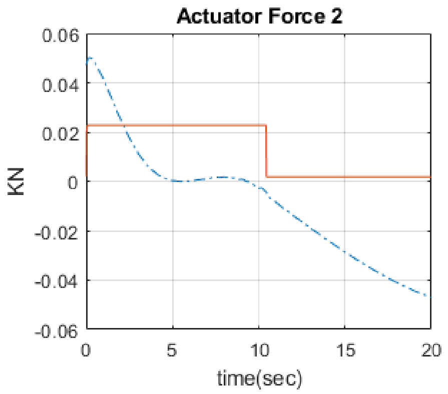
$$K_p = diag[1,1,1], \quad K_d = diag[1,1,1]$$

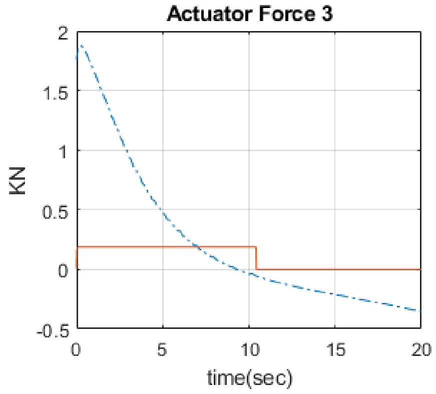


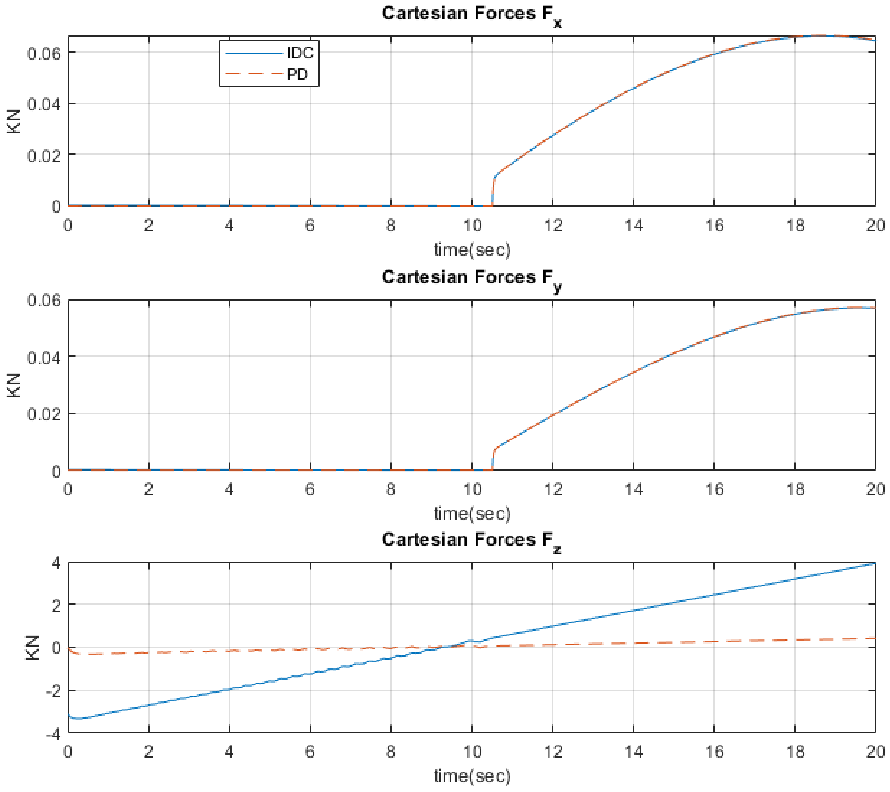


Stiffness Control

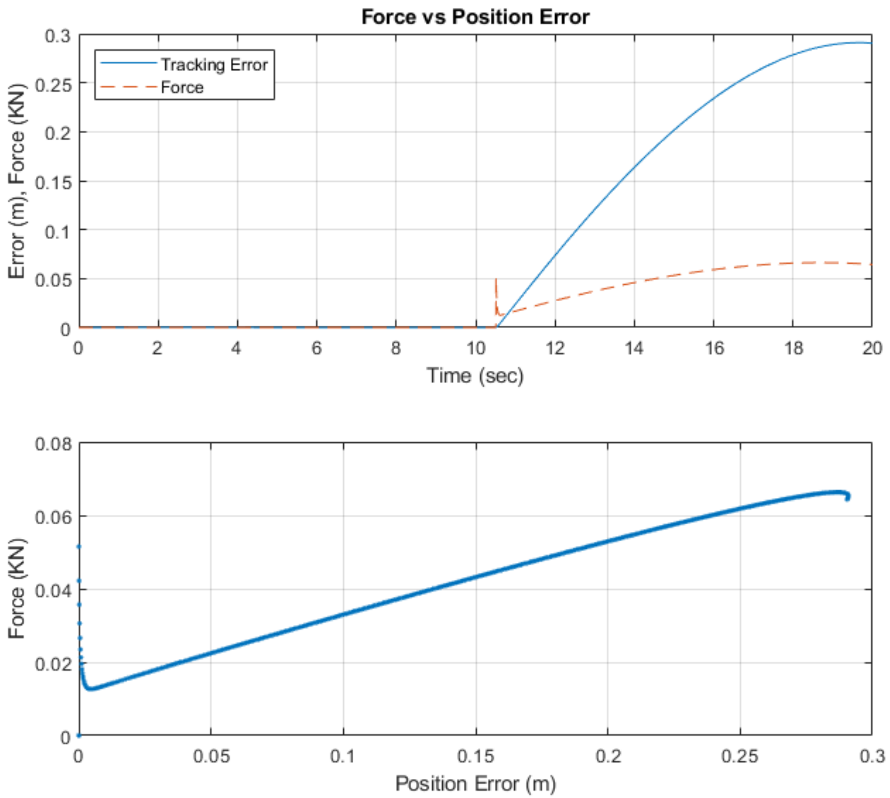






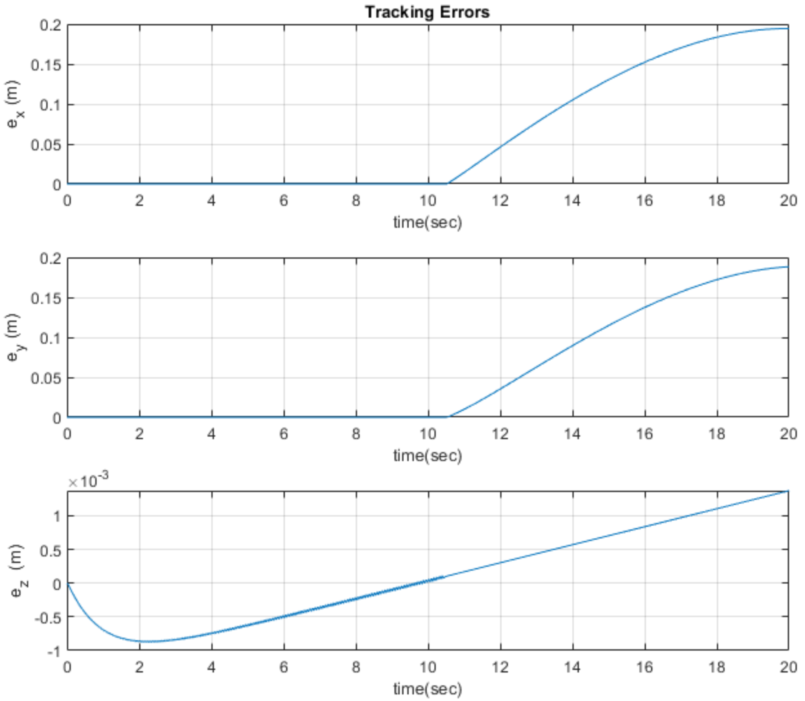


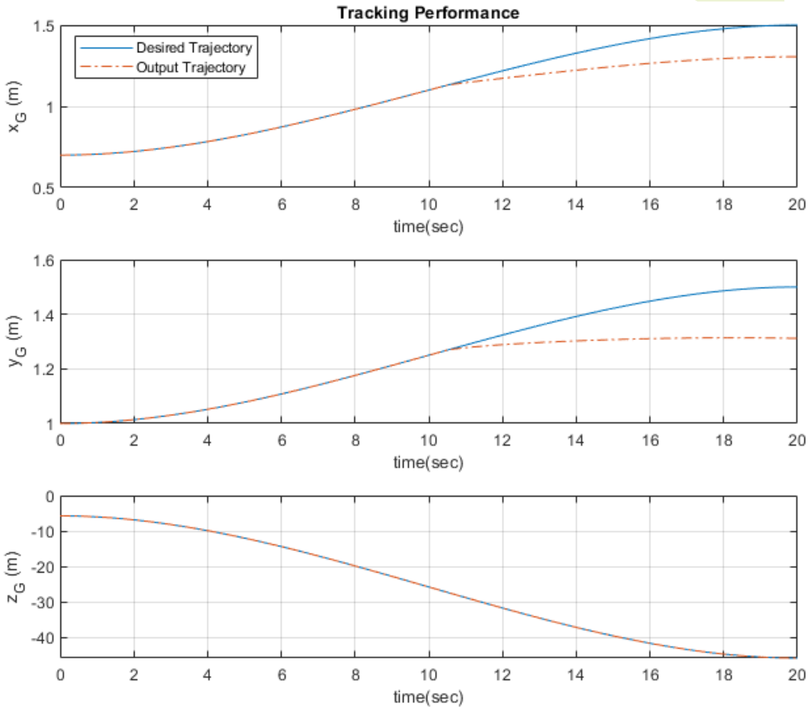
Stiffness Control



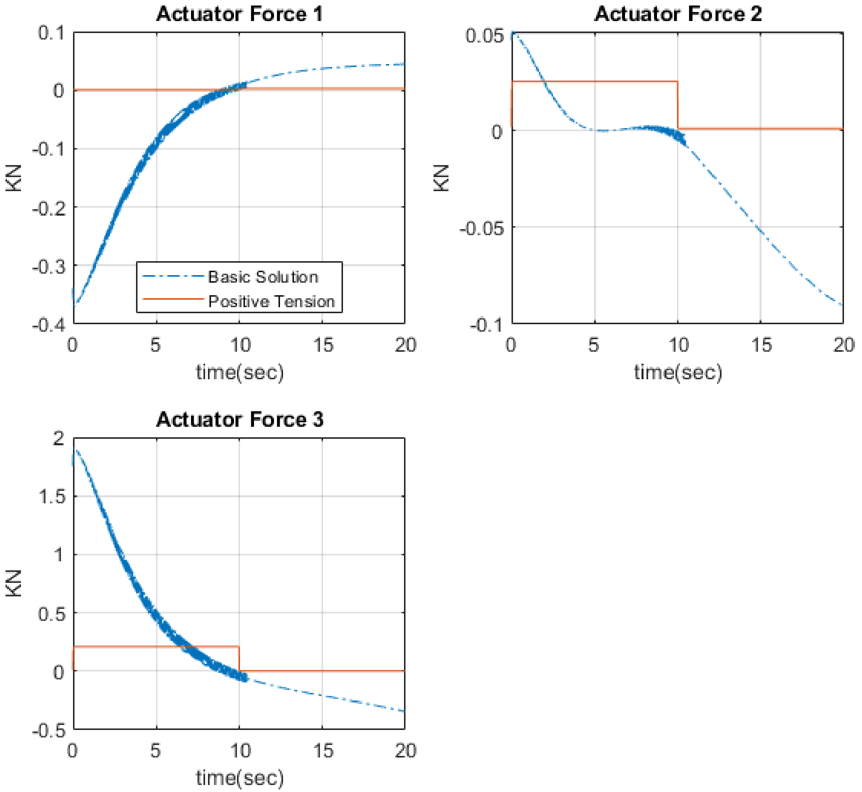
Stiffness Control (5x)

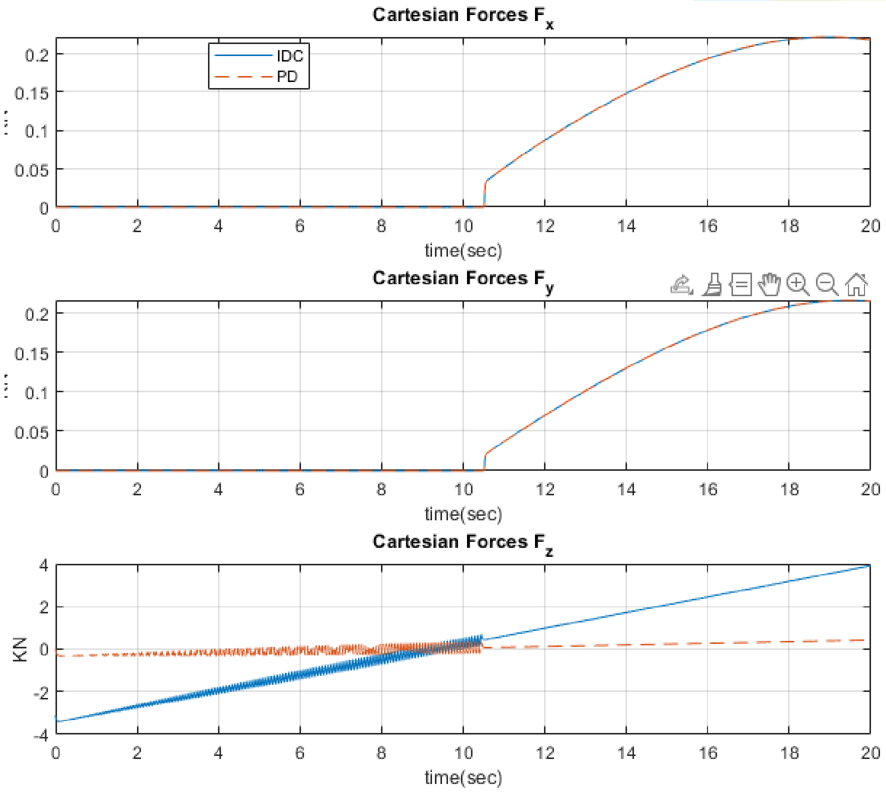
$$K_p = 5 \times diag[1,1,1], \quad K_d = 5 \times diag[1,1,1]$$



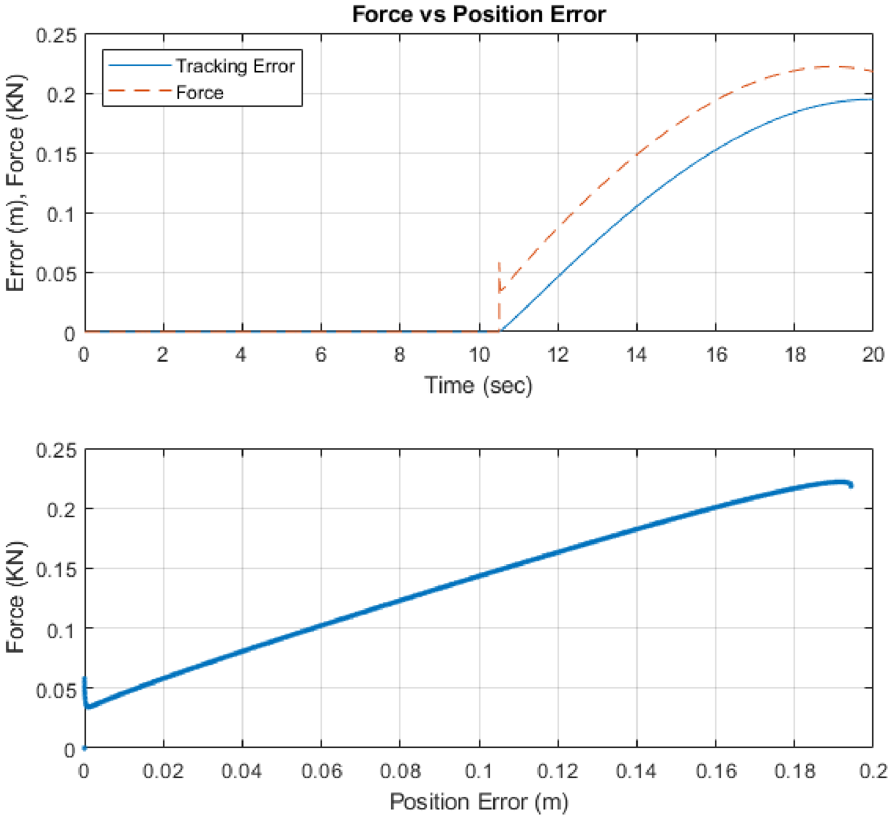


Stiffness Control (5x)



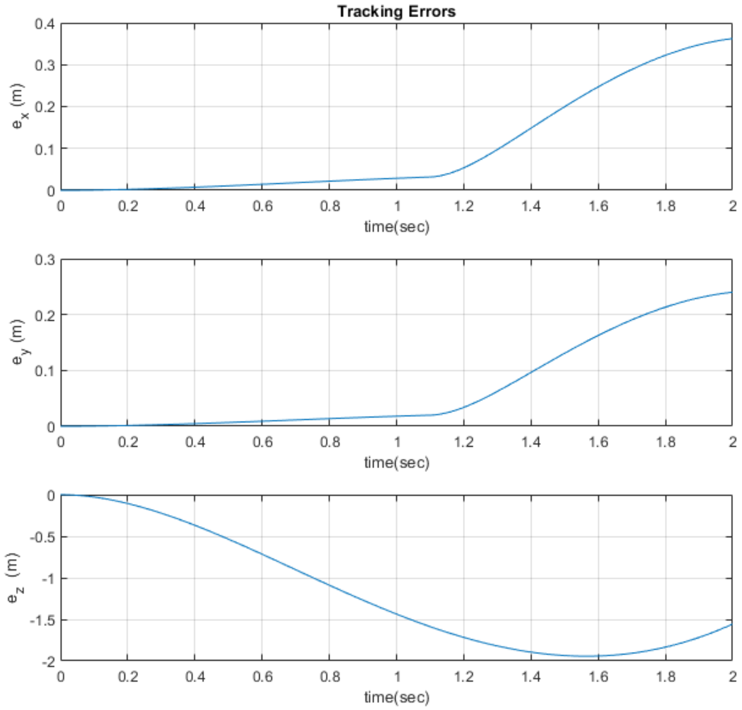


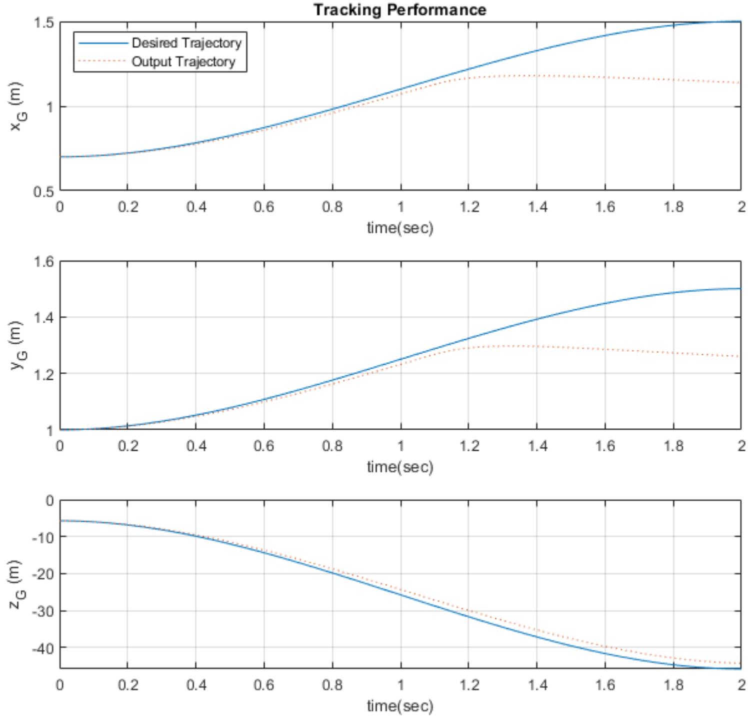
Stiffness Control (5x)



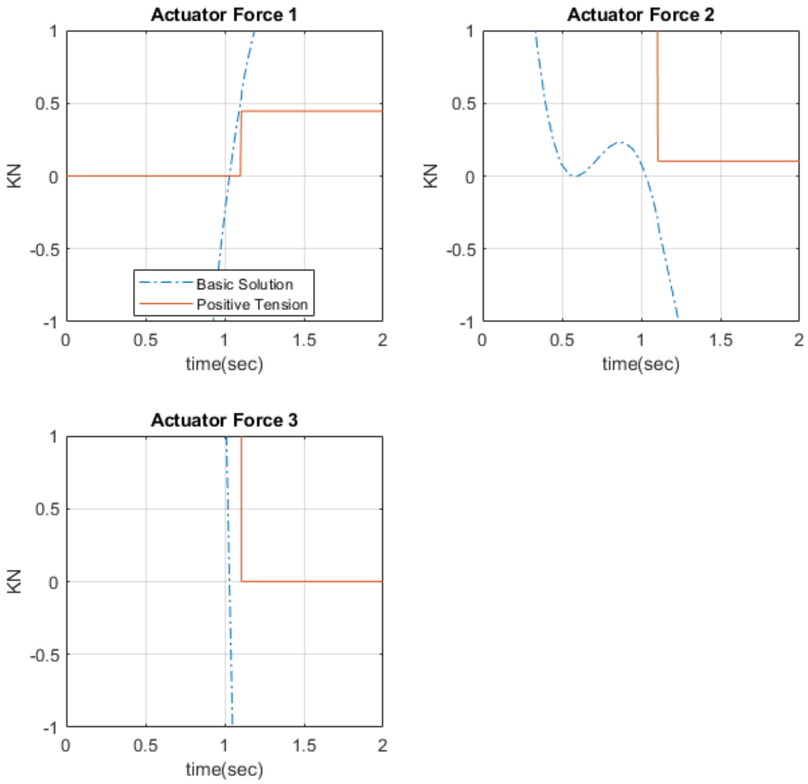
Impedance Control

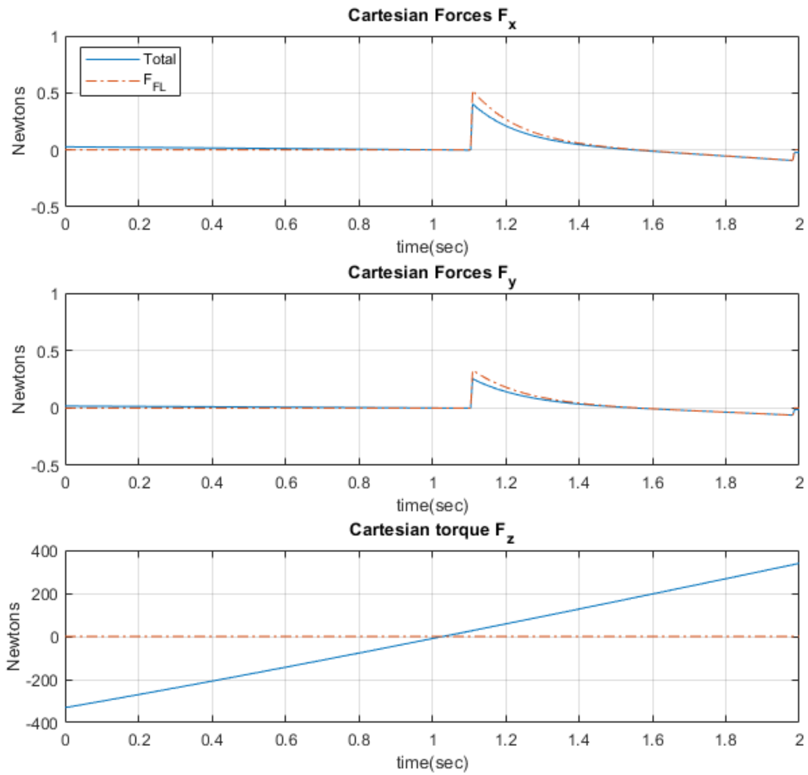
$$K_d = 100 \times diag[1,1,1], C_d = 20 \times diag[1,1,1], M_d = 100 \times diag[1,1,1]$$





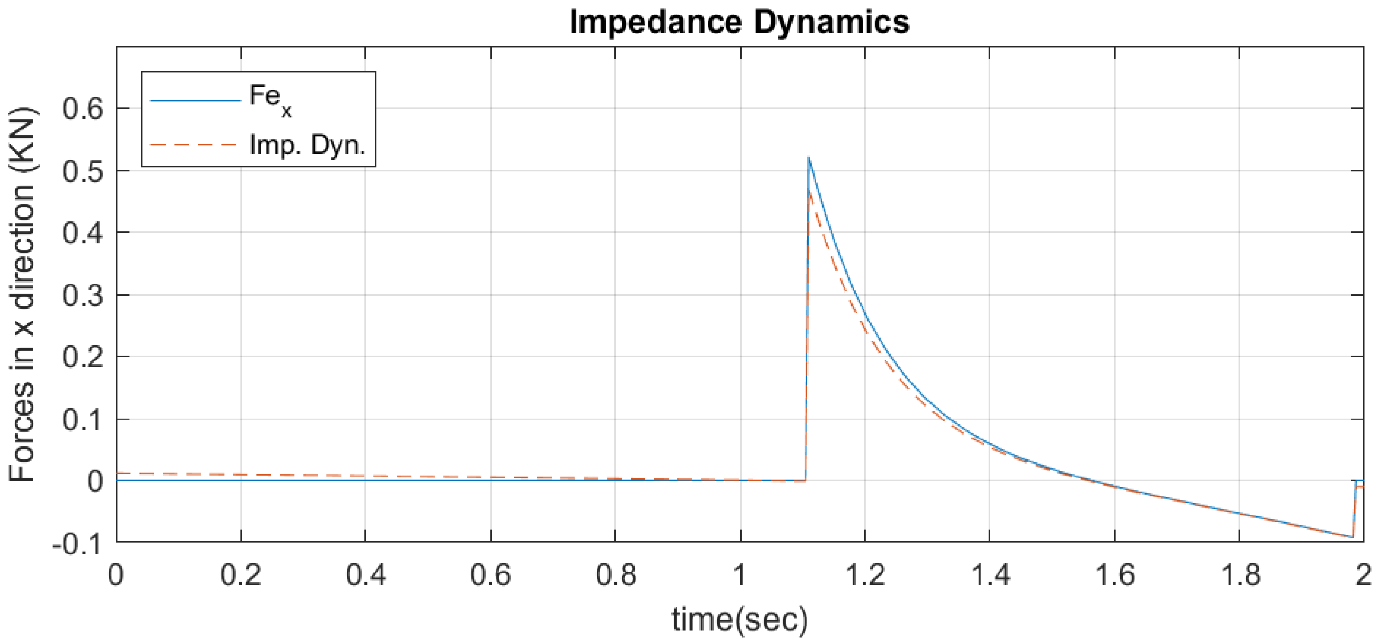
Impedance Control





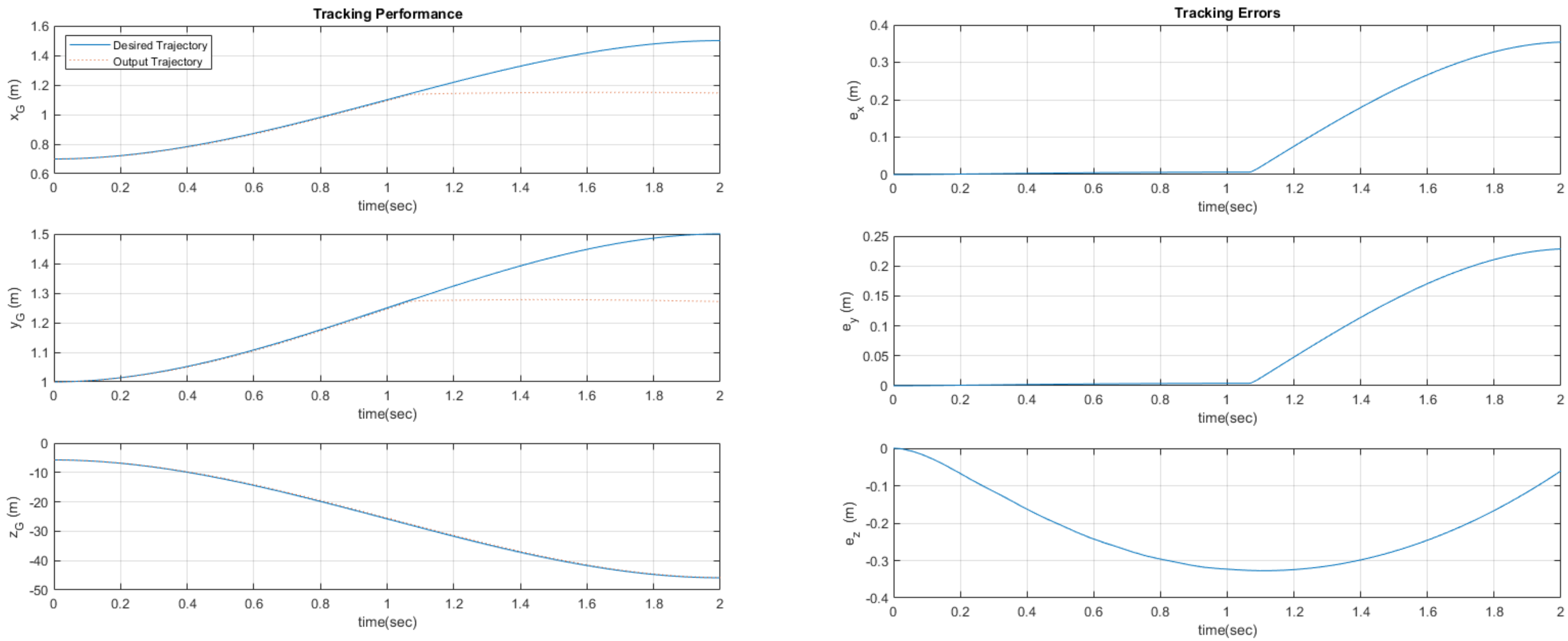
Impedance Control

$$M_d \ddot{e}_x + C_d \dot{e}_x + K_d e_x = \mathcal{F}_e$$

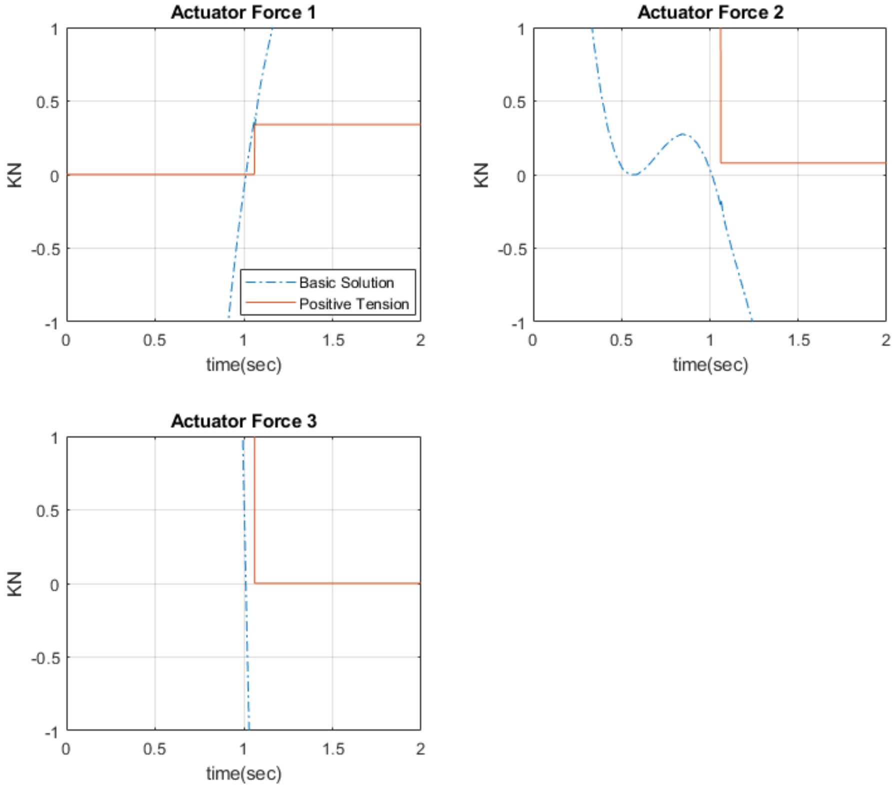


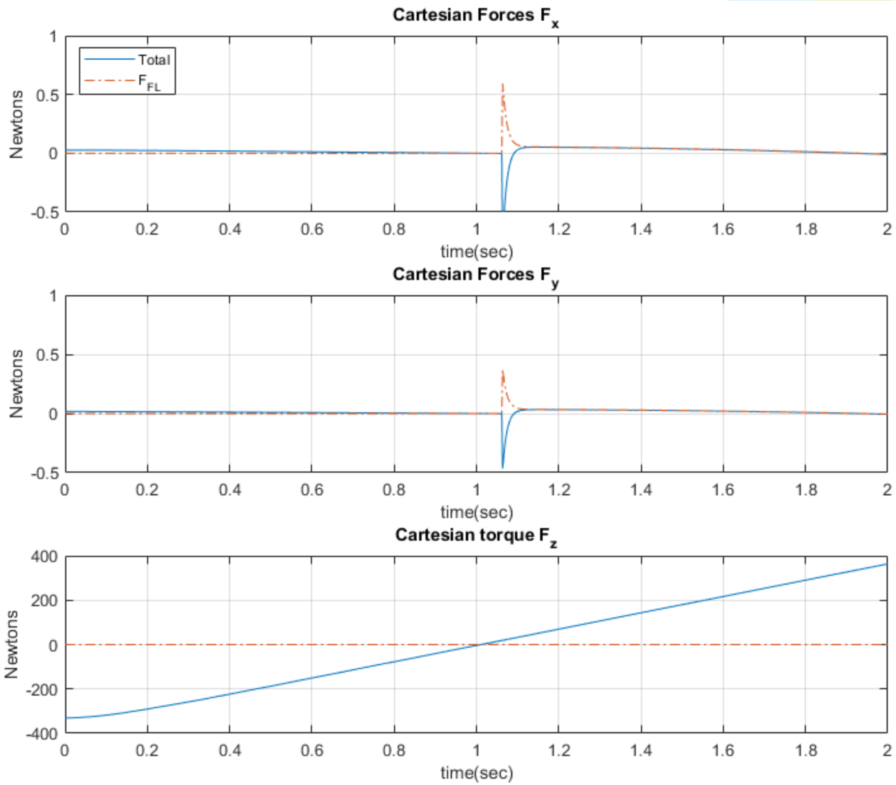
Impedance Control (2)

$$M_d = 100 \times I_{3\times3}, C_d = I_{3\times3}, K_d = 10 \times I_{3\times3}$$



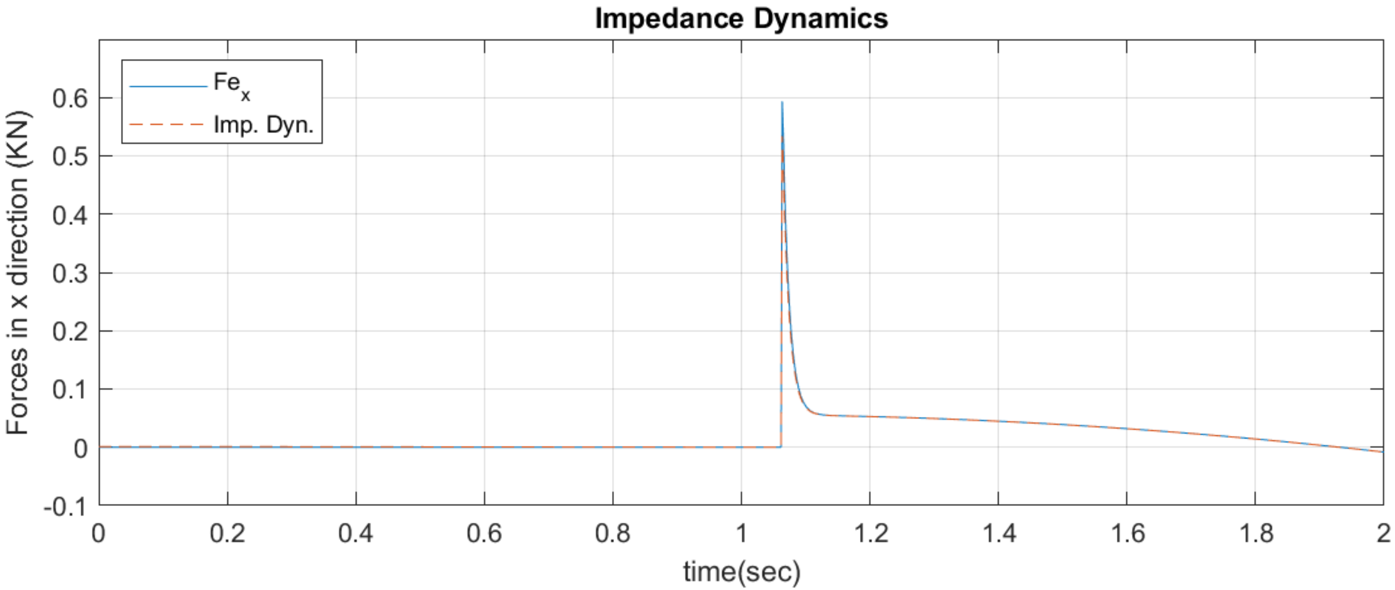
Impedance Control (2)





Impedance Control (2)

$$M_d \ddot{e}_x + C_d \dot{e}_x + K_d e_x = \mathcal{F}_e$$



Thank You