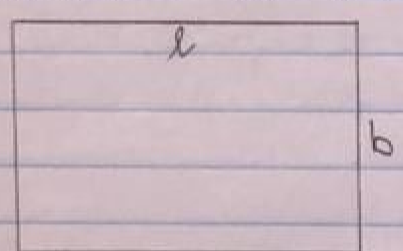
Application of Calculus

{Optimization}

Memo: ex. 13

1) Perimeter of rectangle $= 100$ cm



$P = 2l + 2b$

$100 = 2(l + b)$

$\therefore 50 = l + b$

$\therefore l = (50 - b)$ cm

a) Area $= l \times b$

$A = (50 - b)b$

$A = 50b - b^2$

$A' = 50 - 2b$

$0 = 50 - 2b$

$2b = 50$

$b = 25$ cm

$\therefore b = 25$ cm

$l = 25$ cm

b) Area $= 25 \times 25$
Area $= 625\ cm^2$

c) square.

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3) $s(t) = 2t + 1$

a) $s(0) = 1\ m$

b) i) $s(2) = 5\ m$
ii) $s(5) = 11\ m$

c) $v = s'(t) = 2\ m/s$
i) $s'(2) = 2\ m/s$
ii) $s'(5) = 2\ m/s$

d) There is constant velocity of $2\ m/s$
$\therefore$ constant increase of $2\ m/s$.

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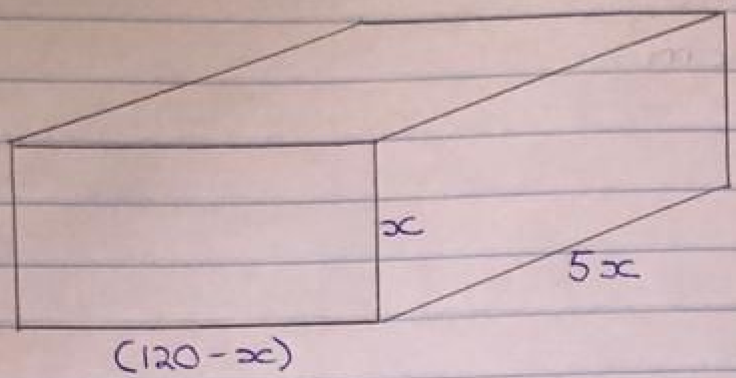
4)



a) $V = l \times w \times h$

$V = (5x)(120-x)(x)$

$V = 5x^2(120-x)$

$V = 600x^2 - 5x^3$

b) $V' = 1200x - 15x^2$

$0 = 15x(80-x)$

$x = 0$ or $x = 80$ m

N.A

6a) $h(t) = 292{,}8t - 4{,}88t^2$

$h'(t) = 292{,}8 - 9{,}76t$

$0 = 292{,}8 - 9{,}76t$

$9{,}76t = 292{,}8$

$t = 30$ sec.

b) $h'(0) = 292{,}8$ m/s.

c) $0 = 292{,}8t - 4{,}88t^2$

$0 = 4{,}88t(60-t)$

$t = 0$ or $t = 60$ $\therefore 60$ sec.

d) $h'(60) = -292{,}8 \text{ m/s}$

e) $h''(t) = -9{,}76 \text{ m/s}^2$

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8) a) $f(x) = x^3 - 11x^2 + 28x$

$0 = x(x^2 - 11x + 28)$

$0 = x(x-7)(x-4)$

$\therefore x = 0 \quad , \quad x = 7 \quad , \quad x = 4$

b) $f(x) - g(x)$

$x^3 - 11x^2 + 28x - 9x$

$x^3 - 11x^2 + 19x$

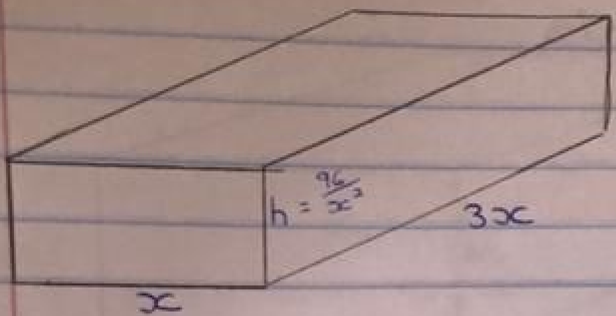
c) $3x^2 - 22x + 19 = 0$

$(3x - 19)(x - 1) = 0$

$x = \frac{19}{3} \qquad x = 1$

d) AB = 9 units.

10)



$V = 288\ cm^3$

a) $V = l \times w \times h$

b) $288 = (3x)(x)(h)$

$288 = 3x^2 \times h$

$\therefore h = \frac{96}{x^2}$

$\therefore \cancel{V = 3x^2 \times \frac{96}{x^2}}$

c) $SA = 2(x)(3x) + 2(3x)(\frac{96}{x^2}) + 2x(\frac{96}{x^2})$

$S = 6x^2 + \frac{576}{x} + \frac{192}{x}$

$S = 6x^2 + \frac{768}{x}$

d) $S = 6x^2 + 768x^{-1}$

$S' = 12x - 768x^{-2}$

$0 = 12x - \frac{768}{x^2}$

$\frac{768}{x^2} = \frac{12x}{1}$

$768 = 12x^3$

$x^3 = 64$

$x = 4$

e) $SA = 6(4)^2 + \frac{768}{4}$

$SA = 288 \text{ cm}^2$

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11) $s(t) = 2t^3 - 18t^2 + 48t$.

a) i) $s(1) = 32$

ii) $s(2) = 40$

iii) $s(3,5) = 33,25$

b) it moves away from A and then it moves back towards it

c) y-intercept $= 0$

$0 = 2t(t^2 - 9t + 24)$

$t/x = 0$

$$t/x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(24)}}{2(1)}$$

$= \text{undf}$.

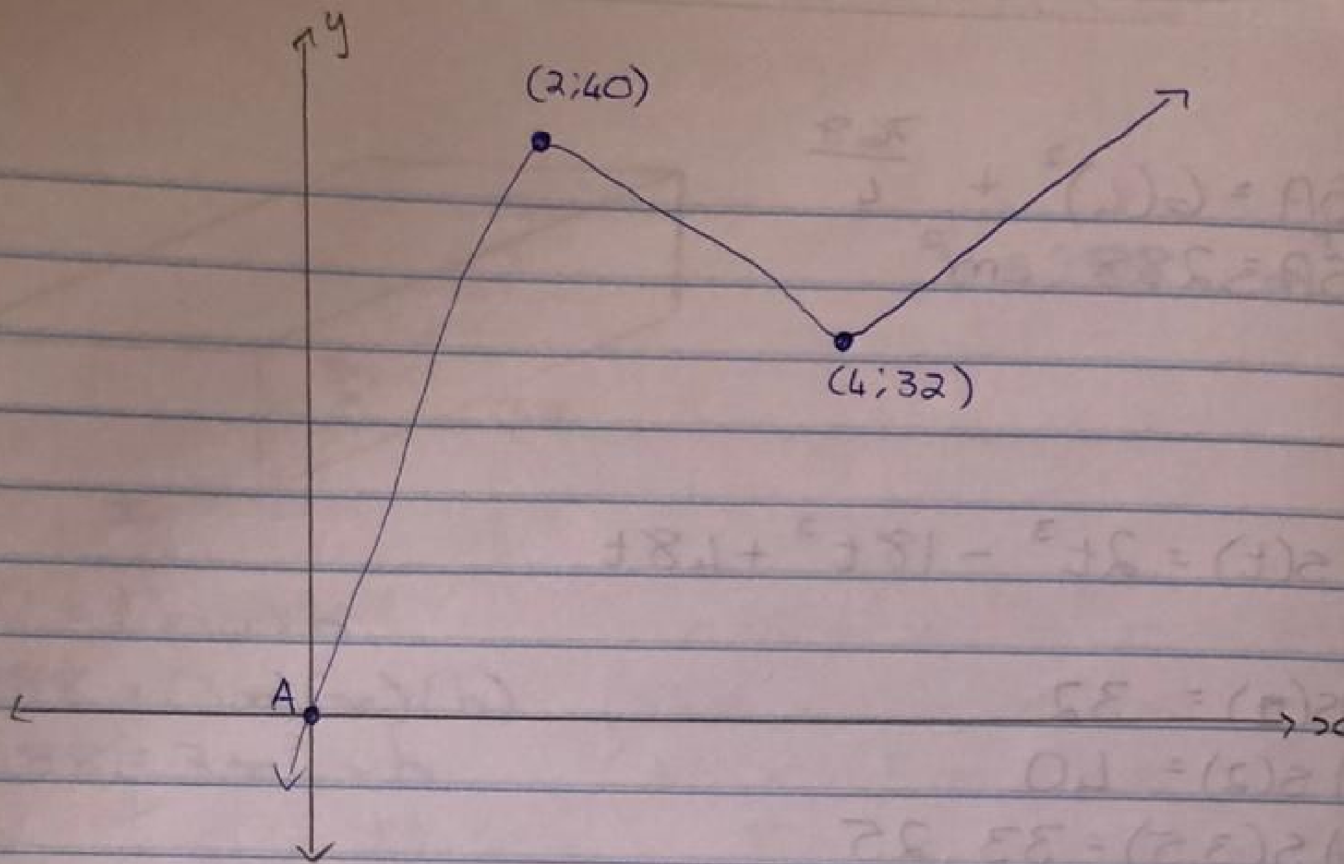
TP's: $s'(t) = 6t^2 - 36t + 48$

$0 = t^2 - 6t + 8$

$0 = (t-4)(t-2)$

$t = 4 \qquad t = 2$

$s(4) = 32 \qquad s(2) = 40$



d) $x \in (0;2)$
$x \in (4;\infty)$

e)

12) $1\,l = 1\,m^3$
$500\,ml = 0{,}5\,m^3$

a) $V = \pi r^2 \times h$
$0{,}5 = \pi r^2 \times h$
$\therefore h = \frac{0{,}5}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r \times h$
$SA = 2\pi r^2 + 2\pi r \left(\frac{0{,}5}{\pi r^2}\right)$

$SA = 2\pi r^2 + \frac{1}{r}$
$SA = 2\pi r^2 + r^{-1}$
$SA' = 4\pi r - r^{-2}$

$0 = \frac{4\pi r}{1} - \frac{1}{r^2}$

$\frac{1}{r^2} = \frac{4\pi r}{1}$
$1 = 4\pi r^3$

$\frac{1}{4\pi} = r^3$
$\therefore r = 0{,}43\ m$

$h = 0{,}86\ m$

b) $r = 43\ cm$
$h = 86\ cm$
$\therefore SA = 2\pi(43)^2 + 2\pi(43)(86)$
$SA = 34\,852{,}8289\ cm^2 \quad \times 0{,}5c$

$\therefore 17426{,}4145\,c \qquad = R\,174{,}26$

13) a) $s = 12$ cm

b) $r^2 + h^2 = s^2$ (pythag)
$r^2 = 144 - h^2$

$r = \sqrt{144 - h^2}$

c) $V = \frac{1}{3}\pi r^2 \times h$
$V = \frac{1}{3}\pi(\sqrt{144 - h^2})^2 \times h$
$V = \frac{1}{3}\pi h\,(144 - h^2)$
$V = 48\pi h - \frac{1}{3}\pi h^3$

d) $V' = 48\pi - \pi h^2$
$0 = \pi(48 - h^2)$
$0 = 48 - h^2$
$h^2 = 48$
$h = 4\sqrt{3}$

e) $V = 48\pi(4\sqrt{3}) - \frac{1}{3}\pi(4\sqrt{3})^3$
$V = 696{,}5$ cm^3