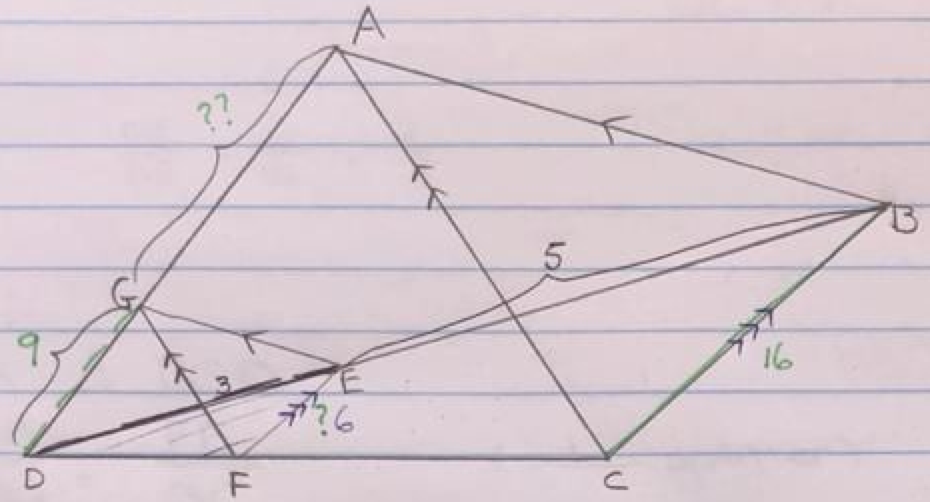
2.1.1) the other two sides proportionally

2.1.2)



a) $\frac{AG}{GD} = \frac{BE}{ED}$ (side-splitter, with AB // GE in Δ ABD)

$\frac{AG}{GD} = \frac{CF}{FD}$ (side-splitter with AC // GF, in Δ ACD)

b) since, $\frac{AG}{GD} = \frac{BE}{ED} = \frac{CF}{FD}$ (proven)

in Δ BCD;
FE // CB (sides prop.)

c) in Δ DEF and Δ DBC;
i) $\hat{D} = \hat{D}$ (common)
ii) $\hat{E} = \hat{B}$ } corresp. $\angle$'s
iii) $\hat{F} = \hat{C}$ } FE // BC.
$\therefore$ Δ DEF ||| Δ DBC ($\angle, \angle, \angle$)

d) if $\frac{DE}{BE} = 0{,}6$ $\quad BC = 16$ $\quad DG = 9$ $\quad \left\{ \begin{array}{c} \text{calculate:} \\ EF \\ AG \end{array} \right\}$

$\downarrow$

$\frac{DE}{BE} = \frac{6}{10} = \frac{3}{5}$

$\therefore$

in Δ BCD;

$\frac{DE}{DB} = \frac{FE}{BC}$ (side splitter)

$\frac{3}{(3+5)} = \frac{FE}{16}$

$48 = 8\ FE$

$\therefore EF = 6$ units

in Δ ABD;

$\frac{DG}{AG} = \frac{DE}{EB}$

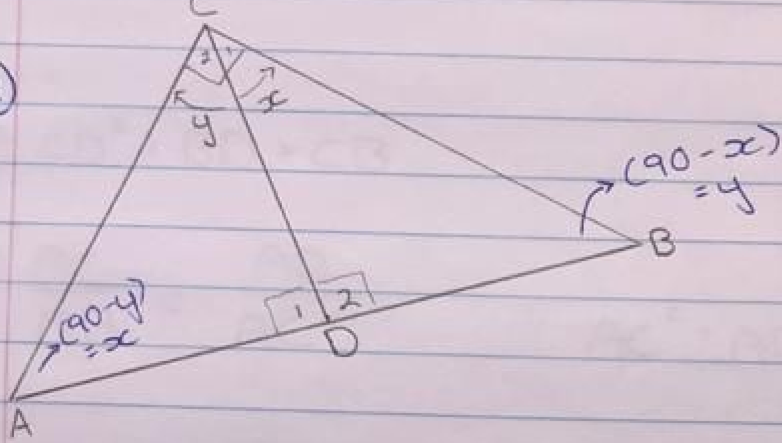
$\frac{9}{AG} = \frac{3}{5}$

$45 = 3\ AG$

$\therefore AG = 15$ units

2.2.1) a) corresponding angles are equal
b) corresponding sides are proportional.

2.2.2)



a) $\hat{C} = x + y$

b) in ΔDCB:
$\hat{D}_2 = 90^\circ$ (CD $\perp$ AB)
$\therefore \hat{B} = 180^\circ - 90^\circ - x$ (int $\angle$s Δ)
$\hat{B} = (90^\circ - x)$

c) in ΔDCA;
$\hat{D}_1 = 90^\circ$ (CD $\parallel$ AB)
$\therefore \hat{A} = (90 - y)$ (int $\angle$s Δ)

d) $\hat{C} = x + y$
$\therefore 90^\circ = x + y$ (given $\hat{C} = 90^\circ$)
So, $x = (90^\circ - y)$ and $y = (90^\circ - x)$

in Δ ACD and Δ CBD;
i) $\hat{D}_1 = \hat{D}_2 = 90^\circ$ (CD $\perp$ AB)
ii) $\hat{C}_2 = \hat{B} = y$
iii) $\hat{A} = \hat{C}_1 = x$
$\therefore \Delta ACD \parallel\!| \Delta CBD$ ($\angle, \angle, \angle$)

e) $\triangle ABC$

f) $\frac{AC}{CB} = \frac{CD}{BD} = \frac{CB}{CD}$

g) $CD^2 = BD \times CB$

h) $\frac{AC}{AB} = \frac{AD}{AC}$ $\therefore AC^2 = AB \times AD$

i) in $\triangle BCD$ and $\triangle BAC$:

i) $\hat{B} = \hat{B}$ (common)

ii) $\hat{D} = \hat{C} = 90^\circ$ (given)

iii) $\hat{C} = \hat{A}$ (3rd $\angle$ of $\triangle$)

$\triangle BCD \parallel\!| \triangle BAC$ ($\angle, \angle, \angle$)

j) $\frac{BC}{BA} = \frac{BD}{BC}$ $\therefore BC^2 = BA \times BD$

k) i) $CD^2 = BD \times CB$

$CD^2 = 2 \times 4$

$CD^2 = 8$

$CD = 2\sqrt{2}$ units

ii) $\frac{BC}{AB} = \frac{BD}{BC}$

$\therefore BC^2 = AB \times BD$

$(4)^2 = AB(2)$

$\therefore AB = 16/2$

$AB = 8$ units

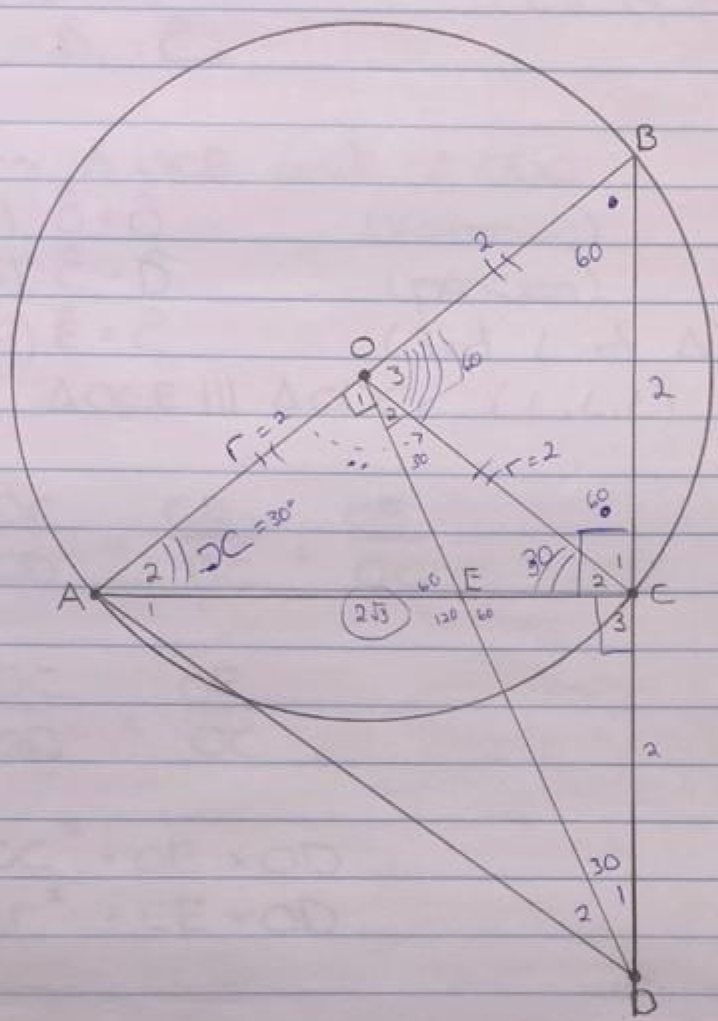
iii) $\frac{AC}{CB} = \frac{CD}{BD}$

$\frac{AC}{4} = \frac{2\sqrt{2}}{2}$

$\therefore 2AC = 8\sqrt{2}$

$\therefore AC = 4\sqrt{2}$

2.3.1) the angles will be equal.



a) $A\hat{C}B = 90^\circ$ (∠s in semi ⊙)

$\hat{C}_3 = 90^\circ$ (∠s on straight line)

$E\hat{O}B = 90^\circ$ (∠s on straight line)

b) $\hat{O}_1 = \hat{C}_3 = 90^\circ$

∴ OADC is cyclic quad.

(∠s in same segment)

c) ∠s opp = radii

d) $\hat{A}_2 = \hat{D}_1$ (∠s subtended = chords

$\hat{A}_2 = \hat{C}_2$ (∠s opp = sides)

$\therefore \hat{D}_1 = \hat{C}_2$

e) in $\triangle OCE$ and $\triangle ODC$:

i) $\hat{O} = \hat{O}$ (common)

ii) $\hat{C} = \hat{D}$ (proven)

iii) $\hat{E} = \hat{C}$ (3rd ∠ of △)

$\therefore \triangle OCE \,|||\, \triangle ODC$ (∠,∠,∠)

f) $\dfrac{OC}{OD} = \dfrac{CE}{DC} = \dfrac{OE}{OC}$

g) $\dfrac{OC}{OD} = \dfrac{OE}{OC}$

$OC^2 = OE \times OD$

$r^2 = OE \times OD$

h) in $\triangle OAE$:

$\tan x = \dfrac{OE}{r}$

$\therefore OE = r \cdot \tan x$

i) $\sin 30^\circ = \dfrac{BC}{AB}$ } in $\triangle ABC$, $AB = 4$

$4 \sin 30 = BC$

$\therefore BC = 2$

Also $\hat{A}_2 = \hat{D}_1 = 30^\circ$

$\hat{B} = 60^\circ$ (int ∠s of △ ABC)

$\therefore \hat{B} = \hat{C}_1$ (∠s opp = sides)

$\therefore \hat{O}_3 = 60^\circ$ (int ∠'s △

$\therefore \hat{O}_2 = 30^\circ$

Now $\hat{O}_2 = \hat{D}_1 = 30^\circ$

$\therefore OC = CD = 2$ (sides opp. = $\angle$s)

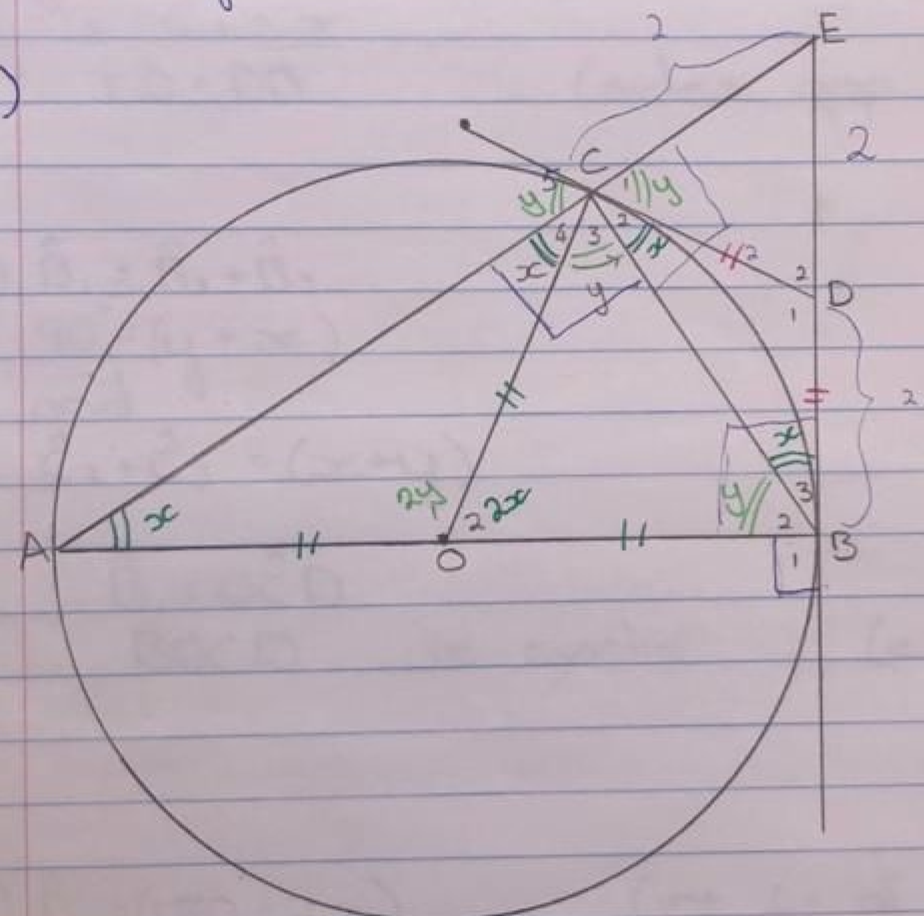
Area $\Delta ABD = \frac{1}{2} AB \cdot BD \sin \hat{B}$

$= \frac{1}{2} (4)(4) \sin 60^\circ$

$= 4\sqrt{3}$ units^2

2.4.1) The two Tangents are equal in length.

2.4.2)



a) $\hat{A} = x$ ($\angle$s opp = radii)

$\hat{B}_3 = x$ (Tan-chord)

$\hat{C}_2 = x$ (Tan-chord)

b) $\hat{B}_2 = y$ ($\angle$s opp = sides)

c) $A\hat{C}B = 90^\circ$ ($\angle$s in semi-$\odot$)
$\hat{B}_1 = 90^\circ$ (Tan-chord)
$B\hat{C}E = 90^\circ$ ($\angle$s on straight line)
$A\hat{B}E = 90^\circ$ ($\angle$s on straight line)
$O\hat{C}D = 90^\circ$

d) $\hat{B}_3 = \hat{C}_2 = x$
$\therefore CD = DB$ (sides opp. = angles)

e) $\hat{B}_1 = \hat{B}_2 + \hat{B}_3$
$90^\circ = (y + x)$
and
$\hat{C}_2 + \hat{C}_3 = (x + y)$

$\therefore \hat{B}_1 = O\hat{C}D$
$\therefore BOCD$ is cyclic (ext $\angle$ cyclic quad)

f) $\hat{D}_1 = (180 - 2x)$ (int $\angle$s of Δ)
$\hat{D}_2 = 180 - (180 - 2x)$ ($\angle$s on straight line)
$\hat{D}_2 = 2x$

$\hat{E} + \hat{D}_2 = A\hat{C}D$ (ext $\angle$ of Δ)
$\hat{E} + 2x = 2x + y$
$\hat{E} = y$
$\hat{C}_5 = y$ (Tan-chord)
$\hat{C}_5 = \hat{C}_1 = y$ (vert opp $\angle$s)

$\therefore \hat{E} = \hat{C}_1$

g) * Think question was supposed to be $CD = DE$.

Then ...

$CD = DE$ because $\hat{C}_1 = \hat{E} = y$

(sides opp $= \angle$s)

h) in Δ EBC and EAB:

i) $\hat{E} = \hat{E}$ (common)

ii) $\hat{B} = \hat{A} = x$ (proven)

iii) $\hat{C} = \hat{B} = 90°$ (proven)

$\therefore \Delta EBC \; ||| \; \Delta EAB$ ($\angle, \angle, \angle$)

i) $\frac{EB}{EA} = \frac{EC}{EB} = \frac{BC}{AB}$

j) $DB = CE = 2$

$\therefore ED = 2$ and $EB = 4$

$\therefore \frac{EC}{EB} = \frac{EB}{EA}$

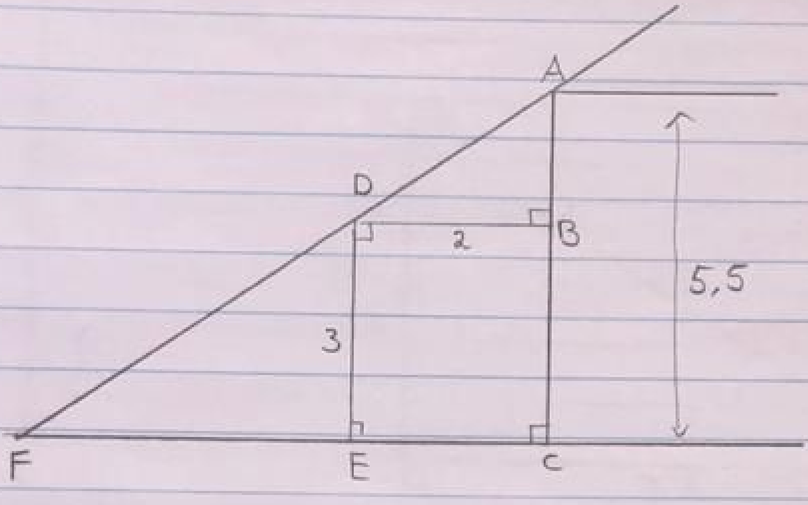
$\frac{2}{4} = \frac{4}{EA}$

$2EA = 16$

$EA = 8$

$\therefore CA = 6$ units.

2.5)



2.5.1) $E\hat{D}B = D\hat{E}C = 90^\circ$

$\therefore DB \parallel EC$ (co-interior $\angle$s $= 180^\circ$)

2.5.2) $= 90^\circ$ ($\angle$s on straight line)

2.5.3) in Δ BAD and Δ EDF

i) $\hat{B} = \hat{E} = 90^\circ$ (given)

ii) $\hat{A} = \hat{D}$ (corresp. $\angle$s, DE // AC)

iii) $\hat{D} = \hat{F}$ (3rd $\angle$ of Δ)

2.5.4) $\dfrac{BA}{ED} = \dfrac{AD}{DF} = \dfrac{BD}{EF}$

2.5.5) $AB = 5,5 - 3 = 2,5$ units

2.5.6) $AD^2 = AB^2 + BD^2$ (pythag)

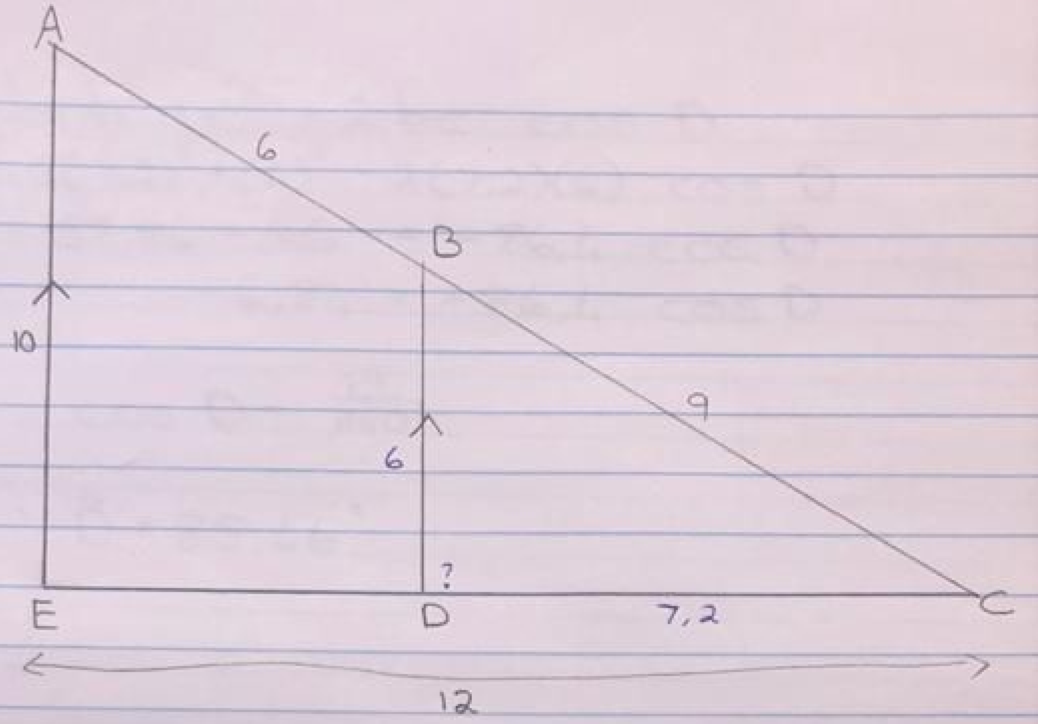
$AD = \sqrt{(2,5)^2 + (2)^2}$

$AD = \sqrt{41}/2$ units $\longrightarrow$ $\dfrac{2,5}{3} = \dfrac{\sqrt{41}/2}{DF}$

$2,5\ DF = 9,60468\ldots$

$\therefore DF = 3,84$

2.6)



2.6.1) Divides the sides proportionally

2.6.2) in $\triangle$ AEC and $\triangle$ BDC :

i) $\hat{A} = \hat{B}$ } (corresp. $\angle$'s, AE $\parallel$ BD)
ii) $\hat{E} = \hat{D}$ }
iii) $\hat{C} = \hat{C}$ (common)

$\therefore$

2.6.3) $\dfrac{AE}{BD} = \dfrac{EC}{DC} = \dfrac{AC}{BC}$

2.6.4) $\dfrac{AC}{BC} = \dfrac{EC}{CD}$

$\therefore \dfrac{15}{9} = \dfrac{12}{CD}$

$\therefore 15\,CD = 108$

$CD = 7{,}2$

$\dfrac{AC}{BC} = \dfrac{AE}{BD}$

$\dfrac{15}{9} = \dfrac{10}{BD}$

$15\,BD = 90$

$BD = 6$

2.6.5) $d^2 = b^2 + c^2 - 2bc \cos D$

$(9)^2 = (7,2)^2 + (6)^2 - 2(7,2)(6) \cos D$

$81 - 51,84 - 36 = -86,4 \cos D$

$-6,84 = -86,4 \cos D$

$\therefore \cos D = \frac{19}{240}$

$\therefore \hat{D} = 85,46^\circ$

2.6.6) Area $\triangle$ BDC $= \frac{1}{2} b.c. \sin D$

$= \frac{1}{2}(7,2)(6) \sin 85,46^\circ$

$= 21,53222603$

Area $\triangle$ AEC $= \frac{1}{2} a.c \sin E$

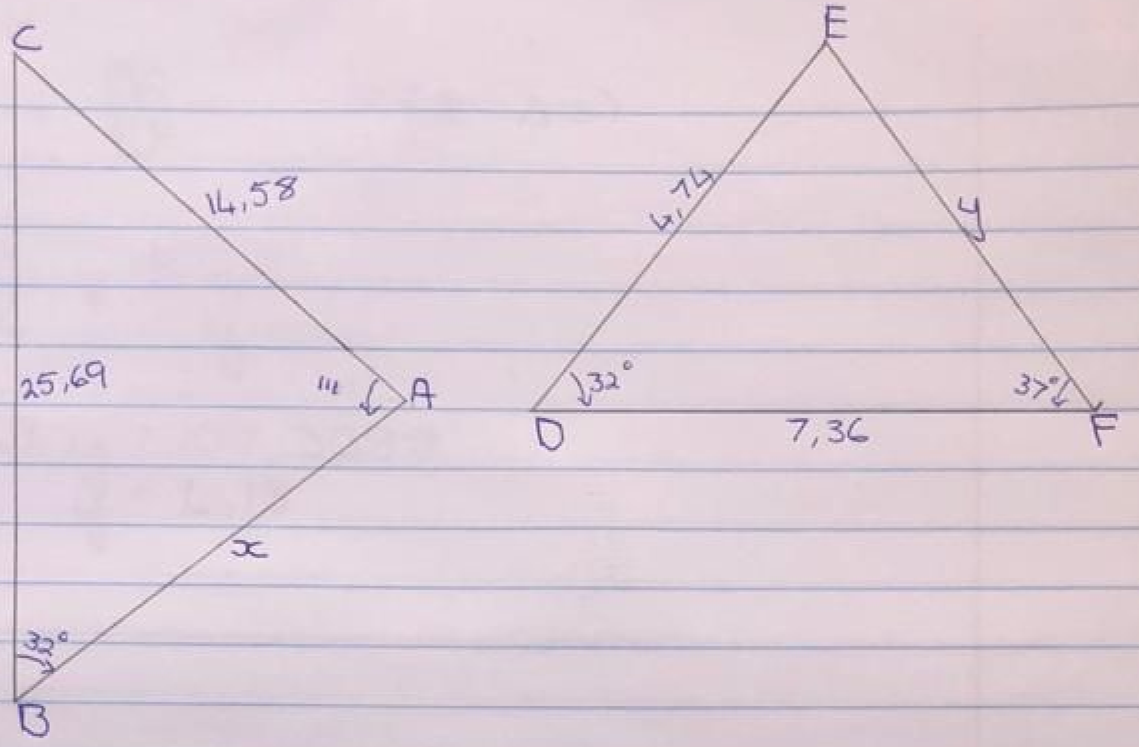
$= \frac{1}{2}(12)(10) \sin 85,46^\circ$

$= 59,81173896$

$\therefore \frac{\text{Area } \triangle \text{ BDC}}{\text{Area } \triangle \text{ AEC}} = \frac{21,53222603}{59,81173896}$

$\therefore 0,36$

2.7)



2.7.1) $\hat{C} = 180° - 111° - 32°$ (int ∠s of Δ)
$\hat{C} = 37°$

$\hat{E} = 180° - 32° - 37°$ (int ∠s of Δ)
$\hat{E} = 111°$

2.7.2) in Δ ABC and Δ EDF
i) $\hat{A} = \hat{E}$ (shown)
ii) $\hat{B} = \hat{D}$ (given)
iii) $\hat{C} = \hat{F}$ (shown
$\therefore$ Δ ABC ||| Δ EDF (∠,∠,∠)

2.7.3) $\frac{AB}{ED} = \frac{BC}{DF}$ (||| Δ's)

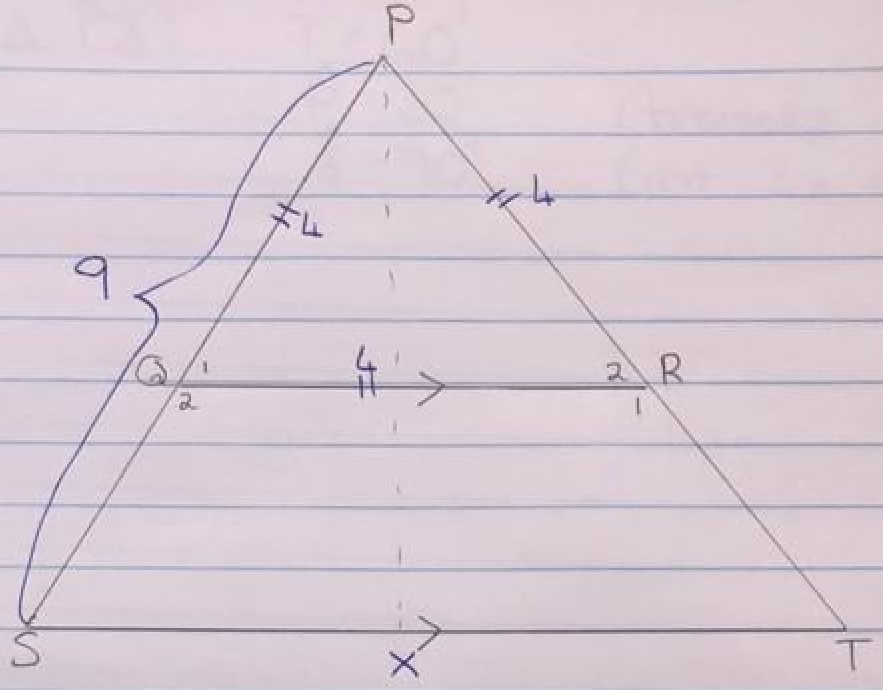
$\frac{x}{4,74} = \frac{25,69}{7,36} \implies 7,36\ x = 121,7706$
$x = 16,54$

$$\frac{BC}{DF} = \frac{AC}{EF} \quad (\parallel\!\mid \ \Delta\text{'s})$$

$$\frac{25{,}69}{7{,}36} = \frac{14{,}58}{y}$$

$$25{,}69\,y = 107{,}3088$$
$$y = 4{,}18$$

2.8)



2.8.1) Δ PQR is equilateral
$\therefore$ all angles $= 60°$

2.8.2) $\hat{S} = \hat{Q}_1 = 60°$ (corresp. $\angle$'s, QR // ST)
$\hat{T} = \hat{R}_2 = 60°$ (corresp. $\angle$'s, QR // ST)

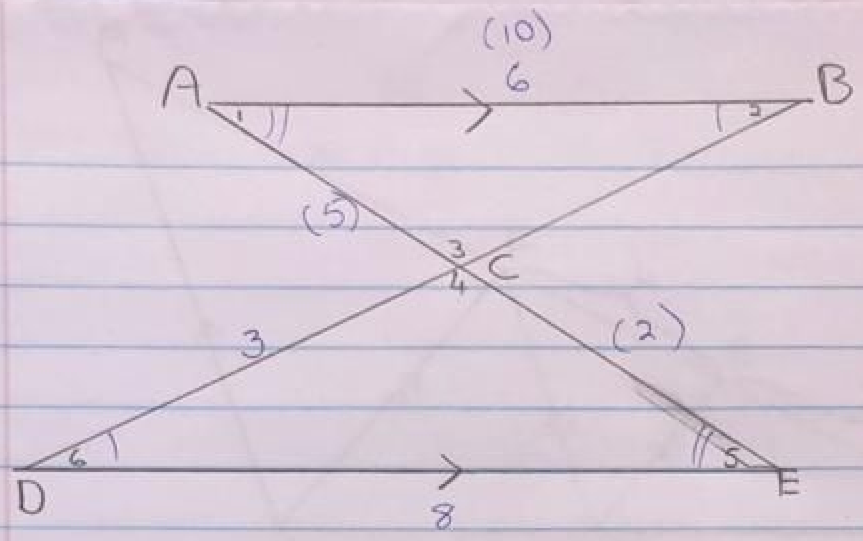
2.8.3) QS = 5 cm

2.8.4) in Δ PQR and Δ PST:
i) $\hat{P} = \hat{P}$ (common)
ii) $\hat{Q} = \hat{S}$ } Proven
iii) $\hat{R} = \hat{T}$
$\therefore \Delta PQR \parallel\!| \Delta PST$ ($\angle, \angle, \angle$)

2.8.5) $\frac{PQ}{PS} = \frac{QR}{ST}$

$\frac{4}{9} = \frac{4}{ST} \Rightarrow$ 4ST = 36
ST = 9 cm

2.9)



2.9.1) in Δ ABC and Δ EDC :

i) $\hat{A}_1 = \hat{E}_5$ } Alternate ∠'s

ii) $\hat{B}_2 = \hat{D}_6$ } AB // DE

iii) $\hat{C}_3 = \hat{C}_4$ (vert opp ∠'s)

$\therefore \Delta ABC \;|||\; \Delta EDC$ (∠, ∠, ∠)

$$\therefore \frac{AB}{ED} = \frac{BC}{DC}$$

$$\frac{6}{8} = \frac{BC}{3}$$

$$18 = 8\,BC$$

$\therefore BC = 2{,}25$ cm

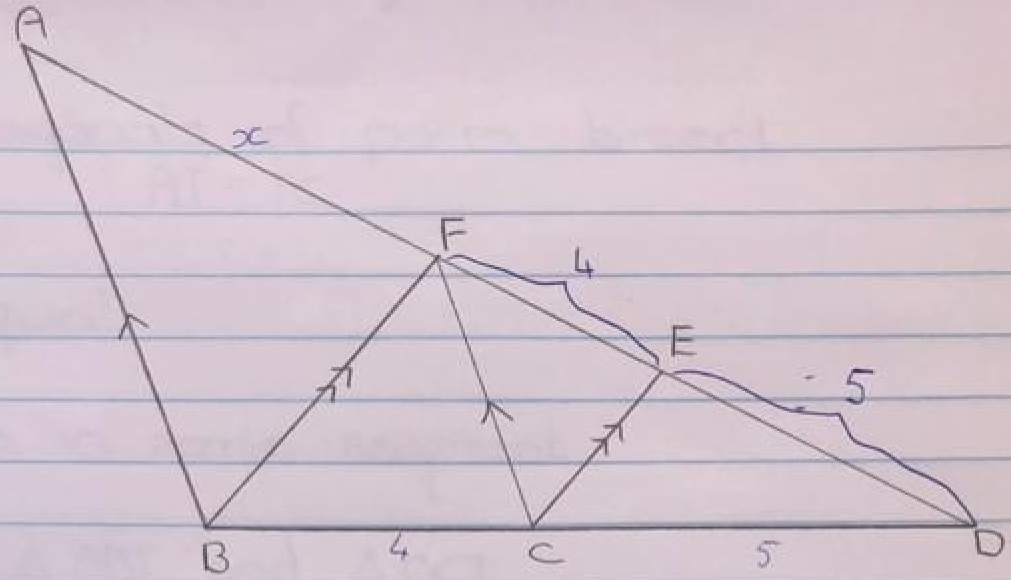
2.9.2) $\dfrac{AB}{DE} = \dfrac{AC}{EC}$

$$\frac{10}{DE} = \frac{5}{2}$$

$$20 = 5DE$$

$\therefore DE = 4$ cm.

2.10)



.10.1) DC : CB

$= \frac{DC}{CB} = \frac{5}{4}$ (side-splitter)

2.10.2) * leave for now

2.10.3) $\frac{DF}{DA} = \frac{CD}{BD}$ (side-splitter)

$\frac{9}{(9+x)} = \frac{5}{9}$

$5(9+x) = 81$

$45 + 5x = 81$

$5x = \frac{81}{45}$

$x = 0,36$

$\therefore \frac{AF}{FD} = \frac{0,36}{9}$

2.11.1) Diagonals of parm bisect

$\therefore AT = TC$

2.11.2) equal

2.11.3) $\angle$-s in same segment

2.11.4) in $\triangle ABT$ and $\triangle OCT$:

i) $\hat{A} = \hat{O}$ ($\angle$s in same segment)

(ii) $\hat{T} = \hat{T}$ (vert opp $\angle$-s)

(iii) $\hat{B} = \hat{C}$ (3rd $\angle$ of $\triangle$)

$\therefore \triangle ABT \parallel\!| \triangle OCT$ ($\angle, \angle, \angle$)

2.11.5) $\dfrac{AB}{BT} = \dfrac{BT}{CT} = \dfrac{AT}{OT}$

2.11.6) $AC = 6$ $\therefore AT = TC = 3$ (diagonals bisect)

$BT = TD = 4$ (")

So $\dfrac{AT}{OT} = \dfrac{BT}{CT}$

$\dfrac{3}{OT} = \dfrac{4}{3}$

$4\,OT = 9$

$OT = \dfrac{9}{4}$

2.11.7) $DO = 4 - \dfrac{9}{4} = \dfrac{7}{4}$

2.12.1) divides the sides proportionally.

2.12.2) Trapezium
- one pair of sides //

2.12.3) in Δ KOP and Δ KLM:

i) $\hat{K} = \hat{K}$ (common)

ii) $\hat{O} = \hat{L}$

iii) $\hat{P} = \hat{M}$

$\left.\begin{array}{l} \\ \end{array}\right\}$ corresp $\angle$'s , OP // LM

$\therefore \Delta KOP \;|||\; \Delta KLM$ ($\angle, \angle, \angle$)

2.12.4) $= 18\ cm^2$