Type 1

max/minima
- → Volume / Surface area } ets
- ∴ measurements

* excamples 1, 2 and 4
- write expression i-to 1 variable using V and SA formulas.
- when word max/min } largest/smallest } used.

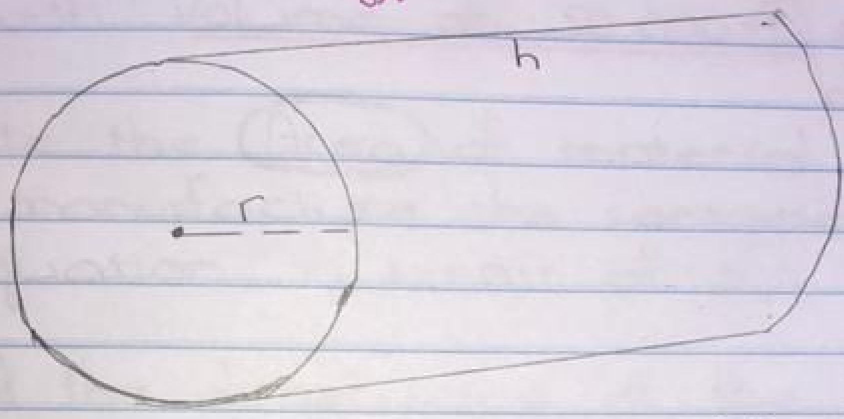
→ get derivative and set $=0$

Examples:

1) Consider the figure, which represents a milk tank with total surface area of $10\ 000\ m^2$.

SA

↳ use this formula!!



a) Write an expression for h in terms of r
↳ get h alone in SA formula!

b) Write down a formula for the Volume in terms of r

c) What should radius of tank be in order for it to hold maximum milk.
$V'(x)$

a) $SA = 2\pi r^2 + 2\pi r . h$

$10\ 000 = 2\pi r(r + h)$

$\frac{5000}{\pi r} = r + h$

$\therefore \underline{h} = \left(\frac{5000}{\pi r} - r\right)$

b) $V = \pi r^2 \times h$

$V = \frac{\pi r^2}{1} \times \left(\frac{5000}{\pi r} - \frac{r}{1}\right)$

$V = 5000r - \pi r^3$

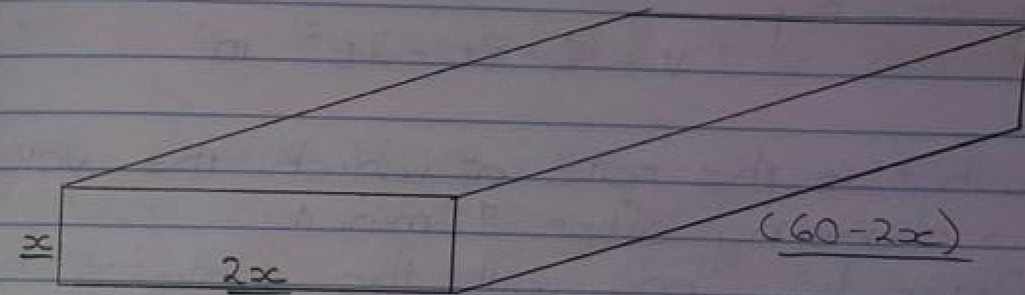
c) $V' = 5000 - 3\pi r^2$

$0 = 5\ 000 - 3\pi r^2$

$3\pi r^2 = 5000$

$\sqrt{r^2} = \sqrt{\frac{5000}{3\pi}}$ $\quad \therefore r = 23,03\ m$

4) A rectangular box has dimensions
$x \times 2x \times (60-2x)$ cm



All dimensions given ito x

a) Determine an expression for the Volume of the box i.t.o x

b) Determine dimensions of box that will give a $\boxed{\text{maximum}}$ volume. get V', set $=0$

c) What is the max Volume?

a) $V = l \times b \times h$

$V = (x)(2x)(60-2x)$ cm^3

$V = 2x^2(60-2x)$ cm^3

$V = 120x^2 - 4x^3$ cm^3

b) $V' = 240x - 12x^2$

$0 = 12x(20-x)$

$x = 0$ or $x = 20$

N.A

$\therefore$ Dimensions:

20 cm × 40 cm × 20 cm

{got these by substituting 20 into dimensions given at very start}

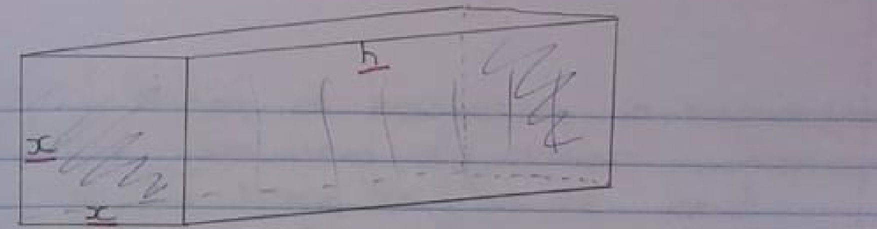
Use the dimensions you found in b) and calculate V.

$\therefore V = l \times b \times h$

$V = 20 \times 40 \times 20$

$V = 16\,000$ cm^3

2)



Start with info given

The diagram represents a container with Volume of 50 000 ℓ * ℓ shows capacity. need to make it meters.

a) Write the Area of material needed to manufacture the container as an equation, in terms of x

b) Find the dimensions of the container that will ensure that the minimum amount of material is used. Get SA' set = 0

$\therefore 1\,000\ \ell = 1\ m^3$ $\qquad \therefore 50\,000\ \ell = 50\ m^3$

$V = l \times b \times h$

$V = (x)(x)(h)$

$50 = x^2 h$

$\therefore h = \frac{50}{x^2}$

we use given info on Volume so we can get height i.t.o x. Now only can we answer a) which deals with surface Area.

a) $SA = 2(x)(x) + 4(x)(h)$

$SA = 2x^2 + \frac{4x}{1}\left(\frac{50}{x^2}\right)$

$SA = 2x^2 + \frac{200}{x}$

before we can derive we need to change term 2 so x isn't in denominator.

b) $\therefore SA = 2x^2 + 200x^{-1}$ * remember we (−) 1 from exponent

$SA' = 4x - 200x^{-2}$

$0 = 4x - \frac{200}{x^2}$ make exponent (+) to solve for x

$\frac{200}{x^2} = \frac{4x}{1}$

$\therefore 200 = 4x^3$

$\therefore \sqrt[3]{x^3} = \sqrt[3]{50}$

$\therefore x = 3{,}68\ m$

Dimensions:
3,68 m and
3,69 m

Type 2

• Rates of change!!

examples 3 and 5.

* check if they ask for avg rate or rate at specific time/ instantaneous rate at t = 'specific time'
  - get derivative
  - subs time into derivative

* min/max smallest/largest } derivative = 0

3) The number of bacteria is represented by the following equation, where t is the time in hours and $s(t)$ the number of bacteria.

$$s(t) = 2t^2 - 12t + 20\,000$$

a) find the average rate at which the bacteria decrease during first 2 hours.

b) calculate the rate at which the bacteria increase / decrease at the 8th hour. → specific !!

c) When is the number of the bacteria the smallest. $s'(t) = 0$

d) What is the minimum number of bacteria that can be found?

a) $s(0) = 20\,000$ $\quad$ $s(2) = 19\,984$

$\therefore$ avg rate $= \dfrac{19\,984 - 20\,000}{2-0} = -8$ bacteria/h

negative shows it's decreasing.

b) $s'(t) = 4t - 12$

$s'(8) = \underline{20}$ bacteria/h increasing

↳ since it's + nr, it shows ↑

c) $0 = 4t - 12$

$12 = 4t$

$\therefore t = 3$ hours

d) $s(3) = 2(3)^2 - 12(3) + 20\,000$

$\therefore$ min of 19 982.

5) The volume of a tank of water at a given time (t in minutes) is given by;

$$V(t) = 10 + 8t - 2t^2 \ m^3$$

a) What is the rate at which the volume is changing after 1 minute specific.

b) After how long will the water's Volume be a maximum? $v'(t) = 0$

c) When will the tank be empty?

a) $v'(t) = 8 - 4t$

$v'(1) = 8 \ m^3/min.$

b) $0 = 8 - 4t$

$4t = 8$

$t = 2$ minutes.

c) $0 = 10 + 8t - 2t^2$

$0 = t^2 - 4t - 5$

$0 = (t-5)(t+1)$

$t = 5$ or $t = -1$

N.A

$\therefore$ 5 minutes

Type 3 (example 6):

$$\begin{cases} \text{Displacement} & (s) \\ \text{Velocity} & s'(t) \\ \text{Acceleration} & s''(t) \end{cases}$$

- displacement → height at given time
  = if (+) it moves ↑
  = if (−) it comes ↓

* max/min → $s'(t) = 0$

* velocity at specific time
  → get $s'(t)$
  → subs t into $s'(t)$

* acceleration
  → $s''(t)$ ~~[illegible]~~.

6) The displacement of a projectile is given by;

$$s(t) = 5t^2 - 20t$$

with s in meters and t in seconds.

a) What is the time when the displacement is a maximum? $s'(t) = 0$

b) What is the velocity $s'(t)$ of the projectile after 5 seconds?

c) What is the acceleration $s''(t)$ of the projectile?

a) $s'(t) = 10t - 20$

$0 = 10t - 20$

$20 = 10t$

$\therefore t = 2$ sec.

b) $s'(5) = 10(5) - 20 = 30$ m/s.

c) $s''(t) = 10$ m/s^2