# Applications of Differential Calculus

* There are 3 types of questions that can be asked:

→ Minima & Maxima
→ Rate of change
→ Calculus of motion.

## 1) Minima & maxima:

→ minimum/maximum is found when derivative equals zero.
$f'(x) = 0$

*Hints / Steps:
→ draw diagram if one isn't given.
→ use info given to set up equation in ONE variable.
→ find $f'(x)$
Solve $f'(x) = 0$
→ tie up loose ends

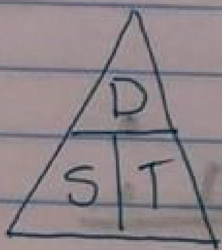
→ know formula's for volume and surface area of 3-d shapes.

## 2) Rate of change :

- if y is a function of $x$, then instantaneous rate of change is $\frac{dy}{dx}$.

$$f'(x) = 0$$

## 3) Calculus of motion :

- 

distance, speed, time

* $f(x) \longrightarrow$ distance / displacement $(m)$

$f'(x) \longrightarrow$ speed / velocity $(m/s)$ / rate of change

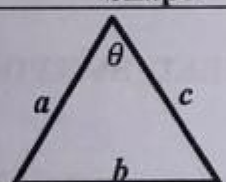
$f''(x) \longrightarrow$ acceleration $(m/s^2)$

- ***SPECIAL CASES***

Suppose TWO moving bodies are involved, one going up and on coming down, and the ask:

- When (at what time) will they pass each other?
  This will happen when the two heights are equal.
  Therefore, equate the two height equations of the bodies and solve for t.
- When (at what time) will their speeds be equal?
  Now you equate the two speed equations (s' = s' ) and solve for t.

- The following formulae will assist you when dealing with optimisation problems involving mensuration

| **Shape** | **Perimeter** | **Area** |
|---|---|---|
|   Triangle | $P = a + b + c$ | $\text{Area} = \frac{1}{2} \times \text{base} \times \text{Perp. Height}$<br>$\text{Area} = \frac{1}{2} ac\sin\theta$ |
| Square | $P = 4s$ | $\text{Area} = s^2$ |
| Rectangle | $P = 2l + 2b$ | $\text{Area} = l \times b$ |
| Parallelogram | $P = 2l + 2b$ | $\text{Area} = \text{base} \times \text{perp. Height}$ |
| Trapezium | P = sum of four sides | $\text{Area} = \frac{1}{2} \times (\text{sum of // sides}) \times h$ |
| Circle | $C = 2\pi r = \pi D$ | $\text{Area} = \pi r^2$ |

| **Object** | **Volume** | **Surface area** |
|---|---|---|
| Rectangular solid | $V = lbh$ | $TSA = 2lb + 2lh + 2bh$<br>(for a closed box) |
| Cube | $V = s^3$ | $TSA = 6s^2$ |
| Sphere | $V = \frac{4}{3}\pi r^3$ | Surface Area $= 4\pi r^2$ |
| Cylinder | $V = \pi r^2 h$ | Area of curved surface $= 2\pi rh$<br>$TSA = 2\pi r^2 + 2\pi rh$ |

- **To determine *where* a function $f$ will have a maximum / minimum, solve for $f'(x) = 0$.**

- **To calculate the *actual* minimum / maximum value itself, substitute the answer of $f'(x) = 0$ into $f(x)$.**