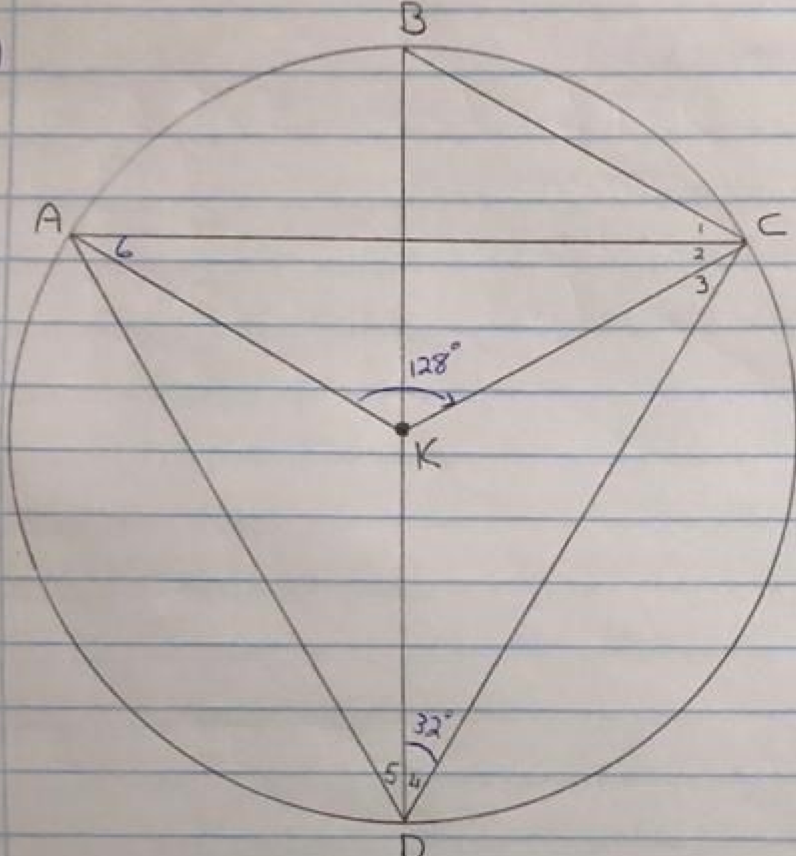
Topic 3: Geometry → memo

1.1)



1.1.1) KA, KB, KC, KD

1.1.2) ∠s in semi ⊙

1.1.3) isosceles Δ , AK = CK (radii)

1.1.4 a) $\hat{C}_2 = \hat{A}_6$ (∠s opp = sides

$\hat{C}_2 = \dfrac{(180° - 128°)}{2}$ (int ∠s of Δ)

$\hat{C}_2 = 26°$

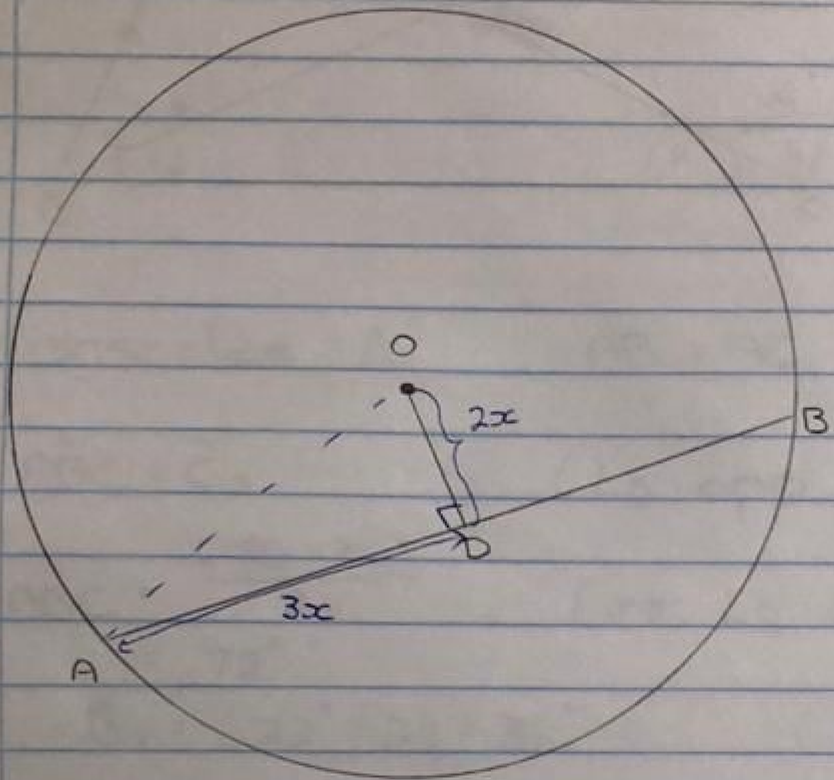
b) $A\hat{D}C = \frac{1}{2}(128^\circ)$ ($\angle$ centre $= 2 \times \angle$ circumf)

$= 64^\circ$

$\therefore \hat{D}_5 = 32^\circ$

c) $\hat{C}_1 = 32^\circ$ ($\angle$s in same segment)

1.2.1) ... bisects the chord.



$AB = 6x$
$OD = 2x$

1.2.2.1) $AD = \frac{6x}{2} = 3x$ (line centre $\odot \perp$ chord)

1.2.2.2) in Δ OAD:

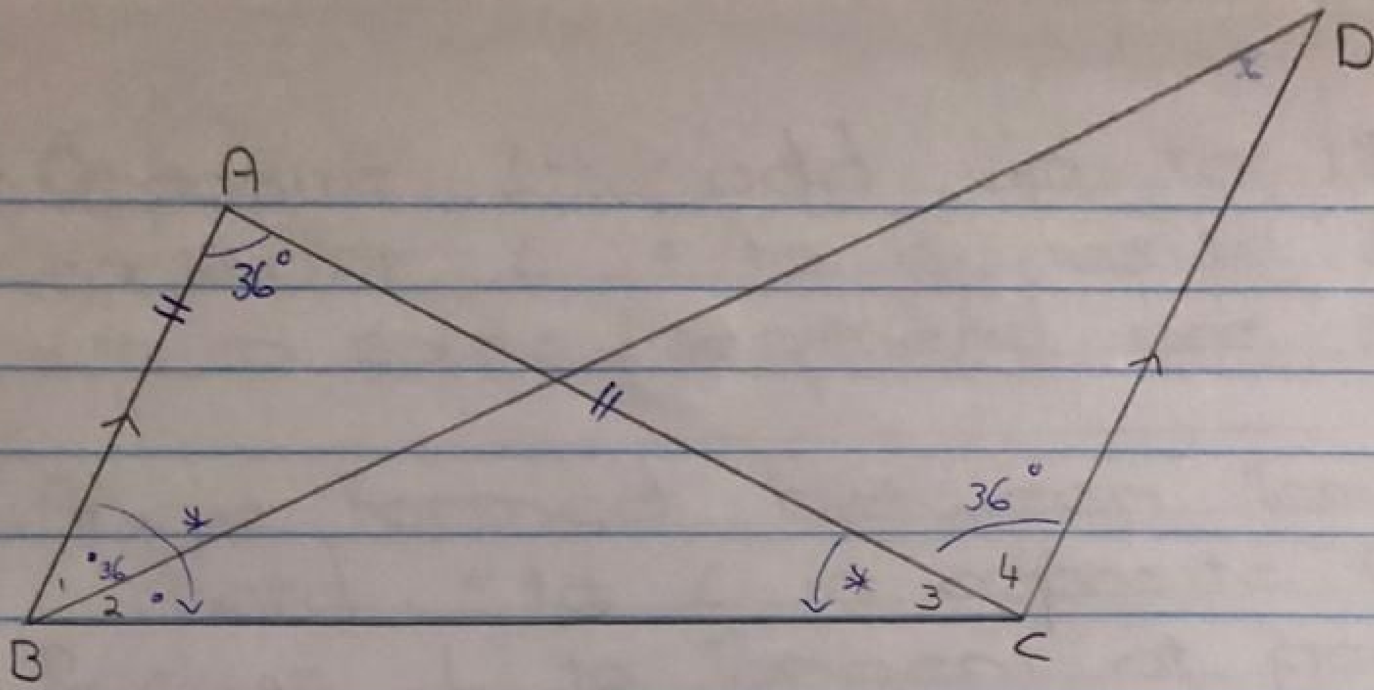
$AD^2 + OD^2 = OA^2$ (pythag)

$(3x)^2 + (2x)^2 = OA^2$

$13x^2 = OA^2$

$\therefore OA = \sqrt{13}\ x$ units.

1.3)



1.3.1) isosceles Δ , $AB = AC$

1.3.2) $A\hat{B}C = \hat{C}_3$ (∠s opp = sides)

$A\hat{B}C = \frac{(180-36)}{2}$ (int ∠s of Δ)

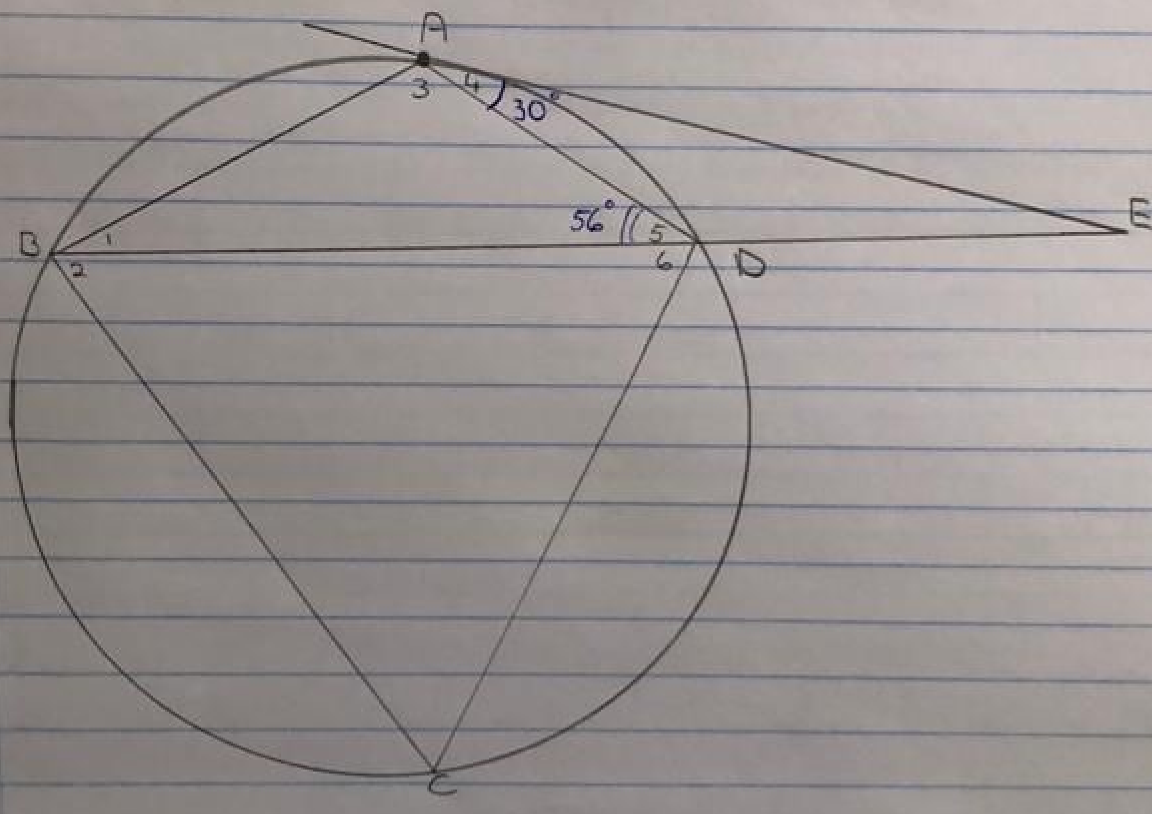
$= 72°$

$\therefore \hat{B}_1 = (72° \div 2) = 36°$ (bisector)

1.4.1) • Opposite ∠'s add up to 180°
• exterior ∠ = to opposite interior ∠
• ∠'s in same segment are =

1.4.2) • Angle formed between Tangent and chord = to ∠ in opposite ⊙ segment
• Radius ⊥ to Tangent at point of contact, thus makes 90° ∠.

1.4.3.1)



a) $\hat{B}_1 = 30°$ (Tan-chord)
b) $\hat{A}_3 = 94°$ (int. ∠s of Δ)
c) $B\hat{C}D = 86°$ (opp ∠s cyclic quad)
d) $A\hat{D}E = 124°$ (∠s on straight line)
$\hat{E} = 26°$ (int ∠s of Δ)

or

(ext ∠ of Δ)

1.4.3.2) No.

$\hat{A}_3 = 94^\circ$

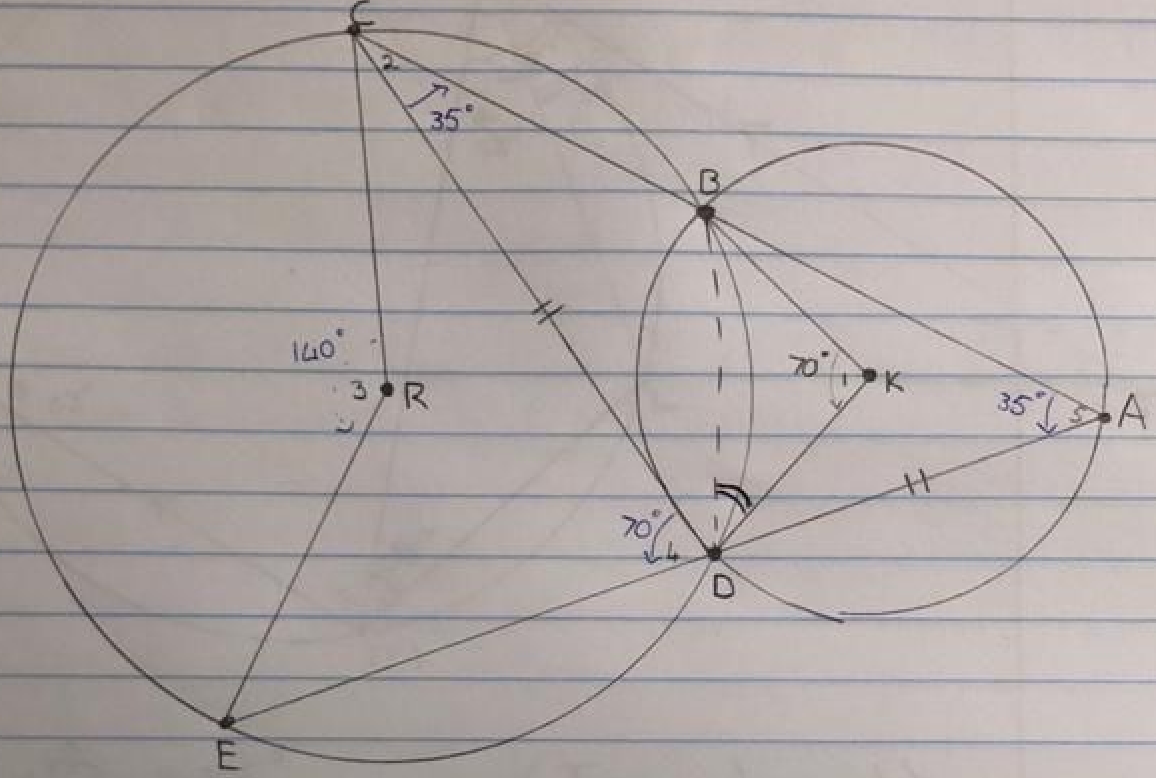
if BD were diameter, $\hat{A}_3$ would $= 90^\circ$

1.4.3.3) $\hat{E} \neq \hat{C}$ and $B\hat{A}E + \hat{C} \neq 180^\circ$

So ABCE is not cyclic.

1.5.1) will be twice the size of the angle formed at the circumference.

1.5.2)



i) $\hat{A}_5 = 35°$ ($\angle$ at centre $= 2 \times \angle$ at circumf.)

ii) $\hat{A}_5 = \hat{C}_2 = 35°$ ($\angle$s opp. = sides)
$\therefore \hat{D}_4 = 70°$ (ext $\angle$ of Δ)

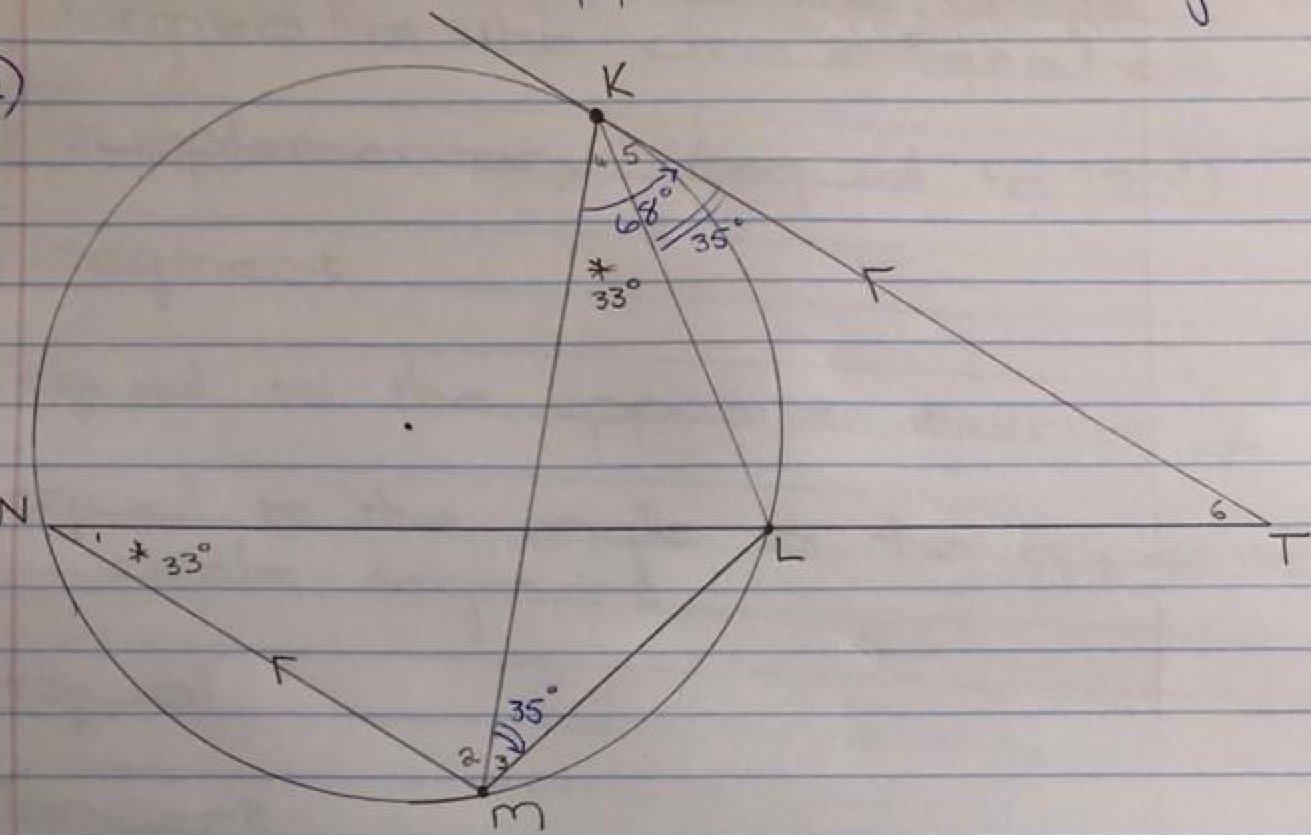
iii) $\hat{R}_3 = 140°$ ($\angle$ at centre $= 2 \times \angle$ at circumf.)

iv) connect B and D.
in Δ BDK:
$\hat{B} = \hat{D} = \dfrac{(180-70)}{2}$ ($\angle$s opp = radii) (int $\angle$s of Δ)

$\therefore B\hat{D}K = 55°$

1.6.1) the $\angle$ in the opposite circle segment

1.6.2)



1.6.2.1) $\hat{K}_5 = \hat{M}_3 = 35°$ (Tan - chord)

1.6.2.2) $\hat{K}_4 = 33°$

$\hat{K}_4 = \hat{N}_1 = 33°$ ($\angle$s in same segment)

1.6.2.3) $\hat{T}_6 = 33°$ (alternate $\angle$'s, KT // NM)

1.7.1) twice the size of the angle formed at the circumference.

1.7.2) supplementary (equal to 180°)

1.7.3) segment

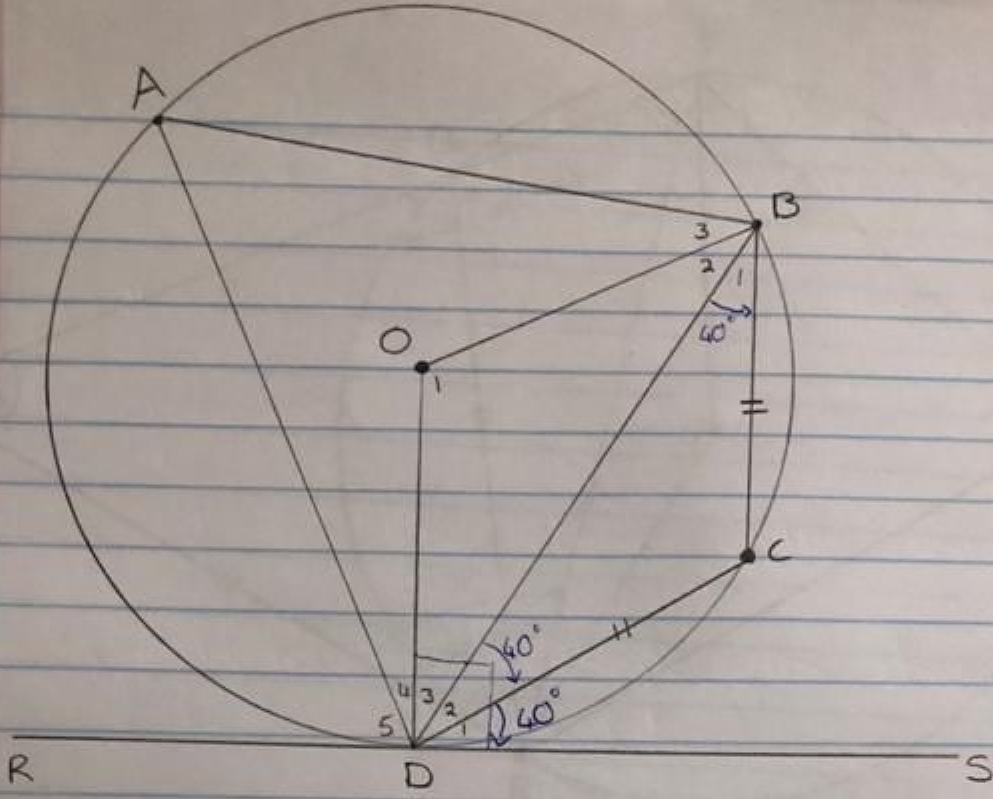
1.7.4) equal to the opposite interior $\angle$

1.7.5) equal to the angle in the opposite circle segment.

1.7.6) equal

1.7.7) Tangent

1.8)



1.8.1.1) A, B, C, D lie on circumference of ⊙

1.8.1.2) BC = DC

1.8.1.3) radius ⊥ Tangent

1.8.1.4) radii of ⊙

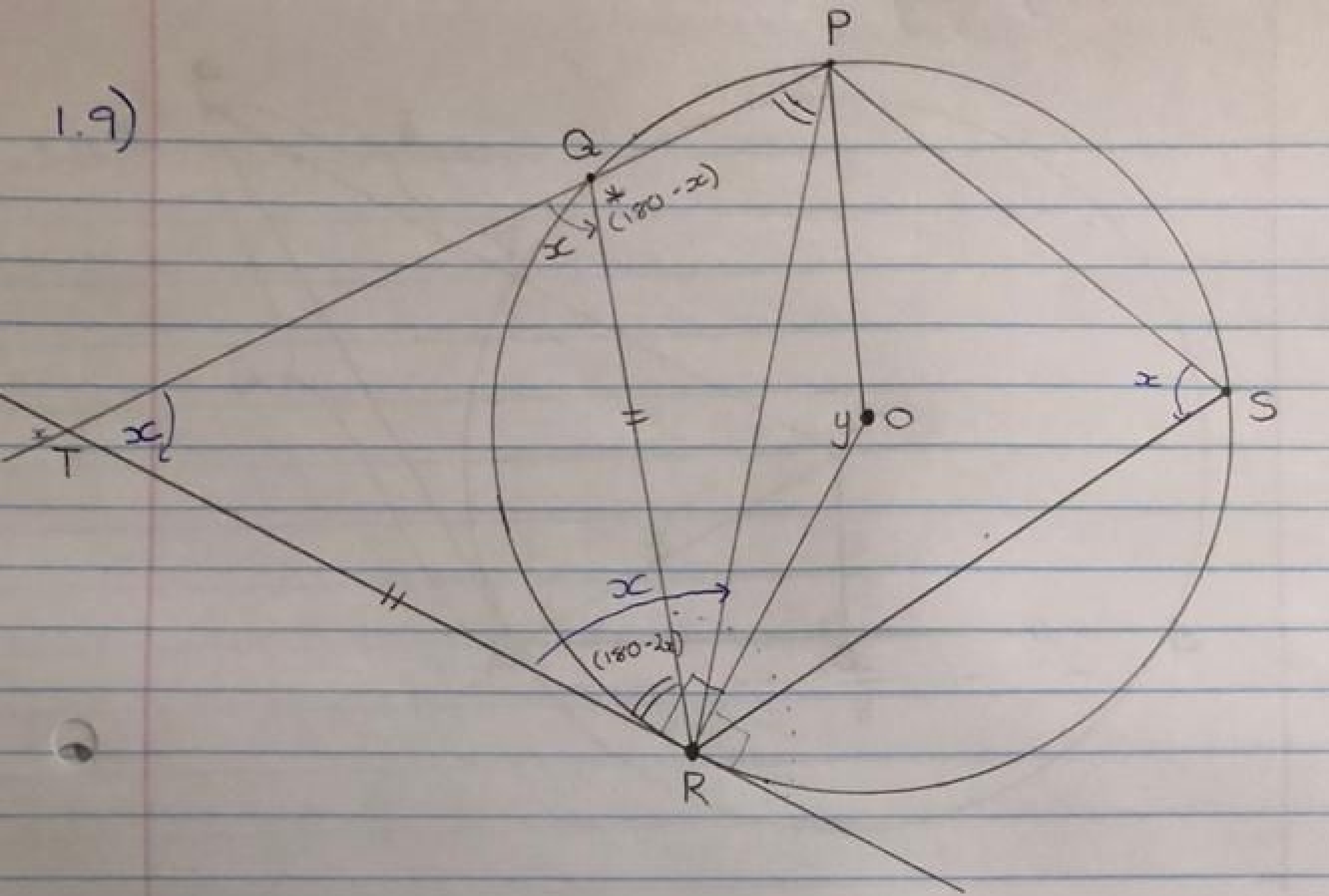
1.8.2.1) $\hat{D}_1 = \hat{B}_1 = 40°$ (Tan-chord)
$\hat{B}_1 = \hat{D}_2 = 40°$ (∠s opp = sides)

1.8.2.2) $\hat{C} = 100°$ (int ∠s of Δ)

1.8.2.3) $\hat{A} = 80°$ (opp ∠s cyclic quad)

1.8.2.4) $\hat{O}_1 = 160°$ (∠ at centre = 2 × ∠ at circumf)

1.9)



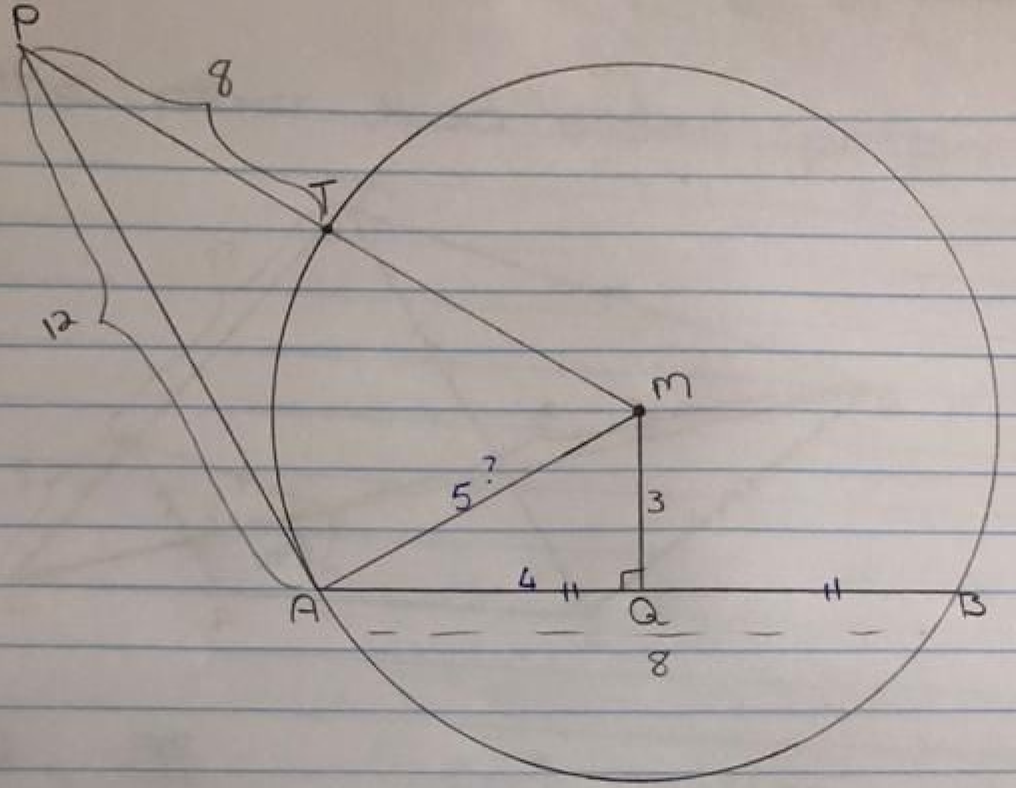
1.9.1) $Q\hat{T}R = x$ (∠s opp = sides)
$\hat{S} = x$ (ext ∠ cyclic quad)
$T\hat{R}P = x$ (Tan - chord)

1.9.2) $P\hat{O}R = 2x$ (∠ at centre = 2 × ∠ at circumf)

1.9.3) PTRO will be cyclic if
$P\hat{O}R + \hat{T} = 180^\circ$
$\therefore 2x + x = 180^\circ$
$3x = 180^\circ$
$\therefore x = 60^\circ$

∴ it won't be for all x-values other than $x = 60^\circ$

1.10)



1.10.1) AQ = 4 units (line centre ⊙ ⊥ chord)

1.10.2) equal to the sum of the right angled sides, both squared indivi-dually.

1.10.3) $MQ^2 + AQ^2 = AM^2$ (pythag)

$(3)^2 + (4)^2 = AM^2$

$\therefore AM = 5$ units.

1.10.4) $AM = TM = 5$ (radii)

$\therefore PM = 8 + 5 = 13$.

So, in Δ PAM:

$(5)^2 + (12)^2 = (13)^2$

$169 = 169$.

$\therefore P\hat{A}M = 90^\circ$ (pythag converse)

$\therefore PA \perp MA$

$\therefore$ PA is tangent.