CUBIC FUNCTION: SECTION B

QUESTION 1:

The function $f(x)=ax^3+bx+1$ has a local minimum at $(2;-15)$.
Determine the values of a and b. (6)

QUESTION 2:

A cubic graph is given with the following equation: $f(x)=ax^3+bx^2+cx+d$
It is known that:

$f(0)=12$; $f(1)=f(6)=f(-2)=0$; $f'(-\frac{2}{3})=f'(4)=0$

$f(-\frac{2}{3})=14{,}81$; $f(4)=-36$

2.1 Sketch $f(x)$ and indicate clearly the intercepts with the axes as well as the coordinates of the turning points. (6)

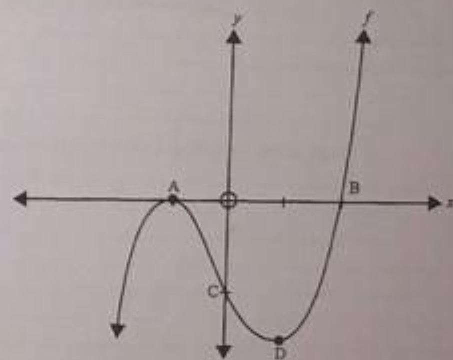
2.2 Write down the equation of $f(x)$. (5)

2.3 Identify for which values of x, $f(x)$ is concave downwards. (3)

2.4 Determine the values of x for which $f(x).f'(x)>0$ (3)

QUESTION 3:

The sketch below shows $f(x)=x^3-12x-16$



3.1 Determine the lengths of OA, OB and OC. (4)

3.2 Determine the coordinates of the turning point D. (4)

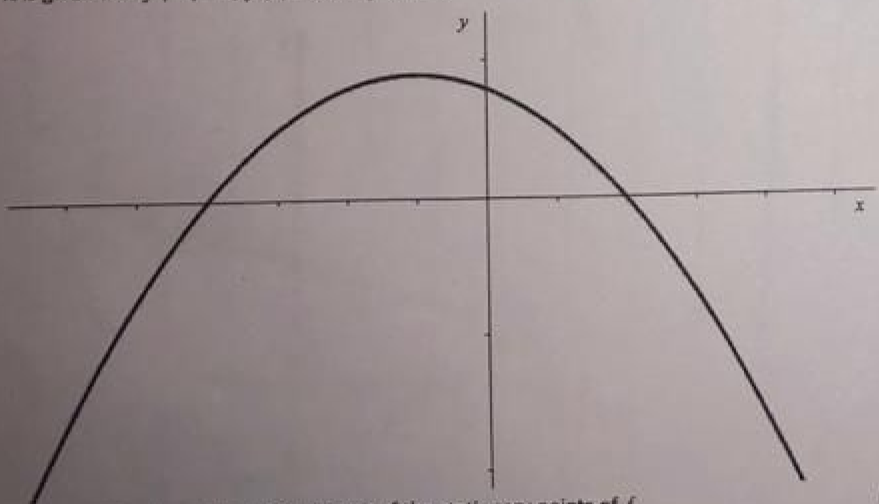
3.3 Determine the equation of the tangent line passing through point C. (4)

QUESTION 4

Given: $f(x) = ax^3 + bx^2 + cx + d$.
Draw a possible sketch of $f'(x)$ if a, b and c are all **negative** real numbers. (4)

QUESTION 5

The graph of $y = ax^2 + bx + c$ below represents the derivative of $f(x)$.
It is given that $f'(-4) = 0$, $f'(2) = 0$ and $f'(0) = 8$.



5.1 Write down the x-coordinates of the stationary points of f (2)

5.2 For which value(s) of x is f strictly decreasing? (2)

5.3 Explain at which value of x the stationary point of f will be a local minimum. (1)

5.4 Determine the x-coordinate of the point of inflection of f. (3)

5.5 For which value(s) of x is f concave up? (2)

5.6 Find the equation of $f(x)$ if $P(1\,;2\frac{2}{3})$ is a point on $f(x)$ (8)

Q1

$$f(x) = ax^3 + bx + 1$$
local min $(2; -15)$

$f(2) = a(2)^3 + 2b + 1$
$-15 = 8a + 2b + 1$
$0 = 8a + 2b + 16$
$0 = 4a + b + 8$

$f'(x) = 3ax^2 + b$
$f'(2) = 12a + b$
$0 = 12a + b$

$\therefore\ 4a + b + 8 = 12a + b$
$8 = 8a$
$1 = a$

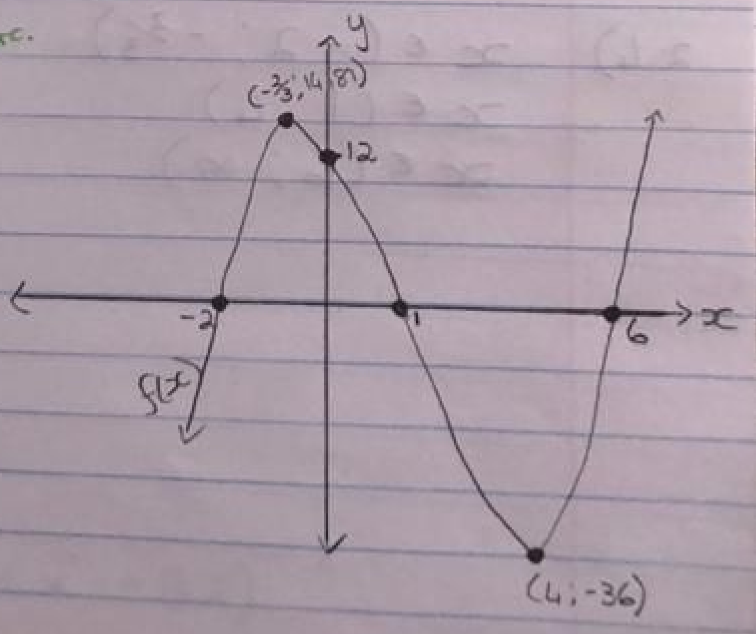
$\therefore\ 0 = 12(1) + b$
$\therefore b = -12$

Q2

2.1) $f(0) = 12$ → y-interc.
$f(1) = f(6) = f(-2) = 0$ → x-interc.
$f'(-\frac{2}{3}) = f'(4) = 0$
x-values of TP's

Y-values of TP's
$f(-\frac{2}{3}) = 14{,}81$
$f(4) = -36$

↳ Given info ---- you sketch it.



2.2) $f(x) = ax^3 + bx^2 + cx + 12$

$f(x) = a(x-1)(x-6)(x+2)$

through $(0;12)$

$12 = a(-1)(-6)(2)$

$12 = 12a$

$1 = a$

$\therefore f(x) = (x-1)(x^2-4x-12)$

$f(x) = x^3 - 4x^2 - 12x - x^2 + 4x + 12$

$f(x) = x^3 - 5x^2 - 8x + 12$

2.3) $f'(x) = 3x^2 - 10x - 8$

$f''(x) = 6x - 10$

$0 = 6x - 10$

$10 = 6x$

$\frac{5}{3} = x$

$\therefore x < \frac{5}{3}$

2.4) $x \in (-2; -\frac{2}{3})$

$x \in (1; 4)$

$x \in (6; \infty)$

Q3

3.1) $f(x) = x^3 - 12x - 16$

OA and OB: (needs x-intercepts)

OC (needs y-intercept)

$\therefore C(0; -16)$

$\therefore OC = 16$ units.

A and B:

$f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & +0 & -12 & -16 \\ & & -2 & 4 & 16 \\ \hline & 1 & -2 & -8 & \end{array}$$

$\therefore 0 = (x+2)(x^2 - 2x - 8)$

$0 = (x+2)(x-4)(x+2)$

$\therefore x = -2$ and $x = 4$

$\therefore A(-2; 0)$

$B(4; 0)$

$\therefore OA = 2$ units

$OB = 4$ units

3.2) $f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$x = \pm 2$

$f(2) = -32$

$\therefore D(2; -32)$

3.3) $f'(0) = -12$

$\therefore y = -12x + c$, through $(0; -16)$

$-16 = c$

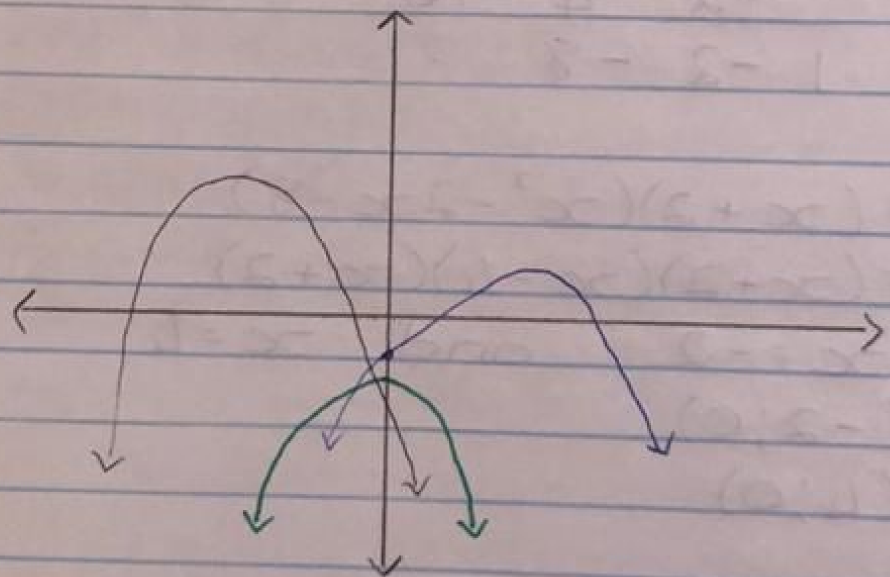
$\therefore y = -12x - 16$

Q4

$f(x) = -ax^3 - bx^2 - cx + d$

$f'(x) = \underline{\underline{-3}}ax^2 - 2bx - c$ — (-) y-intercept

∩



Q5

5.1) $x = -4$ and $x = 2$

5.2) $x \in (-\infty; -4)$
$x \in (2; \infty)$

5.3) $x = -4$

5.4) TP of parabola, x-value

$$= \frac{-4+2}{2} = -1$$

$\therefore x = -1$

5.5) $x < -1$

5.6) $f(x) = ax^3 + bx^2 + cx + d.$

$f'(x) = 3ax^2 + 2bx + c$
$f'(x) = k(x+4)(x-2)$
through $(0; 8)$
$8 = k(4)(-2)$
$8 = -8k$
$-1 = k$

$\therefore f'(x) = -x^2 - 2x + 8$

$3a = -1$ $\quad$ $-2 = 2b$ $\quad$ $c = 8$
$a = -\frac{1}{3}$ $\quad$ $b = -1$

$\therefore f(x) = -\frac{1}{3}x^3 - x^2 + 8x + d$
through $(1; \frac{8}{3})$
$\frac{8}{3} = -\frac{1}{3} - 1 + 8 + d$
$d = -4$