Textbook ex's Memo..

Ex 11

13 a) F → the y-intercept !! So where $x=0$.

$\therefore F(0;3)$

b) $f(x) = 2x^3 + px^2 + qx + 3$

$f'(x) = 6x^2 + 2px + q$
$f'(2) = 6(2)^2 + 2p(2) + q$
$0 = 24 + 4p + q$ ①

we have TP...
- so get derivative of f
- subs x-value of TP and set = 0

$f(2) = 2(2)^3 + p(2)^2 + q(2) + 3$
$-9 = 16 + 4p + 2q + 3$
$0 = 28 + 4p + 2q$ ②

subs E into original

$q = -24 - 4p$ ③

we subs E into original so we can do simultaneous equations.

$0 = 28 + 4p + 2(-24 - 4p)$
$0 = 28 + 4p - 48 - 8p$
$4p = -20$
$\underline{p = -5}$

$\therefore \underline{q = -4}$

c) $f(x) = 2x^3 - 5x^2 - 4x + 3$

$\left\{ \begin{array}{l} \text{A, B and C are } x\text{-interc.} \\ \text{so } y=0 \text{, solve for } x \end{array} \right\}$

$\therefore f(-1) = 0$, $(x+1)$ is factor.

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & +3 \\ & & -2 & +7 & -3 \\ \hline & 2 & -7 & +3 & \end{array}$$

$f(x) = (x+1)(2x^2 - 7x + 3)$

$0 = (x+1)(2x-1)(x-3)$

$x = -1 \qquad x = \frac{1}{2} \qquad x = 3$

$\therefore A(-1;0)$

$B(\frac{1}{2};0)$

$C(3;0)$

d) D → is TP !! get $f'(x)$, set $=0$

$f'(x) = 6x^2 - 10x - 4$

$0 = 3x^2 - 5x - 2$

$0 = (3x+1)(x-2)$

$x = -\frac{1}{3} \qquad x = 2$

N.A → one given on sketch.

$\therefore f(-\frac{1}{3}) = \frac{100}{27}$

$\therefore D(-\frac{1}{3} ; \frac{100}{27})$

e) $f'(x) = 6x^2 - 10x - 4$

$f'(0) = -4$

$\therefore m = -4$

So $y = -4x + c$

through $(0;\ \ )$

$3 = c$

$\therefore y = -4x + 3$

f) $\left\{ \begin{array}{l} \text{will again intersect ?? (where they will} \\ \text{cut each other) } \therefore \text{ simultaneous} \end{array} \right.$

{set graphs = to each other}

$-4x + 3 = 2x^3 - 5x^2 - 4x + 3$

$0 = 2x^3 - 5x^2$

$0 = x^2(2x - 5)$

$x = 0$ $\qquad$ $x = \frac{5}{2}$

N A

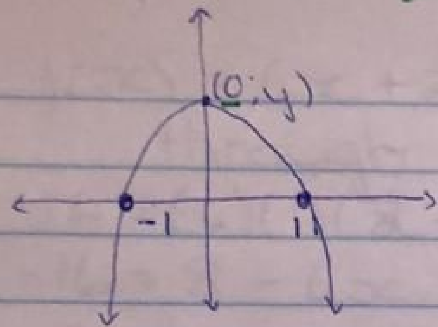
$y = -4\left(\frac{5}{2}\right) + 3$

$y = -7$

$\therefore \left(\frac{5}{2}; -7\right)$

$POI = \frac{-1+1}{2} = 0$

14 a)



→ x-values of turning points of f are x-intercepts of f'.

→ x value of point of inflection is x value of TP of f'

b) $f(x) = a(x+2)(x-1)^2$ → x-intercepts.

through $(0;-2)$

$-2 = a(2)(-1)^2$

$-2 = 2a$

$\therefore a = -1$

$\therefore f(x) = -1(x+2)(x^2-2x+1)$

$f(x) = -(x^3 - 2x^2 + x + 2x^2 - 4x + 2)$

$f(x) = -x^3 + 3x - 2$

c) $x \in (-\infty; -1)$ and $x \in (1; \infty)$

d) $x \in (-1; 1)$

16 a) $f(x) = (x+2)(x-1)(x - \boxed{x_1})$

two of the 3 x-intercepts are given.

we don't have this root, so I make it a variable to solve.

through $(2;-16)$

$-16 = (4)(1)(2 - x_1)$

$-16 = 8 - 4x_1$

$-24 = -4x_1$

$6 = x_1$

subs root.

$\therefore f(x) = (x+2)(x-1)(x-6)$

$f(x) = (x+2)(x^2 - 7x + 6)$

$f(x) = x^3 - 7x^2 + 6x + 2x^2 - 14x + 12$

$f(x) = x^3 - 5x^2 - 8x + 12$

b) $C(6;0)$

c) $g(x) = ax + q$

gradient of line

$m = \dfrac{0-(-16)}{6-2} = 4$

$g(x) = 4x + q$

through $(6;0)$

$0 = 24 + q$

$\therefore q = -24$

$\therefore g(x) = 4x - 24$

d) A → where graphs intersect.
∴ where they are =

$$\therefore\ x^3 - 5x^2 - 8x + 12 = 4x - 24$$
$$x^3 - 5x^2 - 12x + 36 = 0$$

{ we know that other points of intersection is at $x = 2$ and $x = 6$ } → divide with one of these. (Any one)

$$\therefore\ \begin{array}{c|cccc} 6 & 1 & -5 & -12 & +36 \\ & & 6 & 6 & -36 \\ \hline & 1 & 1 & -6 & \end{array}$$

$$\therefore\ (x-6)(x^2+x-6) = 0$$
$$(x-6)(x+3)(x-2) = 0$$

$x = 6$ N.A $x = -3$ $x = 2$ N.A

$$y = 4(-3) - 24$$
$$y = -36$$

$\therefore\ A(-3; -36)$

e) $x \in (-\infty; -3]$
$x \in [2; 6]$ } intervals where g is above f

f) $x \in (-2; 1)$
$x \in (6; \infty)$ } $f(x) \cdot g(x) \boxed{<0}$ gives negative answer.

∴ one above and other below x-axis

17 a) $p = 4$ - y-intercept

b) $f(x) = x^3 + mx^2 + nx + 4$

through $(-1;0)$

$0 = (-1)^3 + m(-1)^2 + n(-1) + 4$

$0 = -1 + m - n + 4$

$0 = m - n + 3$

$f'(x) = 3x^2 + 2mx + n$ → C(0;4) is TP.

$f'(0) = n$

$0 = n$

at TP gradient always = 0 and gradient at a point is found by subs. x-value into f' is gradient.

$\therefore 0 = m + 3$

$m = -3$

c) B → TP !!

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$\underset{N.A}{x = 0}$ $\qquad$ $x = 2$

$\therefore B(2;0)$

d) $f'(x) = 3x^2 - 6x$

$f'(3) = 9$

$\therefore y = 9x + c$

through $(3;4)$

$4 = 9(3) + c$

$\therefore c = -23$

point of contact:

$x = 3$

$y = f(3) = 4$

$\therefore (3;4)$

$\therefore y = 9x - 23$

e) line throug A and C;

$m = \frac{4-0}{0-(-1)} = 4$

$y = 4x + c$, through $(0;4)$

$4 = c$

$\therefore y = 4x + 4$

now,

$4x + 4 = x^3 - 3x^2 + 4$

$0 = x^3 - 3x^2 - 4x$

$0 = x(x^2 - 3x - 4)$

$0 = x(x-4)(x+1)$

$x = 0$, $x = 4$, $x = -1$

N.A   ↓   N.A

$y = 4(4) + 4$

$y = 20$

$\therefore$ at $(4;20)$

f) $c > 4$

$c < 0$

g) shifts 2 units down

h) 3 $\mathbb{R}$ roots.

Ex 12

7) a) $m = -9$

b) $x = 1$
$x = 5$

c) $x \in (1;5)$

d) x-value of TP of $f'(x)$
$= \frac{1+5}{2} = 3$

$\therefore x = 3$

e) $f'(x) = a(x-1)(x-5)$
through $(4;-9)$
$-9 = a(3)(-1)$
$-9 = -3a$
$\therefore a = 3$

$\therefore f'(x) = 3(x^2 - 6x + 5)$
$f'(x) = 3x^2 - 18x + 15$

f) $f'(x) = 3ax^2 + 2bx + c$
$\therefore 3a = 3$ and $2b = -18$ and $c = 15$
$a = 1$ $b = -9$

g) $f(x) = x^3 - 9x^2 + 15x + d$

$f(3) = (3)^3 - 9(3)^2 + 15(3) + d$

$0 = -9 + d$

$9 = d$

$\therefore f(x) = x^3 - 9x^2 + 15x + 9$

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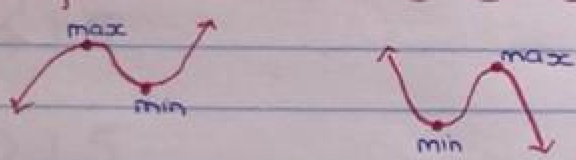
9 $h(x) = -2x^3 - 6x^2 + 48x$

{local max and local min refers to TP's of graph.}

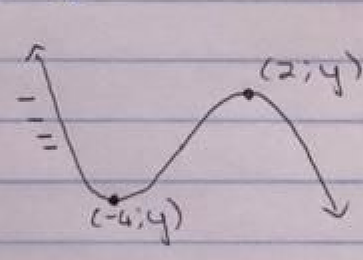


a) $h'(x) = -6x^2 - 12x + 48$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4$ and $x = 2$



$\therefore$ local max at $x = 2$

b) local min at $x = -4$

c) $h''(x) = -12x - 12$

$0 = -12x - 12$

$12x = -12$

$x = -1$

$\therefore x < -1$

d) $x > -1$

10 a) $x = -1$

b) $x = 4$

c) $x = 1,5$

d) $x < 1,5$

e) $x > 1,5$

f) $x = 1,5$

g) $x \leqslant 1,5$

h) $x > 1,5$

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11 a) $x = -4$
$x = 0$

b) $x = -4$

c) $x = 0$

d) $x \in (-4; 0)$

e) $x \in (-\infty; -4)$ and $x \in (0; \infty)$

f) $m = 0$
g) $x = -2$
h) $x < -2$
i) $x > -2$

12a) $h(1) = 0$, $(x-1)$ is factor

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -24 & +26 \\ & & 1 & -2 & -26 \\ \hline & 1 & -2 & -26 & \end{array}$$

$h(x) = (x-1)(x^2 - 2x - 26)$

$0 = x - 1$ and $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-26)}}{2(1)}$

$x = 1$

$x = 6{,}20$

$x = -4{,}20$

$(1;0)$

$(6{,}20;0)$

$(-4{,}20;0)$

b) $h'(x) = 3x^2 - 6x - 24$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4$ $x = -2$

$h(4) = -54$ $h(-2) = 54$

c) $(4;-54)$ local min

$(-2;54)$ local max

d) $h''(x) = 6x - 6$ → $h(1) = 0$

$0 = 6x - 6$

$6 = 6x$ $\therefore (1;0)$

$1 = x$