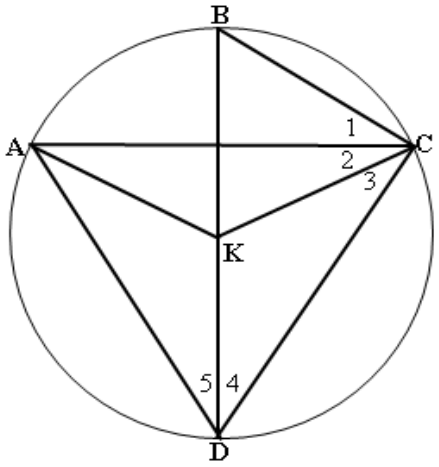
TOPIC 3: EUCLIDEAN GEOMETRY

QUESTION 1

1.1 In the accompanying diagram, BD is a diameter of the circle with centre K.
$A\hat{K}C = 128°$ and $\widehat{D_4} = 32°$.



1.1.1 Name four radii in the given diagram. (4)

1.1.2 Why is $B\hat{C}D = 90°$? (1)

1.1.3 What type of triangle is Δ AKC ? Provide a reason for your answer. (2)

1.1.4 Determine, with reasons, the sizes of the following angles

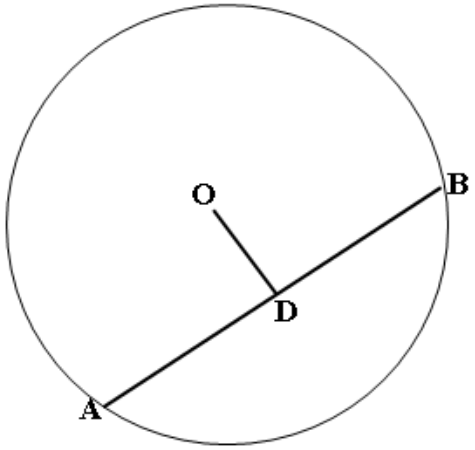
a) $\hat{C}_2$ (3)

b) $\hat{D}_5$ (3)

c) $\hat{C}_1$ (2)

1.2 1.2.1 Complete the following statement: If a line segment is drawn from the centre of the circle, perpendicular to a chord, then it…… (1)

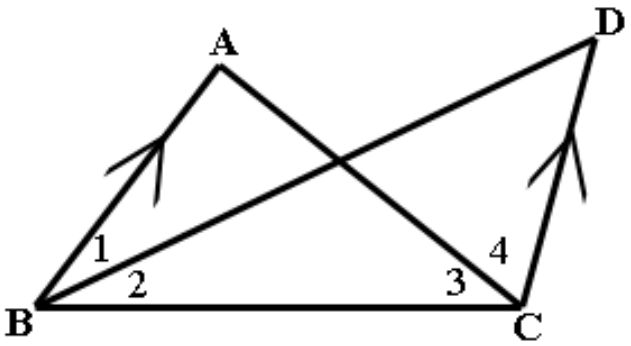
1.2.2 In the diagram alongside, O is the centre of the circle with OD $\perp$ AB. AB = 6*x* units and OD = 2*x* units.



1.2.2.1 Express , with reason, AD in terms of *x* (2)

1.2.2.2 Hence, calculate the radius, OA in terms of *x*. (4)

1.3 In the accompanying diagram, A, B and C are the vertices of Δ ABC. The straight line through C, parallel to BA, meets the bisector of $A\widehat{B}C$ at D. AB = AC and $B\widehat{A}C = 36°$



1.3.1 What type of triangle is Δ ABC ? Give a reason for your answer. (2)

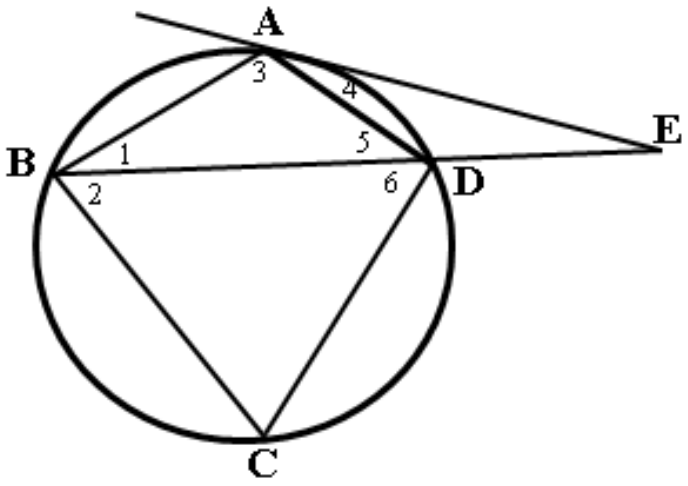
1.3.2 Calculate, with reasons, the size of $\widehat{B}_1$ (4)

1.3.3 Are the points A, B, C and D concyclic? Explain. (4)

1.4 1.4.1 List three ways to prove a quadrilateral is cyclic. (4)

1.4.2 List two ways to show that a line is a tangent to a circle at a given point. (2)

1.4.3.1 In the given diagram, ABCD is a cyclic quadrilateral. The tangent to the circle at A meets BD produced at E. $\widehat{A}_4 = 30°$ and $\widehat{D}_5 = 56°$.



Determine, with reasons, the magnitude of the following angles.

a) $\widehat{B}_1$ (2)

b) $\widehat{A}_3$ (2)

c) $B\widehat{C}D$ (2)

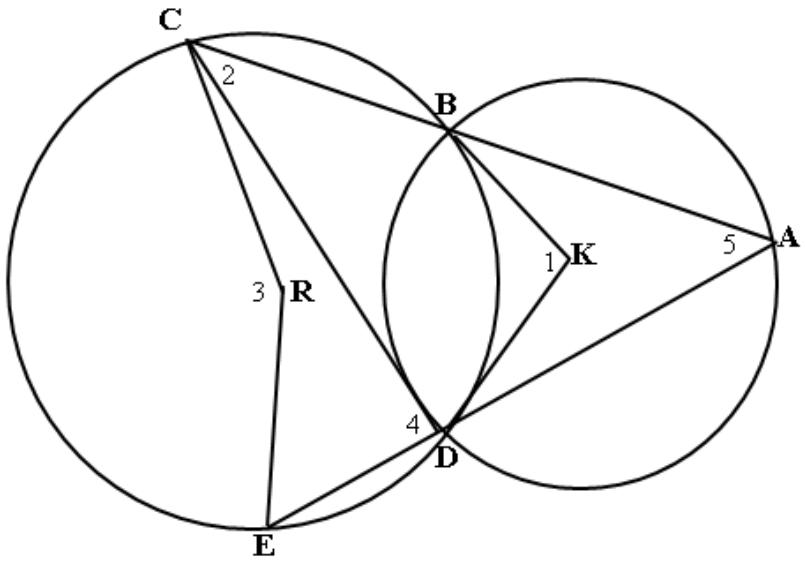
d) The interior angles of Δ AED (2)

1.4.3.2 Is BD a diameter of the circle? Explain. (2)

1.4.3.3 Show, using appropriate calculations that ABCE is **not** a cyclic quadrilateral. (3)

1.5 1.5.1 Complete the statement of the following theorem in your answer book: The angle which an arc of a circle subtends at the centre ……. (1)

1.5.2 In the given diagram, R and K are centres of two unequal circles, which intersect at B and D. CBA and EDA are double chords such that CD = DA. $\widehat{K}_1 = 70°$



Calculate, with reasons, the sizes of the following angles:

i) $\widehat{A}_5$ (3)

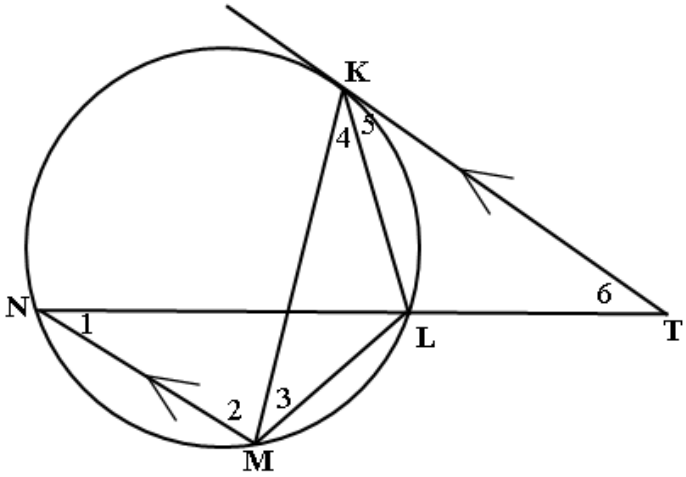
ii) $\widehat{D}_4$ (3)

iii) $\widehat{R}_3$ (2)

iv) $B\widehat{D}K$ (3)

1.6 1.6.1 Complete the statement of the following theorem in your answer book: (1)
The angle between a tangent to a circle and the chord drawn from the point of contact is equal to……..

1.6.2 In the accompanying figure, the points K, N , M and L lie on the circle. The tangent KT to the circle at K is parallel to the chord NM. The chord NL is produced to T.
$\widehat{M}_3 = 35°$ and $M\widehat{K}T = 68°$.



Calculate, with reasons, the sizes of the following angles:

1.6.2.1 $\widehat{K}_5$ (2)

1.6.2.2 $\widehat{N}_1$ (3)

1.6.2.3 $\widehat{T}_6$ (2)

1.7 Complete the following statements by filling in the missing word(s) so that the statements are correct.

1.7.1 The angle subtended by a chord at the centre of a circle is…… (1)

1.7.2 The opposite angles of a cyclic quadrilateral are…….. (1)

1.7.3 Angles subtended by a chord of a circle in the same……..are equal. (1)

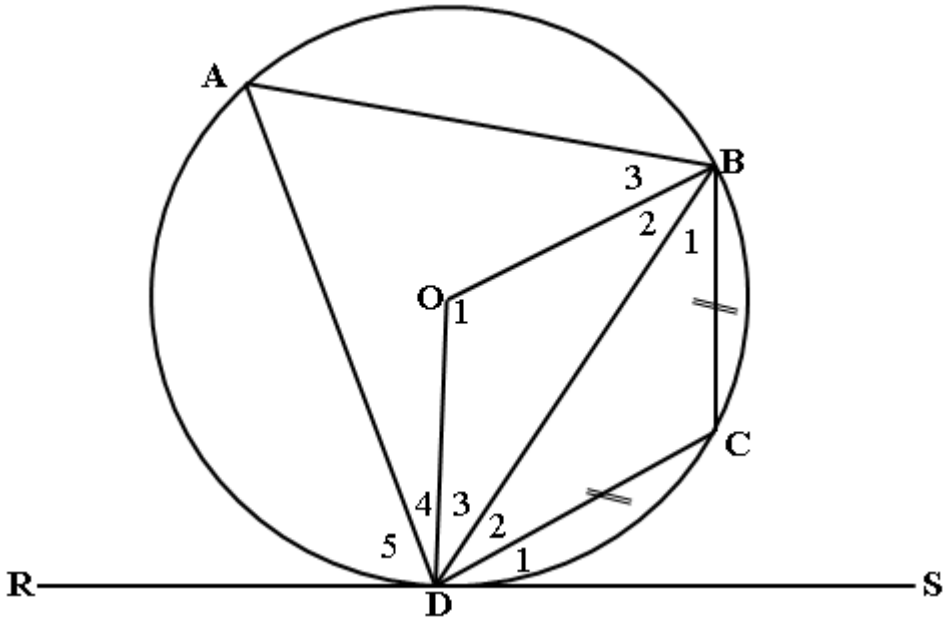
1.7.4 The exterior angle of a cyclic quadrilateral is…….. (1)

1.7.5 The angle between the tangent to a circle and a chord at the point of contact is…… (1)

1.7.6 Tangents drawn to a circle from a common point are……. (1)

1.7.7 If a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a……..to the circle. (1)

1.8 In the given diagram, the points A, B, C and D lie on the circle with centre O. RDS is a tangent to circle at D. BC = DC; $C\widehat{D}S = 40°$.



1.8.1 Provide reason(s) to make the following statements TRUE.

1.8.1.1 ABCD is a cyclic quadrilateral. (1)

1.8.1.2 Δ BCD is an isosceles triangle. (1)

1.8.1.3 $O\widehat{D}S = 90°$ (1)

1.8.1.4 OB = OD (1)

1.8.2 Calculate, with reasons, the sizes of the following:

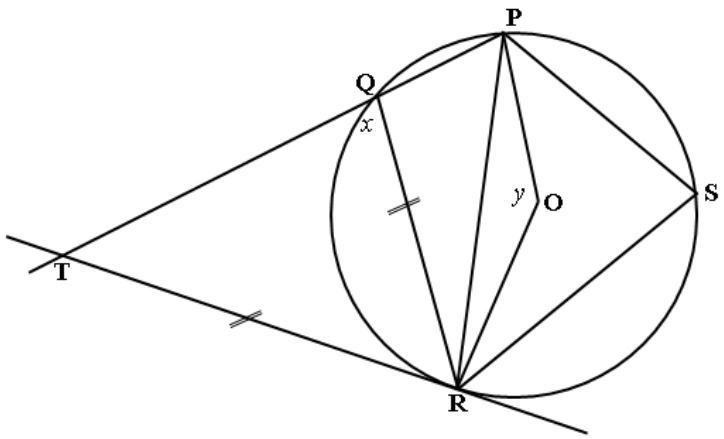
1.8.2.1 $\widehat{D}_2$ (3)

1.8.2.2 $\widehat{C}$ (2)

1.8.2.3 $\widehat{A}$ (2)

1.8.2.4 $\widehat{O}_1$ (2)

1.9 In the given diagram, O is the centre of the circle. The points S, P, Q and R lie on the circle. TQP is a straight line. QR = TR. $T\widehat{Q}R = x$; $P\widehat{O}R = y$. TR is a tangent to the circle at R.

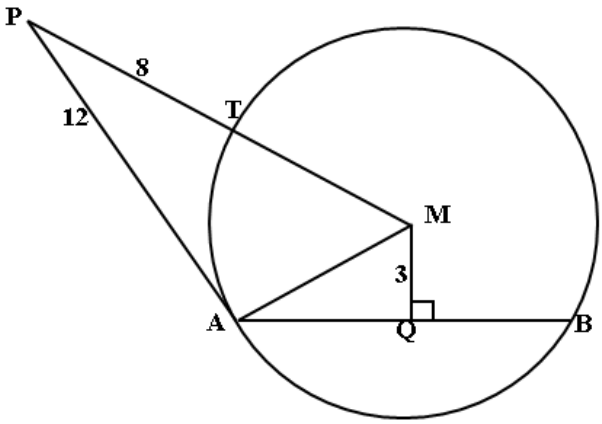


1.9.1 Name, with reason(s), three other angles equal to *x* (5)

1.9.2 Express $P\widehat{O}R$ in terms of *x*. (2)

1.9.3 Determine the value(s) of *x* for which PTRO **will not** be a cyclic quadrilateral. (4)

1.10 In the accompanying diagram, M is the centre of the circle. AB is a chord with length 8 units. MQ ⊥ AB. PA = 12 units and PT = 8 units, with T a point on the circle. MQ = 3 units. AB = 8 units



1.10.1 Write down, with reason the length of AQ. (2)

1.10.2 Complete: In a right-angled triangle, the square on the hypotenuse is…… (1)

1.10.3 Determine the length AM (3)

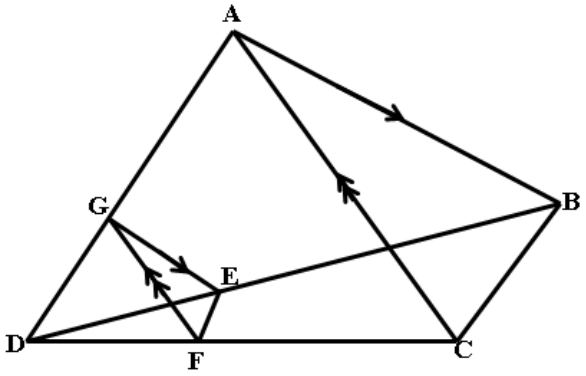
1.10.4 Hence, use appropriate calculations to show that PA is a tangent to the circle at A. (4)

Ratio, Proportion and Similarity

QUESTION 2

2.1 2.1.1 Complete the following statement: A line drawn parallel to one side of triangle, divides …… (1)

2.1.2 In the given diagram, ABCD is a quadrilateral with AB ∥ GE and FG ∥ CA.



a) Write down, with reasons, TWO ratios each equal to AG:GD (3)

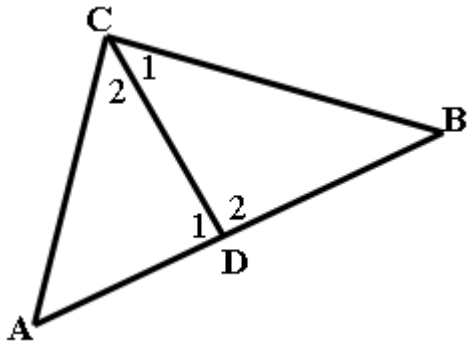
b) Hence, prove that EF ∥ BC (2)

c) Prove ΔDEF ||| ΔDBC (3)

d) If $\frac{DE}{BE} = 0,6$; BC = 16 units and DG = 9 units, calculate the lengths of EF and AG (in this order). (6)

2.2 2.2.1 Complete the following: Two triangles are similar if:
a)…….
b)……. (2)

2.2.2 In the accompanying diagram, ABC is a right –angled triangle with $\hat{C} = 90°$. The point D lies on AB such that CD is perpendicular to AB. Let: $\hat{C}_1 = x$ and $\hat{C}_2 = y$



a) Express $\hat{C}$ in terms of x and y (1)

b) In Δ DCB , express $\hat{B}$ in terms of x (1)

c) In Δ DCA , express $\hat{A}$ in terms of y (1)

d) Hence, prove $\Delta ACD \,|||\, \Delta CBD$ (4)

e) Name, in order of corresponding letters, another triangle in the figure which is similar to ΔACD (2)

f) If $\Delta ACD \,|||\, \Delta CBD$ complete the following ratios:

$$\frac{AC}{CB} = \frac{CD}{\quad} = \frac{\quad}{CD}$$ (2)

g) Hence complete: $CD^2 =$ ... ... × ... ... (1)

h) If $\Delta ACD \,|||\, \Delta ABC$, show that $AC^2 = AB \times AD$ (2)

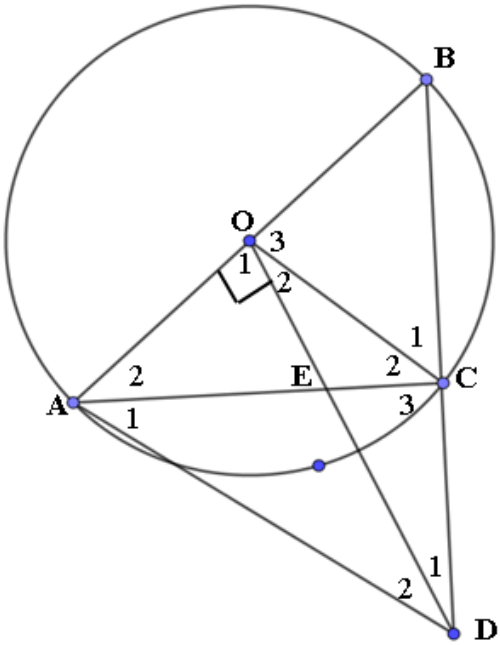
i) Prove $\Delta BCD \,|||\, \Delta BAC$ (3)

j) Hence, deduce that $BC^2 = BD \times AB$ (2)

k) If BC = 4 units, and BD = 2 units, determine the lengths of , in simplified surd form:
   i) CD
   ii) AB
   iii) AC (8)

2.3 2.3.1 Complete the following statement: If a line segment joining two points subtends equal angles at two other points on the same side of the line segment, …… (2)

2.3.2 In the given diagram, AB is a diameter of the circle with centre O. DO is perpendicular to AB at O. The chord BC meets OE produced at D. OD and AC intersects at E.



a) If $\widehat{O}_1 = 90°$, name, with reasons THREE other angles equal to 90° (4)

b) Hence, deduce that OADC is a cyclic quadrilateral. (2)

c) Why is $\widehat{A}_2 = \widehat{C}_2$? (1)

d) Hence, show that $\widehat{D}_1 = \widehat{C}_2$? (2)

e) Show $\Delta OCE \,|||\, \Delta ODC$ (3)

f) Hence, complete, $\frac{OC}{OD} = \frac{CE}{DC} = \ -$ (1)

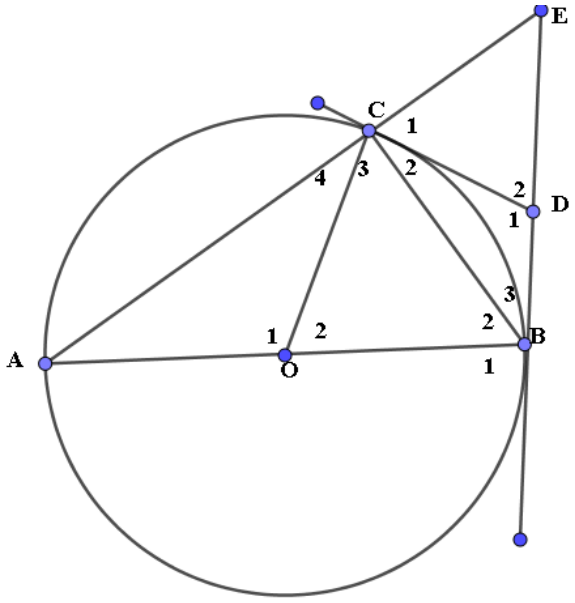
g) If r is the radius of the circle, determine $OE \times OD$ in terms of r (2)

h) If $\widehat{A}_2 = x$, show that OE = r.tan x (2)

i) If $\hat{A}_2 = 30°$, and r = 2 units, determine the area of Δ ABD, in simplified surd form. (3)

2.4 2.4.1 Complete the following statement: If two tangents are drawn to a circle from a common point, …… (1)

2.4.2 In the given figure, O is the centre of the circle. AB is a diameter of the circle. The tangent to the circle at B meets the AC produced at E. The tangent to the circle at C meets EB at D. Let $\hat{C}_4 = x \; and \; \hat{C}_3 = y$



a) Name with reasons, THREE angles equal to x (5)

b) Name with reason, ONE angle equal to y. (1)

c) In the given diagram, name with reasons, FIVE angles equal to 90^o (7)

d) Why is CD = DB? (1)

e) Show that BOCD is a cyclic quadrilateral. (3)

f) Prove that $\hat{E} = \hat{C}_1$ (2)

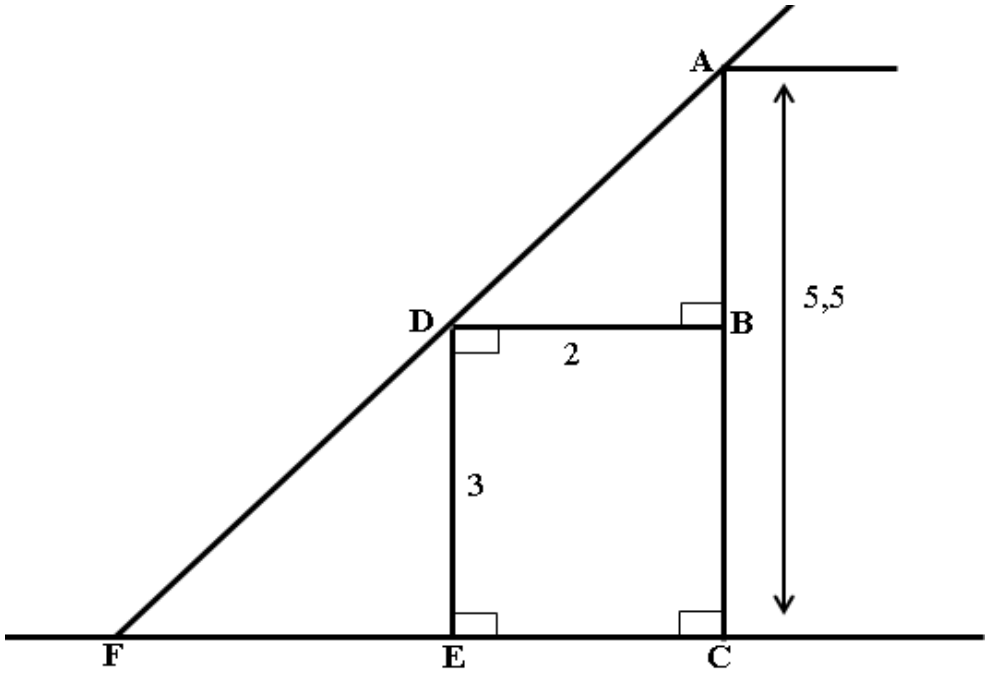
g) Hence, deduce that DB = DE (2)

h) Prove $\Delta EBC \;|||\; \Delta EAB$ (3)

i) Complete: $\frac{EB}{EA} = \; - = \; \frac{BC}{AB}$ (1)

j) If BD = CE = 2 units, determine the length of CA. (7)

2.5 In the given diagram, CEDB represents a rectangular shed 2 metres wide and 3 metres high. The shed stands against a perpendicular building of height 5,5 metres. A ladder, FA, is used to gain access to the roof of the building. AF is the minimum length of the ladder. The diagram below shows a side view of the shed and a ladder leaning against a building AC. EF is the minimum distance of the ladder to the shed.



2.5.1 Why is DB ∥ EC? (1)

2.5.2 Write down, with reason, the size of $D\widehat{E}F$ (1)

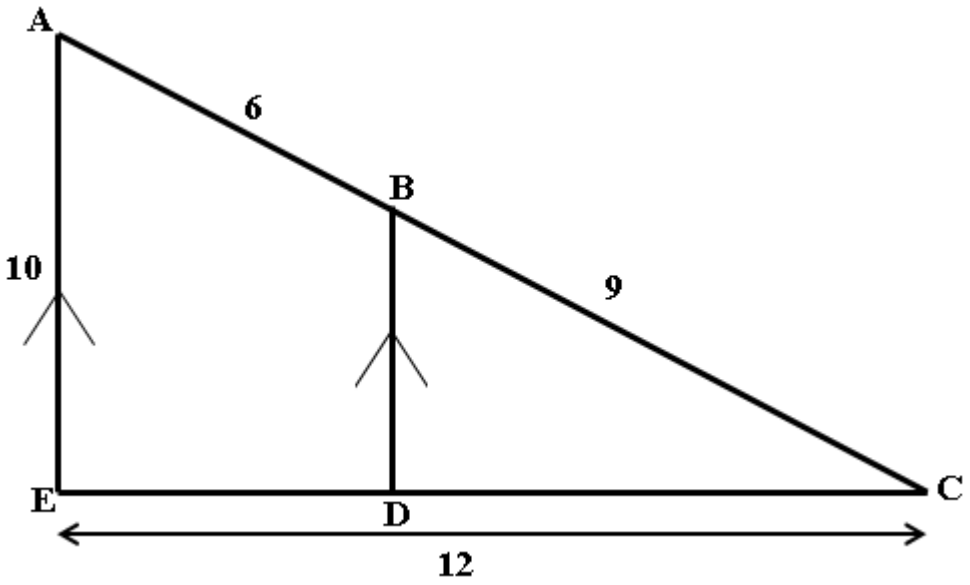
2.5.3 Prove $\Delta BAD \;|||\; \Delta EDF$ (4)

2.5.4 Complete: $\frac{BA}{ED} = \frac{AD}{\quad} = \frac{\quad}{EF}$ (2)

2.5.5 Determine the length of AB (1)

2.5.6 Show, that the minimum distance from the bottom of the ladder to the shed cannot be more than 3 m. (4)

2.6 In the given diagram A, E and C are the vertices of Δ AEC. EA is drawn parallel to DB, with D a point on EC. AE = 10 cm; AB = 6 cm and BC = 9 cm. (**Note**: $A\hat{E}D \neq 90°$; $B\hat{D}C \neq 90°$)



2.6.1 Complete the following statement: A line drawn parallel to one side of a triangle…….. (1)

2.6.2 Prove $\Delta AEC \;|||\; \Delta BDC$ (4)

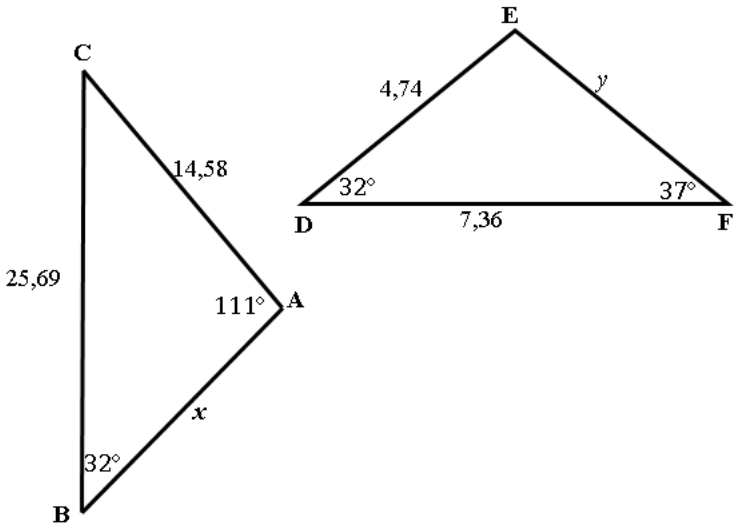
2.6.3 Complete: $\_\_ = \frac{EC}{DC} = \frac{AC}{BC}$ (1)

2.6.4 Hence, show that CD = 7,2 cm and BD = 6 cm. (3)

2.6.5 Hence, determine the size of $B\hat{D}C$, correct to TWO decimal places. (3)

2.6.6 Determine the numerical value of $\frac{\text{Area of } \Delta BDC}{\text{Area of } \Delta AEC}$. (4)

2.7 In the given diagram, two triangles are shown, namely Δ ABC and Δ DEF.
In ΔABC $\hat{A} = 111°$; $\hat{B} = 32°$; AB = *x* mm ; AC = 14,58 mm and CB = 25,69 mm.
In ΔDEF $\hat{F} = 37°$; $\hat{D} = 32°$; DE = 4,74 mm ; EF = *y* mm and FD = 7,36 mm

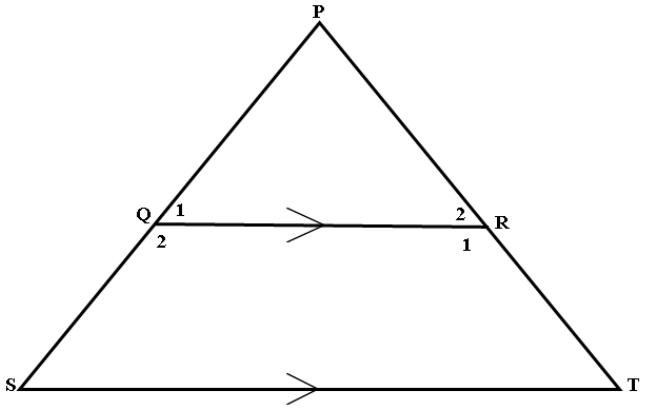


2.7.1 Determine the size of the remaining interior angles of the given triangles. (2)

2.7.2 Show that ΔABC ||| ΔEDF (3)

2.7.3 Hence, determine the numerical values of *x* and *y*. (4)

2.8 In the given diagram, PQR is an equilateral triangle with sides 4 cm. PQ and PR are produced to meet in S and T respectively. ST is parallel to QR. PS = 9 cm.



2.8.1 Write down, with reason, the sizes of the interior angles of Δ PQR (2)

2.8.2 Write down, with reason the size of $\hat{S}$ and $\hat{T}$ (2)

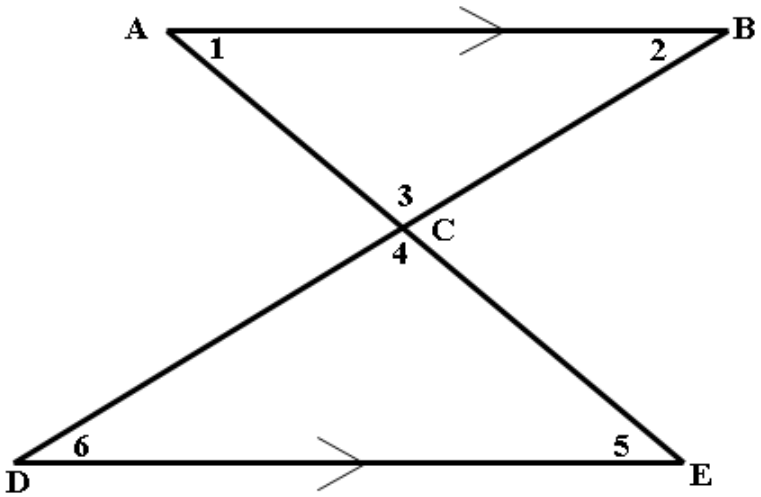
2.8.3 Write down the length of QS (1)

2.8.4 Prove $\Delta PQR \;|||\; \Delta PST$ (3)

2.8.5 Hence, determine the length of ST. (3)

2.8.6 X is an arbitrary point on ST, such that PX is the bisector of $S\widehat{P}T$. Determine the length of PX, correct to TWO decimal places. (3)

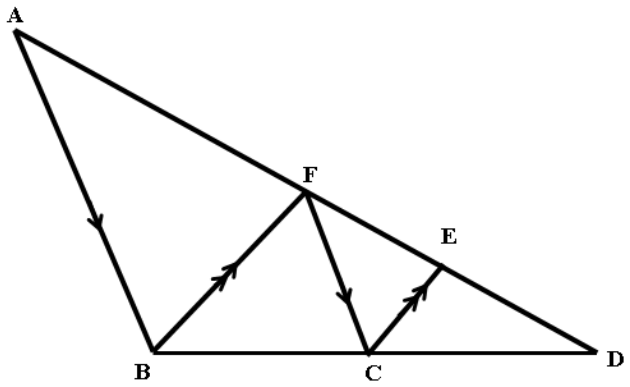
2.9 In the given diagram AB ∥ DE. AE and DB intersect at C.



2.9.1 Determine the length of BC if AB = 6 cm ; DE = 8 cm and DC = 3 cm (4)

2.9.2 Determine the length of DE when EC = 2 cm ; AC = 5 cm and AB = 10 cm (4)

2.10 In the given diagram AB ∥ CF; BF ∥ CE and, $\frac{DE}{FE} = \frac{5}{4}$.



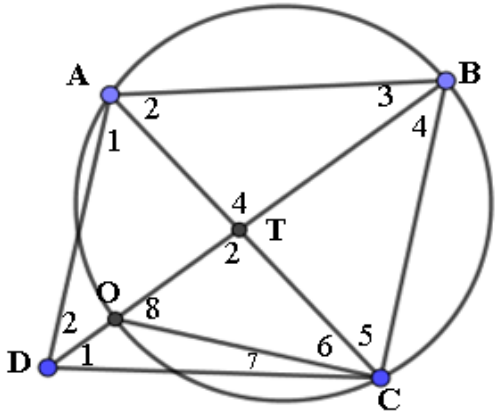
Determine the following ratios with reasons.

2.10.1 DC: CB (2)

2.10.2 $\frac{\text{Area } \Delta \text{ FBC}}{\text{Area } \Delta \text{ FCD}}$ (3)

2.10.3 $\frac{AF}{FD}$ (5)

2.11 In the given diagram ABCD is a parallelogram with the diagonals intersecting at T. The circle passing through the points A, B and C cuts DB at O. [Note: T is **not** the centre of the circle.]



2.11.1 Why is AT = TC? (1)

2.11.2 Complete the following statement: Vertically opposite angles are…… (1)

2.11.3 Why is $\hat{A}_2 = \hat{O}_8$? (1)

2.11.4 Prove $\Delta ABT \,|||\, \Delta OCT$ (3)

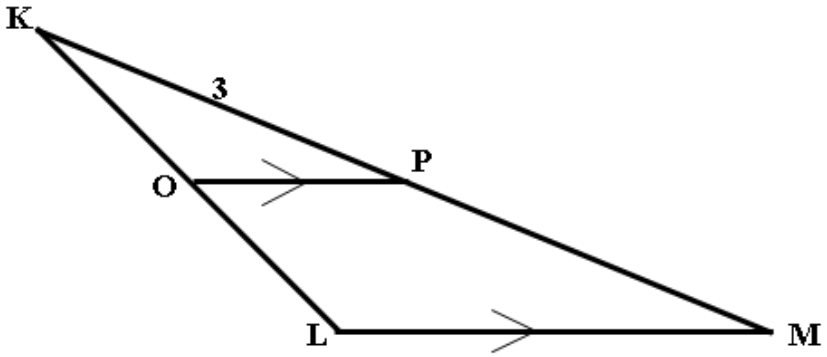
2.11.5 Complete: $\frac{AB}{BT} = \frac{BT}{\quad} = \frac{\quad}{OT}$ (2)

2.11.6 If AC = 6 units and BT = 4 units

a) Show that OT = $\frac{9}{4}$ units (2)

b) Hence, determine the length of DO. (4)

2.12 In the given diagram, K, L and M are the vertices of ΔKLM , with O on KL and P on KM. OP is parallel to LM. KP = 3 cm ; area of $\Delta KOP = 2\ cm^2$ and area of $OPLM = 16\ cm^2$



2.12.1 Complete the following statement: A line drawn parallel to one side of a triangle…… (1)

2.12.2 Provide the geometrical name for the quadrilateral LOPM and provide a reason for your answer. (2)

2.12.3 Prove $\Delta KOP \,|||\, \Delta KLM$ (4)

2.12.4 Write down the area of ΔKLM (1)

2.12.5 Hence, determine the length of PM (4)