Test 2: exponents & surds

Question 1

2.1.1) $\frac{(3x)^{-2}}{3x^{-3}} = \frac{3^{-2}x^{-2}}{3x^{-3}}$ ✓

$= \frac{x^3}{3.3^2x^2}$

$= \frac{x^3}{3^3}$ ✓

$= \frac{x^3}{27}$ ✓

2.1.2) $\frac{x^{-1}+y^{-1}}{x^{-1}y-y^{-1}x} = \frac{\left(\frac{1}{x}+\frac{1}{y}\right)}{\left(\frac{y}{xy}-\frac{x}{xy}\right)}$ ✓ ✓

$= \frac{(y+x)}{xy} \div \frac{(y^2-x^2)}{xy}$ ✓

$= \frac{(y+x)}{xy} \times \frac{xy}{(y+x)(y-x)}$ ✓

$= \frac{1}{(y-x)}$ ✓

2.1.3) $= 6\sqrt{3}\,x^6 + 9\sqrt{3}\,x^6$
$= 15\sqrt{3}\,x^6$

2.2) $2^{3x-6} = (8)^{\frac{1}{2}}$
$2^{3x-6} = (2^3)^{\frac{1}{2}}$
$3x - 6 = \frac{3}{2}$
$3x = \frac{15}{2}$
$6x = 15$
$x = \frac{15}{6}$

1.3.1) $(3^2)^{x^2+x} = (3^3)^{x+1}$
$2x^2 + 2x = 3x + 3$
$2x^2 - x - 3 = 0$
$(2x - 3)(x + 1) = 0$
$2x - 3 = 0$ or $x + 1 = 0$
$2x = 3$ $\quad x = -1$
$x = \frac{3}{2}$

1.3.2) $2^{x+1} + 2^{x+2} = 48$
$2^x(2^1 + 2^2) = 48$
$2^x(6) = 48$
$2^x = 8$
$2^x = 2^3$
$x = 3$

1.33) $3^{2x} - 3 \cdot 3^{x} + 2 = 0$

let $3^x = k$ ✓

$k^2 - 3k + 2 = 0$

$(k-2)(k-1) = 0$

$k = 2$ ✓ or $k = 1$ ✓

$\therefore 3^x = 2$ NA ✓

$3^x = 1$

$3^x = 3^0$

$x = 0$ ✓

Question 2

2.1.1) $\frac{x^{\frac{1}{3}} x^{\frac{1}{4}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{7}{12}}}{x^{\frac{1}{6}}} = x^{\frac{5}{12}}$ ✓ ✓ ✓

2.1.2) $= 8\sqrt{2}\, x^3 + 7\sqrt{2}\, x^3$ ✓

$= 15\sqrt{2}\, x^3$ ✓

2.1.3) $\frac{\sqrt{x}}{x} + \frac{y}{\sqrt{x}}$ LCD: $x\sqrt{x}$

$= \frac{(\sqrt{x})(\sqrt{x}) + xy}{x\sqrt{x}}$ ✓

$= \frac{x + xy}{x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$ ✓

$= \frac{x\sqrt{x} + xy\sqrt{x}}{x \cdot x}$

$= \frac{x\sqrt{x}(1+y)}{x^2}$ ✓ $= \frac{\sqrt{x}(1+y)}{x}$ ✓

2.2.1) $= \frac{3 \times 3\sqrt{2} - 5\sqrt{2}}{2 \times 6\sqrt{2}}$

$= \frac{9\sqrt{2} - 5\sqrt{2}}{12\sqrt{2}}$

$= \frac{4\sqrt{2}}{12\sqrt{2}}$

$= \frac{1}{3}$

2.2.2) $= \frac{2 \times 3\sqrt{2} - 4\sqrt{2}}{2\sqrt{2} + \sqrt{2}}$

$= \frac{6\sqrt{2} - 4\sqrt{2}}{3\sqrt{2}}$

$= \frac{2\sqrt{2}}{3\sqrt{2}}$

$= \frac{3}{3}$

2.2.3) $= \frac{(5\sqrt{3} - 3\sqrt{2})}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5(3) - (3\sqrt{2} \times \sqrt{3})}{2(3)}$

$= \frac{15 - 3\sqrt{6}}{6} = \frac{3(5 - \sqrt{6})}{6} = \frac{5 - \sqrt{6}}{6}$