Test: Algebra, equations & inequalities

Question 1

1.1) $7 - 5x = x^2$

$0 = x^2 + 5x - 7$ ✓

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✓

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-7)}}{2(1)}$ ✓

$x = 1,14$ ✓ and $x = -6,14$ ✓

1.2) LCD: $(x-2)(x-1)$ ✓

$(x-2)(x-1) + (x+1)(x-1) + 2(x-2) = 0$ ✓

$x^2 - 3x + 2 + x^2 - 1 + 2x - 4 = 0$

$2x^2 - x - 3 = 0$ ✓

$(2x - 3)(x + 1) = 0$ ✓

$2x - 3 = 0$ or $x + 1 = 0$

$2x = 3$ $x = -1$ ✓

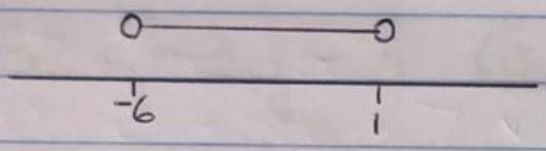
$x = 3/2$ ✓

1.3) $x^2 + 5x - 6 < 0$ ✓

$(x+6)(x-1) < 0$

CV's:

$x = -6$ ✓ and $x = 1$ ✓



$\therefore x \in (-6; 1)$ ✓✓

1.4) $-x(x-9) \leq 14$

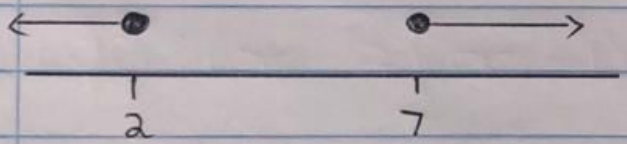
$-x^2 + 9x - 14 \leq 0$ ✓

$x^2 - 9x + 14 \geq 0$

$(x-7)(x-2) \geq 0$ ✓

CV's:

$x = 7$ ✓ and $x = 2$



$\therefore x \in (-\infty; 2]$ ✓ and $x \in [7; \infty)$ ✓

Question 2

2.1) $x + 3y + 3 = 0$ (1)
$xy + 6 = 0$ (2)

$x = -3y - 3$ (3) ✓

$y(-3y - 3) + 6 = 0$
$-3y^2 - 3y + 6 = 0$
$y^2 + y - 2 = 0$ ✓
$(y + 2)(y - 1) = 0$
$y = -2$ ✓ and $y = 1$ ✓

$\therefore x = -3(-2) - 3$
$x = 3$ ✓

$x = -3(1) - 3$
$x = -6$ ✓

2.2.1) $x^2 + 5xy + 6y^2 = 0$
$(x + 3y)(x + 2y) = 0$ ✓

$x + 3y = 0$
$x = -3y$
$\frac{x}{y} = -3$ ✓

or

$x + 2y = 0$
$x = -2y$
$\frac{x}{y} = -2$ ✓

2.2.2) $x + y = 8$
$x = 8 - y$

$\frac{8 - y}{y} = \frac{-3}{1}$ ✓ and $\frac{8 - y}{y} = \frac{-2}{1}$ ✓

$8 - y = -3y$
$8 = -2y$ ✓
$-4 = y$ $\therefore x = 12$

$8 - y = -2y$
$8 = -y$
$8 = y$ ✓ $\therefore x = 0$

Question 3

3.1.1) $\Delta = b^2 - 4ac$
$\Delta = (3)^2 - 4(2)(-4)$
$\Delta = 41$ ✓ ✓
$\therefore$ roots are $\mathbb{R}$, unequal, Q1

3.1.2) $\Delta = (-12)^2 - 4(4)(9)$
$\Delta = 0$ ✓
$\therefore$ roots are $\mathbb{R}$, equal, Q1 ✓

All three must be given to get mark.

3.2) $x(2x+p) = -3$
$2x^2 + px + 3 = 0$
$\Delta = p^2 - 4(2)(3)$
$\Delta = p^2 - 24$ ✓
non real, $\Delta < 0$ ✓
$p^2 - 24 < 0$
$p^2 < 24$
$p < \pm\sqrt{24}$ ✓

3.3) $4x - 3 = k(x-1)(x-1)$
$4x - 3 = k(x^2 - 2x + 1)$
$0 = kx^2 - 2kx - 4x + k + 3$ ✓

$\Delta = (-2k-4)^2 - 4(k)(k+3)$
Real, $\Delta \geq 0$
$(-2k-4)(-2k-4) - 4k(k+3) \geq 0$ ✓
$4k^2 + 16k + 16 - 4k^2 - 12k \geq 0$
$4k + 16 \geq 0$
$4k \geq -16$
$k \geq -4$ ✓

3.4) $x^2 - 2(p-1)x - 4p = 0$

$\Delta = (-2(p-1))^2 - 4(1)(-4p)$ ✓
$\Delta = (-2p+2)^2 + 16p$
$\Delta = (-2p+2)(-2p+\ )+16p$
$\Delta = 4p^2 - 8p + 4 + 16p$
$\Delta = 4p^2 + 8p + 4$
$\Delta = (2p+2)(2p+2)$
$\Delta = (2p+2)^2$ ✓

- $\sqrt{(2p+2)^2} = (2p+2)$ ✓
  ∴ Since Δ is perfect square
  Roots are Q

3.5) $ax(x+1) + 2x(x-1) = x+5$
$ax^2 + ax + 2x^2 - 2x - x - 5 = 0$
$ax^2 + 2x^2 + ax - 3x - 5 = 0$
$(a+2)x^2 + (a-3)x - 5 = 0$ ✓

- $\Delta = (a-3)^2 - 4(a+2)(-5)$
  $\Delta = (a-3)^2 + 20(a+2)$
  $\Delta = (a-3)(a-3) + 20a + 40$
  $\Delta = a^2 - 6a + 9 + 20a + 40$
  $\Delta = a^2 + 14a + 49$
  $\Delta = (a+7)(a+7)$
  $\Delta = (a+7)^2$ ✓

  $(a+7)^2 \geq 0$
  ∴ roots $\mathbb{R}$ ✓