

**SOLUTIONS**

**EXERCISE 1 (Remainder and Factor theorems)**

1.1  $f(x) = x^3 + x^2 - 1$  is divided by  $2x + 3$

$$\begin{aligned}\text{Remainder: } f\left(-\frac{3}{2}\right) &= \left(-\frac{3}{2}\right)^3 + \left(-\frac{3}{2}\right)^2 - 1 \quad (\text{use your calculator}) \\ &= -\frac{17}{8}\end{aligned}$$

1.2  $g(x) = x^3 + 4x^2 - 11x - 31$  is divided by  $x + 5$

$$\begin{aligned}\text{Remainder: } g(-5) &= (-5)^3 + 4(-5)^2 - 11(-5) - 31 \quad (\text{use your calculator}) \\ &= -1\end{aligned}$$

1.3  $p(x) = 2x^3 - x^2 + 3x - 8$  is divided by  $s(x) = x - 2$

$$\begin{aligned}\text{Remainder: } p(2) &= 2(2)^3 - (2)^2 + 3(2) - 8 \quad (\text{use your calculator}) \\ &= 10\end{aligned}$$

1.4  $h(x) = 8x^4 - 4x^2 - 5$  is divided by  $(2x - 1)$

$$\begin{aligned}\text{Remainder: } h\left(\frac{1}{2}\right) &= 8\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^2 - 5 \quad (\text{use your calculator}) \\ &= -\frac{11}{2}\end{aligned}$$

$$\begin{aligned}2.1.1. \text{ Remainder: } f(-1) &= (-1)^3 - 2(-1)^2 - 4(-1) + 3 \\ &= 4\end{aligned}$$

$$\begin{aligned}2.1.2 \text{ Remainder: } f(3) &= (3)^3 - 2(3)^2 - 4(3) + 3 \\ &= 0\end{aligned}$$

2.2  $(x - 3)$  is a factor of  $f(x)$ , and  $(x + 1)$  is not a factor.

3.  $f(x) = x^3 + 5x^2 - 17x - 21$ , divided by  $x + 1$

$$\begin{aligned}\therefore f(-1) &= (-1)^3 + 5(-1)^2 - 17(-1) - 21 \\ &= 0\end{aligned}$$

The remainder = 0, therefore  $x + 1$  is a factor of  $f$ .

4.  $g(x) = x^3 + px + 6$  ;  $(2 - x)$  is a factor.  $(2 - x = 0 ; \therefore 2 = x)$

$$\begin{aligned}\therefore g(2) &= 0 \\ (2)^3 + p(2) + 6 &= 0 \\ 8 + 2p + 6 &= 0 \\ 2p &= -14 \\ p &= -7\end{aligned}$$

5.1  $p(x) = x^3 + ax^2 + bx + 6$  ;  $x - 2$  is a factor.

$$\begin{aligned}\therefore (2)^3 + a(2)^2 + b(2) + 6 &= 0 \\ 8 + 4a + 2b + 6 &= 0 \\ 4a + 2b &= -14 \\ \therefore 2a + b &= -7 \dots\dots\dots(1)\end{aligned}$$

5.2  $p(x) = x^3 + ax^2 + bx + 6$  ; divided by  $x - 3$  gives a remainder of 48

$$\begin{aligned}\therefore (3)^3 + a(3)^2 + b(3) + 6 &= 48 \\ 27 + 9a + 3b + 6 &= 48 \\ 9a + 3b &= 15 \\ 3a + b &= 5 \dots\dots\dots(2)\end{aligned}$$

Subst. (1)  $b = -7 - 2a$  into (2)

$$\begin{aligned}3a + (-7 - 2a) &= 5 \\ 3a - 2a &= 5 + 7 \\ a &= 12 \quad \text{into } b = -7 - 2a \\ & \quad b = -7 - 2(12) \\ & \quad b = -31\end{aligned}$$

**EXERCISE 2: (Factorise cubic expressions and functions)**

- 1     a      $27x^3 - 8 = (3x - 2)(9x^2 + 6x + 4)$
- b      $5x^3 + 40 = 5(x^3 + 8) = 5(x + 2)(x^2 - 2x + 4)$
- c      $x^3 + 3x^2 + 2x + 6$   
 $= x^2(x + 3) + 2(x + 3)$   
 $= (x + 3)(x^2 + 2)$

$$\begin{aligned} \text{d} \quad & 4x^3 - x^2 - 16x + 4 \\ &= x^2(4x - 1) - 4(4x - 1) \\ &= (4x - 1)(x^2 - 4) \\ &= (4x - 1)(x - 2)(x + 2) \end{aligned}$$

$$\begin{aligned} \text{e} \quad & 4x^3 - 2x^2 + 10x - 5 \\ &= 2x^2(2x - 1) + 5(2x - 1) \\ &= (2x - 1)(2x^2 + 5) \end{aligned}$$

$$\begin{aligned} \text{f} \quad & x^3 + 2x^2 + 2x + 1 \\ &= (x^3 + 1) + (2x^2 + 2x) \\ &= (x + 1)(x^2 - x + 1) + 2x(x + 1) \\ &= (x + 1)(x^2 + x + 1) \end{aligned}$$

$$\begin{aligned} \text{g} \quad & x^3 - x^2 - 22x + 40 \\ &= (x - 2)(x^2 + x - 20) \\ &= (x - 2)(x + 5)(x - 4) \end{aligned}$$

$$\begin{aligned} \text{h} \quad & x^3 + 2x^2 - 5x - 6 \\ &= (x - 2)(x^2 + 4x + 3) \\ &= (x - 2)(x + 3)(x + 1) \end{aligned}$$

$$\begin{aligned} \text{i} \quad & 3x^3 - 7x^2 + 4 \\ &= (x - 1)(3x^2 - 4x - 4) \\ &= (x - 1)(3x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} \text{j} \quad & x^3 - 19x + 30 \\ &= (x - 2)(x^2 + 2x - 15) \\ &= (x - 2)(x + 5)(x - 3) \end{aligned}$$

$$\begin{aligned} \text{k} \quad & x^3 - x^2 - x - 2 \\ &= (x - 2)(x^2 + x + 1) \end{aligned}$$

2      a       $x(x^2 + 2x - 4) = 0$

$$x = 0 \text{ or } x = -1 \pm \sqrt{5}$$

b       $(x - 2)(x^2 - x - 3) = 0$

$$x = 2 \text{ or } x = \frac{1 \pm \sqrt{3}}{2}$$

c       $(2x^3 - 12x^2) - (x - 6) = 0$

$$2x^2(x - 6) - (x - 6) = 0$$

$$(x - 6)(2x^2 - 1) = 0$$

$$x = 6 \text{ or } x = \pm \sqrt{\frac{1}{2}}$$

d       $(2x^3 - x^2) - (8x - 4) = 0$

$$x^2(2x - 1) - 4(2x - 1) = 0$$

$$(x^2 - 4)(2x - 1) = 0$$

$$x = 2 \text{ or } x = -2 \text{ or } x = \frac{1}{2}$$

e       $(x - 1)(x^2 + 2x + 2) = 0$

$$x = 1$$

f       $(x + 2)(x^2 - 2x - 8) = 0$

$$(x + 2)(x - 4)(x + 2) = 0$$

$$x = -2 \text{ or } x = 4$$

g       $(x^3 - 20x) + (3x^2 - 60) = 0$

$$x(x^2 - 20) + 3(x^2 - 20) = 0$$

$$(x^2 - 20)(x + 3) = 0$$

$$x = \pm 2\sqrt{5} \text{ or } x = -3$$

$$\begin{aligned} 3 \quad f(3) &= 3^3 - 3^2 - 5(3) - 3 \\ &= 27 - 9 - 15 - 3 = 0 \end{aligned}$$

$(x - 3)$  is a factor

$$(x - 3)(x^2 + 2x + 1) = 0$$

$$(x - 3)(x + 1)^2 = 0$$

$$x = 3 \text{ or } x = -1$$

$$\begin{aligned} 4 \quad g\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 2 \\ &= \frac{1}{2} - 2 - \frac{1}{2} + 2 = 0 \end{aligned}$$

$$(2x - 1)(2x^2 - 3x - 2) = 0$$

$$(2x - 1)(2x + 1)(x - 2) = 0$$

$$x = \frac{1}{2} \text{ or } x = -\frac{1}{2} \text{ or } x = 2$$