P O L Y N O M I A L S

REMAINDER & FACTOR THEOREMS

THE REMAINER THEOREM

If a polynomial $f(x)$ is divided by a linear polynomial $(ax + b)$, then the remainder is given by $f\left(-\dfrac{b}{a}\right)$.

- If a function $f(x)$ is divided by $x - 3$, the remainder is $f(3)$
- If a function $f(x)$ is divided by $2x - 5$, the remainder is $f\left(\dfrac{5}{2}\right)$
- If a function $f(x)$ is divided by $5x + 1$, the remainder is $f\left(-\dfrac{1}{5}\right)$

Example

If $2x^2 + 5x - 1$ is divided by $x + 4$, determine the remainder.

Solution

Remainder $= f(-4)$

$f(-4) = 2(-4)^2 + 5(-4) - 1 = 11$

$\therefore$ the remainder is 11.

THE FACTOR THEOREM

If $f(x)$ is a polynomial, and $f\left(-\dfrac{b}{a}\right) = 0$, then $ax + b$ is a factor of $f(x)$.

Example

Determine if $x - 1$ is a factor of $f(x) = 3x^4 + 3x^2 - 5x - 1$.

Solution

Remainder $= f(1)$

$f(1) = 3(1)(1)^4 + 3(1)^2 - 5(1) - 1 = 0$

$\therefore x - 1$ is a factor of $f(x)$

The FACTOR THEOREM is mostly used to:

1. **Factorize cubic (third degree) expressions**
2. **Find solutions (x – values) of cubic equations**
3. **Find the x – intercepts of the graph of the cubic function**