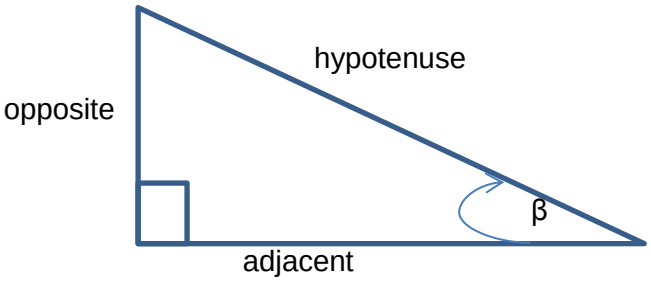
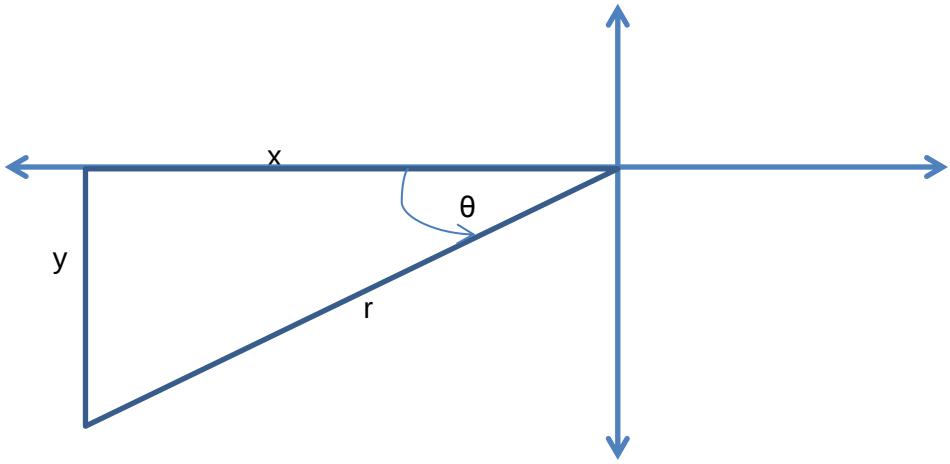
Worksheet 7 Memo- Trigonometry

Grade 10 Mathematics

1. a)



b)



3. a) $\sin\theta = \frac{AC}{AB}$ b) $\sin\alpha = \frac{BC}{AB}$

c) $\cos\theta = \frac{BC}{AB}$ d) $\cos\alpha = \frac{AC}{AB}$

e) $\tan\theta = \frac{AC}{BC}$ f) $\tan\alpha = \frac{BC}{AC}$

4. a) $\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$

c) $\tan 15° = 2 - \sqrt{3}$ d) $\sin 30° = \frac{1}{2}$

e) $\cos 30° = \frac{\sqrt{3}}{2}$ f) $\tan 30° = \frac{\sqrt{3}}{3}$

g) $\sin 45° = \frac{\sqrt{2}}{2}$ h) $\cos 45° = \frac{\sqrt{2}}{2}$

i) $\tan 45° = 1$ j) $\sin 60° = \frac{\sqrt{3}}{2}$

k) $\cos 60° = \frac{1}{2}$ l) $\tan 60° = \sqrt{3}$

m) $\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$ n) $\cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}$

o) $\tan 75° = 2 + \sqrt{3}$ p) $\sin 90° = 1$

q) $\cos 90° = 0$ r) $\tan 90° = undefined$

5. a) sin 50° = 0.77 b) cos 40 ° = 0.77 c) tan 50° = 1.19

d) sin 40° = 0.64 e) cos 50° = 0.64 f) tan 40° = 0.84

g) sin 88° = 0.99≈ 1 h) cos 88° = 0.03 i) tan 88° = 28.64

j) sin 5° = 0.09 k) cos 5° = 0.996 ≈ 1 l) tan 5° = 0.09

m) sin 23° = 0.39 n) cos 23° = 0.92 o) tan 23° = 0.42

6. a) sin α

$= \sin 62°$

$= 0.88$

b) sin β

$= \sin 28°$

$= 0.47$

c) sin(α + β)

$= \sin(62° + 28°)$

$= \sin 90$

$= 1$

d) sin (α – β)

$= \sin(62° - 28°)$

$= \sin 34°$

$= 0.56$

e) sin 2α

$= \sin 2(62°)$

$= \sin 124°$

$= 0.83$

f) $\sin \frac{\alpha}{2}$

$= \sin \frac{62°}{2}$

$= \sin 31°$

$= 0.52$

g) cos α + sin β

$= \cos 62° + \sin 28°$

$= 0.4695 + 0.4695$

$= 0.94$

h) (cos α) × (sin β)

$= (\cos 62°) \times (\sin 28°)$

$= (0.4692) \times (0.4692)$

$= 0.22$

i) $\cos \frac{\beta}{2}$

$= \cos \frac{28°}{2}$

$= \cos 14$

$= 0.97$

j) tan 3(α + β)

$= \tan 3(62° + 28°)$

$= \tan 3(90°)$

$= \tan 270°$

$= undefined$

k) tan(α – β)

$= \tan(62° - 28°)$

$= \tan 34°$

$= 0.67$

l) $\frac{\sin \beta}{\cos \beta}$

$= \frac{\sin 28°}{\cos 28°}$

$= \frac{0.4695}{0.8829}$

$= 0.53$

m) tan β

$= \tan 28°$

$= 0.53$

n) sin β – tan α

$= \sin 28° - \tan 62°$

$= 0.4694 - 1.8807$

$= -1.41$

o) $\sin^2\alpha + \cos^2\alpha$

$= \sin^2 62° + \cos^2 62°$

$= 0.7796 + 0.2204$

$= 1$

7. a) sin θ = 0.574

∴ θ = 35°

b) cos θ = 0.857

∴ θ = 31°

c) tan θ = 0.213

∴ θ = 12°

d) 2 sin θ = 1.2

∴ sin θ = 0.6

∴ θ = 37°

e) 3 cos θ = 1.5

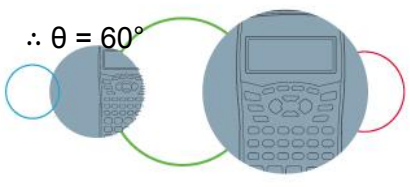
∴ cos θ = 0.5

∴ θ = 60°

f) 2 tan θ = 3.467

∴ tan θ = 1.7335

∴ θ = 60°



g) $2 \sin \theta - 0.4 = 0$

$\therefore 2 \sin \theta = 0.4$

$\therefore \sin \theta = 0.2$

$\therefore \theta = 12°$

h) $2 - 5 \cos \theta = 0$

$\therefore 2 = 5 \cos \theta$

$\therefore 0.4 = \cos \theta$

$\therefore \theta = 66°$

i) $9 \tan \theta - 10 = 4$

$\therefore 9 \tan \theta = 14$

$\therefore \tan \theta = 1\frac{5}{9}$

$\therefore \theta = 57°$

j) $\sin (\theta - 30°) = 0.242$

$\therefore \theta - 30° = 14°$

$\therefore \theta = 44°$

k) $\cos 3(\theta - 10°) = 0$

$\therefore 3(\theta - 10°) = 90°$

$\therefore \theta - 10° = 30°$

$\therefore \theta = 40°$

l) $\tan (2\theta + 15°) = 3$

$\therefore 2\theta + 15 = 72°$

$\therefore 2\theta = 57°$

$\therefore \theta = 28.5$ or $29°$

m) $\sin \frac{\theta}{2} = \frac{\sqrt{2}}{2}$

$\therefore \frac{\theta}{2} = 45°$

$\therefore \theta = 90°$

n) $2 \cos \left(\frac{\theta}{3} - 10°\right) = 0$

$\therefore \cos \left(\frac{\theta}{3} - 10°\right) = 0$

$\therefore \frac{\theta}{3} - 10° = 90°$

$\therefore \frac{\theta}{3} = 100°$

$\therefore \theta = 300°$

o) $\frac{\tan(\theta+20)}{4} = 0.1$

$\therefore \tan (\theta + 20°) = 0.4$

$\therefore \theta + 20° = 22°$

$\therefore \theta = 2°$

8. a) $\sin \delta$

$\therefore \text{cosec } \delta$

b) $\cos \delta$

$\therefore \sec \delta$

c) $\tan \delta$

$\therefore \cot \delta$

9. a) $\cot 34°$

$= \frac{1}{\tan 34°}$

$= 1.48$

b) $\sec 80°$

$= \frac{1}{\cos 80°}$

$= 5.76$

c) $\text{cosec } 53°$

$= \frac{1}{\sin 53°}$

$= 1.25$

d) $\cot 15° \times \frac{1}{\tan 15°}$

$= \frac{1}{\tan 15°} \times \frac{1}{\tan 15°}$

$= (2 + \sqrt{3})(2 + \sqrt{3})$

$= 7 + 4\sqrt{3}$

$= 13.93$

e) $2 \sec 60$

$= 2\left(\frac{1}{\cos 60°}\right)$

$= 4$

f) $\frac{1}{cosec\ 30°}$

$= \frac{1}{\frac{1}{\sin 30°}}$

$= \sin 30°$

$= \frac{1}{2}$

g) $\sec 45° \times \text{cosec } 45°$

$= \frac{1}{\cos 45°} \times \frac{1}{\sin 45°}$

$= \sqrt{2} \times \sqrt{2}$

$= 2$

h) $\cot 15° + \sec 30°$

$= \frac{1}{\tan 15°} + \frac{1}{\cos 30°}$

$= 2 + \sqrt{3} + \frac{2\sqrt{3}}{3}$

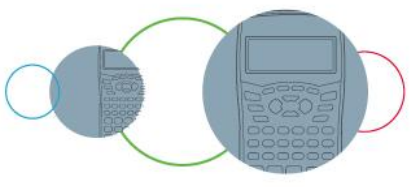
$= \frac{6+5\sqrt{3}}{3} = 4.89$

i) $\text{cosec } 25° - 3$

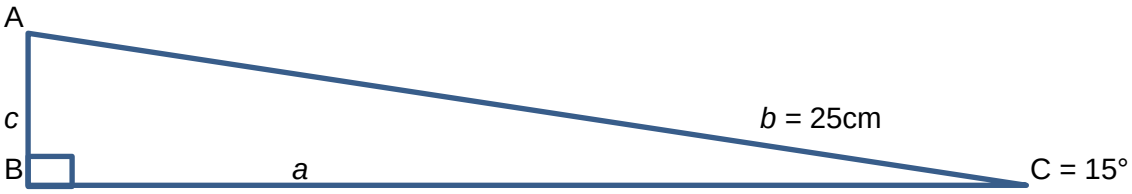
$= \frac{1}{\sin 25°} - 3$

$= 2.37 - 3$

$= -0.63$



10. a)



∴ to find c → $\frac{opp}{hyp} = \sin C$

∴ sin 15°= $\frac{c}{25}$

∴ 25 sin 15° = c

∴ c = 6.47cm

∴ to find a → $\frac{adj}{hyp} = \cos C$

∴ $\cos 15° = \frac{a}{25}$

∴ 25 cos 15° = a

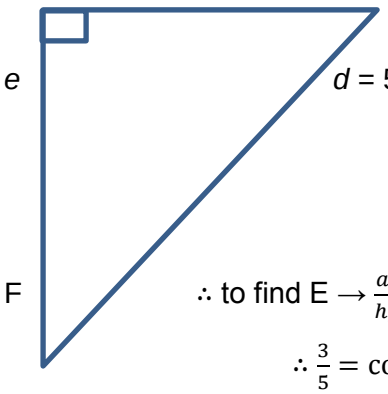
∴ a = 24.15cm

And to find A → A + 15° + 90° = 180°

∴ A = 180° - 105°

∴ A = 75°

b)



∴ to find e → $d^2 = e^2 + f^2$

∴ $(5)^2 = e^2 + (3)^2$

∴ 25 – 9 = e^2

∴ 16 = e^2

∴ 4 cm = e

∴ to find E → $\frac{adj}{hyp} = \cos E$

∴ $\frac{3}{5} = \cos E$

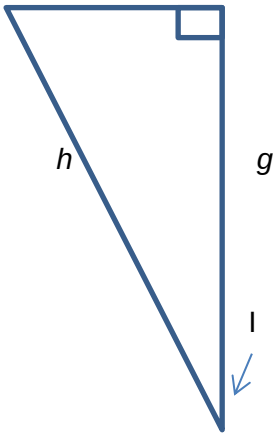
∴ E = 53.13°

∴ to find F → 180° = F + 90° + 53.13°

∴ F = 180° - 143.13°

∴ F = 36.87°

c)



∴ to find i → $\frac{adj}{opp} = \tan G$

∴ $\tan 30 = \frac{i}{16}$

∴ 16 tan 30 = i

∴ $i = 9.24mm$

∴ to find h → $\frac{opp}{hyp} = \sin G$

∴ $\frac{16}{h} = \sin 30°$

∴ 16 = h sin 30°

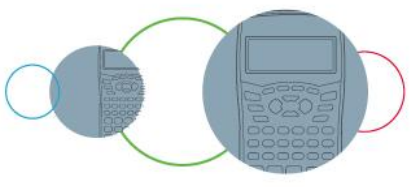
∴ $\frac{16}{\sin 30°} = h$

∴ h = 32mm

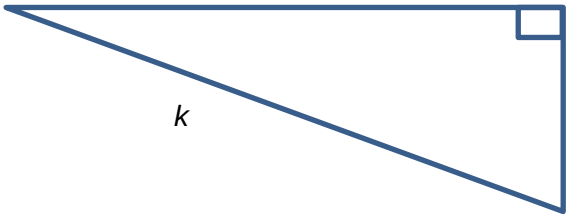
∴ to find I → 180° = I + 90° + 30°

∴ I = 180° - 120°

∴ I = 60°



d) J l = 94m K



L = 41°

∴ to find j → $\frac{opp}{adj} = \tan L$

∴ $\frac{94}{j} = \tan 41°$

∴ $94 = j \tan 41°$

∴ $\frac{94}{\tan 41} = j$

∴ j = 108.13m

∴ to find k → $\frac{opp}{hyp} = \sin L$

∴ $\frac{94}{k} = \sin 41°$

∴ $94 = k \sin 41°$

∴ $\frac{94}{\sin 41°} = k$

∴ $k = 143.28m$

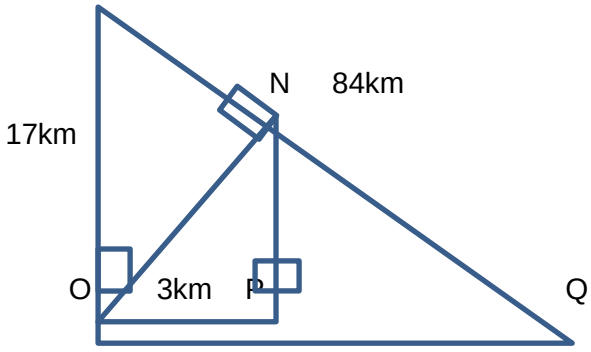
∴ to find J → 180° = J + 90° + 41°

∴ J = 180° - 131°

∴ J = 49°

(or you could use Pythagoras).

e) M = 35°



∴ to find OQ → $MQ^2 = MO^2 + OQ^2$

∴ $(84)^2 = (17)^2 + OQ^2$

∴ 7 056 – 289 = OQ^2

∴ OQ^2 = 6 767

∴ OQ = 82.26km

and ∴ PQ = 82.26 – 3 = 79.26km

∴ to find MN → $\frac{adj}{hyp} = \cos M$

∴ $\frac{MN}{17} = \cos 35°$

∴ MN = 17 cos 35°

∴ MN = 13.93km

∴ to find ON → $\frac{opp}{hyp} = \sin M$

∴ $\frac{ON}{17} = \sin 35°$

∴ ON = 17 sin 35°

∴ ON = 9.75km

∴ to find NP → $ON^2 = NP^2 + OP^2$

∴ $(9.75)^2 = NP^2 + (3)^2$

∴ NP^2 = 95.0625 -9

∴NP^2 = 86.0625

∴ NP = 9.28km

∴ to find NQ → NQ = 84km – MN

∴ NQ = 84km – 13.93km = 70.o7km

∴ to find Q → Q = 180° - (90° + 35°)

∴ Q = 180° - 125° = 55°

∴ to find $Q\hat{N}P$ → 180° = N + 90° + 55°

∴ $Q\hat{N}P = 180° - 145°$

∴ $Q\hat{N}P = 35°$

∴ to find $O\hat{N}P$ → 180° = N + 90° + 35°

∴ $O\hat{N}P = 180° - 125°$

∴ $O\hat{N}P = 55°$

∴ to find $N\hat{O}P$→ 180° = O + 90° + 55°

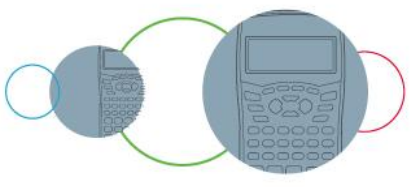
∴ $N\hat{O}P = 180° - 145°$

∴ $N\hat{O}P = 35°$

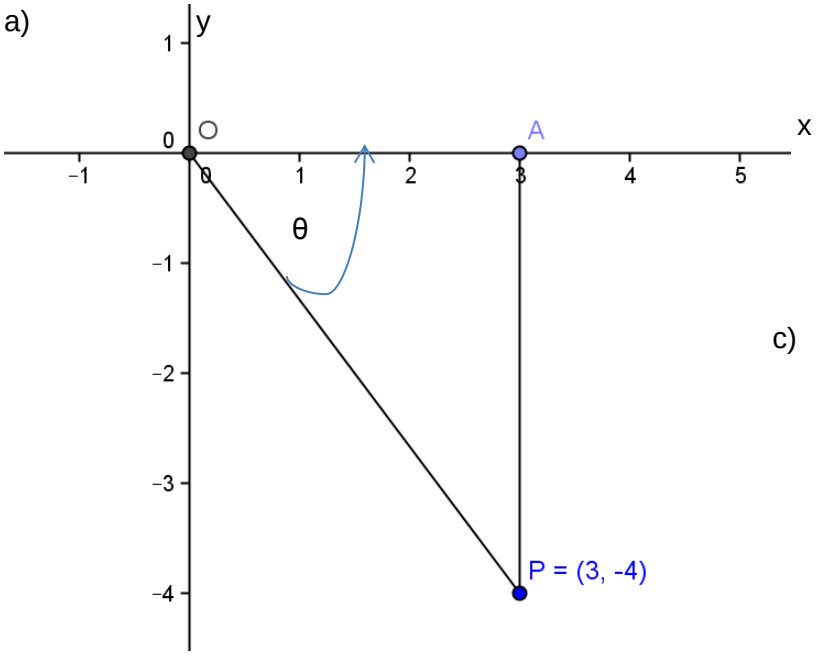
∴ to find $M\hat{O}N$ → 180° = O + 90° + 35°

∴ $M\hat{O}N = 180° - 125°$

∴ $M\hat{O}N = 55°$



11. a)



b) $r^2 = y^2 + x^2$

$\therefore r^2 = (-4)^2 + (3)^2$

$\therefore r^2 = 25$

$\therefore r = 5$

c) i) $\sin\theta = \frac{y}{r} = \frac{-4}{5}$

ii) $\cos\theta = \frac{x}{r} = \frac{3}{5}$

iii) $\tan\theta = \frac{y}{x} = \frac{-4}{3}$

iv) $\sin^2\theta + \cos^2\theta$

$= \left(\frac{-4}{5}\right)^2 + \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} + \frac{9}{25}$

$= 1$

v) $2\sin\theta$

$= 2\left(\frac{-4}{5}\right)$

$= -\frac{8}{5}$

vi) $\sin\theta + \cos\theta$

$= \frac{-4}{5} + \frac{3}{5}$

$= -\frac{1}{5}$

vii) $\tan\theta \times \cos\theta$

$= \frac{-4}{3} \times \frac{3}{5}$

$= \frac{-4}{5}$

viii) $\cos\theta - \sin\theta$

$= \frac{3}{5} - \left(\frac{-4}{5}\right)$

$= \frac{3}{5} + \frac{4}{5}$

$= \frac{7}{5}$

ix) $3\sin\theta + 2\tan\theta$

$= 3\left(-\frac{4}{5}\right) + 2\left(\frac{-4}{3}\right)$

$= -\frac{12}{5} - \frac{8}{3}$

$= \frac{-36-40}{15}$

$= \frac{-76}{15}$

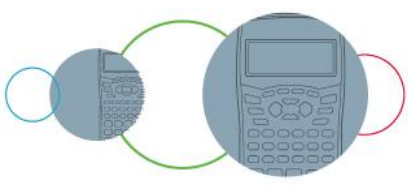
x) $\cos\theta + \sin\theta + \tan\theta$

$= \frac{3}{5} + \left(-\frac{4}{5}\right) + \left(-\frac{4}{3}\right)$

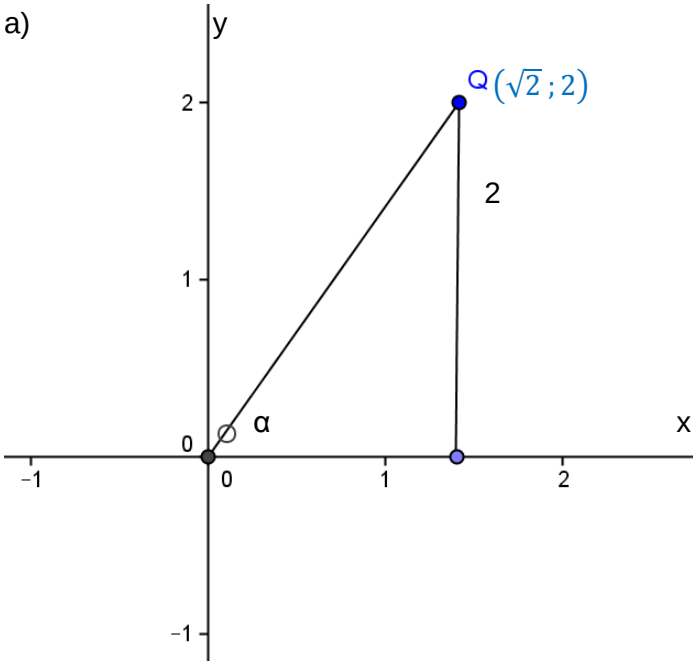
$= \frac{3}{5} - \frac{4}{5} - \frac{4}{3}$

$= \frac{9-12-20}{15}$

$= \frac{-23}{15}$



12. a)



b) $r^2 = y^2 + x^2$

$OQ^2 = (2)^2 + \left(\sqrt{2}\right)^2$

$OQ^2 = 4 + 2$

$OQ^2 = 6$

$OQ = \sqrt{6}$

c) $\tan\alpha = \frac{y}{x}$

$\therefore \tan\alpha = \frac{2}{\sqrt{2}}$

$\therefore \alpha = 54.74°$

d) i) $\sin\alpha = \frac{y}{r} = \frac{2}{\sqrt{6}}$

ii) $cosec\ \alpha$

$= \frac{1}{\sin\alpha}$

$= \frac{r}{y}$

$= \frac{\sqrt{6}}{2}$

iii) $\cos\alpha + \tan\alpha$

$= \frac{x}{r} + \frac{y}{x}$

$= \frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{2}}$

$= \frac{2+2\sqrt{2}}{2\sqrt{3}}$

$= \frac{1+\sqrt{2}}{\sqrt{3}}$

iv) $2\sin\alpha$

$= 2\left(\frac{2}{\sqrt{6}}\right)$

$= \frac{4}{\sqrt{6}}$

v) $\frac{1}{\sin\alpha}$

$= \frac{r}{y}$

$= \frac{\sqrt{6}}{2}$

vi) $\cos^2\alpha - \sin^2\alpha$

$= \left(\frac{\sqrt{2}}{\sqrt{6}}\right)^2 - \left(\frac{2}{\sqrt{6}}\right)^2$

$= \frac{2}{6} - \frac{4}{6}$

$= -2$

vii) $\cos^2\alpha + \sin^2\alpha$

$= \left(\frac{\sqrt{2}}{\sqrt{6}}\right)^2 + \left(\frac{2}{\sqrt{6}}\right)^2$

$= \frac{2}{6} + \frac{4}{6}$

$= 1$

viii) $\frac{\cos\alpha}{\sin\alpha}$

$= \frac{\frac{\sqrt{2}}{\sqrt{6}}}{\frac{2}{\sqrt{6}}}$

$= \frac{\sqrt{2}}{\sqrt{6}} \times \frac{\sqrt{6}}{2}$

$= \frac{\sqrt{2}}{2}$

ix) $\cot\alpha$

$= \frac{1}{\tan\alpha}$

$= \frac{x}{y} = \frac{\sqrt{2}}{2}$

x) $\tan\alpha \times \sin\alpha$

$= \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{6}}$

$= \frac{4}{2\sqrt{3}}$

$= \frac{2}{\sqrt{3}}$

13. The special angles are: 0°, 30°, 45°, 60° and 90°.

14. a) sin 30° + cos 60°

$= \frac{1}{2} + \frac{1}{2}$

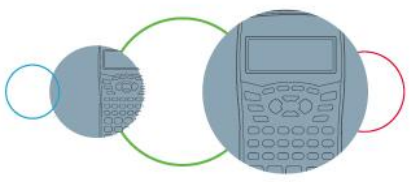
$= 1$

b) sin 45° + cos 45°

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$



c) sin 45° × cos 45°

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{2}{4}$

$= \frac{1}{2}$

d) tan 30° - sin 30°

$= \frac{\sqrt{3}}{3} - \frac{1}{2}$

$= \frac{2\sqrt{3}-3}{6}$

e) tan 30° × sin 60°

$= \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{6} = \frac{1}{2}$

f) cos 90° + sin 90°

$= 0 + 1$

$= 1$

g) cos30°.sin60° + sin30°.cos60°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

h) sin45°.sin45° - cos45°.cos 45°

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{2}{4} - \frac{2}{4}$

$= 0$

i) $\sin^2 30° + \cos^2 30$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

j) $2\cos^2 45° - 1$

$= 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1$

$= 2\left(\frac{2}{4}\right) - 1$

$= \frac{4}{4} - 1$

$= 0$