Cubic functions Revision book Memo

$\boxed{Q1}$

$y = x^3 - 3x^2$

1.1.1) $0 = x^2(x-3)$

$\therefore x = 0$ and $x = 3$

$(0;0)$ $(3;0)$

1.1.2) $y = 0$ $\therefore (0;0)$

1.2) $f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0$ $x = 2$

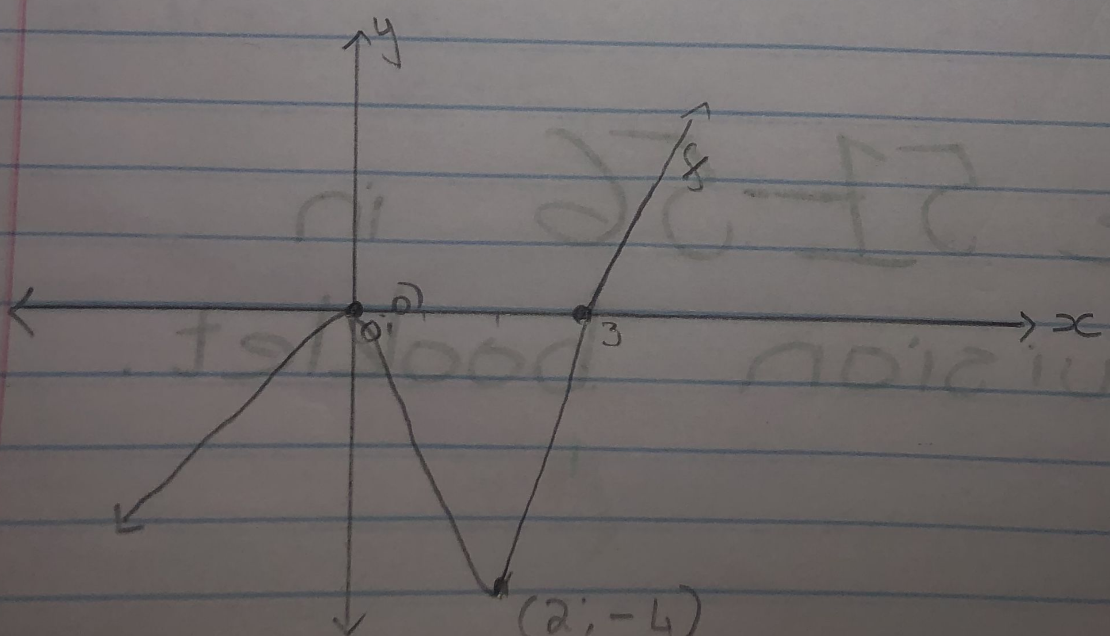
$f(0) = 0$ $f(2) = -4$

$(0;0)$ $(2;-4)$

$\downarrow$ $\downarrow$

local maximum local minimum

1.4)



$\boxed{Q2}$

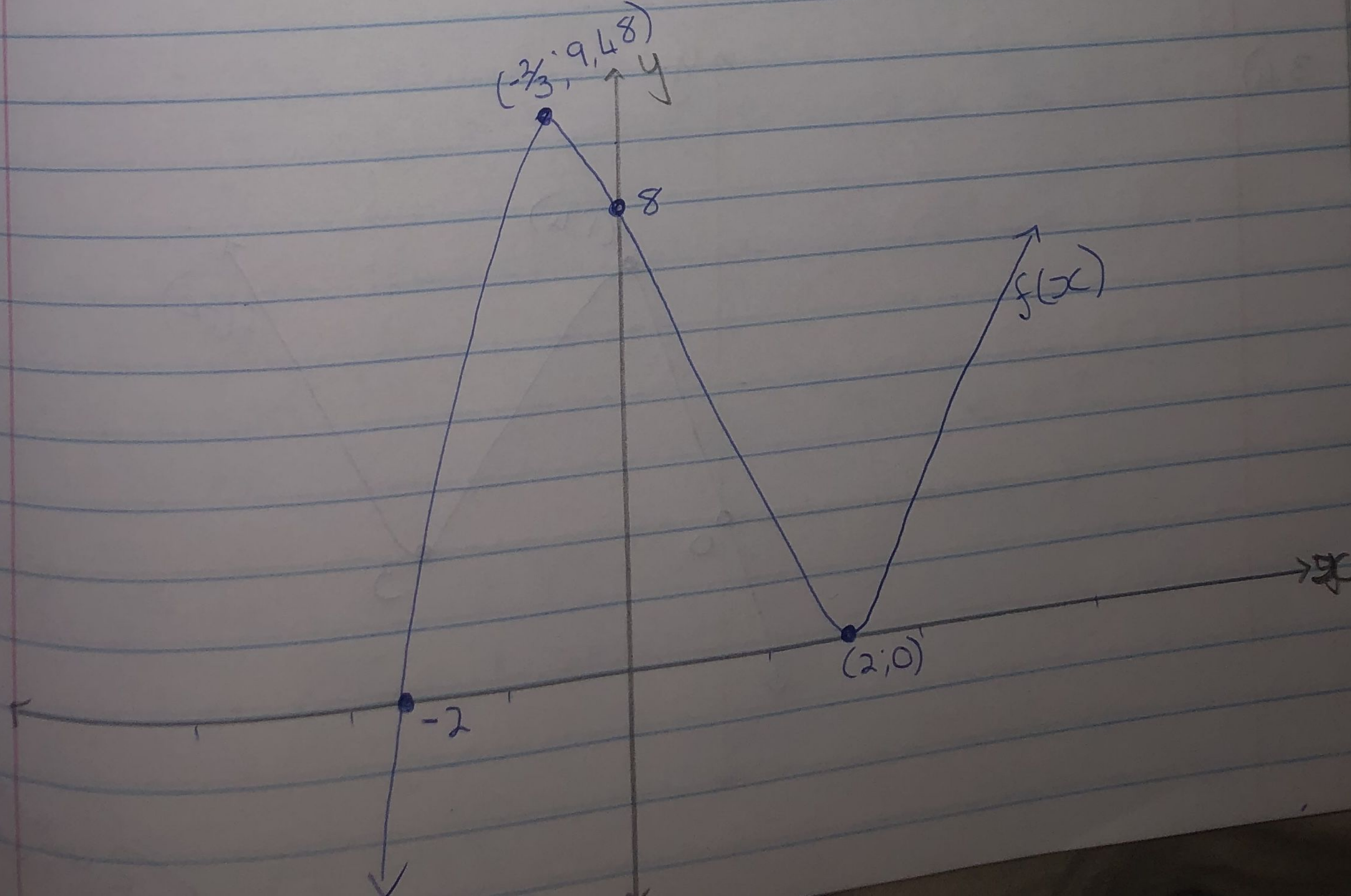
$f(x) = x^3 - 2x^2 - 4x + 8$
$f(x) = (x+2)(x-2)^2$

2.1) $x = -2$ and $x = 2$
$\therefore (-2;0)$ $(2;0)$

$y = 8$
$\therefore (0;8)$

2.2) $f'(x) = 3x^2 - 4x - 4$
$0 = (3x+2)(x-2)$
$x = -\frac{2}{3}$ and $x = 2$
$f(-\frac{2}{3}) = \frac{256}{27}$ / $9{,}48$ $f(2) = 0$
$(2;0)$
local max local min.

2.3)



Q3

$$f(x) = x^3 - 6x^2 + 9x$$
$$f(x) = x(x^2 - 6x + 9)$$

3.1) $f(x) = x(x-3)(x-3)$

3.2) $x = 0$ and $x = 3$

3.3) $f'(x) = 3x^2 - 12x + 9$

$$0 = 3x^2 - 12x + 9$$
$$0 = x^2 - 4x + 3$$
$$0 = (x-3)(x-1)$$

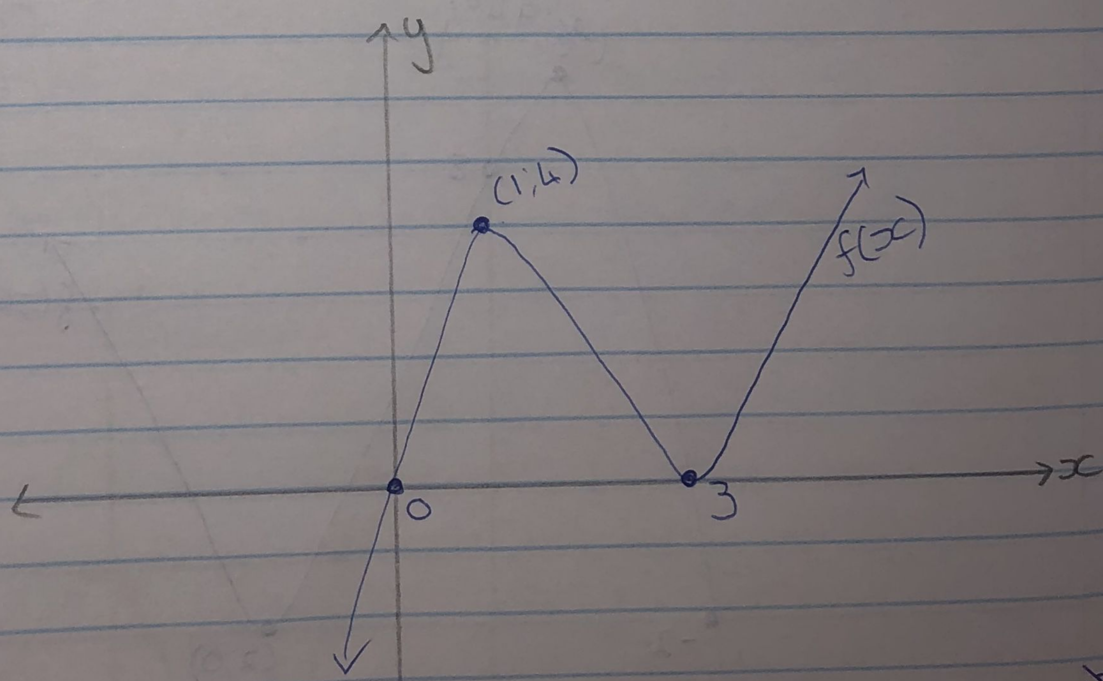
$x = 3$ and $x = 1$

$f(3) = 0$ $\quad$ $f(1) = 4$

$(3;0)$ $\quad$ $(1;4)$

local min. $\quad$ local max

3.4)



$\boxed{3.5}$ will sh
late

Q4

4.1) $f(x) = (x-1)(x^2-5x+4)$
$f(x) = (x-1)(x-4)(x-1)$
$\therefore x = 1$ and $x = 4$
$\therefore A(1;0)$ $B(4;0)$

4.2) $f'(x) = 3x^2 - 12x + 9$
$0 = 3x^2 - 12x + 9$
$0 = x^2 - 4x + 3$
$0 = (x-3)(x-1)$
$x = 3$ and $x = 1$
$f(3) = -4$ $f(1) = 0$

4.3) $C(3;-4)$

4.4) $h(x) = mx + n$
y-intercept $h(x)$ = y-intercept $f(x) = -4$

$\therefore y = mx - 4$, through $(4;0)$
$0 = 4m - 4$
$4 = 4m$
$1 = m$
$\therefore h(x) = x - 4$

4.5) $D(0;-4)$ $C(3;-4)$
$\therefore DC = 3$ units

4.6) $x \in (1;3)$

$\boxed{Q5}$

5.1.1) OM {O is at origin and M is x-intercept of f}

$f(2) = 0$ $\therefore$ $x-2$ a factor

$\therefore$ $\boxed{x = 2}$ $\therefore$ $M(2;0)$

$\therefore$ OM $= 2$ units.

5.1.2) OB {O is origin and B is both x-intercept and local max}

$\therefore$

$$\begin{array}{r|rrrr} 2 & 1 & +2 & -4 & -8 \\ & & 2 & 8 & 8 \\ \hline & 1 & 4 & 4 & \end{array}$$

$0 = (x-2)(x^2+4x+4)$

$0 = (x-2)(x+2)(x+2)$

$\therefore$ $x = 2$ and $\boxed{x = -2}$

$\therefore$ $B(-2;0)$

$\therefore$ OB $= 2$ units

origin

5.1.3) OL $\rightarrow$ L is x-intercept of $g(x)$

$y = 4x + 1$

$0 = 4x + 1$

$-1 = 4x$

$-\frac{1}{4} = x$

$\therefore$ OL $= \frac{1}{4}$ unit

5.1.4) KD $\rightarrow$ {K (y-intercept $g(x)$) and
D (y-intercept of $f(x)$)}

$\left.\begin{array}{l} y = 1 \\ y = -8 \end{array}\right\}$ $\therefore$ KD $= 9$ units

5.2) $f'(x) = 3x^2 + 4x - 4$

$0 = (3x - 2)(x + 2)$

$x = \frac{2}{3}$ and $x = -2$

$f(\frac{2}{3}) = \frac{-256}{27}$ N.A

$\therefore E(\frac{2}{3} ; -\frac{256}{27})$

5.3) $x^3 + 2x^2 - 4x - 8 = 4x + 1$

$x^3 + 2x^2 - 8x - 9 = 0$

$\searrow f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & +2 & -8 & -9 \\ & & -1 & -1 & 9 \\ \hline & 1 & 1 & -9 & \end{array}$$

$(x+1)(x^2 + x - 9) = 0$

$\therefore C(-1; y)$

$\therefore y = 4(-1) + 1$

$y = -3$

$\therefore C(-1; -3)$

5.4) $f'(x) = 3x^2 + 4x - 4$

$f'(2) = 16$

$m = 16$

$\boxed{Q6}$

6.1) $f(1) = -(1)^3 + 5(1)^2 + 8(1) - 12 = 0$

6.2) $f(x) = -(x^3 - 5x^2 - 8x + 12)$
$= -(x-1)(ax^2 + bx + c)$

$$\begin{array}{c|cccc} \therefore\ 1 & 1 & -5 & -8 & +12 \\ & & 1 & -4 & -12 \\ \hline & 1 & -4 & -12 & \\ & a & b & c & \end{array}$$

$\therefore f(x) = -(x-1)(x^2 - 4x - 12)$

6.3) $0 = -(x-1)(x-6)(x+2)$

6.4.1) from (6.3) $\rightarrow x = 1$, $x = 6$, $x = -2$
$\therefore A(-2;0)$
$B(1;0)$
$C(6;0)$

$\therefore OA = 2$ units
$OB = 1$ unit
$OC = 6$ units

6.4.2) $OD = 12$ units
*y-intercept

6.5) $f'(x) = -3x^2 + 10x + 8$

6.6) $0 = -3x^2 + 10x + 8$

$0 = 3x^2 - 10x - 8$

$0 = (3x + 2)(x - 4)$

$x = -2/3$ and $x = 4$

N.A

$f(4) = 36$

$\therefore E(4;36)$

6.7) $f'(0) = 8$

$\therefore m = 8$ at D

$\therefore m = 8$ at F

$f'(x) = -3x^2 + 10x + 8$

$8 = -3x^2 + 10x + 8$

$0 = -3x^2 + 10x$

$0 = x(-3x + 10)$

$x = 0$ or $x = \frac{10}{3}$

$f\left(\frac{10}{3}\right) =$

$\therefore F\left(\frac{10}{3}; \frac{896}{27}\right)$

6.8) *We still need to do integration.

Q7

7.1) $h'(x) = 3x^2 + 4x - 7$

$0 = (3x + 7)(x - 1)$

$x = -\frac{7}{3}$ and $x = 1$

$\therefore$ 2 stationary points since 2 different x-values.

7.2) Root {x-intercept}

$k(x) = x^3 + 3x^2 + 3x \cancel{+8}$ -> make this -7

Then $k(1) = 0$

$$\therefore \begin{array}{r|rrrr} 1 & 1 & +3 & +3 & -7 \\ & & 1 & 4 & 7 \\ \hline & 1 & 4 & 7 & \end{array}$$

$k(x) = (x - 1)(x^2 + 4x + 7)$

$\therefore x = 1$ $\quad$ $x = \text{undef}$ {if subs into quadratic formula}

7.3) $f'(x) = 3x^2 + 6x + 6$

$0 = x^2 + 2x + 2$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$

$x = \text{undef}$.

$\therefore$ no stationary points

$\boxed{Q8}$

8.2) $f(x) = x^3 - 3x + 2$
$f(1) = 0$

$\therefore$

$$\begin{array}{c|cccc} 1 & 1 & +0 & -3 & +2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \end{array}$$

$f(x) = (x-1)(x^2 + x - 2)$
$f(x) = (x-1)(x+2)(x-1)$
$f(x) = (x+2)(x-1)^2$

8.3) $x = -2$ and $x = 1$
$(-2;0)$ $(1;0)$

and $y = 2$ $\therefore (0;2)$

8.4) $f'(x) = 3x^2 - 3$

8.5) $0 = 3(x^2 - 1)$
$0 = (x-1)(x+1)$
$x = 1$ and $x = -1$
$f(1) = 0$ $f(-1) = 4$

$(1;0)$ $(-1;4)$

8.6) local min. local max

8.7)