GRADE 12 MATHEMATICS

SOLUTIONS

ANAYTICAL GEOMETRY MIND MAP

ANALYTICAL GEOMETRY

1. REVISION

   Distance
   Midpoint
   Gradient
   Straight line
   inclination

2. EQUATION OF A CIRCLE

   Circle centred at the origin
   Circle centred off the origin

3. EQUATION OF THE TANGENT TO A CIRCLE

SOLUTIONS

SECTION A

Question 1

1.1.1	$E = \left[\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right]$ $= \left[\frac{3+5}{2}; \frac{5+3}{2}\right]$ $E = (4; 4)$	✓substitution into correct formula ✓ co-ordinates of E.	(2)
1.1.2	$m_{AB} = m_{DC}$ $\frac{y-y_1}{x-x_1} = \frac{3-1}{5+1}$ $\frac{y-5}{x-3} = \frac{2}{6}$ $\therefore y-5 = 2 \ \& \ x-3 = 6$ $\therefore y = 7 \ \& \ x = 9$ B(9; 7) OR $\frac{-1+x}{x} = 4$, $\frac{1+y}{x} = 4$ $x = 9$, $y = 7$ B(9; 7)	✓equating two gradients. ✓ simplification ✓co-ordinates for B ✓$\frac{-1+x}{x} = 4$ ✓ $\frac{1+y}{x} = 4$ ✓co-ordinates for B	(3)
1.1.3	$F = \frac{-1+5}{2}; \frac{1+3}{2}$ $F = [2; 2]$ $y - y_1 = m(x - x_1)$ $y - 2 = 1(x-2)$ $y = x - 2 + 2$ $y = x$	✓ co-ordinates of F ✓ formula ✓ correct value of m ✓correct substitution into formula ✓ Answer	(5)
1.2	$m_{DE} = m_{EG}$ $\frac{2,5-4}{t+1-4} = \frac{3}{5}$ $\frac{-1,5}{t-3} = \frac{3}{5}$ $3t - 9 = -7,5$ $3t = 1,5$ $t = 0,5$	✓ equating gradients ✓ correct substitution ✓ simplification ✓ answer	(4)

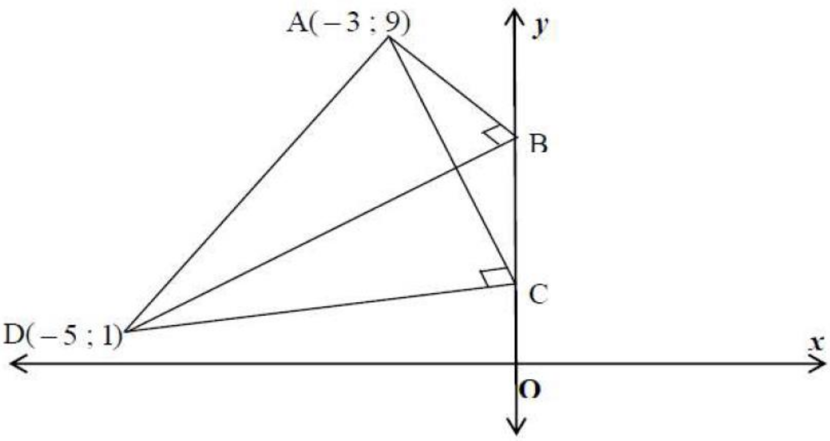
1.3	$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(9-3)^2+(7-5)^2}$ $= \sqrt{40}$ $= \sqrt{(-1-3)^2+(1-5)^2}$ $= \sqrt{32}$ $\therefore$ ABCD is NOT a rhombus because $AB \neq AD$ OR $m_{AC} = \frac{y_2-y_1}{x_2-x_1}$ $= \frac{5-3}{3-5}$ $= -\frac{2}{3}$ $m_{DB} = \frac{3}{5}$ $m_{AC} \times m_{DB} = -\frac{2}{3} \times \frac{3}{5}$ $= -\frac{2}{5}$ $\neq -1$ $\therefore$ ABCD is not a rhombus because: $m_{AC} \times m_{DB} \neq -1$	✓ substitution in formula ✓ answer ✓ answer ✓ statement ✓ reason ✓ substitution in formula ✓ answer ✓ answer ✓ statement ✓ reason	(5)
			[19]

Question 2

2.1	$m_{PR} = \frac{2+4}{9+4} = \frac{6}{13}$	✓✓vervang met die formule antwoord✓(3)
2.2	$\tan\theta = \frac{6}{13}$ $\theta = 24.78°$ $\alpha = 90° - 24.78°$ $= 65.22°$	✓$\tan\theta = \frac{6}{13}$ ✓$\theta = 24.78°$ ✓$\alpha = 90° - 24.78°$ ✓antwoord (4)
2.3	$(a-9)^2 + (10-2)^2 = (4\sqrt{5})^2$ $a^2 - 18a + 81 + 64 = 80$ $a^2 - 18a + 65 = 0$ $(a-13)(a-5) = 0$ $a = 13$ N/A or $\therefore a = 5$	✓vervang met die formule ✓$4\sqrt{5}$ ✓$a^2 - 18a + 65 = 0$ ✓$(a-13)(a-5)$ ✓$a = 13$ N/A ✓antwoord (6)

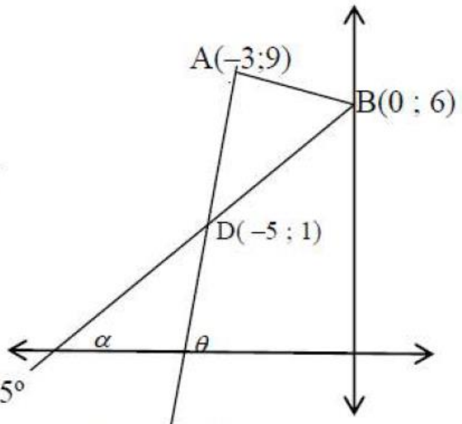
2.4	$m_{PR} = \frac{6}{13}$ and Q (5 ; 10) $y - y_1 = m(x - x_1)$ $y - 10 = \frac{6}{13}(x - 5)$ $y = \frac{6}{13}x + \frac{100}{13}$	✓$m_{PR} = \frac{6}{13}$ ✓Vervang met die korrekte formule ✓antwoord (3)

Question 3



3.1	$M\left(\frac{-3-5}{2};\frac{9+1}{2}\right)$ M(– 4 ; 5)	✓$x = -4$ ✓ $y = 5$ (2)
3.2	$AM^2 = (-4+3)^2 + (5-9)^2$ **OR/OF** $DM^2 = (-5+4)^2 + (1-5)^2$ $= 1 + 16$ $= 1 + 16$ $= 17$ $= 17$ $\therefore r = \sqrt{17}$ **OR/OF** $AD^2 = (-5+3)^2 + (1-9)^2$ $= 4 + 64$ $= 68$ $\therefore AD = \sqrt{68}$ $\therefore \text{radius} = \frac{\sqrt{68}}{2}$ $= \sqrt{17}$	✓correct substitution into distance formula *korrekte vervanging in afstand formule* ✓ $r = \sqrt{17}$ ✓correct substitution into distance formula *korrekte vervanging in afstand formule* ✓ $r = \sqrt{17}$ (2)
3.3	Yes, the circle will pass through point C *Ja, die sirkel gaan deur punt C* $\hat{B} = \hat{C} = 90°$, AD is the diameter line subtends equal ∠s. $\hat{B} = \hat{C} = 90°$, AD die middellyn *lynstuk onderspan gelyke ∠e*	✓yes / *ja* ✓reason / *rede* (2)

3.4	B(0 ; y) $m_{AB} \times m_{BD} = -1$ $\left(\frac{9-y}{-3-0}\right)\left(\frac{1-y}{-5-0}\right) = -1$ $(9-y)(1-y) = -15$ $9 - 10y + y^2 = -15$ $y^2 - 10y + 24 = 0$ $(y-6)(y-4) = 0$ $\therefore y = 6$ or $y = 4$ $\quad \therefore B(0;6)$ **OR/OF** $AB^2 + BD^2 = AD^2$ $(9-y)^2 + (-3-0)^2 + (-5-0)^2 + (1-y)^2 = (-5+3)^2 + (1-9)^2$ $81 - 18y + y^2 + 9 + 25 + 1 - 2y + y^2 = 4 + 64$ $2y^2 - 20y + 48 = 0$ $y^2 - 10y + 24 = 0$ $(y-6)(y-4) = 0$ $y = 6$ or $y = 4$ $\therefore B(0;6)$	✓ $m_{AB} \times m_{BD} = -1$ ✓ $\left(\frac{9-y}{-3-0}\right)$ / $\left(\frac{1-y}{-5-0}\right)$ ✓standard form *standaardvorm* ✓ factors / faktore ✓ B(0 ; 6) (5) ✓ $(9-y)^2 + (-3-0)^2 +$ $(-5-0)^2 + (1-y)^2$ ✓ $(-5+3)^2 + (1-9)^2$ ✓standard form/ *Standaardvorm* ✓factors / faktore ✓ B(0 ; 6) (5)
3.5	$m_{AB} = \frac{9-6}{-3-0}$ $= -1$ $m_{\parallel} = -1$ $y - y_1 = m(x - x_1)$ **OR/OF** $y = -x + c$ $y - 1 = -1(x+5)$ $\quad -5 = -1 + c$ $y - 1 = -x - 5$ $\quad c = -4$ $y = -x - 4$ $\quad y = -x - 4$	✓ $m_{\parallel} = -1$ ✓substitution of (–1 ; 5) *vervanging van* (–1 ; 5) ✓ $y = -x - 4$ (3)

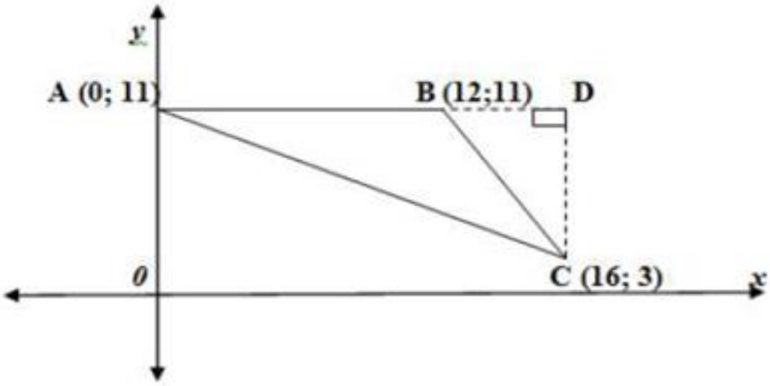
3.6	$m_{AD}=\dfrac{1-9}{-5+3}=4$ $\tan\theta=4$ $\theta=76^\circ$ $m_{DB}=\dfrac{1-6}{-5-0}$ $=1$ $\tan\alpha=1$ $=45^\circ$ $B\hat{D}A=76^\circ-45^\circ$ $=31^\circ$ ext $\angle$ of Δ  **OR/OF** $AB^2=(-3-0)^2+(9-6)^2$ $AB=\sqrt{18}$ $BD^2=(-5-0)^2+(1-6)^2$ $BD=\sqrt{50}$ $\tan B\hat{D}A=\dfrac{\sqrt{18}}{\sqrt{50}}=0{,}6$ $B\hat{D}A=31^\circ$	✓ $m_{AD}=\dfrac{1-9}{-5+3}=4$ ✓ $\tan\theta=4$ ✓76° ✓ $\tan\alpha=1$ ✓45° ✓ 31° (–1 if NOT rounded) 31° (–1 as NIE afgerond) ✓ $(-3-0)^2+(9-6)^2$ ✓ $AB=\sqrt{18}$ ✓ $(-5-0)^2+(1-6)^2$ ✓ $BD=\sqrt{50}$ ✓using a correct trig ratio *gebruik korrekte trig verh* ✓ 31° (–1 if NOT rounded) 31° (–1 as NIE afgerond) (6) **[20]**

Question 4

4.1	$\dfrac{x_D-1}{2}=2$ $\dfrac{y_D+0}{2}=2$ $x_D=5$ $x_D=4$ D(5 ; 4)	✓ $x_D=5$ ✓ $y_D=4$ (2)
4.2	$m_{CD}=\dfrac{4-(-2)}{5-2}$ $=2$ $\tan\ \alpha\ =\ 2$ $\therefore\alpha=63{,}4^\circ$	✓ substitution into gradient formula ✓ $\tan\alpha=2$ ✓answer (3)

4.3	$m_{AB} = m_{CD} = 2$ AB‖CD, equal gradients $y = 2x + c$ $0 = 2(-1) + c$ $c = 2$ $y = 2x + 2$	✓ $m_{AB} = 2$ ✓ subst $(-1\ ;\ 0)$ ✓ answer (3)
4.4	$m_{AD} = \frac{4-(0)}{5-(-1)}$ $= \frac{2}{3}$ tan (∠ of inclination of AD) $= \frac{2}{3}$ ∠ of inclination of AD = 33,7° $\therefore \theta = 63{,}4^\circ - 33{,}7^\circ$ $= 29{,}7^\circ$	✓ $m = \frac{2}{3}$ ✓33,7° ✓29,7° (3)
4.5	$3AB = DC$ $\therefore 9AB^2 = DC^2$ $9\left[(x+1)^2 + (y-0)^2\right] = (5-2)^2 + (4+2)^2$ $9\left[(x+1)^2 + y^2\right] = 45$ $AB // DC, \therefore \frac{y-0}{x+1} = 2$ $(x+1)^2 + y^2 = 5$ ……………(1) $y = 2x + 2$ ……….. (2) Substitute (2) in (1) $(x+1)^2 + (2x+2)^2 = 5$ $x^2 + 2x + 1 + 4x^2 + 8x + 4 - 5 = 0$ $5x^2 + 10x = 0$ $5x(x+2) = 0$ $x \neq 0\ ;\ x = -2$ Substitute $x = -2$ in (1) y = –2 <u>B(–2: –2)</u>	✓ substitution ✓ substitute $y = 2x + 2$ ✓ standard form ✓ $x = -2$ ✓ $y = -2$ (5) **[16]**

Question 5



5.1	$y = 11$ AB = 12	✓ ✓ y = 11 ✓ AB = 12 (3)
52	D(16;11)	✓ ✓ (2)
5.3	M(8; 7)	✓ ✓ (2)
5.4	$m_{AC} = \frac{3-11}{16} = -\frac{8}{16} = -\frac{1}{2}$ $m_{line} = 2$ $y - 7 = 2(x - 8)$ $y = 2x - 9$	✓ $-\frac{1}{2}$ ✓ $m_{line} = 2$ ✓ substitution/*vervanging* ✓ equation /*vergelyking* (4)
5.5	$y = 2(12) - 9$ $= 15$ $\neq 11$ No, it does not pass through B/ /*Nee, dit gaan nie deur B*	✓ substitute/*vervang* ✓ $\neq 11$ No, it does not pass through B/ *Nee, dit gaan nie deur B* (2)
5.6	$\tan\theta = m_{BC} = \frac{11-3}{12-16}$ $\tan\theta = -2$ $\theta = 116{,}57°$	✓ $\tan\theta$ ✓ -2 ✓ $116{,}57°$ (3)

5.7	$m_{\text{new line}} = -\frac{1}{2}$ $y-11 = -\frac{1}{2}(x-16)$ $y = -\frac{1}{2}x + 19$	✓ $-\frac{8}{13}$ ✓ substitute/*vervang* ✓ equation /*vergelyking* (3)
		[19]

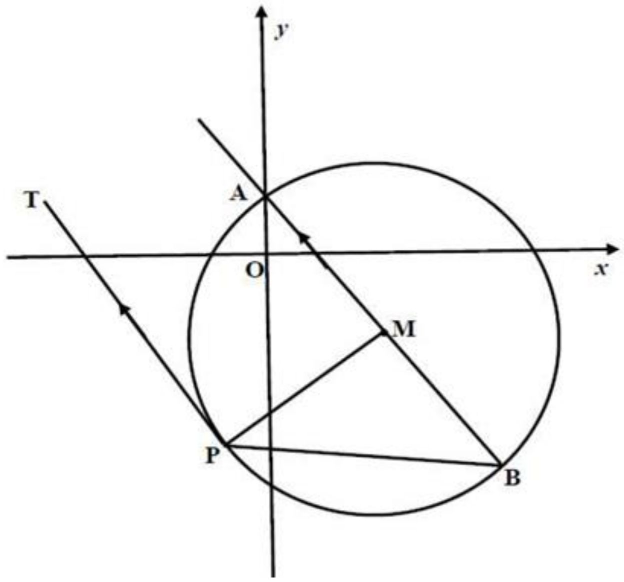
SECTION B

Question 1

1.1	$3x-4y+8=0$ $3(0)-4y+8=0$ $y=2$ B(0;2)	✓ $x=0$ ✓ answer (2)
1.2	$BC^2 = (0-3)^2+(2+2)^2$ $= 25$ $(x-3)^2+(y+2)^2 = 25$ $x^2-6x+9+y^2+4y+4=25$ $x^2-6x+y^2+4y-12=0$	✓ substitution ✓ answer ✓ equation ✓ expansion (4)
1.3	$x=8$ *or* $x=-2$ $t=-8$ *or* $t=2$	✓✓equations ✓✓t-values Answer only full marks (4)
1.4.1	$BD = BC = 5$	✓ answer (1)
1.4.2		

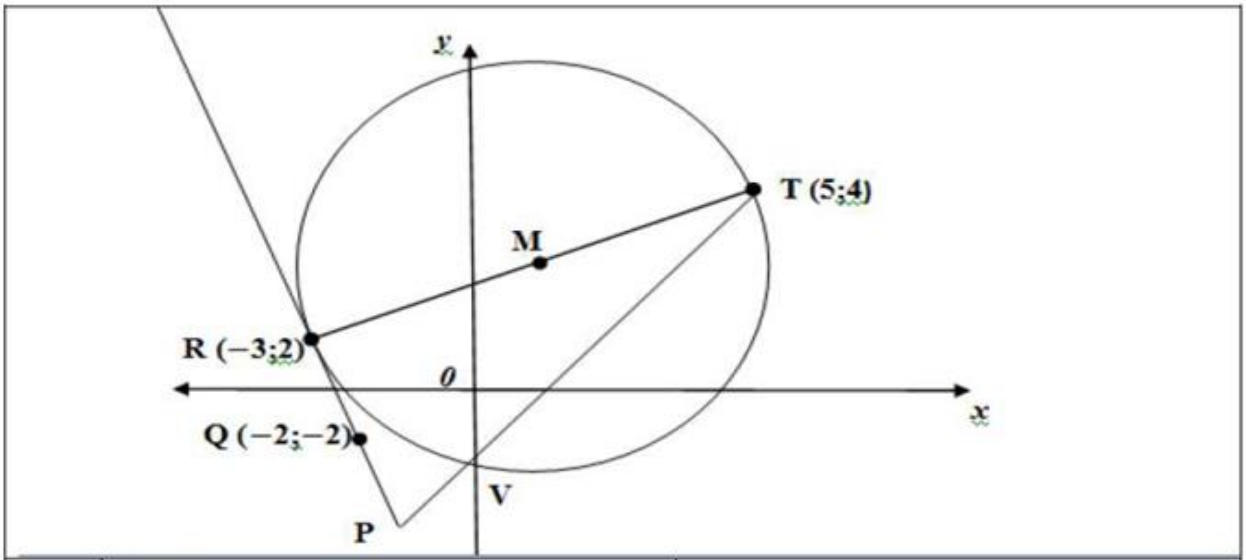
	B(0;2) and D(x;y) BD: $3x-4y+8=0$ $25=(x-0)^2+(y-2)^2$ $25=x^2+y^2-4y+4$ $x=\frac{4y-8}{3}$ $25=\left(\frac{4y-8}{3}\right)^2+y^2-4y+4$ $25=\frac{16y^2-64y+64}{9}+y^2-4y+4$ $225=16y^2-64y+64+9y^2-36y+36$ $0=25y^2-100y-125$ $0=y^2-4y-5$ $0=(y-5)(y+1)$ $y=5$ *or* $y=-1$ $\therefore y=5$ D(4;5) **OR** $25=x^2+y^2-4y+4$ $y=\frac{3x+8}{4}$ $25=x^2+\left(\frac{3x+8}{4}\right)^2-4\left(\frac{3x+8}{4}\right)+4$ $125=\frac{9y^2+48x+64}{16}-3x-8$ $0=25x^2-400$ $0=x^2-16$ $0=(x-4)(x+4)$ $x=4$ *or* $x=-4$ $\therefore x=4$ D(4;5)	✓ distance formula ✓ substitution ✓ expansion ✓ standard form ✓ y-value ✓ x- value (6) ✓ distance formula ✓ substitution ✓ expansion ✓ standard form ✓ x-value ✓ y- value (6)
1.5	$E(2;-9)$ $C(3;-2)$ $D(4;5)$ $m_{EC}=\frac{-2+9}{3-2}=7$ $m_{CD}=\frac{5+2}{4-3}=7$ $m_{ED}=\frac{-9-5}{2-4}=7$ $\therefore E, C$ *and* D are collier	✓ gradient of EC ✓ gradient of CD ✓ conclusion (3) Any two gradients
		[20]

Question 2



2.1	$x^2 - 2x + y^2 + 4y - 5 = 0$ $x^2 - 2x + (1)^2 + y^2 + 4y + (2)^2 = 5 + 1 + 4$ $(x-1)^2 + (y+2)^2 = 10$ $M(1;-2)$	✓$(x-1)^2 + (y+2)^2$ ✓10 ✓✓answer (4)
2.2	$T\hat{P}M = 90°$ (radius $\perp$ tangent) $P\hat{M}B = T\hat{P}M$ (alternate $\angle s$) $= 90°$	✓ $T\hat{P}M = 90°$ ✓ radius $\perp$ tangent ✓ answer (3)
2.3	PM: $3y - x + 7 = 0$ $y = \frac{1}{3}x - \frac{7}{3}$ $m_{AB} = -3$ Equation of AB: $y - y_1 = m(x - x_1)$ $y + 2 = -3(x-1)$ $y = -3x + 1$	✓ $m_{PM} = \frac{1}{3}$ ✓ $m_{AB} = -3$ ✓ sub. $(1; -2)$ ✓answer (4)
2.4	A (0; 1)	✓x- coordinate ✓y-coordinate (2)
2.5	$TM = \sqrt{80}$; $PM = \sqrt{10}$ $PT = \sqrt{TM^2 - PM^2}$ (Pythagoras thm) $= \sqrt{80-10}$ $= \sqrt{70}$ or 8,37	✓ $\sqrt{10}$ ✓ $\sqrt{80-10}$ ✓ $\sqrt{70}$ or 8,37 (3) **[16]**

Question 3



3.1	M(1; 3) $r^2 = (5-1)^2 + (4-3)^2$ $r^2 = 16 + 1 = 17$ $(x - 1)^2 + (y - 3)^2 = 17$	✓ ✓M ✓ substitute/***vervanging*** ✓ $r^2 = 17$ ✓ $(x - 1)^2 + (y - 3)^2 = 17$ (5)
3.2	$m_{PR} = \frac{-2-2}{-2+3} = -4$ $m_{RT} = \frac{4-2}{5+3} = \frac{1}{4}$ $m_{PR} \times m_{RT} = -1$ PR is a tangent/*PR is 'n raaklyn*	✓ m_{PR} ✓ m_{RT} ✓ product/***produk*** $= -1$ (3)
3.3	Y int: $(0-1)^2 + (y-3)^2 = 17$ $1 + y^2 - 6y + 9 = 17$ $y^2 - 6y - 7 = 0$ $(y-7)(y+1) = 0$ $y = -1$ or/*of* $y = 7$ $V(0; -1)$	✓ let $x = 0$ ✓ standard/***standaard vorm*** ✓ $y = -1$ or/*of* $y = 7$ ✓ $V(0; -1)$ (4)
3.4	$m_{PT} = \frac{4+1}{5-0} = 1$ $\tan\alpha = 1$ $\alpha = 45°$ $\tan\beta = -4$ $\beta = 104°$	✓ m_{PT} ✓ $\tan\alpha = 1$ ✓ $\alpha = 45°$ ✓ $\tan\beta = -4$
	$\theta = 59°$	✓ $\beta = 104°$ ✓ $\theta = 59°$ (6)
		[18]

Question 4

4.1	$r^2 = (2-4)^2 + (3-5)^2$ $= 8$ $(x-2)^2 + (y-3)^2 = 8$		✓ subst into distance formula ✓ 8 ✓ $(x-2)^2$ ✓ $(y-3)^2$ (4)
4.2	$m_{NP} = \frac{5-3}{4-2} = 1$ $m_{PT} = -1$ $y = -x + c$ $5 = -4 + c$ $c = 9$ $y = -x + 9$ $0 = -x + 9$ $x = 9$ $\therefore$ T(9 ; 0)	NP $\perp$ PT, product of gradients = – 1	✓ $m_{NP} = 1$ ✓ $m_{PT} = -1$ ✓ subst (5; 4) ✓ $c = 9$ ✓ $y = -x + 9$ ✓coordinates of T (6)
4.3	PT $= \sqrt{(9-4)^2 + (5-0)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$		✓ substitution into distance formula ✓ $\sqrt{50}$ or $5\sqrt{2}$ (2)
4.4	Area $= \pi \times \text{PT}^2$ $= \pi \times 50$ $= 157$		✓ substitution into area formula ✓ 157 (2)
4.5	$\tan N\hat{P}T = \frac{\sqrt{8}}{\sqrt{50}}$ $N\hat{P}T = 21{,}8°$		✓ $\tan N\hat{P}T = \frac{\sqrt{8}}{\sqrt{50}}$ ✓21,8° (2)
4.6	NP = NM PT = TM $\therefore$ MNPT is a kite	radii radii two pairs of adjacent sides equal in length	✓ S/R ✓ S/R ✓ reason (3)
			[23]