Revision Exercise p.54

3) 14; 14; 12; 8; 2; ...; −96

0 −2 −4 −6 −8 −10

−2 −2 −2 −2 −2

3.1) −6; −16

3.2) $2a = -2$
$a = -1$

$3a + b = 0$
$3(-1) + b = 0$
$b = 3$

$a + b + c = 14$
$-1 + 3 + c = 14$
$c = 14 - 2$
$c = 12$

$\therefore T_n = -n^2 + 3n + 12$

3.3) $T_{51} = -(51)^2 + 3(51) + 12$
$= -2436$

3.4) $-96 = -n^2 + 3n + 12$
$0 = -n^2 + 3n + 12 + 96$
$0 = -n^2 + 3n + 108$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(-1)(108)}}{2(-1)}$$

$n = -9$ or $\underline{n = 12}$
N/A

4) $-9; -6; 1; x; 27; \ldots$

$3 \quad 7 \quad x-1 \quad 27-x$

$4 \quad 4 \quad 4$

4.1) 4

4.2) $27-x-(x-1)$

$=27-x-x+1$

$=28-2x$

4.3) $4=28-2x$

$2x=28-4$

$2x=24$

$x=12$

4.4) $2a=4$

$a=2$

$3a+b=3$

$3(2)+b=3$

$b=3-6$

$=-3$

$a+b+c=-9$

$2-3+c=-9$

$c=-9+1$

$=-8$

$\therefore T_n=2n^2-3n-8$

$T_9=2(9)^2-3(9)-8$

$=127$

4.5) $397=2n^2-3n-8$

$0=2n^2-3n-405$

$$n=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-405)}}{2(2)}$$

$n=15$ or $n=\frac{-27}{2}$ N/A $\therefore T_{15}=397$

Exercise 1

1.1) 3; 7; 11; 15; ...

a) $T_n = an + b$

$a = 4$

$b = 3 - 4 = -1$

$\therefore T_n = 4n - 1$

b) $T_{30} = 4(30) - 1$

$= 119$

c) $1011 = 4n - 1$

$1012 = 4n$

$\therefore n = 253$

2 2 2 2

2 4 6 8 10

1.2) 0; 2; 6; 12; ...

a) 20, 30

b) $T_n = an^2 + bn + c$

$2a = 2 \therefore a = 1$

$3a + b = 2$

$b = 2 - 3(1) = -1$

$a + b + c = 0$

$1 - 1 + c = 0 \quad \therefore c = 0$

$\therefore T_n = n^2 - n$

c) $1980 = n^2 - n$

$0 = n^2 - n - 1980$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$- \frac{(-1) \pm \sqrt{(-1)^2 - 4(1)(-1980)}}{2(1)}$

$n = 45$ or $n \neq -44$ N/A

$\therefore$ The 45th term

$-2 \quad -2 \quad -2$

1.3) $7; 5; 3; 1; \ldots$

a) $T_n = an + b$

$a = -2$

$b = 7 - (-2)$

$= 9$

$\therefore T_n = -2n + 9$

b) $T_{100} = -2(100) + 9$

$= -191$

c) $-3999 = -2n + 9$

$-4008 = -2n$

$n = 2004$

$4 \quad 4$

$9 \quad 13 \quad 17 \quad 21 \quad 25$

1.4) $6; 15; 28; 45; \ldots$

a) $66; 91$

b) $T_n = an^2 + bn + c$

$2a = 4 \quad \therefore a = 2$

$3a + b = 9$

$3(2) + b = 9$

$b = 9 - 6 = 3$

$a + b + c = 6$

$2 + 3 + c = 6$

$c = 6 - 5 = 1$

$\therefore T_n = 2n^2 + 3n + 1$

c) $561 = 2n^2 + 3n + 1$

$0 = 2n^2 + 3n - 560$

$$n = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-560)}}{2(2)}$$

$n = -17,5$ or $n = 16$

N/A

$\therefore$ the 16th term

$+3 \quad +3$

1.5) $-8; -5; -2; \ldots$

a) $T_n = an + b$

$a = 3$

$b = -8 - 3$

$= -11$

$\therefore T_n = 3n - 11$

b) $T_{40} = 3(40) - 11$

$= 109$

c) $2680 = 3n - 11$

$2691 = 3n$

$n = 897$

$-6 \quad -6$

$-9 \quad -15 \quad -21 \quad -27 \quad -33$

* 1.6) $-2; -11; -26; -47; \ldots$

a) $-74; -107$

b) $T_n = an^2 + bn + c$

$2a = -6 \quad \therefore a = -3$

$3a + b = -9$

$3(-3) + b = -9 \quad \therefore b = 0$

$a + b + c = -2$

$-3 + 0 + c = -2$

$\therefore c = 1$

$\therefore T_n = -3n^2 + 1$

c) $-121\,202 = -3n^2 + 1$

$0 = -3n^2 + 121\,203$

$0 = -3(n^2 - 40401)$

$0 = n^2 - 40\,401$

$\sqrt{40401} = \sqrt{n^2}$

$\therefore n = 201$

Exercise 2

2.1) 2; **18**; 7; **12**; 12; **6**; 17; ...

a) 0; 22

b) $T_n = an + b$

$a = 5$

$b = 2 - 5 = -3$

$\therefore T_n = 5n - 3$

c) 10^{th} term

d) $2 + 18 + 7 + 12$

$= 39$

$+2 \quad +2$

$+7 \quad +9 \quad +11$

2.2) $-7; 0; 9; 20; \ldots$

a) $T_n = an^2 + bn + c$

$2a = 2 \quad \therefore a = 1$

$3a + b = 7$

$3(1) + b = 7 \quad \therefore b = 4$

$a + b + c = -7$

$1 + 4 + c = -7$

$c = -12$

$\therefore T_n = n^2 + 4n - 12$

b) $128 = n^2 + 4n - 12$

$0 = n^2 + 4n - 140$

$$n = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-140)}}{2(1)}$$

$n = -14$ N/A or $n = 10$

∴ the 10^{th} term

c) $T_n = an + b$

$a = 2$

$b = 7 - 2 = 5$

$\therefore T_n = 2n + 5$

d) $599 = 2n + 5$

$594 = 2n$

$n = 297$

∴ between T_{297} & T_{298}

	1	2	3	4
2.3) Total Squares:	9 3^2	25 5^2	49 7^2	81 9^2
Grey:	5	13	25	41
White:	4	12	24	40

a) ∴ 40 White squares in 4th Pattern

b) ∴ 4 ; 12 ; 24 ; 40; ...

8 12 16

4 4

$\therefore T(157) = 2(157)^2 + 2(157$

$= 49\,612$

$T_n = an^2 + bn + c$

$2a = 4 \quad \therefore a = 2$

$3a + b = 8$

$3(2) + b = 8$

$\therefore b = 2$

$a + b + c = 4$

$2 + 2 + c = 4$

$\therefore c = 0$

$\therefore T_n = 2n^2 + 2n$

c) 5 ; 13 ; 25 ; 41

8 12 16

4 4

$a = 2$

$b = 2 \qquad \therefore T_n = 2n^2 + 2n + 1$

$c = 1$

$613 > 2n^2 + 2n + 1$

$0 > 2n^2 + 2n - 612$

$$n = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-612)}}{2(2)}$$

$n = 17$ or $n = -18$
N/A

$\therefore n = 16$

d) Total Squares: $9 ; 25 ; 49 ; 81 ; \ldots$

$16 \quad 24 \quad 32$

$8 \quad 8$

$\therefore T_n = an^2 + bn + c$

$2a = 8 \quad \therefore a = 4$

$3a + b = 16$

$3(4) + b = 16$

$b = 4$

$a + b + c = 9$

$4 + 4 + c = 9$

$c = 1$

$\therefore T_n = 4n^2 + 4n + 1$

$4n^2$ will always be even

$4n$ will always be even

even + 1 = odd

$\therefore$ Total number of squares will always be odd.

2.4) $-1 ; -7 ; -11 ; p ; \dots$

$-6 \quad -4 \quad -2$

$+2 \quad +2$

a) $p = -13$

b) $T_n = an^2 + bn + c$

$2a = 2 \therefore a = 1$

$3a + b = -6$

$3 + b = -6$

$\therefore b = -9$

$\therefore T_n = n^2 - 9n + 7$

$a + b + c = -1$

$1 + (-9) + c = -1$

$c = 7$

c) $-6 ; -4 ; -2 ; \dots$

$T_n = an + b$

$a = 2$

$b = -6 - 2 = -8$

$\therefore T_n = 2n - 8$

$96 = 2n - 8$

$104 = 2n$

$n = 52$

$\therefore T_{52}$ & T_{53}

$T_{52} = (52)^2 - 9(52) + 7$

$= \underline{2243} \rightarrow$

$T_{53} = (53)^2 - 9(53) + 7$

$= \underline{2339} \rightarrow$