

Revision Exercise p.54

$$\begin{array}{ccccccccc} 3) & 14 & ; & 14 & ; & 12 & ; & 8 & ; & 2 & ; & \dots & ; & -96 \\ & \swarrow & & \swarrow & & \swarrow & & \swarrow & & \swarrow & & & & \\ & 0 & & -2 & & -4 & & -6 & & -8 & & -10 & & \\ & \swarrow & & \swarrow & & \swarrow & & \swarrow & & \swarrow & & & & \\ & -2 & & -2 & & -2 & & -2 & & -2 & & & & \end{array}$$

$$3.1) -6; -16$$

$$\begin{array}{lll} 3.2) & 2a = -2 & 3a + b = 0 & a + b + c = 14 \\ & a = -1 & 3(-1) + b = 0 & -1 + 3 + c = 14 \\ & & b = 3 & c = 14 - 2 \\ & & & c = 12 \end{array}$$

$$\therefore T_n = -n^2 + 3n + 12$$

$$\begin{aligned} 3.3) \quad T_{51} &= -(51)^2 + 3(51) + 12 \\ &= -2436 \end{aligned}$$

$$\begin{aligned} 3.4) \quad -96 &= -n^2 + 3n + 12 \\ 0 &= -n^2 + 3n + 12 + 96 \\ 0 &= -n^2 + 3n + 108 \end{aligned}$$

$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{(3)^2 - 4(-1)(108)}}{2(-1)} \end{aligned}$$

$$\begin{array}{l} n = -9 \quad \text{or} \quad \underline{n = 12} \\ \text{N/A} \end{array}$$

$$4) \quad -9; -6; 1; x; 27; \dots$$

$$\begin{array}{ccccccc} & \swarrow & & \swarrow & & \swarrow & & \swarrow \\ & 3 & & 7 & & x-1 & & 27-x \\ & \swarrow & & \swarrow & & \swarrow & & \\ & 4 & & 4 & & 4 & & \end{array}$$

$$4.1) \quad 4$$

$$4.2) \quad 27 - x - (x - 1)$$

$$= 27 - x - x + 1$$

$$= 28 - 2x$$

$$4.3) \quad 4 = 28 - 2x$$

$$2x = 28 - 4$$

$$2x = 24$$

$$x = 12$$

$$4.4) \quad \begin{array}{lll} 2a = 4 & 3a + b = 3 & a + b + c = -9 \\ a = 2 & 3(2) + b = 3 & 2 - 3 + c = -9 \\ & b = 3 - 6 & c = -9 + 1 \\ & = -3 & = -8 \end{array}$$

$$\therefore T_n = 2n^2 - 3n - 8$$

$$T_9 = 2(9)^2 - 3(9) - 8$$

$$= 127$$

$$4.5) \quad 397 = 2n^2 - 3n - 8$$

$$0 = 2n^2 - 3n - 405$$

$$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-405)}}{2(2)}$$

$$n = 15 \quad \text{or} \quad n = \frac{-27}{2} \text{ N/A} \quad \therefore T_{15} = 397$$

Exercise 1

1.1) $3; 7; 11; 15; \dots$

a) $T_n = an + b$

$a = 4$

$b = 3 - 4 = -1$

$\therefore T_n = 4n - 1$

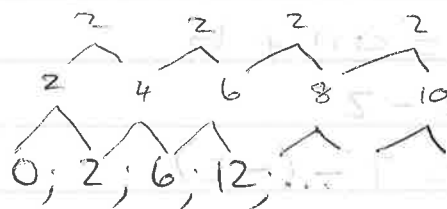
b) $T_{30} = 4(30) - 1$
 $= 119$

c) $1011 = 4n - 1$

$1012 = 4n$

$\therefore n = 253$

1.2)



a) $20; 30$

b) $T_n = an^2 + bn + c$

$2a = 2 \therefore a = 1$

$3a + b = 2$

$b = 2 - 3(1) = -1$

$a + b + c = 0$

$1 - 1 + c = 0 \therefore c = 0$

$\therefore T_n = n^2 - n$

c) $1980 = n^2 - n$

$0 = n^2 - n - 1980$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1980)}}{2(1)}$$

$n = 45 \quad \text{or} \quad n = -44$
 N/A

$\therefore$ The 45th term

$$1.3) \overset{-2}{7}; \overset{-2}{5}; \overset{-2}{3}; 1; \dots$$

$$a) T_n = an + b$$

$$a = -2$$

$$b = 7 - (-2) \\ = 9$$

$$\therefore T_n = -2n + 9$$

$$b) T_{100} = -2(100) + 9 \\ = -191$$

$$c) -3999 = -2n + 9$$

$$-4008 = -2n$$

$$n = 2004$$

$$1.4) \overset{9}{6}; \overset{13}{15}; \overset{17}{28}; \overset{21}{45}; \dots$$

$$a) 66; 91$$

$$b) T_n = an^2 + bn + c$$

$$2a = 4 \therefore a = 2$$

$$3a + b = 9$$

$$3(2) + b = 9$$

$$b = 9 - 6 = 3$$

$$a + b + c = 6$$

$$2 + 3 + c = 6$$

$$c = 6 - 5 = 1$$

$$\therefore T_n = 2n^2 + 3n + 1$$

$$c) 561 = 2n^2 + 3n + 1$$

$$0 = 2n^2 + 3n - 560$$

$$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-560)}}{2(2)}$$

$$n = -17.5 \text{ or } n = 16$$

N/A

$\therefore$ the 16th term

$$1.5) \quad -8; \overset{+3}{-5}; \overset{+3}{-2}; \dots$$

$$a) \quad T_n = an + b$$

$$a = 3$$

$$b = -8 - 3$$

$$b = -11$$

$$\therefore T_n = 3n - 11$$

$$b) \quad T_{40} = 3(40) - 11 \\ = 109$$

$$c) \quad 2680 = 3n - 11$$

$$2691 = 3n$$

$$n = 897$$

$$* 1.6) \quad \begin{array}{ccccccc} & & -6 & & -6 & & -6 \\ & \swarrow & & \swarrow & & \swarrow & \\ -9 & & -15 & & -21 & & -27 \\ \swarrow & & \swarrow & & \swarrow & & \swarrow \\ -2 & & -11 & & -26 & & -47 \end{array}$$

$$a) \quad -74; -107$$

$$b) \quad T_n = an^2 + bn + c$$

$$2a = -6 \therefore a = -3$$

$$3a + b = -9$$

$$3(-3) + b = -9 \therefore b = 0$$

$$a + b + c = -2$$

$$-3 + 0 + c = -2$$

$$\therefore c = 1$$

$$\therefore T_n = -3n^2 + 1$$

$$c) \quad -121202 = -3n^2 + 1$$

$$0 = -3n^2 + 121203$$

$$0 = -3(n^2 - 40401)$$

$$0 = n^2 - 40401$$

$$\sqrt{40401} = \sqrt{n^2}$$

$$\therefore n = 201$$

Exercise 2

2.1) 2; 18; 7; 12; 12; 6; 17; ... 2.2) -7; 0; 9; 20; ...

a) 0; 22

b) $T_n = an + b$

$a = 5$

$b = 2 - 5 = -3$

$\therefore T_n = 5n - 3$

c) 10th term

d) $2 + 18 + 7 + 12$
 $= 39$

a) $T_n = an^2 + bn + c$

$2a = 2 \therefore a = 1$

$3a + b = 7$

$3(1) + b = 7 \therefore b = 4$

$a + b + c = -7$

$1 + 4 + c = -7$

$c = -12$

$\therefore T_n = n^2 + 4n - 12$

b) $128 = n^2 + 4n - 12$

$0 = n^2 + 4n - 140$

$$n = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-140)}}{2(1)}$$

$n = -14$ or $n = 10$
N/A

$\therefore$ the 10th term

c) $T_n = an + b$

$a = 2$

$b = 7 - 2 = 5$

$\therefore T_n = 2n + 5$

d) $599 = 2n + 5$

$594 = 2n$

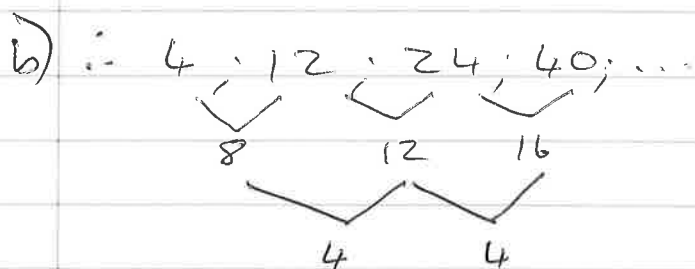
$n = 297$

$\therefore$ between T_{297} & T_{298}

2.3) Total Squares:

	1	2	3	4
Total Squares:	9 3^2	25 5^2	49 7^2	81 9^2
Grey:	5	13	25	41
White:	4	12	24	40

a) $\therefore$ 40 White squares in 4th Pattern



$$\therefore T_{(157)} = 2(157)^2 + 2(157) \\ = 49612$$

$$T_n = an^2 + bn + c$$

$$2a = 4 \therefore a = 2$$

$$3a + b = 8$$

$$3(2) + b = 8$$

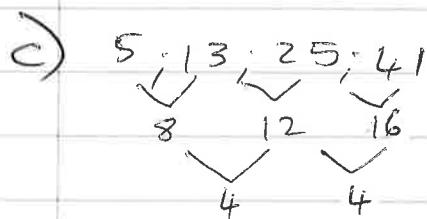
$$\therefore b = 2$$

$$a + b + c = 4$$

$$2 + 2 + c = 4$$

$$\therefore c = 0$$

$$\therefore T_n = 2n^2 + 2n$$



$$a = 2$$

$$b = 2 \therefore T_n = 2n^2 + 2n + 1$$

$$c = 1$$

$$613 > 2n^2 + 2n + 1$$

$$0 > 2n^2 + 2n - 612$$

$$n = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-612)}}{2(2)}$$

$$n = 17 \quad \text{or} \quad n = -18$$

N/A

$$\therefore n = 16 \rightarrow$$

d) Total Squares: $9, 25, 49, 81, \dots$

Diagram showing differences:

$$\begin{array}{ccccc} & 9 & 25 & 49 & 81 & \dots \\ & \swarrow & \searrow & \swarrow & \searrow & \\ & 16 & 24 & 32 & & \\ & \swarrow & \searrow & \swarrow & \searrow & \\ & 8 & 8 & & & \end{array}$$

$$\therefore T_n = an^2 + bn + c$$

$$2a = 8 \quad \therefore a = 4$$

$$3a + b = 16$$

$$3(4) + b = 16$$

$$b = 4$$

$$a + b + c = 9$$

$$4 + 4 + c = 9$$

$$c = 1$$

$$\therefore T_n = 4n^2 + 4n + 1$$

$4n^2$ will always be even

$4n$ will always be even

even + 1 = odd

$\therefore$ Total number of squares will always be odd.

$$\begin{array}{ccccccc}
 2.4) & -1 & ; & -7 & ; & -11 & ; & p & ; & \dots \\
 & \swarrow & & \swarrow & & \swarrow & & \swarrow & & \\
 & -6 & & -4 & & -2 & & & & \\
 & \swarrow & & \swarrow & & \swarrow & & \swarrow & & \\
 & +2 & & +2 & & & & & &
 \end{array}$$

a) $p = -13$

b) $T_n = an^2 + bn + c$

$$2a = 2 \therefore a = 1$$

$$3a + b = -6$$

$$3 + b = -6$$

$$\therefore b = -9$$

$$\therefore T_n = n^2 - 9n + 7$$

$$a + b + c = -1$$

$$1 + (-9) + c = -1$$

$$c = 7$$

c) $-6, -4, -2, \dots$ $T_n = an + b$

$$a = 2$$

$$b = -6 - 2 = -8$$

$$\therefore T_n = 2n - 8$$

$$96 = 2n - 8$$

$$104 = 2n$$

$$n = 52$$

$$\therefore T_{52} \neq T_{53}$$

$$\begin{aligned}
 T_{52} &= (52)^2 - 9(52) + 7 \\
 &= \underline{2243} \rightarrow
 \end{aligned}$$

$$\begin{aligned}
 T_{53} &= (53)^2 - 9(53) + 7 \\
 &= \underline{2339} \rightarrow
 \end{aligned}$$