* Exercise, mixed.

Formulas:

- distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- midpoint: $M = \left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2} \right)$

→ Also: $m_1 = m_2$ → // lines and collinear

$m_1 \times m_2 = -1$ → $\perp$ lines.

Examples:

1) Given: $P(-4; -1)$, $Q(6; 3)$, $R(6; b)$ and $S(-4; -3)$:

a) determine gradient of PQ
b) if PQ // SR, determine the value of 'b'.
c) Show that PQ = SR
d) Is PQRS a parm? Give a reason for your answer.
e) Calculate the midpoint of:
 i) PR
 ii) SQ
f) What can you deduce from the diagonals of a parm?

1a) $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{-4-6} = \frac{-4}{-10} = 2/5$

You could've also done it this way....

$= \frac{3-(-1)}{6-(-4)} = \frac{4}{10} = 2/5$

b) // lines: $m_1 = m_1$

Calculated in (a)

$\boxed{m_{PQ}} = m_{SR}$

Just substitute your answer

$\frac{2}{5} = \frac{(-3-b)}{\underbrace{(-4-6)}_{-10}}$

$-20 = 5(-3-b)$

$-20 = -15 - 5b$

$-5 = -5b$

$1 = b$

c) $PQ = \sqrt{(-4-6)^2 + (-1-3)^2}$

$PQ = \sqrt{100 + 16}$

$PQ = \sqrt{116}$

$PQ = 2\sqrt{29}$

$SR = \sqrt{(-4-6)^2 + (-3-1)^2}$

$SR = \sqrt{100 + 16}$

$SR = \sqrt{116}$

$SR = 2\sqrt{29}$

$\therefore SR = PQ = 2\sqrt{29}$

end your answer with conclusion

d) Yes PQRS is a parm.
→ One pair of sides (opposite sides) are = and //

e) i) $M = \left(\frac{-4+6}{2} ; \frac{-1+1}{2} \right)$
$M(1 ; 0)$, midpoint PR

ii) $N = \left(\frac{6-4}{2} ; \frac{3-3}{2} \right)$
$N = (1 ; 0)$, midpoint SQ

f) Since both midpoints are $(1;0)$
The diagonals bisect each other.

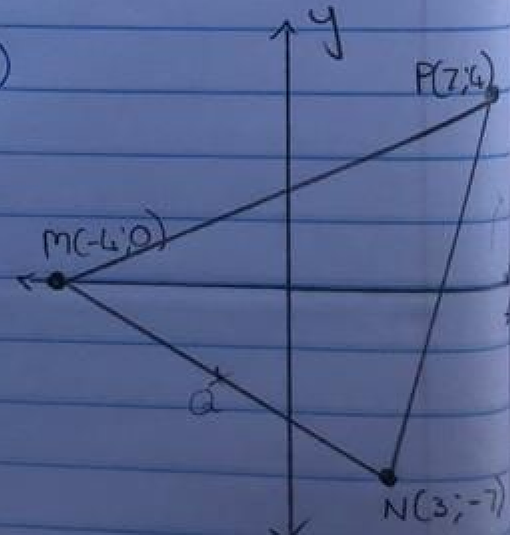
Do the following mixed exercise:

1) BD and AC are diagonals of a parm. If $B=(2;3)$, $D=(6;0)$, $C=(7;5)$, $A=(x;y)$ find values of x and y

2) Given: $A(3;7)$ $B(5;11)$ $C(6;3)$, find;

a) length of AB, in simplest surd form
b) length of BC, to 2 decimals.
c) midpoint of AC
d) gradient of AB and BC.

3) The points $M(-4;0)$, $N(3;-7)$ and $P(7;4)$ are shown in the diagram: Calculate;



a) midpoint Q, of MN
b) gradients of MN and PQ
c) the products of m_{MN} and m_{PQ}
d) What can you state about MN and PQ.
e) the lengths of PM and PN
f) Draw a conclusion about Δ PMN.

4) Given: G(3;7), H(-5;1), K(1;-3) and D(x;y) are vertices of a parm.

a) Calculate the length HK.

b) Calculate the co-ordinates of D

c) Find midpoint M, the midpoint of HD

5) Are the points A(1;8), B(4,17) and C(-2;-1) co-linear? Explain your answer.

6) DEFG is a quadrilateral with vertices D(-1;2), E(3;6), F(7,2) and G(3;-2).

a) find the length of EF

b) find the slope of DG

c) Calculate the midpoint of DF.

d) What kind of quad is DEFG? Give a reason for your answer.

Mixed excercise Memo

Parallelogram properties!!

1) **Diagonals bisect:**

$\therefore$ Midpoint BD = Midpoint AC

$$m\left(\frac{2+6}{2} ; \frac{3+0}{2}\right) = m\left(\frac{\cancel{x}+7}{2} ; \frac{y+5}{2}\right)$$

$$m\left(4 ; \frac{3}{2}\right) = \left[\frac{(x+7)}{2} ; \frac{(y+5)}{2}\right]$$

$\therefore \frac{4}{1} = \frac{(x+7)}{2}$ and $\frac{3}{2} = \frac{(y+5)}{2}$

$8 = x+7$

$1 = x$

$6 = 2(y+5)$

$6 = 2y+10$

$-4 = 2y$

$-2 = y$

$\therefore A(1 ; -2)$

* For this question the properties of a parm was very important.
→ If you didn't know the ~~t~~theory this question could've been tricky

2a) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(5-3)^2+(11-7)^2}$

They asked length $\therefore$ distance formula

$AB = \sqrt{4+16}$

$AB = \sqrt{20} \quad = 2\sqrt{5}$ units

(Simplest surd)

b) $BC = \sqrt{(6-5)^2+(3-11)^2}$

$BC = \sqrt{1+64}$

$BC = \sqrt{65} \quad = 8{,}06$ units (2 dec.)

c) $\boxed{M\left[\frac{x_1+x_2}{2}\ ;\ \frac{y_1+y_2}{2}\right]}$ 'midpoint' in question, $\therefore$ this formula.

$M = \left(\frac{6+3}{2}\ ;\ \frac{3+7}{2}\right)$

$M\left(\frac{9}{2}\ ;\ 5\right)$

d) $m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{11-7}{5-3} = \frac{4}{2} = 2$

$m_{BC} = \frac{11-3}{5-6} = \frac{+8}{-1} = -8$

↳ 'gradient' in question, so write it's formula !!

3a) *"midpoint" in question

$Q = \left[\frac{x_1+x_2}{2} ; \frac{y_1+y_2}{2}\right]$ ↳ This formula

$Q = \left[\frac{-4+3}{2} ; \frac{0+(-7)}{2}\right]$ ← Subs M and N

$Q\left(-\frac{1}{2} ; -\frac{7}{2}\right)$

→ word 'gradient' in question

b) $m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-0}{3-(-4)} = \frac{-7}{7} = -1$

$m_{PQ} = \frac{-\frac{7}{2} - 4}{-\frac{1}{2} - 7} = -\frac{15}{2} \div -\frac{15}{2} = 1$

c) $m_{MN} \times m_{PQ}$
$= (-1)(1)$
$= -1$

*keyword in question → Product, which means ×

d) MN ⊥ PQ → Conclusion of gradient theory

e) $PN = \sqrt{(7-3)^2 + (4-(-7))^2}$
$PN = \sqrt{16+121}$
$PN = \sqrt{137}$

$PM = \sqrt{(7-(-4))^2 + (4-0)^2}$
$PM = \sqrt{121+16}$
$PM = \sqrt{137}$

the word "length" = distance formula.

f) Δ is isosceles, PN = PM → Triangle properties

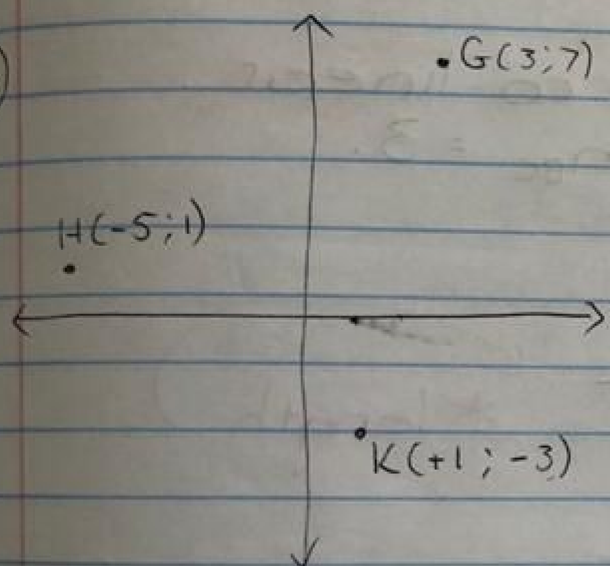
4a) $HK = \sqrt{(-5-1)^2 + (1-(-3))^2}$ * "length"

$HK = \sqrt{36 + 16}$

$HK = \sqrt{52}$

$HK = 2\sqrt{13}$

b)



*This is my shortcut :)

D( )??

→ You could've used = distance or = m'

$\therefore m_{HK} = \frac{-3-1}{1-(-5)} = \frac{-4}{6}$

or both of them & then do simultaneous equations.

$\therefore D(3+6; 7-4)$

$D(9;3)$

c) $m = \left(\frac{-5+9}{2}; \frac{1+3}{2}\right)$ * "Midpoint" :)

$m(2;2)$

5) *Co-linear $\{m_i = m_s\}$

$m_{AB} = \frac{17-8}{4-1} = \frac{9}{3} = 3$

$m_{BC} = \frac{-1-17}{-2-4} = \frac{-18}{-6} = 3$

$\therefore$ Points are co-linear.
since $m_{AB} = m_{BC} = 3$.

6a) $EF = \sqrt{(7-3)^2 + (2-6)^2}$ *length

$EF = \sqrt{16+16}$

$EF = \sqrt{32}$

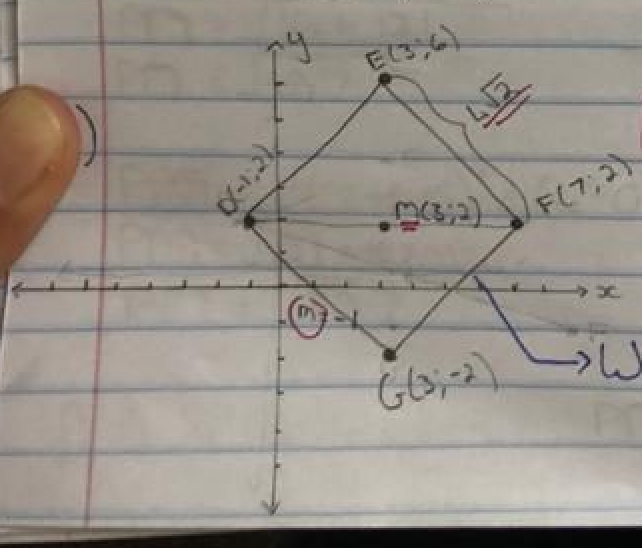
$EF = 4\sqrt{2}$

b) $m_{DG} = \frac{2-(-2)}{-1-3} = \frac{4}{-4} = -1$ * 'Slope' another word for gradient

c) $m = \left[\frac{7-1}{2}; \frac{2+2}{2}\right]$ *"midpoint"

$\therefore m(3;2)$

)



Drawing a rough sketch of given info helps to figure out what needs to be done.
Need more info to draw conclusion.

We have a Midpoint, m_{DG}, length of EF .... 3 random things can't make conclusion

* in sketch it 'looked' like a square,
so check if it really is.

$DE = \sqrt{(-1-3)^2 + (2-6)^2}$

$DE = \sqrt{16 + 16}$

$DE = \sqrt{32}$

$\boxed{DE} = 4\sqrt{2}$

↳ I check DE to see if it's square.

$\therefore DE = EF$

If you did DG it can be rectangle, square or parm.

also $m_{DE} = \dfrac{6-2}{3-(-1)}$

$\boxed{m_{DE}} = \dfrac{4}{4} = \underline{\underline{1}}$

↳ used DE to show 90°∠

$\therefore m_{DE} \times m_{DG} = -1$

$\therefore DE \perp DG$ → if we did m_{EF} we might have said it's either □, ▭ or parm

$\therefore$ DEFG is square

sides are =

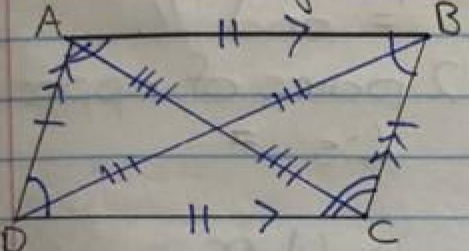
Angles at vertices $\{\hat{D}, \hat{E}, \hat{F}$ and $\hat{G}\}$ $= 90^\circ$

Properties of quadrilaterals:

→ Can be asked in Analytical Geometry.

→ Forms the main/central feature of Gr 10 Euclidian Geometry.

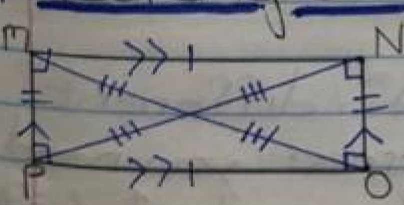
*Parallelogram:



To show/prove it's a parm: (any one of following)

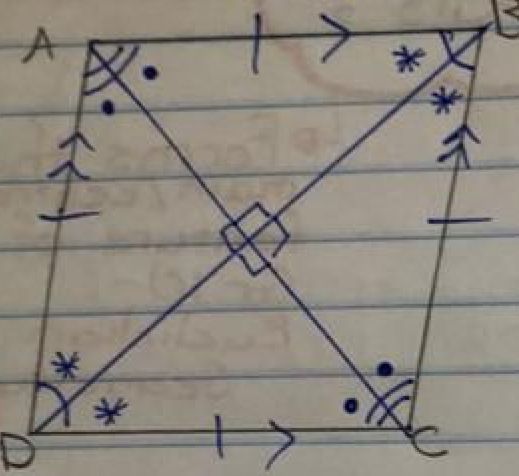
- → 1 pair of opposite sides are = and //.
- → 2 pairs of opposite sides are =
- → 2 pairs of opposite sides are //
- → diagonals bisect each other.
- → 2 pairs of opposite angles are =

*Rectangle:



- → Special parm: 2 pairs of opposite sides are = and //.
  - what makes it different from parm.
- → ∠'s are 90°
- → Diagonals are =, and bisects.

* Rhombus:



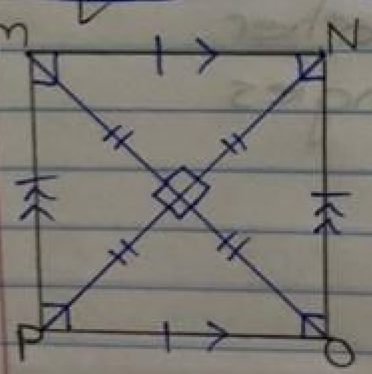
→ Also a special parm:

Parm
- 2 pairs of sides = and //.
- 2 pairs of opposite ∠'s =

→ What makes it 'special' / different from parm?

- The diagonals cut at 90°
- All sides are =
- diagonals bisect the angles.

* Square:



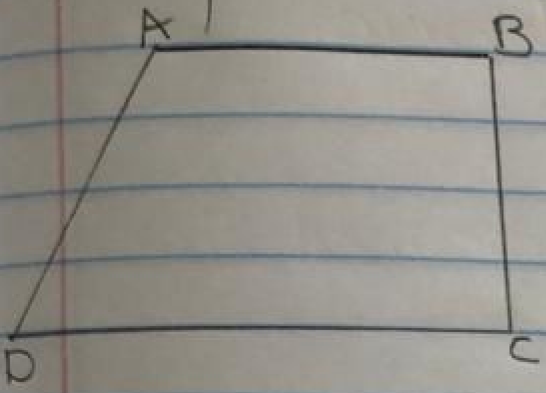
→ Special parm & also special rhombus:

- 2 pairs of opposite sides are // and =

→ What makes it "special" / different?

- all ∠'s are 90° {in prove you need to show only 1 ∠ = 90°}
- diagonals are =
- diagonals bisect at 90°
- all sides are =

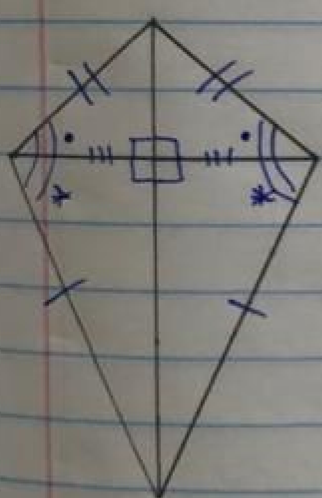
* Trapezium:



→ 1 pair of opposite sides //

→ it's only characteristic/property.

* Kite:



{To prove/show:}

→ 1/2 pair of adjacent sides are

→ diagonals intercept at 90°

{Other characteristics}

- long diagonal bisects short one
- 1 pair of opp. ∠s are = and bisected by short diagonal